Chapter 1
Modern Mathematics: An International Movement Diversely Shaped in National Contexts

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Abstract We reflect on the non-parallel origins and development of modern mathematics, as an educational movement, and its American counterpart New Math. The 1959 Royaumont Seminar played a decisive role in bringing together American and European reformers, acted as a catalyst, but did not lead to substantial reform cooperation on either side of the Atlantic. We pay attention to the pluriform nature of the movement(s), shaped by national traditions, existing educational systems, and societies at large. Moreover, we characterize the modern mathematics reform movement and list some of its main features. Adri Treffers' and Hans Freudenthal's model of classifying different approaches to mathematics education into four *ideal* types proved helpful. We conclude with some reflections on the rapid demise of modern mathematics, which in our view should not be regarded as a total failure, but was a breeding ground for thorough reflection on mathematics education, nationally and internationally, and was the basis for the emergence of *mathematics education* as an autonomous scientific discipline.

Keywords American movement · CIEAEM · Educational reform · European movement · Dissemination of reform · Hans Freudenthal · Internationalization · Modern mathematics · National developments · New Math · OECD · Reform movement · Royaumont seminar · School mathematics · SMP · SMSG · Structuralist mathematics education

Introduction

Modern mathematics (or New Math(s), or new mathematics, etc.) refers to a rather short but drastic change in the way mathematics was understood and taught in Europe, the United States of America, and in various other countries around the globe (Kilpatrick 2012). "'New Maths' perhaps more than any other curriculum reform caught the imagination of the world at large" (Moon 1986, p. 8). The roots of modern mathematics, as an educational movement, were in the early 1950s, the peak of its influence was in the 1960s, and the enthusiasm for it, declined from the mid-1970s onward. A main feature of the movement was the introduction of new teaching content, and new teaching materials and practices as a response to the perceived poor state of mathematics education after World War II. Inspiration for new approaches to mathematics teaching came not only from developments in

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pre-War research in pure mathematics, especially from the actions of the French "Bourbaki" group, but also, particularly in the Anglo-Saxon world, from new applications that had emerged or were being further developed during the War (Zwaneveld and De Bock 2019).

Modern mathematics had various national faces, but in general, the emphasis shifted from developing technical–computational skills to insights into mathematical structure, often but not exclusively pursued by the study of abstract concepts such as sets, relations, algebraic structures, number bases other than 10, etc. Other characteristics were the replacement of traditional synthetic Euclidean geometry by an algebraic, affine, or vector-based approach (or combinations thereof), with a special focus on geometric transformations as objects of study in their own right and as tools for proving. The teaching of calculus (or analysis) became theoretically more rigorous by building it on the concepts of limit and continuity, defined in a topological environment. The changes in mathematical content were accompanied by pedagogical innovations, moving away from lecturing and focusing on students' self-activity. New "structural" materials, such as the Cuisenaire rods, Dienes' logic blocks, and Papy's minicomputer, were carefully designed to stimulate students toward guided discovery and enhanced conceptual understanding. Outstanding, charismatic reformers (such as Max Beberman, Caleb Gattegno, Zoltán Dienes, Frédérique Lenger, Georges Papy, etc.) gave the so-called "model lessons" in their countries and beyond, convincingly showing how modern mathematics could be successfully taught to all students.

In many countries, the reform affected primary, secondary, and tertiary education, albeit to varying degrees and in top–down order (Moon 1986). Typically, curriculum reform first entered universities and then was advocated for the scientific strands of upper secondary education (or high school), with the argument of reducing the gap between school and university mathematics. Next, the "non-scientific" strands of general education, technical education, and middle school came into the reformers' spotlight. Finally, within a space of three to five years, arithmetic-mathematics instruction in primary (or "elementary") schools, and even kindergarten became involved in the reform.

An American Cradle and a European Cradle

Regarding the origins of modern mathematics (New Math) as an educational model, different views circulate. Nadimi Amiri (2017b), reporting her doctoral research on the modern mathematics reform in Luxembourg, has posited that the movement started in the United States of America during the 1950s, as "a series of new reform programs, known as the 'New Math reform'," and later "travelled to Europe through the support of the Organisation for Economic Cooperation and Development (OECD)" (p. 738), unambiguously identifying the 1959 Royaumont Seminar as "the first event that officially started the New Math reform movement in Europe" (Nadimi Amiri 2017a, p. 89).[1] Barrantes and Ruiz (1998) seem to endorse the thesis that modern mathematics in Europe was an import from the American New Math, although they situated the "crossing of the Atlantic" one year earlier when they stated:

> Even though in Europe, in the 1950s, there was intellectual concern regarding the teaching of pre-university Mathematics, the initial drive towards reform was given in Edinburgh at the *International Congress of Mathematicians* in 1958. After a report by five American participants representing various groups in the United States, a wave of opinion gave voice to the need for a reform in the methods of teaching Mathematics in Europe. (p. 1)

[1] In 1959, at the time of the Royaumont Seminar, the OECD was called the Organisation for European Economic Cooperation, or OEEC. It was formed in 1948 to administer American and Canadian aid under the Marshall Plan for the reconstruction of Europe after World War II. As an OEEC initiative, the Royaumont Seminar was partly funded by U.S. money (OEEC 1961a). Once the Marshall Plan was complete, Canada and the U.S. joined the OEEC nations, which created the OECD on December 14, 1960. The OECD entered into force on September 30, 1961 (OECD 2020).

According to Moon (1986), however, the claim that “a ‘wave’ of development in the USA ‘crossed over’ to Europe, although it is oft repeated, may be too simplistic a picture” (p. 46). Instead, Moon argues that the preface of the Royaumont report (OEEC 1961a) shows that “a pattern of ‘parallel’ innovation would be a more appropriate characterization” (pp. 46–47). Kilpatrick (2012) and Vanpaemel and De Bock (2019) endorsed Moon’s thesis. Moreover, Vanpaemel and De Bock (2019) have argued that the American and European movements did not have quite the same background motives and developed largely independently during the 1950s. They met briefly at the 1959 Royaumont Seminar and at subsequent international meetings, but once again evolved largely independently thereafter.

In the first part of this volume, pre-Royaumont developments in the United States of America (Chap. 2) and in Western Europe (Chap. 3) are described and analyzed. These analyses provide further support for the thesis of two parallel movements.

According to many popular accounts, mathematics reform in the United States of America began in 1958 when, in the wake of the Sputnik scare which gripped the American public, a School Mathematics Study Group (SMSG) was established and massively funded with the goal of producing a new mathematics curriculum and accompanying study materials for American students. The real story, however, is more complicated and nuanced. David Lindsay Roberts (Chap. 2), retracing this story to the end of 1959, shows that there were in fact a variety of curricular experiments, commonly referred to by the unifying term New Math. Complaints about the sorry state of mathematics in American schools and calls for modernization appeared as early as 1950. The first response was given in 1952 when the University of Illinois Committee on School Mathematics (UICSM) drafted and implemented a curriculum that logically developed mathematical ideas and emphasized deeper levels of understanding. In the mid-1950s, the UICSM project was followed by several other efforts to reform school mathematics. The main aim of these early efforts was to ensure that secondary schools would be encouraged to offer high-quality mathematical education, which would lead to more students enrolling in university mathematics and science courses. Therefore, school mathematics had to be brought into line with twentieth-century mathematical thinking, both in content and style, i.e., with attention to axiomatic structure, logic, rigor, and precision of language. The launch of Sputnik in October 1957 created the political climate for a significant strengthening of efforts, leading to the creation of SMSG led by Edward G. Begle who organized a nationwide network of authors to write curricular materials for each grade of secondary school and later for elementary school as well. Many curriculum modernizers of the late 1950s and 1960s, including Begle, believed that, given the right framework conditions, the new (improved) mathematics was suitable for *all* students, not just those destined for university studies (Begle 1971; Kilpatrick 2012).

Beginning in 1950, a similar reform movement emerged in Europe. Although it coincided with what was happening at the same time in the United States of America, mutual influence is unlikely, though both movements were influenced to some extent by the work of Bourbaki. The organization that initiated the European reform movement was the International Commission for the Study and Improvement of Mathematics Teaching (CIEAEM), formally established in 1952 after Caleb Gattegno, an Egyptian-born mathematician and psychologist, had paved the way for it in the previous two years. In Chap. 3, the editor of this volume reconstructs the debates at the first meetings of this group, particularly the meeting in 1952 where the Bourbakists met the Swiss psychologist Jean Piaget. Debates resulted in the assumption of an alignment of mathematical and mental structures, which became a main argument for the reform of mathematics education in Europe: The Bourbaki model for the science of mathematics became a model for mathematics education. In the mid-1950s, CIEAEM debates also moved to national levels, particularly in France and Belgium. In Europe, the first systematic experiment with modern mathematics in the classroom was not carried out until the 1958–1959 school year (De Bock and Vanpaemel 2018)—shortly before the Royaumont Seminar took place in the fall of 1959.

Compared to the American SMSG, CIEAEM operated on a much smaller scale, was not funded and not affiliated with any official body or government, and had no interest in implementing a large-scale reform. Despite its impressive name, CIEAEM was and always remained a small informal group, an inside group with little reach. Some of its members would take a leading role in the subsequent modern mathematics reform, but before 1959 this was not visible to outside observers. It is unlikely that many American reformers were aware of the existence of the CIEAEM during the 1950s.

Dissemination of the Reform

The 1959 Royaumont Seminar was a crucial gathering for the modern mathematics (New Math) movement for two main reasons. First, for the first time in history, European mathematicians such as Jean Dieudonné, Gustave Choquet, and André Lichnerowicz, who were members of or had a strong link with Bourbaki, as well as American reformers such as Edward G. Begle, were actually brought together to engage in an in-depth discussion about future avenues for school mathematics. Second, for most OECD countries, Royaumont marked the launch of the movement; in others, such as France and Belgium, Royaumont accelerated a reform that had been emerging during the 1950s. In several non-OECD countries, efforts to reform the school mathematics curriculum resembling those taken by OECD countries were undertaken during the 1960s and early 1970s. In the second part of this volume (Chaps. 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, and 24), the post-Royaumont implementation of modern mathematics is documented, including case studies of countries in all continents. These studies show the different faces of the reform as shaped by national traditions in mathematics education, educational systems in place, and societies at large.

The Royaumont Seminar also motivated communication and internationalization in the world of mathematics education. In Chap. 4, Fulvia Furinghetti and Marta Menghini retrace the reform debate at international meetings during the 1960s. In the first half of that decade, the position and approach to geometry in schools were central to this debate. At Royaumont, Jean Dieudonné had initiated this debate with a proposal to free geometry from the legacy of Euclid, but this led to controversy rather than agreement among the participants. In a follow-up meeting to Royaumont, in Zagreb-Dubrovnik (1960), a group of experts who had been appointed to draft a modern mathematics program for secondary education agreed on the introduction of set theory, algebra, analysis, probability theory, and statistics, but regarding geometry education, the outcome was an ambiguous compromise. For the final years (15- to 18-years-olds), an axiomatic and structural approach was recommended, while in the early years (11- to 15-year-olds), the emphasis would be put on a more intuitive approach (OEEC 1961b).

From the mid-1960s, the first classroom experiments with modern mathematics received considerable attention in international forums. In particular, the audacious approaches of Caleb Gattegno, Zoltán Dienes, and the Belgian Georges Papy, combining mathematical rigor with innovative pedagogies, received considerable attention and appreciation. At that time, modern mathematics was in full preparation in most Western European countries, especially those which belonged to the OECD.

During the second half of the 1960s, modern mathematics spread rapidly worldwide. The reform debate had already reached a number of countries in Eastern Europe, Latin America, Africa, and Asia in the early 1960s, countries that were not part of the original OECD. In 1978, the International Commission on Mathematical Instruction published a report "Change in Mathematics Education since the late 1950s," which included 16 countries around the globe, but no reference was made to changes in any South American nation (Freudenthal 1978). The report documented the reform efforts in each participating country in the preceding two decades, the different directions, the varying degrees of success, and the influences of educational systems. Many reform projects in African and Asian countries took inspiration from either the American SMSG or from the British counterpart of SMSG, the School Mathematics Project (SMP). Best known is an SMP offshoot, the African

Mathematics Program, commonly known as the "Entebbe Project" because in 1962 it organized a workshop in Entebbe, Uganda, in which 11 English-speaking African countries participated (Swetz 1975). In Latin America, where reform was promoted from the First Inter-American Conference on Mathematics Education (1961), the movement was a concoction of ideas from the SMSG, of input from European reformers such as Dienes, Lucienne Félix, Gattegno, and Papy, and of work by mathematicians and mathematical educators from the home continent (D'Ambrosio 1991).

Characterization of the Reform

The country- and region-specific chapters in the second part of this volume (Chaps. 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, and 24), as well as other review publications (Kilpatrick 2012; Moon 1986; Phillips 2015; Servais 1975; Vanpaemel and De Bock 2019), illustrate the pluriform nature of the international modern mathematics/New Math movement. Anglo-Saxon interpretations, for example, differed quite essentially from continental European ones (see also Karp's (2008) interview with Geoffrey Howson). And even within the same parts of the world, implementations differed, depending on national cultures and local educational systems. In some countries, the reform of mathematics education was restricted to a limited number of experimental classes driven by highly motivated individuals; in others, the curriculum was strictly determined by a central authority, which left little room for teachers' own initiatives (Vanpaemel et al. 2012). In an extensive survey of the various reform movements of the 1960s and early 1970s in continental Europe, Willy Servais (1975) concluded that

> continental Europe seems more homogeneous than it actually is. In the evolution of their mathematics education some countries have been bold, even rash. Others are advancing more cautiously, more patiently, more deeply. (p. 55)

The question arises whether a unique and comprehensive characterization of *the* modern mathematics movement is possible at all. Was there something like a common core to which all of these national (and subnational) reform efforts adhered?

At the risk of being one-sided and partly betraying a well-intentioned reform movement, we make an attempt anyway. Evidently, there was the naive theory of sets, a "new" framework for a unified presentation of mathematics, and a starting point for teaching, both in content and in method. Many people associated the new mathematics primarily with the language of sets and its iconic representations of Venn and arrow diagrams that began to dominate textbooks from the mid-1960s onward (De Bock and Vanpaemel 2019). More essential than "sets and arrows," however, seems to us the determination to shape mathematics education from the standpoint of mathematical structure(s), starting from poor structures and gradually constructing more rich structures (see also Chap. 3 in this volume). The latter led, among other things, to a kind of global algebraization of mathematics education, especially at the secondary level, to the prioritization of the affine viewpoint in geometry, and to the deletion of most of the synthetic (Euclidean) geometry of figures (Rouche 1984). The concept of structure was central to Bourbaki's attempts, beginning in the 1930s, to reorganize mathematical science, but after World War II, it became a key instrument and a privileged language of "modern" science in general, both in the natural and social sciences (see, e.g., Gispert 2010). Bourbakists often advocated the role of structures as tools for mathematical discovery (see, e.g., Bourbaki 1948). Although structures, as used by this prominent group of research mathematicians to organize and advance their science, could not have the same meaning for learners, structures were seen in the 1950s and 1960s as tools to organize and advance school mathematics, more specifically to achieve a better conceptual understanding of basic mathematical concepts and methods.

Adri Treffers (1987), and later Hans Freudenthal (1991), proposed a model to characterize different types of mathematics education in terms of four *ideal* types—mechanistic, empiristic, structuralist, and realistic mathematics education—created with a view to a global orientation, although susceptible to nuances. The model was based on a double dichotomy (presence versus absence) of horizontal and vertical mathematization (roughly "mathematizing reality" versus "mathematizing mathematics," respectively) in actual or intended learning processes. Modern mathematics approaches were labeled structuralist—the horizontal component was absent, but the vertical component was cultivated.

> In the nineteen sixties and seventies of our century, under the name of New Math, the structuralist view was advertised and propagated. … On behalf of the prestructured mathematics to be taught, a correspondingly structured world was invented of Venn diagrams, arrow schemes, "games" and so on, to be mathematised by the learner. This was, indeed, a kind of horizontally mathematising activity, yet it started from an *ad hoc* created world, which had nothing in common with the learner's living world. It was mathematics taught in the ivory tower of the rational individual, far from world and society. (Freudenthal 1991, p. 135, italics in original)

The emphasis on mathematical structure (rather than the mastery of specific knowledge or technical–computational skills) implied the introduction of new content, materials, and practices in school mathematics. Although specific features varied across countries and regions, and obviously with the level of education, we attempt to list some main features of mathematics curricula in the 1960s and 1970s.

- The so-called "fundamental" concepts like sets and relations became the starting points; "richer" concepts were described in terms of these concepts (e.g., geometrical objects were defined as "sets of points").
- Concept and problem representation were directed to the use of Venn diagrams and arrow graphs; students were discouraged from making their own problem visualizations.
- Recommended curricula were oriented from abstract to concrete (e.g., in geometry: first points, then lines, and finally "rich" geometrical figures).
- Precise formulations, exact definitions, and correct symbol use received much attention in verbal and written explanation and communication (e.g., a clear distinction was made between "numbers" and "numerals").
- There was a focus on conceptual mathematical understanding (e.g., by studying number systems with bases other than 10) and basic laws (commutativity, associativity, …) rather than on computational fluency and number facts.
- Linear algebra became a "royal road" (Choquet 1964, p. 11) to affine and thereafter Euclidean geometry; much attention was given to geometric transformations and their underlying structures, less to geometric problem solving.

A Failed Reform?

The introduction of modern mathematics/New Math curricula was not without controversy, but in the 1960s critiques had little impact. A most outspoken "early" opponent was Morris Kline, who, along with 64 other renowned American and Canadian mathematicians, signed a memorandum denouncing the abstract tenor of modern mathematics and the neglect of practical applications (Ahlfors et al. 1962). The memorandum was well-noticed, also in Europe (see, e.g., De Bock and Vanpaemel 2019; Guitart 2020), but could not stop the modern mathematics movement, which was then in full swing. In the first half of the 1970s, the movement reached its peak internationally, but at the same time, clear signs of an upcoming decline appeared (Moon 1986). In 1973, Kline's book *Why Johnny Can't Add: The Failure of the New Math* (Kline 1973) was published, in which the author severely criticized the roles of mathematicians in propagating the New Math and associated recommended teaching practices.

Kline's (1973) book quickly acquired an iconic status. A major European critical voice of that period was that of the distinguished French mathematician René Thom. Thom wrote an oft-quoted opinion article in *American Scientist* (Thom 1971) with a response from Jean Dieudonné (1973), and was invited to deliver a plenary address at the Second International Congress on Mathematical Education (ICME-2) held in Exeter (UK) in 1972 (Thom 1973). Eight years after the Exeter Congress, at ICME-4 held in Berkeley (USA) in 1980, modern mathematics was no longer an issue at all (Zweng et al. 1983). In this international shortlist of famous New Math opponents, the figure Hans Freudenthal, President of the International Commission on Mathematical Instruction (ICMI) between 1967 and 1970, should not be left out. Freudenthal converted early to the critical camp (Freudenthal 1963) and helped ensure that a "different version" of modern mathematics could take root in the Netherlands (see also Chap. 11 in this volume). Critical voices with mainly national influence are discussed in Chaps. 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, and 24 of this volume.

In retrospect, modern mathematics/New Math occupied a rather short period in the history of mathematics education: In most countries, the innovation was stopped already one or two decades after its introduction. Even in countries that were mostly affected by modern mathematics, such as Belgium, France, and the United States, it had disappeared by the end of the previous century, being replaced by more constructivist or realistic approaches to mathematics education. In particular, in countries in which modern mathematics was also introduced at the primary level, it was typically a period of very short duration. Moreover, one may question whether, especially at the primary school level, daily classroom practice was affected by modern mathematics as much as reform documents and official curricula of that period suggest. It is apparent that computational and measurement techniques as well as word problem solving—key parts of the "pre-New Math curricula" for that level—were not dropped by primary school teachers during the period of modern mathematics (De Bock and Vanpaemel 2019). Although modern mathematics disappeared rather quickly from the international scene, in some countries isolated elements of it were kept (e.g., in Belgian primary education, Venn diagrams are still used to explain the relationships between the various regular plane figures). In countries in which modern mathematics was implemented in a less radical way, such as the United Kingdom, this was, even more, the case ("Not all of the 'new' mathematics found an established place in the school curriculum, but much did," Howson 2013, p. 647). Moreover, regardless of the country considered, structural elements as well as forms of abstraction and formalization, began to play a greater role in mathematics education in one way or another. Today, however, these elements are no longer seen as starting points for teaching, but rather as final stages in students' developing mathematical culture.

The modern mathematics movement also generated national and international momentum in the community of mathematics teachers (and other people involved in mathematics education). In countries such as the United Kingdom and Belgium, (new) associations of mathematics teachers were founded in which the debate about an upcoming reform of mathematics was held. Committed teachers felt connected as collaborators in an ambitious and valuable educational project that transcended ideological boundaries (De Bock and Vanpaemel 2019). Internationally, at the end of the 1960s, partly from the ashes of the modern mathematics movement, the amount of international collaboration grew and *mathematics education* became recognized as an autonomous scientific discipline (see also Chaps. 4 and 18 in this volume). Under Freudenthal's presidency, ICMI adopted an agenda that fostered a new and refreshing dynamic in thinking about and researching mathematics education. The anchor points were the establishment of the ICMEs (the first took place in Lyon, France, in 1969) and the creation of the journal *Educational Studies in Mathematics* (launched in 1968), both at Freudenthal's initiative. It was followed by the *Journal for Research in Mathematics Education* (launched in 1970) and other research journals in mathematics education. Modern mathematics was by no means an overall success, but it was at the root of developments that probably would not have occurred without it.

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