Chapter 12
Nordic Cooperation on Modernization of School Mathematics, 1960–1967

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Abstract After a seminar on new thinking in school mathematics held in Royaumont in 1959, four Nordic countries, Denmark, Finland, Norway, and Sweden, agreed to cooperate on school mathematics reform. A joint committee, the Nordic Committee for the Modernization of School Mathematics, declared a need for revising aims and content. Concepts from set theory, and the function concept, as well as greater precision in presentation, could promote interest, insight, and understanding of the subject. Working teams for three school levels: grades 1–6, 7–9, and 10–12, wrote directives for joint experimental texts and teacher guides. A total of 1310 classes in the four countries took part in experimental instruction. More than 180 000 copies of experimental texts were produced. The Nordic cooperation on modernizing mathematics teaching was a remarkable experiment on the cooperation of independent nations. Gradually, each nation went its own way in grades 1–9, where comprehensive 9-year compulsory education was underway in each country. The experiments initiated a long-needed discussion about curriculum, stagnated in certain routines and topics, and had an impact on curriculum development in the redefined school systems. In grades 10–12, steps were taken to create coherence between the gymnasia and university level.

Keywords Agnete Bundgaard · Axioms of the number field · Bent Christiansen · Comprehensive 9-year compulsory education · Experimental instruction · Experimental texts · Function concept · Lennart Sandgren · Matts Håstad · Modern mathematics · Nordic Committee for the Modernization of School Mathematics · Nordic cooperation · Place value notation systems · Probability · School mathematics reform · Set concept · Statistics · Vectors

Introduction

An influential seminar on new thinking in school mathematics was held in Royaumont, France in 1959 by the OEEC, the Organization for European Economic Cooperation, later OECD. One of the seminar's final recommendations was that each country would have its own unique way of making a reform—of introducing new material, and of experimenting with possible programs. Channels were to be provided for communicating the results of these programs and of experiments between countries to utilize the best thinking of all countries in stimulating new ideas (OEEC 1961, p. 125).

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International school mathematics reform movements were parts of reconsiderations of diverse humanitarian values following World War II, such as access to education for all, the democratization of mathematics, the relevance of psychology in mathematics education, and the need to take all school levels from primary to university level into consideration and ensure coherence.

Economic concerns were the main driving force of OEEC in its engagement in the Royaumont Seminar. The premises of the Royaumont Seminar were that changes in cultural, industrial, and economic patterns called for a basic change in educational patterns. More people must be better trained in scientific knowledge (OEEC 1961, p. 107). Education, in particular technical education, would contribute substantially to economic and social progress. This argument was well-received in a world that was recovering from the calamities of war.

The Nordic participants in the Royaumont Seminar agreed to cooperate on reforming their school mathematics. Three Nordic countries were represented at the seminar: Denmark, Norway, and Sweden. Nordic cooperation between the three countries and Finland operated during 1960–1967. An official report was published by the end of the project, written in Nordic languages (NR 1967b), while some sections were also published in English (NR 1967a). The final report gave an account of the results, but not necessarily of the operating process. Documents on the activities of the joint Nordic reform movement, used in this account, are preserved in the Swedish National Archives in Stockholm.

This chapter contains an examination of the Nordic cooperation in modernizing school mathematics and the driving forces behind it. The research method draws on archived documents about the cooperation, the final report, and scholars' accounts of the introduction and implementation of modern mathematics in the Nordic countries. The question is to which degree four independent states with some heritage in common, but different situations in other respects could create a common policy in the field of mathematics education, in particular at the compulsory school level.

The Nordic Countries

The Nordic countries are a well-defined group. They cooperate at the Nordic Council, an official body for formal inter-parliamentary cooperation. The Nordic Council was founded in 1952 by Denmark, Iceland, Norway, and Sweden whose languages are of North-Germanic origin. The four countries were also members of OEEC, later OECD. Finland, not a member of OEEC, joined the Nordic Council in 1955.

The Nordic countries have a long history in common. Iceland was settled from Norway around the year 900 and became its tributary in 1262. Denmark, Norway, and Sweden joined in the Kalmar Union from 1397 until 1523 when Sweden separated from the union, and Norway and Iceland became under Danish rule. Finland was part of Sweden from around 1150 to 1809. From the twelfth century until the 1350s, large-scale migration from Sweden to Finland resulted in Swedish settlements in southern and western coastal areas of Finland. Swedish became an official language in Finland, presently spoken by 5% of the population, while Finnish, spoken by the majority, is a Uralic language, related to Estonian and Hungarian.

In the sixteenth century, Lutheranism, the evangelic Lutheran protestant religion, became formally established in various principalities, including the kingdoms of Sweden and Denmark. The Protestant Reform, according to its general approach of assuming the population was literate, set out to issue school ordinances in subsequent decades to establish *gymnasia* in larger towns that would prepare its students for university studies. The gymnasium school system was introduced in Sweden in 1626. In Denmark, Norway, and Iceland, the former catholic cathedral schools were gradually converted to gymnasia. In the 1960s, gymnasia in all the Nordic countries began at grade 10 (i.e., at the age of 16).

All the Nordic countries had their share of World War II. Denmark and Norway were occupied by the Germans with considerable resistance. Finland was in war with the Soviet Union during 1939–

1940, followed by conflicts, both against and alongside the Allies. Iceland was first occupied by the British and later protected by the United States. Sweden was neutral but received refugees from its neighboring countries. There was a need to restore the societies after the war. Denmark, Norway, and Finland needed economic restoration while Sweden was least affected due to its metal resources and steel industry.

When the Nordic countries took up formal cooperation by establishing the Nordic Council in 1952, the premises were that each nation could use its own North-Germanic language. This was a hindrance to the Finns of whom 95% had a language of different origin as a mother tongue, even if Swedish was also an official language taught at schools. It also hindered Icelanders whose language had developed differently from the others in its isolation. To counteract this, Danish was the first foreign language at school in Iceland until 1999. By 1960, when a Nordic cooperation on reforming school mathematics was formed, the total population in the five countries was around 20 million inhabitants, where Sweden was the most populous, with 7.5 million, Denmark, Norway, and Finland with around 4 million each, and Iceland with 0.2 million having only 1% of the total population.

In the 1960s, the Nordic countries were reorganizing their education systems and extending their compulsory education to a uniform 9-year program. Grade 1 of compulsory education began in the year when a pupil became 7 years old. Denmark had new school legislative acts by which compulsory education became stepwise a 9-year homogeneous school: The 1958 Act, followed by curriculum guidelines—popularly called *Blå betænkning*, the *Blue Memorandum*—and the 1975 Act. Earlier, a decision had to be made at the age of 13 if the pupil would prepare for technical education or for gymnasium, leading to university entrance (Skolelovgivningen i Danmark, efter 1521). Similar developments, which aimed at the democratization of the school systems, took place in Norway, Sweden, Finland, and Iceland. In all the countries, the debates extended over many years, coinciding with the period of the implementation of modern mathematics. The grounds for reforms were fertile.

Modern Mathematics

In the mid-1960s, when modern mathematics was spreading around the world, the arguments were that school mathematics was considered fallen into a rigid system and needed new thinking. The basic premise of modern mathematics in schools was to enhance understanding and place less emphasis on training skills and rote learning. A part of the ideology was that new concepts could unite the many topics of mathematics, providing increased clarity and exactness, and reducing the gap between university mathematics and school mathematics.

A central factor of the modern mathematics curriculum was the set concept. Another important concept was function, which had been introduced at the gymnasium level in the early 1900s with the Meraner reform movement, led by Felix Klein (Krüger 2019). Some elements of logic were often included. Attached to the new concepts in school mathematics was a new symbolic language. There was an emphasis on structure, such as basing the arithmetic operations on the axioms of the rational and real numbers. Geometry was built up more as an axiomatic system than in terms of mensuration.

New topics were also introduced. Elementary statistics, elementary probability, and the Cartesian coordinate system were introduced at the compulsory school level, as were elementary combinatorics, modular arithmetic, and place-value notation systems with bases different from ten, such as five or seven, and the binary notation system. This could lead to the group concept, another important structure.

Nordic Cooperation in School Mathematics

Soon after the Royaumont Seminar, Lennart Sandgren, representing the Swedish Ministry of Education at the seminar, wrote letters to a Danish guest speaker at Royaumont, Svend Bundgaard (U 2)[1], and the Norwegian delegate Ingebrigt Johansson (U 3). In the letters, dated December 31, 1959, Sandgren suggested that it would be most suitable to organize the cooperation under the Cultural Commission of the Nordic Council. The letters were accompanied by a memorandum (U 1) with five points about a prospective Nordic cooperation:

1. At the Royaumont Seminar, it was assumed that modernizing mathematics teaching demanded new textbooks and support to teachers, in addition to groups of experts who would prepare syllabuses, etc. It was suggested that OEEC would support groups of regional cooperation.
2. The participants at the seminar, among them Svend Bundgaard and Ole Rindung from Denmark, Ingebrigt Johansson and Kay Piene from Norway, and Lennart Sandgren from Sweden, agreed that there were good reasons for cooperation within Scandinavia, for example the school systems and syllabuses were relatively similar. The benefits were many: More possibilities to engage experts, any duplication of work could be avoided, cooperation was economically preferable, and more experimental results could be obtained.
3. A regional committee would be established with members from Denmark, Iceland, Norway, and Sweden, and, if finances would not be a hindrance [from OEEC], Finland should be added. The regional committee should be established as soon as possible. Its tasks would be to work out syllabuses, create textbooks for the pupils and teachers' guides, evaluate the need for reforms in teacher education and for in-service courses for teachers, and initiate experiments with new textbooks. Furthermore, based on achieved experience, to propose desirable changes in the school mathematics programs could be formulated and proposed to governmental bodies in the various countries.

   The committee's concern would be all school levels, while the first steps would be at grades 7–9 and the gymnasium level, grades 10–12. The committee could make use of material from other bodies, such as the School Mathematics Study Group (SMSG) at Yale University in the United States. The material produced by the committee would be provisional and be made available to whoever would use it as a basis for textbooks produced by textbook publishers.
4. The composition of the committee was specified. It should include university mathematicians, mathematics teachers, inspectors, and users of mathematics, such as engineers. There were also instructions on the committee's working teams for each school level, and their experts; the frequency of their meetings; and secretarial staff to begin with.
5. A provisional budget was presented. Denmark, Finland, Norway, and Sweden were to share costs in the proportion 4 : 4 : 4 : 7 as Sweden had the largest population. Iceland was not mentioned. The share of Iceland with its tiny population would be small, while transport to and from Iceland was expensive at that time. No documentation has been found at the National Archives of Iceland about the matter.

[1] References marked U# denote numbered outgoing correspondence, while those marked I# denote incoming correspondence from and to the central office of the NKMM committee in Stockholm. The documents are preserved at the Swedish national archives (SE/RA/2717).

Nordic Committee for the Modernization of School Mathematics

A meeting of the new committee, appointed by the Nordic Cultural Commission, was held on October 3–4, 1960, in Stockholm, chaired by Sandgren, with four representatives from each country, Denmark, Finland, Norway, and Sweden (U 8; U 9). The 16 representatives comprised the committee, which became called, in Swedish, *Nordiska kommittén för modernisering av matematikundervisningen* (NKMM) [Nordic Committee for the Modernization of School Mathematics]. The members of the committee were as follows:

- From Denmark (D): Erik Kristensen (1922–2006), Bent Christiansen (1921–1996) (see Figure 12.1), Ole Rindung (1921–1984), and Agnete Bundgaard (1909–1995) (sister of Svend Bundgaard (1912–1984)).
- From Finland (F): Matti Koskenniemi (replaced by Paavo Malinen in 1962), Yrjö Juve (1919–1967), Harkko Helvelahti, and Inkeri Simola (1930–2012).
- From Norway (N): Ingebrigt Johansson (1904–1987), Kay Piene (1904–1968), Henrik Halvorsen (replaced in 1965 by Ragnar Solvang (1930–2018)), and Torgeir Bue (1912–1995) (replaced by Karsten Kjelberg in 1963).
- From Sweden (S): Lennart Sandgren (1926–2009) (see Figure 12.2), Sixten Thörnquist, Göran Holmström, and Thure Öberg (resigned in 1963).

The Swede Matts Håstad (1931–2019) was appointed as secretary, to be understood as executive officer, and the committee's address was decided within the Swedish Ministry of Education in Stockholm. The costs were to be shared in the proportions 1 : 1 : 1 : 2, Sweden carrying the largest share.

The committee decided to set up three working teams, each working on a specified school level. They were to write preliminary drafts of syllabuses. The teams consisted of the following members:

- Grades 1–6: Torgeir Bue (N), Agnete Bundgaard (D), Veikko Heinonen (F), and Charles Hultman (S). Erik Kristensen (D) was appointed as an expert in clear mathematical questions.
- Grades 7–9: Bent Christiansen (D), Kay Piene (N), and Inkeri Simola (F).
- Grades 10–12: Ingebrigt Johansson (N), Ole Rindung (D), and Sixten Thörnquist (S).

Plans for writing proposals for course syllabuses and teacher guides were discussed. The teams for grades 7–9 and grades 10–12 were expected to present their proposals in January 1961. More time

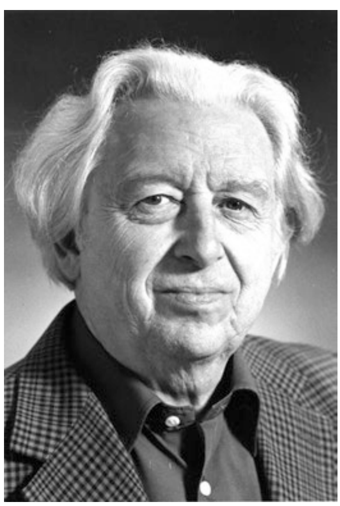

Figure 12.1 Bent Christiansen

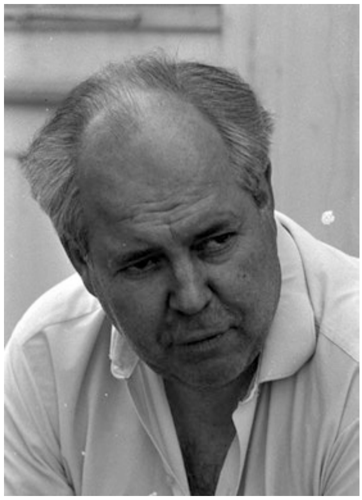

Figure 12.2 Lennart Sandgren. (Blekinge museum, Sweden)

was needed for the team for grades 1–6, as it was necessary to start by analyzing the present material with respect to pedagogical, psychological, and mathematical aspects, so proposals on course syllabuses for that level were to be expected in the autumn 1961. Small-scale teaching experiments could be performed in the academic years following the presentations of the proposals.

On November 23, 1960, Sandgren wrote to Rindung (U 23), reporting that the possibilities of grants from the OEEC were more than expected.

Experimental Texts and Experimental Teaching

The initial working premises of the committee were that there was a need for revising aims and developing a new content, and secondly that better results might be achieved by changing the way mathematics was presented in the schools. Simple concepts from set theory, and the general concept of function, as well as greater precision in presentation, could promote interest, insight, and understanding of the subject alike (NR 1967a, p. 45).

The working teams for the different levels laid down general principles in their directives to be followed in writing experimental texts. The texts were to be given a relatively broad structure so that pupils—except at the lowest level—could study them on their own. Brief comments to teachers were to be provided with hints on methods. Teams of writers (2–3 persons each team) to work out and prepare the texts were appointed. They received a set fee, but the copyright for the texts reverted to the writers after a given period, usually 4 years. Some texts were ready as early as the summer of 1961, others were successively completed through the beginning of 1966. According to the final report, 90 classes took part in experimental instruction at grades 1–6, 450 classes at grades 7–9, and 770 classes at grades 10–12. In most classes, more than one experimental text was used, and more than 180 000 copies of each experimental texts were produced. It was assumed that activities at grades 1–6 would be for unstreamed classes. Work at higher grades was restricted mainly to streams taking more extensive courses in mathematics. Information was collected from the teachers, various tests were made, but for practical and theoretical reasons it was hardly possible to make any detailed comparison with traditional teaching. For example, in most cases only a minor part of the new teaching matter was in common with traditional teaching (NR 1967a, pp. 45–46).

The teachers taking part in these experiments were assumed to comprise a positive selection of skilled and experienced staff. However, they often faced entirely new material, and, in many cases, they had no opportunity to study the entire series of texts in advance. Their reports show that when teaching was repeated for a second year, results were better than in the first year. Experiments started simultaneously in several grades to acquire a wide experience in a reasonable time. The same concepts belonging to modern mathematics, such as the set concept and its derived concepts, had therefore to be treated from scratch at several different levels. The comparison with traditional teaching was influenced by the fact that the experimental texts—unlike most school textbooks—were not based on extensive practical experience in teaching a known body of matter (NR 1967a, pp. 46–47).

In the following, the work process is expounded in some detail according to archived documents about the activities of the NKMM (SE/RA/2717).

Grades 1–6

The working team for grades 1–6, Bundgaard (D), Bue (N), Heinonen (F), and Hultman (S), began its activity in a meeting during December 9–11, 1960. Agnete Bundgaard wrote to Sandgren (I 39) on January 2, 1961, reporting on the meeting as the contact person of the team to the committee. The members were becoming acquainted and there were different opinions. They would like to meet next time for 8 days to do some proper work. Language problems emerged. The Finn, Dr. Heinonen, did not understand much Swedish and no Danish. Matts Håstad replied on January 11 (U 42), regretting Heinonen's case, but Mats Björkman (S) was willing to assist the team as expert. Håstad recommended that the team would meet during 3 days before the NKMM meeting in February.

A directive to the writers of experimental texts for grades 1–6 seems to have been sent out in October 1961 but is documented in a revised form in early November 1961 (U 213). The main document had 14 pages and was written in Swedish. The stated main goal was to make the children confident with the number concept. To that end, children were to become confident with what it means when two [finite] sets contain the same number of elements, and when two sets contain a different number of elements. In connection to that, the concepts "more than" and "less than" were to be practiced. This would be made clear by pairing elements in the two sets. The addition was to be presented as a union of disjoint sets where the commutative law would emerge, as well as the associative law. Multiplication was to be introduced as repeated addition. Children were expected to create their own addition table and the multiplication table. Subtraction was introduced as a "fill-in" method, such as $3 + \square = 7$. The number 0 would be introduced that way, and eventually, the unsolvability of the problem $3 + \square = 1$. The algorithms for multi-digit multiplication and division would be more detailed and different from what was commonly used. Decimal fractions were to be introduced before common fractions which were to be introduced in practical situations. Algorithms for common fractions were to be confined to simple cases and revisited in grades 7 and 8.

Attached were three appendices. There was a 12-page appendix from Torgeir Bue in Norwegian where sets were not mentioned, and the mathematics was put in context. An emphasis was on differentiating the content, related to children's experiences, for example, using the same operations in different situations; differentiating the difficulty level; and differentiating the forms of exercises, for example, by self-controlling exercises, fill-in exercises, situation-exercises where the children must find facts themselves; exercises where children would make up texts themselves with given facts; exercises with texts without numbers; variations of picture-exercises; detective-exercises with errors for children to find; exercises in interpreting graphs, etc.

Another appendix, written in Danish by Agnete Bundgaard, had 11 pages plus three pages on instruction. In the first school year, emphasis was to be laid on building up the set concept and the number concept, using related concepts, such as pairs, disjoint sets, subsets, sets of sets, mapping into and onto a set, and one-to-one correspondence mapping. This was to be taught by using pins, balls, apples, etc. The third appendix consisted of three pages of sample problems, written in Swedish. Some were taken from the journal *Arithmetic Teacher*, and 25 problems were on combinatorics.

There were reactions to the main document. Torgeir Bue (I 278) had little to say, simply that the work had gone fast and well. He did not want to object to details but referred to his own document. The mathematics expert Erik Kristensen (I 285) doubted that he was knowledgeable enough about primary-level mathematics teaching. He sent, however, several comments from which document U 213 had been revised. Among his comments, he found the goals vague: "To provide understanding of mathematical connections" and "to make the mathematics teaching enjoyable." The latter was a method rather than a goal. He expressed a pity for the children, hoping that they could endure receiving such a thorough teaching as was presented for the number concept. He commented also on new algorithms for subtraction and division, wondering if they were better than the old ones, even if it reflected a true idealism to have children understand long division in detail.

In a meeting of the NKMM on October 12–13, 1961 (U 253), it was decided to continue the work for grades 1–6 in two teams and to write experimental texts for grades 1–3 and grades 4–6, respectively. Each team was to include three pedagogues and one mathematician. Experimental teaching was to begin during the school year 1962–1963 if possible. The former working team was asked to complete its directive to the textbook authors. Bue, Bundgaard, and Hultman as well as Björkman were present at the meeting, but not Heinonen.

Agnete Bundgaard wrote to Sandgren on November 16, 1961 (I 286), quite upset. She found it desirable to unite the two writing teams for grades 1–6. It would not be possible to create comprehensive material if one team was to begin at grade 4 without knowledge about the material prepared for the first three years. Perhaps it would be opportune to unite the former and the new teams. She found it very important to choose topics systematically. Or maybe, she should just be quiet, and things would be organized in Sweden? She was not in agreement with Bue about including addition- and multiplication tables and the metric system. Her pupils would not like self-controlled exercises either. She wondered who was responsible for instructions to the authors. And was the original team for grades 1–6 dissolved? Seven pages of comments on the directive were attached to Bundgaard's letter. Matts Håstad replied promptly (U 230), saying that her contribution would be included as an appendix to the directive.

The original team for grades 1–6 was dissolved. Bue disappeared as did Heinonen. The only members left were Bundgaard together with Hultman and Björkman who were both titled as experts. In January 1962, a list was sent out with 13 persons (U 268) proposed to the two writing teams for grades 1–6 textbooks. The list contained four Swedes, one of them Hultman, and three Danes, one of them Agnete Bundgaard. There were two Norwegians who declined the invitation to join the teams (I 333–335). Norway had by then withdrawn from the cooperation of writing joint Nordic texts for grades 1–6. Four Finns were listed in the writing teams. According to Paavo Malinen in June 1962 (I 440), they met in Åbo (in the Swedish-speaking area of Finland) in May 1962. They had not been active earlier within the NKMM at that level. Now two members, one of them Eeva Kyttä, were ready to start a preliminary experiment on grades 1–2 the following autumn.

In November 1962, Agnete Bundgaard wrote to Sandgren (I 647) and gave her opinion of the American SMSG material. She found it somewhat disjointed and had not been aware that it was being translated into Danish. She described her ideas about introducing the four arithmetic operations. Discussion of the set concept, and the concepts of subsets, mappings, and disjoint sets, created the basis for the addition of two numbers, and the determination of the number of elements in a given set's complementary set created the basis for subtraction. By concrete examples, made visible by sets, the laws and rules for addition, subtraction, multiplication, and division could be treated; the derived operations [subtraction and division] though by solutions to the equations $a + x = b$ and $a \cdot x = b$ respectively. Thereafter, the [decimal] number notation would be introduced, and then the usual calculations would be explained. Bundgaard had tested her ideas in cooperation with a young teacher at her school and the pupils responded positively (Figure 12.3). Old cards with pictures of numbers had been discarded by the children as they preferred the one-to-one correspondence to the number line.

Bundgaard's ideas were realized in textbooks by her and Eeva Kyttä for grades 1 and 2 which were published and later translated into Icelandic. The axioms of the number field, clearly expressed, were introduced step by step in this series, beginning with the commutative law of addition in its first volume at age 7 (see Figure 12.4), after having presented subsets and ordering (see Figure 12.5). The commutative law was followed by the associative law for addition later in grade 1 (see Figure 12.6). The distributive law was presented in grade 2 (see Figure 12.7).

Additive and multiplicative identities were presented in grade 3 (at age 9) in textbooks written by Agnete Bundgaard only. Multiplicative inverses were introduced in connection with the division of fractions at age 12. The impossibility of dividing by 0 was presented already in grade 4 by discussing the impossibility of solving fill-in examples such as $7 = 0 \cdot \Upsilon$. The additive inverse was not presented, as negative numbers were not presented in the series.

On August 26, 1964 (I 1164), Agnete Bundgaard reported that 11 classes in Denmark would test the text for grade 1, and 7 classes test the text for grade 2. She had heard nothing from Hultman.

On September 10, 1964 (U 839), Håstad informed the Norwegian Experiment Council (Forsøksrådet for Skolverket) that for grades 1 and 2, texts were available in Swedish, Danish, and Finnish, while the Swedish text was partly different from the others. Texts for grade 3 were being processed and would be available in autumn 1965. In October 1964, Håstad (U 866) thanked Bundgaard for her texts for grades 1 and 2 and informed her that Charles Hultman, Margareta Kristiansson, and himself were preparing to write a text for grade 3. Håstad asked which part of that text Bundgaard had thought to work on. In her reply, Bundgaard asked Håstad (I 1247) if they would like to consider if the material should be less ready-made, but rather prepared in such a way that the children could use well-defined objects to proceed themselves and discover simplifications. In other words, to let them discover algorithms on their own.

On January 5, 1965 (I 1314), Agnete Bundgaard informed Matts Håstad that she had started to prepare the text for grade 3 and was excited to know what they (probably Håstad, Hultman, and Kristiansson) thought about a new version of the text for grade 1. She had also written 30 pages of

Figure 12.3 Teachers at the school on Niels Ebbesen Road, Frederiksberg, Denmark, in 1965; Agnete Bundgaard is the second from right in the middle row. (Frederiksberg Stadsarkiv)

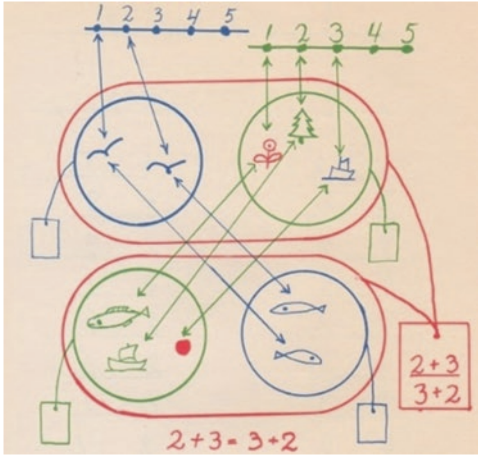

Figure 12.4 The commutative law. The first introduction of addition and the “+” symbol. Age 7. (Bundgaard and Kyttä 1967, Vol. 1, p. 32)

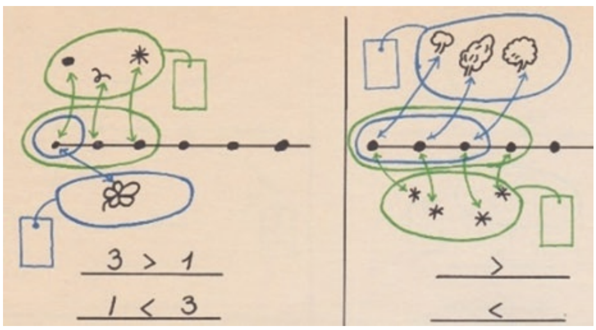

Figure 12.5 Subsets and ordering with one-to-one correspondence to the number line. Age 7. (Bundgaard and Kyttä 1967, Vol. 1, p. 29)

notes for a course to prepare teachers. Very active teachers were testing the material for first grade in two classes.

In April 1965, it was clear that there were two different versions for grade 3: Swedish and Danish. Charles Hultman wrote (U 1052) about the Swedish experimental texts for the lowest levels. He said that teachers who had used the Swedish text for grades 1 and 2 expressed doubts about the use of set-parentheses, {…}. Goals could be reached without these symbols; less formalities were desired. The symbols <, >, and ≠ had been well received. The insecurity that teachers felt might be due to their unfamiliarity with modern mathematics—a critique of the education of primary-level teachers.

Agnete Bundgaard continued her work alone in 1966. In February 1966, she reported (I 1633) that she had contacted a publisher, Gyldendal, who offered to send material to 60 classes for free. In return, the teachers were to attend a course and a couple of meetings. In late summer 1966, she recounted (I 1734) that tests were being run in two classes in grade 5, four classes in grade 4, eight classes in grade 3, 18 classes in grade 2, and 73 classes in grade 1. Furthermore, there were eight classes in Iceland, and one class in Greenland to take care of. In October, she was preparing new material, revising texts for all these grades, and writing teacher guides, in addition to giving talks and courses, and taking care of her own teaching (I 1780). In January 1967, she had prepared a report on the Danish-Finnish version of the grade 1 text for which Matts Håstad suggested some cuts and alterations (U 1676). The final report indicated the number of classes in grades 1–3 that had trialled the

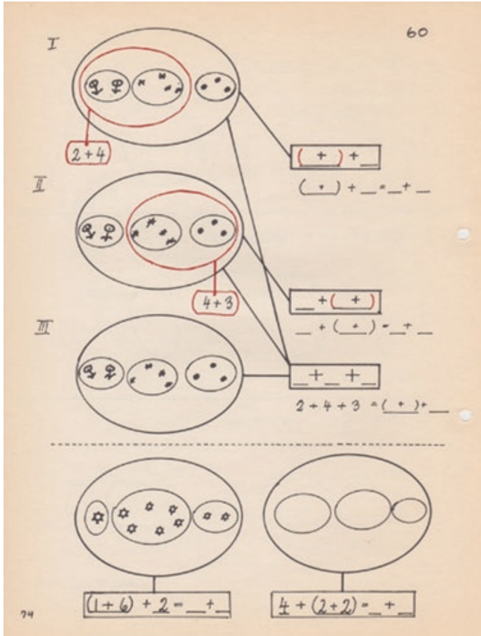

Figure 12.6 Associative law for addition. Age 7. (Bundgaard and Kyttä 1967, Vol. 1, p. 74)

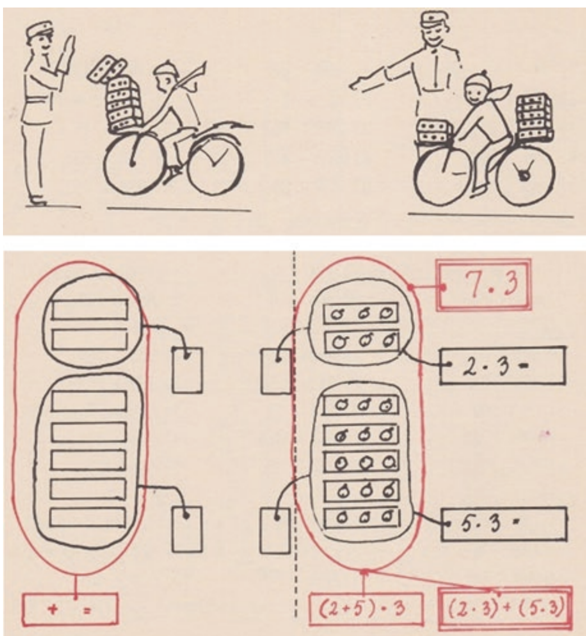

Figure 12.7 Distributive law. Age 8. (Bundgaard and Kyttä 1968, Vol. 2b, p. 72)

materials during 1966–1967, and that the material had been used in Finland in a total of 34 classes but did not mention the classes in Iceland and Greenland. The experimental texts by Bundgaard for grades 4–5, mentioned above, and their tests were not mentioned in the final report of the NKMM activities (NR 1967b, p. 104).

The accounts for Iceland and Greenland were the first to mention experiments outside the four countries comprising the NKMM. The tests in Iceland were the beginning of translations of the Bundgaard-series for grades 1–6 into Icelandic during 1966–1972. According to Gíslason (1978), the initial number of classes in Iceland were seven. In 1966, when the Nordic project was ending, Matts Håstad reported (U 1536) that the authors of the experimental material for grades 1–3 were Agnete Bundgaard and Eeva Kyttä for the Danish-Finnish version, while in fact only the texts translated into Icelandic for grades 1 and 2 are attributed to Eeva Kyttä. Authors of the Swedish version were Margareta Kristiansson, Carin Klaesson, and Matts Håstad. Hultman was not mentioned there, but in the final NKMM report he is listed as an expert for grades 1–6 (NR 1967b, p. 222).

Grades 4–6—SMSG Material

The NKMM committee discussed early (U 1) the possibility of using material from the American School Mathematics Study Group (SMSG). In November 1962 (I 615), permission to translate the SMSG texts for grades 4–6 into Nordic languages was granted upon a request from Edward G. Begle by Stanford University, which was where Begle was now based (after moving from Yale University). Experimental teaching of SMSG texts, translated into Swedish, began in 1963 (U 1545). In September 1963, the Norwegian Halvorsen (I 917) wrote comments on the first part of the Swedish translation for grade 4 with respect to a possible translation into Norwegian. According to Halvorsen, the material was not consistent with the Norwegian plan for experiments on the 9-year compulsory school. Some topics, such as sets and place-value notation in base five, would also take time. However, it could be adjusted to the new Norwegian experimental curriculum. He proposed that a team would take the translation and adjust it to use in a unified compulsory school. The Norwegian Experiment Council was asked to decide if it could be used in a number of schools. The result was that the SMSG material was translated verbatim into Norwegian (Gjone 1983, Vol. II, p. 80; Vol. III, p. 17).

The SMSG text was tested during 1964–1966 in about 20 classes in Sweden. During that period, comparisons showed that this text was the least liked in Sweden but reactions to it did improve over time. The teachers were positively surprised at how well the geometry was going. The final report does not mention that the SMSG material had also been translated into Norwegian (NR 1967b, pp. 108–111).

In April 1965, Hultman (U 1052) reported that the teachers in Sweden felt insecure with the translated material for grades 4–5. That might be the reason why the pupils' results were not as positive as they could have been. The pupils had had difficulties in reading the texts. The texts might not have been translated well enough, and teacher guides were still only available in English. The quest for understanding was more thorough in the experimental texts than in the traditional material. Comparison between the results of the experimental teaching and traditional teaching would not be possible until after grade 6 had been tested. There was, however, a reason to expect that the experimental pupils would be more successful in tasks that demanded independent mathematical thinking, while the other pupils would do better on certain mechanical calculations. The experience acquired was deemed to be so positive that it was highly desirable to continue the experimental teaching with more groups of pupils and more teachers.

Grades 7–9

At the first meeting of the NKMM in October 1960 (U 8, U 9), Bent Christiansen (D), Kay Piene (N), and Inkeri Simola (F) were appointed to a team to make proposals on mathematics contents for grades 7–9. The directive (U 81) to the authors of the experimental texts made it clear in its introduction that the topics were to be presented in terms of the set concept, its derived concepts, and their symbols—for example, the union, $\cup$, and the intersection of two sets, $\cap$, the subset, $\subseteq$, the empty set, $\varnothing$, the symbol $\in$, a complementary set, and some symbols from logic.

The experimental texts were to be on two topics: Algebra and geometry. An algebra text in three volumes was ready for testing during the academic year 1962–1963, written by Bent Christiansen, Matts Håstad, and Ragnar Solvang. Bent Christiansen had the primary responsibility for the first volume, which had three chapters (Christiansen and Christiansen 1965a). The experiences were different in Denmark and Sweden. The committee decided that a somewhat shorter version of algebra, written in a simpler language than the first version, would be used in future experiments in Sweden (NR 1967b, p. 111).

A new Danish version appeared, denoted A 7–9, version D (Christiansen and Christiansen 1965a). It became the basis for experiments in 35 grade 7 classes in Denmark during the academic year 1964–1965. The authors of the Danish version were Allan Malmberg Christiansen and Bent Christiansen. The aim was to research how learning arithmetic and calculations were affected by implementing concepts from set theory, including concepts on relations and functions. Experience showed that the Danish version's first chapter would be suitable for an experiment that would be performed in 65 grade 6 classes during 1965–1966. A report to the Danish experimentation committee, dated June 24, 1965, signed by Allan Christiansen and Bent Christiansen (1965b) on behalf of the Danish department of the NKMM, said that it was of particular interest to study whether understanding and skills might be achieved with less training and more emphasis on insight into the basic concepts of algebra and its interplay with computations.

In a letter by Bent Christiansen to Matts Håstad, dated December 14, 1965 (I 1607), it appears that there was a friction concerning the dissemination of information about the experimental activities. Bent Christiansen, who headed the Mathematics Institute at *Danmarks Lærerhøjskole* [the Royal Danish School of Educational Studies] in Copenhagen and played an important role internationally (Figure 12.8), regretted that the leader and secretary of the NKMM project had replied negatively to a request from UNESCO to publish in its *New Trends* an abstract of a report of the experimental activities in grade 7 in Denmark. Christiansen pointed out that he and his collaborators had written the Danish algebra text. Under no circumstances would he agree that his Institute's possibilities of

Figure 12.8 ICME-3, Karlsruhe 1976, Opening lecture. On the front row from the right: Heinz Kunle, Mrs. Kunle, Shokichi Iyanaga, Mrs. Heinrich Behnke, Bent Christiansen, Hanne Christiansen, Hans-Georg Steiner, Mrs. Steiner. (Courtesy of Gert Schubring and Livia Giacardi)

expression about achieved experiences would again be forfeited. He had arranged to tear the present experiment with 65 classes in grade 6 away from the NKMM committee. The Institute's contribution to that matter was quite formidable. Most of the 65 teachers had attended courses there, often quite long ones, with participants being provided with special literature. Christiansen indicated that he would not dream of having the teaching of the experimental texts made by accidentally chosen teachers without professional insight or real interest in the matter. The choice of teachers was extensive: More than 1000 teachers in Denmark were attending courses and more than 4000 teachers, one-third of the number of Danish teachers, had been in contact with the new material. Furthermore, Bent Christiansen stated that he had noticed that the authors' names had been left out in a new version of volume V of the Swedish *Algebra 7–9*, where many of his own thoughts had been used, while all other volumes were meticulously marked to the authors in concern.

The friction seems to have been settled according to exchanges during 1966 (I 1706; I 1808), where economic transactions were explained. On October 27, 1966, a letter from Håstad (U 1536) shows that a Swedish algebra version had been developed, based on earlier versions. The authors listed were Bent Christiansen, Matts Håstad, and Ragnar Solvang, while the Danish algebra version was attributed to Allan Christiansen and Bent Christiansen only.

Two geometry series of different difficulty levels were made, both written in Swedish (U 839; U 1536). The more extensive one was written by two Swedes, Bertil Nyman and John Amundsson, and the Finn Inkeri Simola. It was translated into Finnish and Norwegian and tested in all three countries. The goal was to provide pupils with an introduction and motivation. Geometrical objects were introduced by set-theoretical concepts, for example, an angle as the union of two rays with the same endpoint. In the second part, a complete axiomatic system was provided with six basic axioms, based on Gustave Choquet's axiom system (Choquet 1969). One complaint from teachers was that some important results were only found in the exercises (NR 1967b, pp. 124–125). The less extensive geometry was written by the Swedes Gunnar Bergendal, Ove Hemer, and Nils Sander. It was used successfully in Sweden and Finland (NR 1967b, p. 126), and its volume for grade 7 was translated into Icelandic (Bergendal et al. 1970).

A summary of replies to a questionnaire from eight Swedish and three Finnish teachers exists about testing geometry in grade 7 during the academic year 1961–1962 (U 376). The general opinion was to continue the experiment; seven teachers had noticed an increased interest by their pupils, some missed logical proofs, the material suited the age level, and most teachers did not feel that there was a lack of in-service courses.

The NKMM experimental texts, probably the one in algebra with Solvang being co-author, and the more extensive one of geometry, were tested in 8 classes of grade 7 in Trondheim, Norway, during 1965–1966. The experiment continued in Oslo and Trondheim for 5 years. There was no formal assessment, but a report was based on the teachers' impressions: "One feels that the pupils attack the tasks in a more independent way […]. The weaker pupils' attitudes may have become positive and had a fortunate effect on their performance." Another experiment of 3-year duration was made in 1969 with the experimental texts revised by Ragnar Solvang. The revision became a basis for an axiomatic, deductive version of the Norwegian experimental national curriculum (Gjone 1983, Vol. III, pp. 12–15), which in its final version in 1976 became balanced between modern and traditional approaches.

Grades 10–12

The gymnasium level was the main concern of the NKMM (Gjone 1983, Vol. II, p. 91) and the cooperation on the material for that level was the least controversial. In October 1966 (U 1536), texts were available, written by six Swedes (S), two Danes (D), and one Norwegian (N). The texts were on the following:

- *Algebra* by Gunnar Bergendal (S) and Per Häggmark (S)
- *Geometry* by Carl Hyltén-Cavallius (S) and Ib Schauffuss (D)
- *Functions and calculus* by Matts Håstad (S) and Haakon Waadeland (N)
- *Statistics and probability* by Björn Ajne (S) and Lennart Råde (S)
- *Differential equations* by Lars Mejlbo (D) and Carl Hyltén-Cavallius (S)

The algebra text was used in experimental teaching, beginning in 1962 (U 1545) at the first gymnasium grade in Norway and Sweden. The teachers found the text quite suitable for that level, but it was theoretical and best suited for the hard-working students. The text was difficult for the students to study on their own. Two thirds of the teacher group found that the student's calculation skills had declined (NR 1967b, pp. 128–130).

Experimental teaching of the geometry text began in 1961 (U 1545). It was used in Denmark, Norway, and Sweden, and in several Swedish-speaking schools in Finland. All the teachers agreed that vectors should be taught at gymnasium level, combining synthetic and analytic geometry, and vectors in three-dimensional space were easy for the students. The treatment of the scalar product was much criticized, while trigonometry was well received, and recommended to be treated before the scalar product (NR 1967b, pp. 130–132).

The text on functions and calculus was used for experimental teaching in Sweden, and some Swedish-speaking schools in Finland from 1963. It started in 1965 in Norway, from where no experiences were reported. The text was used in the two uppermost grades together with other texts, especially in geometry. There were mixed opinions if skills had declined. 50% of the teacher group felt so, while 10% felt that they had improved. Still, students performed well in their final examinations, written and oral. Teachers found, however, the material too voluminous (NR 1967b, pp. 97, 136–137).

The text on statistics and probability was used in the second or third gymnasium grade since 1961–1962 in all four countries, most extensively in Sweden. The students were generally quite interested in these topics, where knowledge from other topics in the mathematics course could be used in a new way (NR 1967b, pp. 138–140).

The text on differential equations was taught in all the four countries, beginning in 1961. The teachers found it useful to work on that topic, where the calculus could be applied and trained, also as there were applications from physics (NR 1967b, p. 138).

The NKMM committee published in its report a proposal on content of mathematical syllabuses, based on the experimental texts and teaching. Compared to the syllabus of grades 1–9, the syllabus for grades 10–12 was more detailed. Looking at the syllabus for grade 12, there were many items from the mathematics studies that were taught at universities in the Nordic countries in the 1960s. This can be interpreted as an experiment to create coherence between mathematics teaching at universities and gymnasia (Gjone 1983, Vol. II, p. 91).

Aftermath

The formal cooperation was concluded in 1967 when the report was published. It was not the role of the committee to publish textbooks, only to write and test experimental texts. In the following years, several of the authors of experimental texts published textbooks that were to make an impact for the following decades. Meanwhile, all four countries, Denmark, Finland, Norway, and Sweden, were working on new legislation and national curricula according to the new 9-year unified compulsory school system, followed by a revision of the gymnasium level, grades 10–12. The experimental texts and teaching were important factors in that development.

Denmark

All four Danish members of the NKMM found channels for their new ideas of a wider pedagogical coherence through a multilateral production of textbooks. At the gymnasium school level, the reform was strongly reflected in the first round by the textbook series *Matematik 1–3* by Erik Kristensen and Ole Rindung (1962–1964) (Skovsmose 1980). This series was published in several editions and reprints until the 1980s by Gads Forlag and had a dominating position on the market. Issue 3–4 of *Nordisk Matematisk Tidskrift* (NMT) in 1967, contained a very positive review of the Kristensen and Rindung series, where this clause appeared:

> There are immensely many things to enjoy when reading the authors' treatment of this voluminous amount of material. The perceptive use of symbols, especially from set theory and logic, contributes to create a clarity in the presentation, which is praiseworthy[2]. (Møller 1967, p. 111)

In 1965, a large proportion of the group of mathematics teachers at the gymnasium level in Denmark attended summer in-service courses on modern mathematics at Aarhus University where Svend Bundgaard was a mathematics professor. The courses were reported as quite demanding (Mogens Niss, personal communication, December 1, 2020; Henrik Stetkær, personal communication, August 24, 2022).

Danmarks Lærerhøjskole [The Royal Danish School of Educational Studies] played an important role in disseminating ideas about modern mathematics under the leadership of Bent Christiansen. Two books were central in this process: *Almene Begreber fra Logik, Mængdelære og Algebra* [General Concepts from Logic, Set Theory and Algebra] by Bent Christiansen, Jonas Lichtenberg, and Johs. Pedersen (1964), and *Matematik 65* by Bent Christiansen and Jonas Lichtenberg (1965), both aimed at teacher students and teachers at in-service training, providing wide and thorough coverage of the new concept world (Skovsmose 1980). Ragnar Solvang (1966) said in his review in NMT that *Matematik 65* was a very fine book that deserved a large circulation. He found it more accessible than *Almene Begreber fra Logik, Mængdelære og Algebra* to teachers who were not well oriented in advance about the topic.

Bent Christiansen was a prolific writer. In his book on goals and methods in mathematics teaching (Christiansen 1967), he laid down a possible approach for grades 6–9, realized in the textbooks *Matematik 7* 1–2 of 1967–1968 in cooperation with Allan Christiansen and Jonas Lichtenberg, and *Matematik 8G* 1–2 of 1969–1971 in cooperation with Johs. Petersen (Skovsmose 1980, p. 37). All the textbooks authored by Christiansen were published by Munksgaard. Bent Christiansen also made a television program about modern mathematics in 1968 (Moon 1986, p. 185).

Gyldendal published Agnete Bundgaard's series, of which the first two volumes were written together with the Finnish Eeva Kyttä. The series came out during 1965–1971. Jens Høyrup (1979)

[2] All translations were made by the author.

deemed the series written strictly according to the rules set by the professorial reform's demands to the Danish school system. The *Blue Memorandum* curriculum guidelines of 1960–1961, following the 1958 Act, could be said to incorporate many things but definitely not modern mathematics. New guidelines, following the 1975 Act, aimed directly at modern mathematics. Agnete Bundgaard had, however, been a consultant to the *Blue Memorandum*.

The Bundgaard series was not the only series on the market for grades 1–6. Indeed, textbook publishing flourished in this period. Skovsmose (1980) mentioned six series, among them the briskly selling *Matematik* by Jørgen Cort and Erik Johannessen, which Jens Høyrup (1979) found marked by imagination, abounding with ideas and practical teaching experience. While the pedagogical problems with the Bundgaard series were that it was too dry and formal, Cort and Johannessen created tasks with a surplus of sprightliness. These authors were only loosely connected to the experimental activities. Cort had been one of the teachers that had taught the algebra experimental text. There was also *Hej Matematik*, by Matts Håstad, Curt Öreberg, and Leif Svensson, translated and adapted from Swedish.

Sweden

The Swedish members of the writing teams also found publishers for their products, in particular for the gymnasium level. In *Nordisk Matematisk Tidskrift,* a review was published of six new series for the first-year mathematics course according to a new plan of 1966 for the gymnasium mathematics in Sweden (Hanner 1967). One of the two best-received series was *Matematik för Gymnasiet*, by Gunnar Bergendal, Matts Håstad, and Lennart Råde (1966–1968), all authors of experimental texts, published by Biblioteksförlaget. It was written both for the mathematics-physics stream in the gymnasium in three volumes, and for its social science stream. The series lasted in print through the 1970s. In his review, Hanner (1967) states:

> This is a very well processed book, which appears to be suitably concise (except possibly the chapter on numbers which is somewhat too extensive and therefore difficult to overview). A detail that elevates the value of the books is the regularly repeating summaries of the previous sections. These provide a good overview and must be of special value at revision. (p. 162)

Carl Hyltén-Cavallius and Lennart Sandgren (1958, 1969) also revised their textbook on mathematical analysis for beginning university studies with respect to modern mathematics, by writing a special introductory volume about set theory, logic, and functions, and adjusting their main text.

Modern mathematics was introduced on a broad scale in grades 1–9 in Sweden when the national curriculum of 1969 took effect. The overall difference from previous curricula was that topics were moved to earlier grades, such as equations, statistics, functions, and the coordinate system. New topics included place-value number notation with bases other than ten in grade 3, vectors in grade 7, and trigonometry in grade 9. A step in the preparation was to educate all teachers in modern mathematics through a distance course called Delta. In total, 47 600 teachers were registered, a great majority of those teaching mathematics (Prytz 2018).

In 1970, only the pupils in grades 1, 4, and 7 were affected by the reform. This meant that the reform involved all students and teachers only in 1972. By 1972 or 1973, the person in charge of mathematics for grades 1–9 at the central school authorities in the period of 1972–1977, Sven-Erik Gode, seems to have given up on central parts of the reform, and from the beginning felt compelled to address deficiencies in modern mathematics. As a motivation for this, teachers had indicated that the syllabus was too comprehensive. Moreover, the results of the first national tests related to the new syllabus, conducted in 1973, showed that the efforts had partially failed; comparisons revealed that students following the former syllabus had better skills in arithmetic. In the materials issued by the central school administration as early as 1973, ideas central to modern mathematics were made peripheral (Prytz 2017).

Several textbook series for the compulsory school were published from 1969, including the modern mathematics ideas, while traditional series were in a clear minority. For example, Matts Håstad, in cooperation with Curt Öreberg and Leif Svensson, wrote a textbook series for grades 1–9, called *Hej Matematik*, published from 1970. It was republished several times, while from 1977, it did not contain set theory for grades 7–9. After 1974, fewer textbooks included set theory, especially for that level (Prytz 2018).

However, from an elaborative analysis of data, collected from the special reports U 1369, U 1371, U 1425, and U 1431, not published in the official report on the NKMM cooperation, Prytz and Karlberg (2016) concluded that it was possible to use the modern mathematics material and attain acceptable results. Pupils in the experimental classes performed at a level equal to those who received traditional instruction. Still, the fact could not be dismissed that the material needed an experienced teacher with a special interest in modern mathematics. This analysis also contradicts a claim that the modern mathematics material per se had negative effects on students in general. Moreover, Prytz and Karlberg found no support for the claim that students who found mathematics difficult were especially disadvantaged by the modern mathematics material.

Finland

In 1968, the Finnish parliament introduced legislation to reform the education system with free comprehensive schools for children between 7 and 16 years old. In 1970, a national modern mathematics curriculum was implemented for all 12 years of comprehensive schools and gymnasia (Malaty 2009).

There were only two Finnish authors in the writing teams authoring experimental publications. Eeva Kyttä's cooperation with Agnete Bundgaard on the material for grades 1–2 has been mentioned. Inkeri Simola, who was a co-author of the experimental text on geometry for grades 7–9, was the first woman to obtain a doctorate in mathematics in Finland. She was appointed as editor of the *Nordisk Matematisk Tidskrift* on behalf of Finland in 1953. She wrote several articles in the journal, but nothing on modern mathematics. Yrjö Juve, a member of NKMM, was the leader of the experimental project in Finland from 1960, while other obligations carried him away from that area, and he died prematurely in 1967 at age 47 (Lyytikäinen 1967).

Norway

In Norway, the atmosphere still supported reforms in 1967, when the SMSG project from the United States, and the Swedish texts for grades 1–3 were adapted to Norwegian situations. The Swedish texts were tested in Oslo in 1967–1969. From 1967 onward, there was a connection between the reform projects and the project of creating a new national curriculum, culminating in 1971 (Gjone 1983, Vol. VIII, pp. 8–9).

Already in 1967, the planning of a television program on modern mathematics began, supplemented by books and a correspondence course. The program was planned as an in-service course for teachers and others interested, such as parents. One of the persons preparing the course was Ragnar Solvang. The program was sent out in 1972–1973 in 35 episodes, of which five were aimed at the lowest level (Gjone 1983, Vol. V, pp. 33–37).

A period of reaction and discussion followed after 1971, terminating in 1973 when the experiments were about to end. After 3 more years, in 1976, the national mathematics curriculum for compulsory level was complete, with a balance being achieved between modern and traditional approaches. The curriculum had swayed away from the most orthodox form of the modern mathematics movement. At the gymnasium level, the reform movement led to a necessary adjustment to reforms implemented at the university level during the 1950s. There had been controversies, mainly at the compulsory level; the mathematical knowledge necessary for an individual leaving school was a central question throughout the whole period (Gjone 1983, Vol. VIII, pp. 7–11).

Publications related to the NKMM-activities include *Matematikk for reallinjen : Algebra og funksjonslære 1*, by Ingebrigt Johansson, Ragnar Solvang, and Ottar Ytrehus, published by Cappelens Forlag in 1968, and *Logikk og mengdelære* by Solvang and Ytrehus, published in 1973. Kay Piene was a member of the NKMM and of the editorial team of *Nordisk Matematisk Tidskrift*. He wrote several articles about modern mathematics and reviews in that journal but died in 1968 while the reform was still in progress.

IMU—An Individual Mathematics Teaching Project

An interesting side effect of the redefinition of school mathematics in the Nordic countries is the *Individualiserad Matematik Undervisning* (IMU) [Individualized Mathematics Teaching Project], which was under development in grades 7–9 in Sweden from 1964. Its goals were as follows:

1. To construct and test self-instructional study material in mathematics;
2. To find suitable teaching methods and ways of work for using this material;
3. To try out different ways of grouping pupils and making use of teachers to achieve the maximal effect for the material and methods; and
4. To measure the effects of individualized teaching (in comparison to conventional teaching).

Since the IMU material was to be introduced in connection with a new national curriculum of 1969, it was adapted to the coming curriculum. Thus, the IMU material included much from the NKMM project. One of the authors of the material was Matts Håstad, a key figure in the NKMM project (Prytz 2017).

The IMU project was trialled in Norway in 1967, and tests of it began there in 1968. Its first version was a direct translation from Swedish, and the project was led by the Norwegian Experiment Council. Ragnar Solvang objected to it in 1969 and said that it would only be testing a Swedish project in Norway. “Whose interest is that for anyone else than the Swedes?” he asked (Gjone 1983, Vol. III, p. 11). The project was revised comprehensively in 1970, and later versions were published commercially by Dreyers Forlag in Oslo (Gjone 1983, Vol. III, p. 15). The IMU project can be classified as behaviorist (Howson et al. 1982, pp. 202–203). Some Norwegians objected to the idea that the IMU project belonged to the modern mathematics movement, but in Sweden it had a close connection to this movement through Matts Håstad (Gjone 1983, Vol. III, p. 16).

A Case Study: Modern Mathematics in Iceland

The Reykjavík Gymnasium belonged to the Danish gymnasium system until 1918. Danish cultural influences persisted as further studies in mathematical sciences were sought at universities and polytechnic schools in Denmark. Mathematics textbooks used at the gymnasium were in Danish, and teachers were educated in Denmark well into the twentieth century. In 1964, Guðmundur Arnlaugsson, a mathematics teacher at the Reykjavík Gymnasium, who had studied mathematics in Copenhagen at the same time as Svend Bundgaard, stayed in Denmark during World War II, and was in contact with

Danish colleagues, began acting toward implementing modern mathematics in Iceland. Becoming a consultant in mathematics teaching at the Ministry of Education, he organized a week-long in-service course for teachers on modern mathematics using *Almene Begreber fra Logik, Mængdelære og Algebra* (Christiansen et al. 1964) as a basis for the course. He subsequently published his own textbook for grade 9, *Tölur og mengi* [Numbers and Sets] (Arnlaugsson 1966), which provided an introduction to numbers and set theory. *Matematik 65* (Christiansen and Lichtenberg 1965) was used for teacher-education students.

Arnlaugsson forwarded news from Svend Bundgaard about the textbook series by Agnete Bundgaard and Eeva Kyttä and suggested that the series be translated. The material for grade 1 was intended for children who could not read, so there was little to translate in that first volume. The series was tested in seven classes of grade 1 in two schools in Reykjavík during 1966–1967, with regular meetings of project leaders and teachers, and meetings with parents. The following spring, the project was presented to school leaders in Reykjavík, who became quite enthusiastic. When the decision was made, only the course material for grade 1 had been finalized in Danish, while the texts for grades 2 and 3 were still in draft form (Gíslason 1978).

In 1967, school leaders, impressed by presentations of the new ideas, decided to send 86 teachers to attend a preparatory course for teaching the new textbooks. The majority of first-grade pupils in Reykjavík primary schools were to be introduced to modern mathematics (NN1 1969). The project leaders did not have the capacity to keep in touch with all the teachers and arrange information for parents to the same extent as for the first group.

The first presentation of modern mathematics to the wider public was characterized by optimism. Articles were written, and Arnlaugsson made a television program, introducing modern ideas (Bjarnadóttir 2011). The television program, supplemented by Arnlaugsson's textbook, was expected to reach a larger number of parents than would otherwise be possible. In an interview, one of the two project leaders remarked that only teaching methods were being changed, not the content, while various topics were introduced earlier than before. Mechanical working methods had been overly emphasized at the cost of the time that teachers had available to discuss basic mathematical concepts and to train logical thinking and accuracy in presentation. The children were to have no homework (Stefánsson 1967).

At the time the decision was made, the content of the latter part of the series was not known. It turned out to be extremely theoretical (Høyrup 1979). The axioms of the number system, the commutative, associative, and distributive laws, place-value number notation in base five, prime numbers, permutation of three digits, and the transverse sum and its relation to the nine times table were all introduced before the end of grade 3. Set theory with pairing, subsets, intersection, and union, more place-value number notation systems, and geometry with points, lines, and planes in a set-theoretical framework were added in grade 4 (Bundgaard 1969–1972; Bundgaard and Kyttä 1967–1968).

Topics such as time unit computations, listed in the Icelandic national curriculum of 1960, were not mentioned in the Bundgaard series, and monetary units were only marginally discussed in connection with the metric system. These topics were probably considered to be applications and thought to emerge naturally as a consequence of the pupils' training in mathematical thinking.

In 1970, a newspaper published an interview with Agnete Bundgaard and her colleague, Karen Plum, who had visited Iceland to give a course to 65 teachers (NN2 1970). By that time, the Bundgaard series was used in 141 classes in grades 1–4. Bundgaard said that the main emphasis was on promoting pupils' understanding of the nature of the tasks and on training them to use their own judgment in solving tasks and problems. Modern mathematics had been introduced in many countries and was influencing how mathematics was taught. Other nations' experiences suggested that its concepts and symbols would be of great use in training pupils in clarity of thinking and communicating. Agnete Bundgaard said:

> Modern mathematics is like a new language, totally different from the mathematics the parents of modern schoolchildren learnt themselves. Many parents have a hard time accepting not being able to know exactly what their children are working on at school and assist them. But it can have very bad consequences for the child if its parents are trying to help, more being willing than able to guide the child. This can only lead to confusion. Therefore, it has been decided not to assign homework to the children and not even allow them to bring their books home. However, to increase the parents' understanding of what their children are working on, special books have been published, admittedly not available in Icelandic, where the new mathematics is explained, and it should pacify the parents until the moment when the children have reached enough understanding of the project to be able to explain to their parents what is happening there. (NN2 1970, p. 23)

Karen Plum continued that in many places modern mathematics had generated dispute and that

> many years will pass until its advantages can be proved statistically, as all comparison is difficult. But surely modern mathematics teaches children to think logically, and however the world will change, logical thinking will always be necessary. Besides, children have proved to like the modern mathematics and they show more interest in it than children at the same age who learn by the old methods, and these two items weigh not so little. (NN2 1970, p. 24)

By the time of Bundgaard's and Plum's visit, authorities had realized that things were not going well, the mathematics teaching experiments in primary schools had become far too voluminous, too difficult to run with respect to guiding teachers, and even in a few cases the effects had been close to being disastrous (Ragnhildur Bjarnadóttir, personal communication, September 16, 2003). The Ministry of Education and Culture had established a school development department that laid down a procedure for adopting school reforms: To set goals, write national curricula and from there, develop learning materials on an experimental basis. In the crisis that had emerged, the department decided in 1971 to skip the step of setting goals and writing a national curriculum in mathematics but go directly ahead to create a new set of homemade mathematics textbooks within the department (Andri Ísaksson, personal communication, March 10, 2003). In their final editions, sets were hardly mentioned. Enthusiasm for the modern mathematics seems to have reached its peak at the primary level in Iceland before 1972 (Bjarnadóttir 2007).

The cohort born in 1965, entering grade 1 in 1972, and completing grade 6 in 1978, was the last large cohort, with about 40% of the Icelandic population studying the Bundgaard material from grade 1 to grade 6 (see Figure 12.9). After that, authorities began to pull it out gradually, while the new material was being introduced after careful testing, keeping in mind the difficulties of the rapid implementation of modern mathematics.

For grades 7–9, the Swedish versions of algebra and the shorter version of geometry (Bergendal et al. 1970) were translated and used for 3–5 years until new Icelandic texts were developed, containing

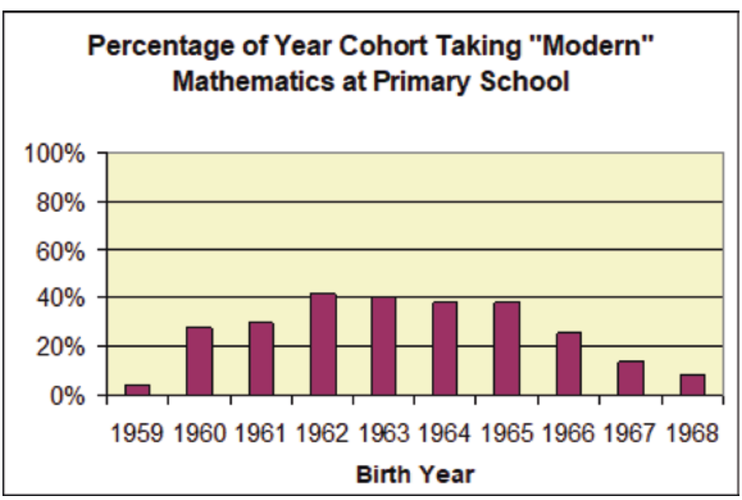

Figure 12.9 Percentage of year cohort taking Bundgaard material up through grade 6

fewer set-theoretical concepts and symbols. Textbooks and syllabuses were on an experimental level until a new national curriculum was published in 1989.

Up to the 1980s, mathematics textbooks in the vernacular were not published in Iceland for the tiny market of grades 11–12 or university. The tradition was textbooks in Danish, considered a window to the international scene of education. For grades 11–12, the Swedish textbook series by Bergendal, Håstad, and Råde (1966–1968), was introduced in 1969. The series endured for most of the 1970s in upper secondary schools. It turned out to be more accessible to Icelandic students than the Danish Kristensen and Rindung series (1962–1964), even if Swedish was less familiar to them than Danish. The Kristensen and Rindung series was used for the mathematics-physics stream of students (Bjarnadóttir 2007).

Discussion

The process of introducing modern mathematics involving cooperation among the Nordic countries was an extraordinary experiment in international collaboration and partnership. As the project went on, the individual countries mostly went their own way. Jeremy Kilpatrick (2012), writing about the international modern mathematics or New Math movement, said that from a distance, school mathematics looked much the same everywhere.

> However, each country has a unique school mathematics—a product of its history, culture, and traditions, and conforming to its social, political, and educational systems. Instructional materials and practices in school mathematics cannot be transported across borders as if they were a common currency. The new math era taught us the paradox of curriculum change: The more school mathematics is internationalized, the more clearly its national character is revealed. (pp. 569–570)

Kilpatrick's observation applies very much to the Nordic countries despite their common heritage. A presumed aim of the Nordic cooperation on modernizing mathematics teaching, namely the preparation of common textbooks that might be translated into the various languages, did not happen to any large degree. Among the obstacles were national situations, such as creating new national curricula around a new 9-year compulsory school period in all the four Nordic countries, was something which would have required lengthy discussions by each country's authorities. This does not mean that the countries were not influenced by the cooperation. The final report recounts, among many things that might be mentioned, effects on the teaching of negative numbers and trigonometry in grades 7–9 in Finland, and approximations in Norway (NR 1967b, p. 218).

Another obstacle was language problems, and they probably hindered the Finns from participating in the cooperative ventures as actively as the others. Only three Finns authored parts of the final product, among them Eeva Kyttä for grades 1–2, and Inkeri Simola on geometry for grades 7–9.

Twelve Swedes were among the writers of the final product. In particular, the executive director Matts Håstad authored three experimental texts: For grades 1–3, on algebra for grades 7–9, and on functions and calculus for grades 11–12. In addition, he was one of the main authors of the IMU project, and he edited the final report on the cooperation (NR 1967b), besides being one of three authors of the textbook series *Hej Matematik*, published in Sweden and Denmark for the compulsory level in the 1970s. Håstad gradually became a central figure in the NKMM cooperation as being most of the time the only employee, who must have had to take many decisions on his own.

In Norway, the experimental activities did not appear to have much effect. The modern mathematics influences, there, seemed to be mainly in the creation of new curricula (Gjone 1983, Vol. III, p. 19). Three authors were Norwegians, one of them Ragnar Solvang, co-author of the Swedish version on algebra for grades 7–9.

In an interview, Mogens Niss (Karp 2015) stated that Denmark was one of the countries that went furthest when it came to introducing the Bourbaki tradition, the modern mathematics approach, into university programs and high school programs. Niss mentioned the influence of the mathematics professor Svend Bundgaard, who said after having spent some time abroad: "This New Math is something we must do in Denmark. We really have to revamp the entire program and modernize it" (p. 59). Svend Bundgaard was an invited guest speaker at the Royaumont Seminar. He organized an international meeting on modern mathematics in Aarhus in 1960 as a follow-up to the Royaumont Seminar (Behnke et al. 1960), but it is not known whether he participated in the Nordic cooperation.

Svend Bundgaard's sister, Agnete, was very active in the NKMM committee in the early years of the project, but her influence diminished as her activities drifted away from the mainstream. Her later works were published commercially and were not recorded among the products of the NKMM cooperation, so they do not seem to have been accepted by the committee. Presented at first as deputy primary school inspector, she was exceptionally well versed in the Bourbaki system of set theory, logic, and structure of number sets. According to her letters, she was teaching young children. She was very definite about how she intended to introduce mathematics from the first grade according to the modern mathematics doctrine. One wonders if Svend Bundgaard cooperated with her and had something to say about her approaches.

Strong personal opinions were conspicuous among the writers of the experimental texts. Agnete Bundgaard and Bent Christiansen clearly had their own opinions which they followed till the end, and these often differed from the track followed by their Nordic colleagues. One of the most noteworthy incidences in the cooperative venture was the disagreement between Agnete Bundgaard and Torgeir Bue on the directive for grades 1–6. Both submitted their own appendices, so neither of them supported the directive fully. Agnete Bundgaard expressed her disagreement on particular items in Bue's appendix, while Bue, who was a leader of a teacher training college, resigned from the working team, and in 1963, from the NKMM committee.

Concluding Remarks

The modern mathematics reform in the Nordic countries began from a perceived need to create coherence between mathematics at the universities and the schools. At the uppermost level, the gymnasia, the reform went fairly well. In addition to making students acquainted with set-theoretical concepts and notations, new topics, such as vectors, statistics, probability, and differential equations, were introduced.

Results of the introduction of modern mathematics at lower school levels were more questionable. Grades 7–9 were under revision for social reasons. Comprehensive 9-year compulsory education was being implemented in all the Nordic countries. Therefore, a new syllabus, suitable for all pupils, was necessary. The suitability of modern mathematics in this respect for that level will be left unanswered.

At the lowest level, grades 1–6, the syllabus had remained unquestioned for ages. The four arithmetic operations with natural numbers, common and decimal fractions, some mensuration, the metric system, and applications from a pre-War agricultural society, were traditionally the topics for those who left compulsory school for work. Modern mathematics for this level proved to be controversial. Not only teachers but also parents and the public had to be informed and convinced. It was only natural that the program would raise questions and discussions. Traditional algorithms were so ingrown that many people did not pay attention to why they were as they were and could not argue logically for them. The aim of modern mathematics was to introduce understanding through logically based arguments. The question remains if that effort was successful. Kristensen's comment (I 285) that it reflected a true idealism to have children understand long division in detail, indicates a doubt that a full understanding of all common algorithms was altogether reachable.

The interplay in the Nordic countries between legislation and national curricula on the one hand, and the implementation of modern mathematics on the other, is notable. In Norway, steps were taken cautiously while new and new drafts of a national curriculum were prepared, until 1973, when a new curriculum, with a balance between the "modern" and the "traditional" approach, was adopted, and the peak of enthusiasm for modern mathematics had been left behind. In Denmark, where the enthusiasm was driven by Bent Christiansen and the Royal School of Educational Studies, the official curriculum guidelines, the *Blue Memorandum*, became increasingly irrelevant, and textbooks were published without adhering to official guidelines (Høyrup 1979; Moon 1986). Sweden may be seen as "in-between": Textbooks were published according to national curricula, first to a modern curriculum, and only a short while later, to a more traditional one. The Swedish experimental texts were not as orthodox at any school level as in Denmark. In Iceland, the modern mathematics materials were used in an experiment involving a large proportion of the compulsory school population. The creation of a national mathematics curriculum was delayed until 1989, long after the experimental period in the 1960s and 1970s.

A positive effect of the modern mathematics reform was that it turned attention to school mathematics and initiated a discussion and a redefinition of what future citizens could and should learn to prepare for a future in an unknown world. Possibly, Agnete Bundgaard went too far in her effort to systematize arithmetic and tie together arithmetic and geometry by set theory. Still, as in Iceland, it turned teachers' attention to the possibility that beginners' mathematics could be presented in many ways. It also awoke an initiative by teachers to create their own teaching materials, incorporating what was noteworthy in the old into the new syllabus.

The interview with Bundgaard and Plum in Iceland had doubtlessly the aim to inform parents and the broader public. However, the idea that information in a foreign language should pacify the parents demonstrated a lack of sense of the situation and respect for the parents, and the same could be said of the decision not to let the pupils bring their textbooks home as it might "have very bad consequences for the child if its parents are trying to help, more by being willing than by being able to guide the child." That, and remarks that "the modern mathematics teaches children to think logically" to a higher degree than earlier, and that parents should wait "until the moment when the children have reached enough understanding of the project to be able to explain to their parents" witness unrealistic convictions of the quality of the program.

Discussions emerged gradually in the Nordic countries on whether implementation of the modern mathematics in schools was a reform, or a disaster resulting in whole cohorts being unable to do the simplest arithmetic (Bjarnadóttir 2006; Gjone 1983; Prytz and Karlberg 2016). However, the emerging pocket calculators were soon to reduce the need for rapid computational skills, mentally or by paper and pencil. In some cases, however, the set-theoretical concepts and their associated symbols became goals in themselves. They were to be learned thoroughly and may have overshadowed the mathematics itself, taking too much time from the number work, and distracting pupils' attention by their distant usefulness.

One wonders if a proper evaluation of the modern mathematics experiments on texts and teaching could not have provided better guidance to authorities about the quality of the new syllabuses and textbooks. One problem was that the goals and contents of the modern mathematics materials were so different from the traditional ones; there were no common standards for an evaluation.

Teachers were extraordinarily important factors in the implementation. Bent Christiansen knew that and put great effort into preparing them. An effort was made in Sweden too. But inevitably, unprepared teachers took over at some point in the implementation process. Prytz and Karlberg (2016) mentioned that the modern material might have required skilled, experienced, and motivated teachers. If the teacher did not share the goals of the new syllabus, teaching might promote learning as merely a meaningless collection of concepts and symbols. This could happen when a new teacher, who had not been prepared or had not internalized the modern ideas, took over teaching in a modern mathematics class but had not acquainted himself/herself with methods and concepts that the pupils had had before. Pupils might also change schools when their families moved to a different area. A new teacher might not have the capacity to follow up on the pupil's experience from the previous school. It became a still more difficult case if the parents were not allowed to help their child as was the case in Iceland. In the Nordic directives for grades 1–6, new algorithms were presented, different from the traditional methods, deeply rooted in the national heritages. There were examples of teachers, trying to turn the pupils back to the conventional algorithms in the middle of the process.

There were also teachers—Agnete Bundgaard and her collaborators, for example—who told about how their little pupils were excited and jubilant about the new items they learned from her text. It is well known that a good teacher can lead pupils into a new topic of almost any kind if he/she knows the topic thoroughly and presents it with sincere enthusiasm. As Jeremy Kilpatrick (2012) said:

> At the crux of any curriculum change is the teacher. The teacher needs to understand the proposed change, agree with it, and be able to enact it with his or her pupils—all situated in a specific educational and cultural context. (p. 569)

Looking away from the implementation problems and critiques on the various details of the modern mathematics movement, it can also be seen as a valuable, long-needed reform. The proponents of the modern mathematics arrived back from Royaumont, enamored by the new ideas, and enthusiastic to create new teaching materials. The enthusiasm spread and opened people's eyes to new possibilities in a reformed curriculum. Ole Skovsmose's (1979) expression about modern mathematics in Denmark has a wider reference in the Nordic countries:

> The 1960s mathematics is a clear and radical breakthrough of the mathematics teaching that over a long period had stagnated around a limited set of methods and problems […]. The mathematics reform has slit the teaching out of a dead and archaic tradition, radically changed its content and enlarged its sphere. (p. 152)

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U 3. Letter from L. Sandgren to I. Johansson, dated December 31, 1959.
U 8. Minutes from the first meeting of the NKMM committee in Stockholm on October 3–4, 1960.
U 9. Memorandum about the NKMM committee, dated October 9, 1960.
U 23. Letter from L. Sandgren to O. Rindung, dated November 15, 1960.
U 42. Letter from M. Håstad to A. Bundgaard, dated January 11, 1961.
U 81. Directives to writers of experimental texts for grades 7–9 from the NKMM.
U 213. Directives to writers of experiment texts for grades 1–6 from the NKMM.
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I 1164. Letter to M. Håstad from A. Bundgaard, dated August 26, 1964.
I 1247. Letter to M. Håstad from A. Bundgaard, dated October 11, 1964.
I 1314. Letter to M. Håstad from A. Bundgaard, dated January 5, 1965.
I 1607. Letter to M. Håstad from B. Christiansen, dated December 14, 1965.
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