Mathematical Modeling of Chiller-Based Refrigeration Systems for Energy Efficiency Optimization

Nadia Nedjah[1(✉)], Luiza de Macedo Mourelle[2], and Marcelo Silveira Dantas Lizarazu[1]

[1] Department of Electronics Engineering and Telecommunications, State University of Rio de Janeiro, Rio de Janeiro, Brazil
{nadia,lizarazu}@eng.uerj.br

[2] Department of Systems Engineering and Computation, State University of Rio de Janeiro, Rio de Janeiro, Brazil
ldmm@eng.uerj.br

Abstract. Chillers are the basis of modern refrigeration systems of large facilities, such as oil refineries, power plants and large commercial buildings. The increasing concerns about the scarcity of water and energy resources require careful optimization processes to achieve energy efficiency in industrial buildings. Optimization require mathematical models of real equipment. In this paper, we present two models for a compression chillers, which are one the main equipment in industrial refrigeration systems. We prove that proposed models are precise and faithful to the real compression chiller used in modern refrigeration system. Moreover, we prove that the model's values model accurately the actual values of the global requirements in terms of power consumption of the whole refrigeration system composed of cooling towers, fans and chillers. The models of the cooling tower and corresponding fans are presented in [5].

Keywords: Energy efficiency · Cooling tower · Chiller · Optimization

1 Introduction

Industrial processes usually generate heat. This heat often must dealt with to be eliminated. In general, water is employed to reduce the generated heat effect. Thus, water is used as a cooling element. However, the used water returns from such cooling process hot. So, it must be cooled down to be reused or discarded. The reuse of water is always the preferred solution because nowadays there is always lack of water and water consumption became absurdly expensive.

A refrigeration system based on cooling towers require a set of equipment that operate interdependently, such as chillers. An undue modification of a certain parameter or operational adjustment in one of these equipment can cause either a positive or negative cascading effects on the operation of the others parts of the system. This can consequently trigger a series of effects that are not necessarily

O. Gervasi et al. (Eds.): ICCSA 2022 Workshops, LNCS 13377, pp. 273–288, 2022.
https://doi.org/10.1007/978-3-031-10536-4_19

satisfactory to the overall system, which includes the reduction of its energy efficiency.

Modern industrial refrigeration systems are based on cooling towers, ventilators and chillers. These kind of refrigeration system are expected to deal with application wherein a large cooling demands is expected. These kind of cooling systems offer a clean and economical approaches to cool down the returning water so it can be reused in the cooling process.

In energy efficiency oriented application regarding modern industrial refrigeration system, models of the composing equipment are required so as simulation van be done *a priori* to find out the setpoint of system that provides such energy efficiency. For this purpose, mathematical models of the included complex equipment are needed.

As a first part, in [5], we propose an accurate model for the tower cell, which composed of a cooling tower and corresponding ventilators. As a second part, this work aims at providing a complete model of modern compression chillers to serve as the basis of optimization of many objectives, such as maximizing the thermal exchange efficiency of the cooling tower and minimizing the energy requirement of the refrigeration system, considering all its composing equipment. For the former, a model for the cooling towers is required. Due to lack of space the model of the cooling tower and underlying processes in the refrigeration system is presented in separate paper [5].

This paper is structured into five sections. First, in Sect. 2, we give a brief description of the structure of the modeled compression chillers. After that, in Sect. 3, we present the two proposed model of the chiller. Later, in Sect. 4, we present, discuss and compare the results obtained from the application of the proposed model to those collected from a real operating refrigeration system. Subsequently, in Sect. 5, we show how the chiller's proposed models as well as the model for the cooling tower and corresponding fans, proposed in [5], to obtain a model of the global required power consumption of the whole refrigeration system. Last but not least, in Sect. 6, the conclusions are drawn together with future work.

2 Chillers in Refrigeration System

Figure 1 illustrates the configuration of a refrigeration system composed of a compression chiller and a cooling tower. Therein, the components that composes the chiller can be verified, as well as its interconnections with the cooling tower and the system as fed with chilled water.

It is noteworthy that the heated water that leaves the chiller's condenser is the one that will be pumped to the cooling tower, and that the water cooled by the tower returns to the chiller's condenser. This is the water condensation circuit, which is also called the primary circuit. Furthermore, note that the chilled water that leaves the chiller's evaporator feeds the fan-coils, which use the chilled water to obtain cooled air and distribute it to the rooms of the installation. After passing through the fan-coils, the initially cold water returns

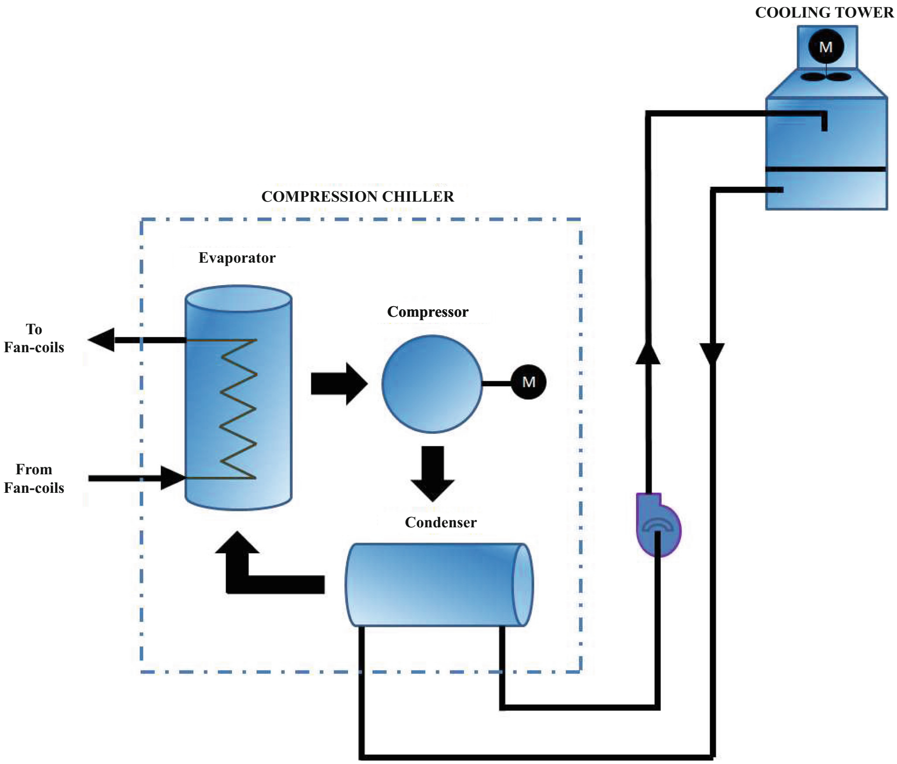


Fig. 1. Cooling system based on compression chiller and cooling tower

to the evaporator with a higher temperature. This is the chilled water circuit, also called the secondary circuit. The fan-coils and other equipment that compose the secondary circuit, such as the water circulation pumps, are not part of the scope of this work. In the fan-coils, the chilled water coming from the evaporator goes to a coil. Therein, the heat exchange is carried out with an induced air flow through forced ventilation occasioned by the fans of the fan-coils. The cooled air resulting from this heat exchange is then distributed.

A water chiller is an equipment whose function is to cool water through a thermodynamic cycle. The two main types of chiller are: compression or electric chiller and absorption chiller. The former uses a physical process, while the latter uses a physical-chemical principle. The system to be optimized in this work uses only compression chillers. Therefore, only this type of chiller will be presented thereafter.

The compression chillers use a mechanical compressor, generally driven by an electric motor, which is used to compress the refrigerant gas that circulates internally to the chiller. The compression of the refrigerant gas is just one of the stages of the thermodynamic cycle necessary for the chiller's operation. The complete thermodynamic cycle of is shown in Fig. 2. Note that in addition to the compressor, the chiller also includes other equipment and constituent parts, the main ones are the condenser, the evaporator and the expansion valve [1].

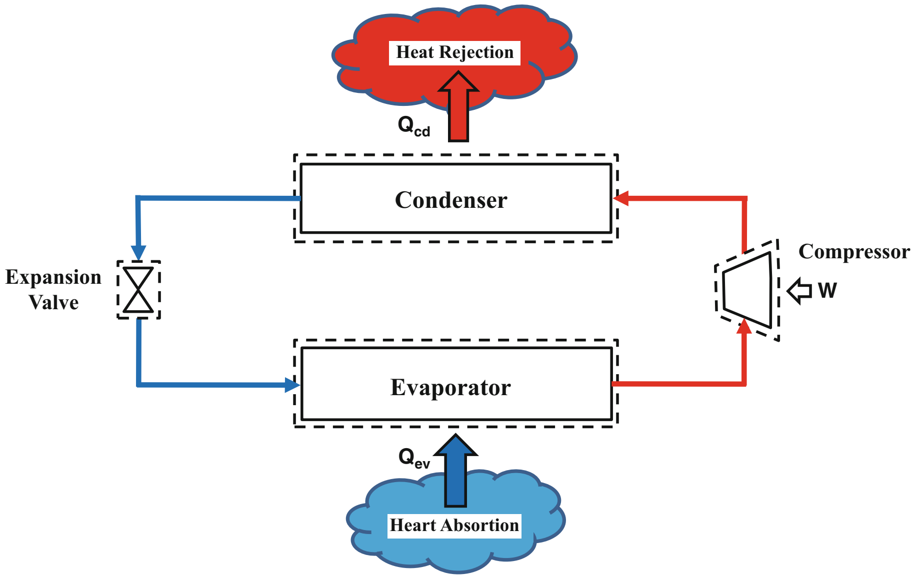


Fig. 2. Chilling cycle by compression

The evaporator, also called cooler, is a heat exchanger composed of finned tubes where, on one side, there is the passage of the refrigerant fluid, and on the other, the circulation of water to be refrigerated. Therefore, the evaporator has the function of cooling the water that circulates through the use of pumps, called pumps of circulation. It is in this equipment that the evaporation of the refrigerant fluid occurs, a phenomenon that occurs after the thermal exchange with the hot water coming from the process, which also circulates in the evaporator, raising the temperature of the refrigerant fluid. Ideally, this process should be isobaric. However, in practice, there is a small pressure reduction in the refrigerant fluid after its passage through the evaporator. The compressor, which can be electric or of combustion, compensates for this reduction in pressure of the refrigerant fluid, sending it to the condenser.

The condenser is a heat exchanger whose function is to condense the refrigerant fluid, which is in a vapor state. This state reached after its passage through the evaporator. The condenser can be of two types: air-cooled or liquid-cooled. In the former, fans are used to carry out the thermal exchange between the forced air and the refrigerant fluid in vapor state, through the circulation of the refrigerant fluid through finned tubes. In the latter, cooling towers are used, so that the water circulating through the tower exchanges heat with the refrigerant fluid in vapor state. This exchange is carried out through coils located inside the condenser, causing the refrigerant fluid to condense. The heated water resulting from this process is raised by means of lifting pumps to the cooling tower. This reduces the temperature of the condensation water, then returns it to the chiller's condenser, in order to allow a continuous thermal exchange with the refrigerant fluid in vapor state coming from the evaporator.

The chilling process also make use of an expansion valve, also called thermostatic valve. This is a direct expansion refrigeration device. So, the expansion of the fluid takes place in the environment to be cooled. Therefore, the thermodynamic cycle of the chiller based on compression occurs as follows: initially, the refrigerant fluid is compressed by the compressor in the super-rheated vapor state, increasing its pressure and temperature. Then, the refrigerant fluid is sent to the condenser, where the heat gained in the compression process is rejected to the outside environment, thus causing the refrigerant fluid to cool down and changing from vapor to liquid. When leaving the condenser in the sub-cooled liquid state, the refrigerant fluid flows through the expansion valve, which causes a pressure drop and consequent temperature drop. Finally, the refrigerant fluid passes through the evaporator, where it absorbs heat from the water to be cooled down, causing the refrigeration effect. Thus, the refrigerant fluid then changes from liquid to vapor, leaving the evaporator as super-heated vapor, returning to the compressor, and the cycle starts again.

3 Proposed Model for the Compression Chiller

In this section, we present the modeling of the process performed by the compression chiller. For the supply of chilled water, the model of a compression chiller can be presented according to Eq. 1:

$$Q_{chiller} = \dot{m}_{water_{evap}} c_{water} (T_{ae_{evap}} - T_{as_{evap}}), \tag{1}$$

where $Q_{chiller}$ represents the instantaneous capacity of the chiller, $T_{ae_{evap}}$ represents the temperature of the water entering the evaporator of the chiller, *i.e.* the temperature of the chilled water that returns from the process to be cooled, $T_{as_{evap}}$ represents the temperature of the water leaving the evaporator, *i.e.* the temperature of the water leaving the chiller and serving the process, c_{water} represents the specific temperature of the water and $\dot{m}_{water_{evap}}$ represents the flow rate of water in the evaporator of the chiller.

The electrical power demanded by the chiller, $P_{chiller}$, can be determined from non-linear regression models as a function of partial load conditions and as a function of temperature values $T_{ae_{evap}}$ and $T_{ae_{cond}}$. The latter represents the temperature of the water that reaches the chiller condenser. According to [6] and [2], the electrical power demanded by the chiller can be determined as indicated in Eq. 2:

$$\begin{aligned} P_{chiller} = {} & Q_{chiller_{nom}} EIR_{nom} Z_{CAP}(T_{ae_{cond}}, T_{as_{evap}}) \\ & \times Z_{EIR}(T_{ae_{cond}}, T_{as_{evap}}) Z_{EIR}(PLR), \end{aligned} \tag{2}$$

where $Q_{chiller_{nom}}$ represents the nominal capacity of the chiller, EIR_{nom} represents the nominal value of the Energy Input Ratio, which is the ratio between the electrical power and the nominal thermal capacity informed by the chiller manufacturer, function $Z_{CAP}(T_{ae_{cond}}, T_{as_{evap}})$ represents the correction factor for the capacity of the chiller, which depends on the temperatures input to the condenser and output to the evaporator and function $Z_{EIR}(T_{ae_{cond}}, T_{as_{chiller}})$ stands for the correction factor of EIR, which also depends on the temperature of condenser input and evaporator output, and function $Z_{EIR}(PLR)$ represents the correction factor of EIR, which depends on the thermal load factor of the chiller. The PLR stands for the Part Load Ratio, and in this case, represents the partial load condition of the chiller.

In this modeling, two optimization scenarios are considered. This requires providing two different models for the chillers used in the refrigeration system. The considered scenarios are defined as:

- Scenario 1: we consider that the evaporator outlet temperature of the chiller is fixed, and the optimization can be achieved by varying the speed of the cooling tower fans, *i.e.* by varying the temperature of the condensation water.
- Scenario 2: we consider that the optimization can be achieved by varying the temperature of the evaporator leaving water from the chiller and also by varying the speed of the tower fans.

Regarding the first scenario, based on Eq. 2, obtaining $P_{chiller}$ depends on $T_{as_{evap}}$, which is fixed and previously adjusted in the chillers. Therefore, this variable must be disregarded in the calculation of factors Z_{CAP} and Z_{EIR} employed in Eq. 2. However, by keeping $T_{as_{evap}}$ fixed in Eq. 2, it is assumed that the factors Z_{CAP} and Z_{EIR} are defined only as a function of $T_{ae_{cond}}$, which is not true. IT also requires other information about the evaporator of the chiller to be applied in the modeling of its consumption, since the inlet and outlet temperatures of the evaporator influence the determination of their energy consumption. In this case, the higher the temperature of the water entering the evaporator is, the greater the consumption, and the lower the temperature of the water leaving the evaporator is, the greater the consumption. Thus, for the first scenario, the use of $T_{ae_{evap}}$ is considered instead of $T_{as_{evap}}$ in the consumption model of the chiller through factors Z_{CAP} and Z_{EIR}. The substitution is valid and the motivation is based on the fact that $T_{ae_{evap}}$ represents the thermal load condition of the system, since it is the temperature of the water that returns from the process. As $T_{as_{evap}}$ is fixed, $T_{ae_{evap}}$ becomes the reference for estimating the thermal load of the chiller.

So, to model the electric power demand by chillers considering the first scenario, an approximation of the approach presented by [6] and [2] is used, where factors Z_{CAP} and Z_{EIR} are obtained as a function of $T_{ae_{cond}}$ and $T_{ae_{evap}}$. After the introduction of the aforementioned modifications, we found out that the use of factor $Z_{EIR}(PLR)$ affects the results of the modeling of $P_{chiller}$, which is hence disregarded in the adjusted modeling. Therefore, the model of $P_{chiller}$ for the first scenario is defined by Eq. 3:

$$P_{chiller} = Q_{chiller_{nom}} EIR_{nom} Z_{CAP_1}(T_{ae_{evap}}, T_{ae_{cond}}) Z_{EIR}(T_{ae_{cond}}, T_{ae_{evap}}). \tag{3}$$

Regarding the second scenario, unlike the first one, the evaporator outlet temperature, $T_{as_{evap}}$, must be used to determine the power demanded by the chiller, since in this scenario it will be allowed to vary $T_{as_{evap}}$ in order to obtain energy efficiency for the refrigeration system. Although the model for $P_{chiller}$ as presented in Eq. 2 considers the use of this variable to evaluate factors Z_{CAP} and Z_{EIR}, we decide to include variable $T_{ae_{evap}}$, since the database obtained in the field presents a more significant range of values for $T_{ae_{evap}}$ compared to $T_{as_{evap}}$. The latter remains constant for long periods of time. So it is safe to conclude that in this case, variable $T_{ae_{evap}}$ has a greater influence on the estimate of consumption of the chiller. Thus, variable $T_{ae_{evap}}$ is included in the model of the electric power demanded by the chiller indicated in Eq. 2. Then, we reach the model defined in Eq. 4:

$$P_{chiller} = Q_{chiller_{nom}} EIR_{nom} Z_{CAP_2}(\Delta T_{ag}, T_{ae_{cond}}) Z_{EIR}(T_{ae_{cond}}, T_{ae_{evap}}). \tag{4}$$

wherein, factor Z_{EIR}, differently from the one indicated in Eq. 2, is obtained as a function of $T_{ae_{cond}}$ and $T_{ae_{evap}}$, in order to maintain the similarity to the model adopted for the first scenario. After the implementation of the aforementioned modifications, similarly to what is verified in the modeling of the first scenario, we found out that the use of factor $Z_{EIR}(PLR)$ hindered the results of the modeling of $P_{chiller}$, so we decided to disregarding the use of this factor in the modeling adopted for the second scenario. Thus, the modeling of $P_{chiller}$ for the second scenario is defined in Eq. 4: Note that factor Z_{CAP} is obtained as a function of $T_{ae_{evap}}$ and $T_{as_{evap}}$ using the variable ΔT_{ag}, which represents the temperature variation in the chilled water circuit, and is defined in Eq. 5:

$$\Delta T_{ag} = T_{ae_{evap}} - T_{as_{evap}}. \tag{5}$$

The analysis of the Eqs. 3 and 4 shows that they basically differ as a function of the factors Z_{CAP}. For this reason, this factor is defined in Eq. 3 as Z_{CAP_1}, and in Eq. 4 as Z_{CAP_2}. As the modeling adopted for the chiller did not follow precisely what is described in [6] and [2], it is necessary to define new equations for factors Z_{CAP_1} and Z_{CAP_2}. In this case, it is established that a quadratic approximation involving variables $T_{ae_{cond}}$ and $T_{ae_{evap}}$ for the modeling of Z_{CAP_1}, and variables ΔT_{ag} and $T_{ae_{evap}}$ for Z_{CAP_2} would be enough. This is later verified with the results obtained after implementing the modeling of these factors. In the same way, we proceed to model factor Z_{EIR}, in this case, involving variables $T_{ae_{cond}}$ and $T_{ae_{evap}}$, similarly to the modeling of the factor Z_{CAP_1}. Equations 6, 7 and 8 describe the models adopted for factors Z_{CAP_1}, Z_{CAP_2} and Z_{EIR}, respectively. In this case, coefficients a_0-a_5 and b_0-b_5 must be obtained from nonlinear regression methods considering the models indicated in Eqs. 6, 7 and 8:

$$\begin{aligned} Z_{CAP_1} = b_0 + b_1 T_{ae_{evap}} + b_2 T^2_{ae_{evap}} + b_3 T_{ae_{cond}} + b_4 T^2_{ae_{cond}} \\ + b_5 T_{ae_{evap}} T_{ae_{cond}}, \end{aligned} \tag{6}$$

$$Z_{CAP_2} = b_0 + b_1 \varDelta T_{ag} + b_2 \varDelta T_{ag}^2 + b_3 T_{ae_{cond}} + b_4 T_{ae_{cond}}^2 + b_5 \varDelta T_{ag}^2 T_{ae_{cond}} + b_6 \varDelta T_{ag} T_{ae_{cond}}^2, \tag{7}$$

$$Z_{EIR} = a_0 + a_1 T_{ae_{evap}} + a_2 T_{ae_{evap}}^2 + a_3 T_{ae_{cond}} + a_4 T_{ae_{cond}}^2 + a_5 T_{ae_{evap}} T_{ae_{cond}}. \tag{8}$$

In addition to the modeling of the electrical power demanded by the chiller, $P_{chiller}$, it is necessary to obtain a model to estimate the temperature of the water leaving the condenser. This is due to the fact that the temperature of the water leaving the chiller's condenser corresponds to the new temperature of the water that will enter the cooling tower, disregarding the heat loss of the system in the stretch between chiller and the tower [5].

In the operation of the evaporator, the heat transferred to the chiller is defined by Eq. 1. Once the values of $Q_{chiller}$ and $P_{chiller}$ are obtained, applying the first law of thermodynamics to the chiller as a whole, which includes the heat exchanges occurring in the evaporator, in the condenser and in the electric motor that drives the compressor of the chiller, Q_cond is obtained. It mainly represent the portion of heat or thermal load to be transferred in the condenser, as indicated in Eq. 9:

$$Q_{cond} = Q_{chiller} - P_{chiller}. \tag{9}$$

In this way, it is possible to estimate the value of the outlet water temperature of the chiller's condenser, $T_{as_{cond}}$, as indicated in Eq. 10:

$$T_{as_{cond}} = \frac{Q_{cond} + \dot{m}_{water} c_{water} T_{ae_{cond}}}{\dot{m}_{water_{cond}} c_{water}}. \tag{10}$$

The value obtained for $T_{as_{cond}}$ is used in the optimization constraint, in order to meet the operational limits of the cooling tower, and also to determine the new effectiveness of the cooling tower [5].

4 Model Validation Results

In order to validate proposed model for the compression chillers, we collect data from the refrigeration system. The database considered for carrying out the modeling is composed of data representing different situations of thermal load, as these are observed during different operational days. These days are selected in way so that to obtain representative data of various thermal load conditions present throughout the day and for different climatic conditions, contemplating hot and cool days. Note that these conditions impact the performance of the cooling system considered in this modeling. We collected 21,385 operating points of the refrigeration system, corresponding to 29 h and 42 min of operation.

To obtain the coefficients of the adopted models, 50% of the points belonging to the database are used, and these coefficients are then applied to the totality of points in the database. The evaluation of the final result using this procedure is performed based on the values of the Mean Square Error (MSE) and the obtained determination coefficient (R^2). The coefficient of determination varies between 0 and 1, and allows assessing how much the adopted model is capable of approximating the actual data. In this case, the closer the value of R^2 to unity is, the better the assessment of the model [4].

In order to obtain the full model of the compression chiller, we first need to determine of the coefficients b_0–b_5, referring to the modeling of the factors Z_{CAP_1} and Z_{CAP_2}, as indicated in Eqs. 6 and 7, respectively. Likewise, the coefficients a_0-a_5 need to be obtained, referring to the modeling of the factor Z_{EIR}, as indicated in Eq. 8. In this case, the Levemberg-Marquardt method is used as a non-linear regression technique to obtain these coefficients [3]. Table 1 indicates the coefficients obtained for the factor Z_{CAP_1}, based on Eq. 6.

Table 1. Obtained values for the coefficient required to model factor Z_{CAP_1}.

Coefficient	Value
b_0	−0.8108
b_1	−0.0838
b_2	+0.0133
b_3	+0.0997
b_4	−0.0012
b_5	−0.0032

Table 2 indicates the coefficients obtained for factor Z_{CAP_2}, based on the use of Eq. 7.

Table 2. Obtained values for the coefficient required to model factor Z_{CAP_2}.

Coefficient	Value
b_0	−0.1177
b_1	+0.3381
b_2	−0.0513
b_3	−0.0276
b_4	+0.0022
b_5	+0.0030
b_6	−0.0006479

Table 3 indicates the coefficients obtained for the factor Z_{EIR} based on the use of the modeling indicated in Eq. 8.

Table 3. Obtained values for the coefficient required to model factor Z_{EIR}.

Coefficient	Value
a_0	-1.0405
a_1	$+0.1379$
a_2	-0.0090
a_3	$+0.0840$
a_4	-0.0022
a_5	$+0.0033$

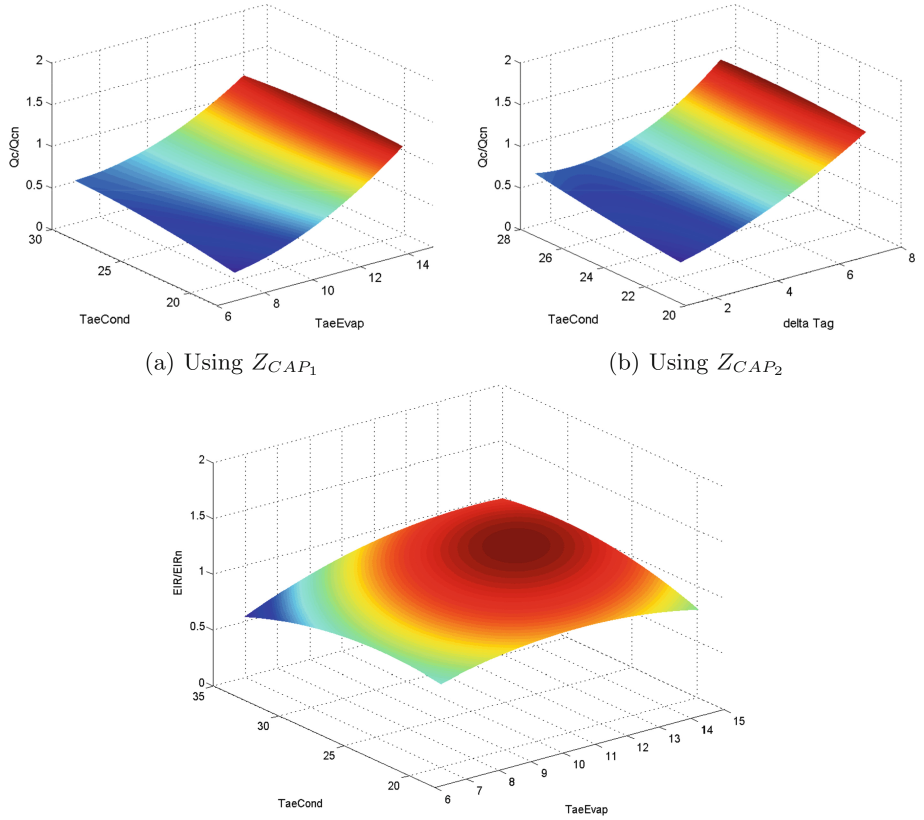
Figures 3(a), 3(b) and 3(c) depict the modeling of factors Z_{CAP_1}, Z_{CAP_2} and Z_{EIR} when applied to the collected data. All indicate the influence of the temperatures of the condensation water, $T_{ae_{cond}}$, and the return water of the secondary circuit, $T_{ae_{evap}}$, on the consumption of chillers. Recall that these factors are used for their calculation in Eqs. 3 and 4.

Modeling Z_{CAP_1} obtained an MSE of 1.25×10^{-3} and an R^2 of 0.9977 for the value of $Q_{chiller}/Q_{chiller_{nom}}$, *i.e.* the instantaneous load factor of the chiller. The modeling of Z_{EIR} occasioned an MSE of 1.54×10^{-4} and an R^2 of 0.9999 for the value of EIR/EIR_{nom} of chiller. Both results are excellent, presenting values of R^2 very close to 1, hence validating the models adopted to estimate these factors.

In Fig. 3(a), we can observe that as $T_{ae_{cond}}$ is reduced, the load factor of the chiller is also reduced, as expected. The result of the modeling factor Z_{EIR} can be seen in Fig. 3(c), where we observe that, as $T_{ae_{cond}}$ and $T_{ae_{evap}}$ decrease, the factor EIR/EIR_{nom} also decrease. This represents a condition of lower electrical consumption of the chiller. Analyzing the curve presented in Fig. 3(a) together with the one presented in Fig. 3(c), we can note that the return temperature of the ice water, $T_{ae_{evap}}$, exerts a greater influence on the load factor of the chiller.

The modeling of Z_{CAP_2} provides an MSE of 1.24×10^{-3} and an R^2 of 0.9965 for the estimated value of $Q_{chiller}/Q_{chiller_{nom}}$. This an excellent result as the value for R^2 is very close to 1, validating thus the adopted model. We can observe in Fig. 3(b) that, as ΔT_{ag} increases, the load factor of the chiller also increases. This is correct, since for a given fixed condition of heat load and inlet and outlet temperatures of the water in the cooling tower, an increase in ΔT_{ag} is only possible by reducing the temperature of chiller water output, represented by $T_{as_{evap}}$, which leads to an increase in the chiller load factor.

The models obtained for factors Z_{CAP_1} and Z_{CAP_2} are considered to have comparable performance, since the first presents a slightly higher R^2, while the second presents a larger MSE. Note that factor Z_{CAP_1} is obtained based on two variables, while the factor Z_{CAP_2} needs three variables. Form this point of view, the model using Z_{CAP_1} is a less complex than that is based on Z_{CAP_2}. However, both Z_{CAP_1} and Z_{CAP_2} based models can be implemented and their impact on the optimization algorithms behavior investigated. So, the best model can be selected based on time and precision requirements of the application at hand.



(a) Using Z_{CAP_1}

(b) Using Z_{CAP_2}

(c) Factor Z_{EIR}

Fig. 3. Model's results for Q_c/Q_{cn} in terms of ΔT_{ag} and $T_{ae_{cond}}$ as well as EIR/EIR_n in terms of $T_{ae_{cond}}$ e $T_{ae_{evap}}$ for a chiller of 1000 TR.

Figure 4 compares the real values collected in the field with the values obtained from the adopted model. This result is obtained using the factor Z_{CAP_1}. The values of the instantaneous total electrical power of the chillers, in MW, obtained from the model defined by Eq. 3, show a faithful representation in relation to the real values collected in the field. We obtained an MSE of 1.102×10^{-3} and an R^2 of 0.9835. Thus, the values of MSE and R^2 obtained validate the model adopted for the consumption of chillers using factor Z_{CAP_1}.

In the same way, Fig. 5 compares the real values of instantaneous total electrical power of chillers, in MW, with the values obtained from the obtained modeling proposed in Eq. 4, which uses the factor Z_{CAP_2}. The results obtained for the modeling also showed a good representation in relation to the real values collected in the field, providing an MSE of 1.633×10^{-3} and a R^2 of 0.9755. The values of MSE and R^2 obtained validate the modeling adopted for the consumption of chillers using the factor Z_{CAP_2}.

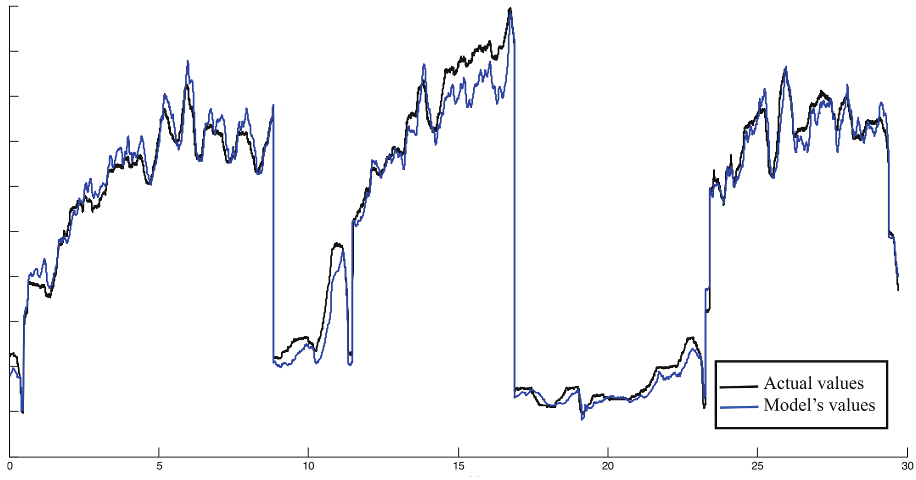


Fig. 4. Actual consumption of chillers *vs.* modeling using the factor Z_{CAP_1}.

We observe that, in terms of modeling the instantaneous total electrical power demanded by the chillers, the use of the factor Z_{CAP_1} presented a lower MSE and a higher R^2 compared to the modeling using the factor Z_{CAP_2}. We conclude, therefore, that the use of the factor Z_{CAP_1} presented more precise results.

Figure 6 compares the real values collected in the field with the values obtained from the modeling of the outlet water temperature of the chillers condenser, obtained from Eq. 10. The result obtained, although a little inferior compared to the other models, showed good representation, with an MSE of 1.51 and a R^2 of 0.8237, thus validating the use of the proposed model.

Recall that the curves represented in Figs. 4, 5 and 6 are the result of applying the models obtained from a database corresponding to 29 h and 42 min of operation of the cooling system, with readings performed every 5 s, corresponding to 21,385 points.

5 Global Energy Demand

The global energy consumption of the considered refrigeration system involves the consumption of the following equipment: cooling tower fans, chillers, condensation water circulation pumps and chilled water circulation pumps. In this case, as the condensed water and chilled water circulation pumps operate at a fixed speed, their energy consumption is therefore constant, regardless of any adjustment in the speed of the cooling tower fans or variation in the temperature of the outlet water of the chiller evaporator. Thus, the inclusion of the consumption of these equipment in the computation of the global consumption is unnecessary. With the modeling of the cooling tower and fans presented in [5] and the compression chillers, presented herein, it is possible to estimate the

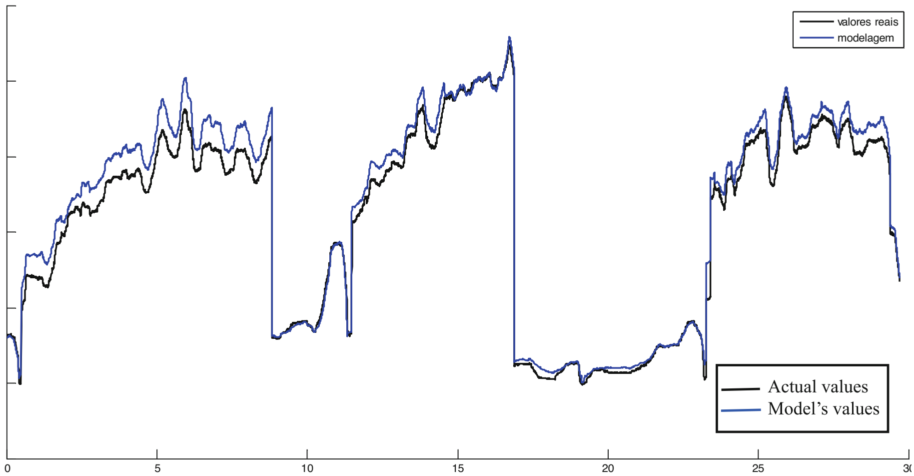


Fig. 5. Actual consumption of chillers *vs.* modeling using the factor Z_{CAP_2}.

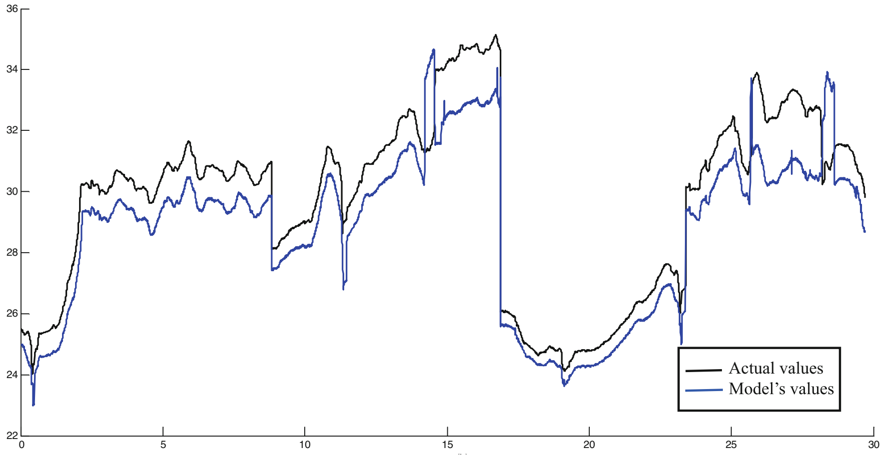


Fig. 6. Actual temperature of the water leaving the chiller condenser *vs.* modeling.

global electric energy savings obtained for the refrigeration system to be used in the energy efficiency oriented optimizations. Therefore, only the consumption of fans and chillers will be included in the modeling of global energy consumption. The instantaneous global power demanded by the cooling system obtained from Eq. 11:

$$P_{global} = n_c P_{chiller} + n_v P_v, \tag{11}$$

where n_c represents the number of chillers that are in operation, and n_v, the number of fans that are in operation. Variable $P_{chiller}$ can be obtained from Eqs. 3 and 4, and P_v is obtained via the model presented in [5].

Figure 7 presents the estimated global electrical power demand for the refrigeration system using the factor Z_{CAP_1} in the modeling of *chillers*. In this case, we obtained an MSE of 1.1×10^{-3} and a R^2 of 0.9830 in the modeling of the global electrical power demand of the refrigeration system, using a database of 21,385 points. This result validates the modeling using the factor Z_{CAP_1}.

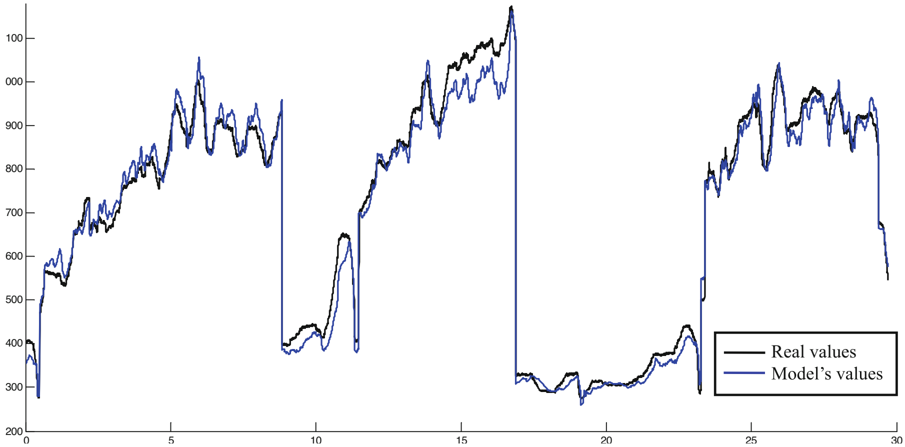


Fig. 7. Real global energy demand *vs.* that obtained via the proposed model using the factor Z_{CAP_1}.

Figure 8 presents the estimated global electrical power using the factor Z_{CAP_2} in the modeling of *chillers*. In this case, we obtained an MSE of 1.6×10^{-3} and a R^2 of 0.9756 in modeling the global electrical power demand of the refrigeration system, using the same database of 21,385 points. This result validates the modeling of global energy consumption using the factor Z_{CAP_2}.

Figures 7 and 8 also show the curves of the electrical power demanded obtained with the real data collected in the field, in order to allow comparison with the results obtained using the adopted models. The total electrical energy consumption of the refrigeration system is numerically equal to the area under each curve shown.

It is clear from the curves and the occasioned error that modeling of the global consumption of the refrigeration system using the factor Z_{CAP_1} presents a more accurate result than the one using factor Z_{CAP_2}. In the former, the actual values are closer to the model's values, as can be noted when comparing the results presented in Figs. 7 and 8.

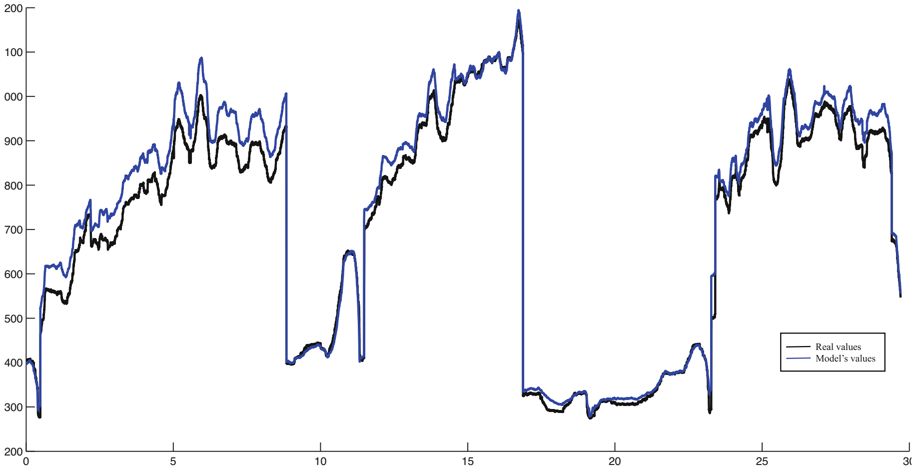


Fig. 8. Real global energy demand *vs.* that obtained via the proposed model using the factor Z_{CAP_2}.

6 Conclusions

In this paper, two models of the compression chiller, which is a main equipment in any industrial refrigeration system based on cooling towers, is presented. Due to lack of space, the proposed model for the cooling tower and the corresponding fans has been be presented in [5]. Both chiller proposed models are validated comparing the model's values to real data, collected from an existing refrigeration system. The models of the chillers show satisfactory results. Moreover, we show that the usage of the proposed model for the whole refrigeration system, which includes several tower cells and several chillers, allows us to model the global demand in terms of energy of the system. We prove that the model's results are accurate and faithful enough to the actual values. This allows their use in successful multi-objective optimization, aiming at energy efficiency.

Acknowledgments. This work is supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq - Brazil) and by Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ - Brazil 203.111/2018). We are most grateful for their continuous financial support.

References

1. ASHRAE: Handbook - HVAC Systems and Equipment, SI edition. American Society of Heating, Refrigerating and Air Conditioning Engineers (2012)
2. Brandemuel, M.J., Gabel, S., Andersen, I.A.: Toolkit for secondary HVAC system energy calculation (1993)

3. Gavin, H.P.: The Levenberg-Marquardt method for nonlinear least squares curve-fitting problems, pp. 1–19 (2020)
4. Leontaritis, I.J., Billings, S.A.: Model selection and validation methods for non-linear systems. Int. J. Control **45**(1), 311–341 (1987)
5. Nedjah, N., Mourelle, L.M., Lizarazu, M.S.D.: Mathematical modeling of cooling towers-based refrigeration systems for energy efficiency optimization. In: Gervasi, O., et al., (eds.) International Conference on Computational Science and Its Applications, pp. 19–27 (2022). accepted
6. Zmeureanu, R., Zelaya, E.M., Giguere, D.: Simulation de la consommation d'énergie d'un aréna à l'aide du logiciel doe-2.1e. In: ESIM2002 Conference, Canada (2002)