Comparison of Influence of Various Proposals of Transforming Nominants into Stimulants on Linear Ordering and Grouping of Listed Companies

Barbara Batóg and Katarzyna Wawrzyniak

Abstract The paper is a continuation of the authors' research on the transformation of nominants with a recommended range of values into stimulants normalised in the interval ⟨0, 1⟩, which is a necessary condition for the use of linear ordering methods for multivariate objects. The present article is empirical in nature. The research was aimed to test to what extent the selected formulas for transformation of nominants into stimulants influence the final result of linear ordering and grouping of objects on the basis of different variable nominants, and therefore whether there are significant differences in the final ranking of the objects obtained using these formulas. The study focused on symmetric nominants and used four formulas of transformation nominants into stimulants: Kukuła's proposal and three authors' proposals. The results of linear ordering were evaluated in terms of the discriminatory properties of the obtained synthetic measures, the consistency of the rankings of objects and the concordance of their grouping. The study uses data on financial ratios characterising the financial situation of companies in the Machinery Industry sector listed on the Warsaw Stock Exchange in 2019.

Keywords Transformation of nominants into stimulants · Linear ordering · Grouping · Financial ratios

1 Introduction

Linear ordering methods (with or without pattern) are used to determine the order of multivariate objects. A necessary condition for the application of almost all these methods is the unification of the nature of the variables constituting the basis for aggregation. Due to the fact that the studied objects are usually ordered from the best

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to the worst from the point of view of the analysed phenomenon, it is necessary to transform destimulants and nominants into stimulants.

In the papers of Batóg and Wawrzyniak (2020, 2021), the authors presented own proposals for the transformation of symmetric and asymmetric nominants into stimulants normalised in the range $\langle 0, 1 \rangle$, and the obtained results of transformation of single indicator nominants were compared with the results of their transformation obtained on the basis of formulas known in the literature and proposed by Kukuła (2000) and Kowalewski (2002, 2006). It turned out that in the case of single indicator nominants, the authors' proposals of transformation of nominants into stimulants made it possible to obtain greater consistency in the ordering of the examined objects according to the value of a given indicator nominant before and after the transformation than the formulas known from the literature. Therefore, in further research in this area, the authors decided to conduct empirical research, the results of which are presented in the current paper. This research aimed to test to what extent the proposed formulas for transforming nominants into stimulants affect the final result of linear ordering and grouping objects on the basis of several variable nominants, and thus whether there are significant differences in the final ranking of the objects obtained using the proposed formulas and those known in the literature. Moreover, the authors tried to find an answer to the question whether there is a rationale for recommending a specific formula for transforming nominants into stimulants.

At this stage of the research, the focus was on symmetric nominants, i.e., those for which the situation of the given object with values of the indicator nominant below the lower and above the upper limit of the recommended range of values is evaluated equally. Therefore, the transformation proposed by Kukuła (2000) and the authors' proposals based on linear and nonlinear concave and convex transformations (Batóg and Wawrzyniak 2020, 2021) were used to transform the symmetric nominants into stimulants normalised in the range $\langle 0, 1 \rangle$. Then the values of synthetic measures, calculated on the basis of the values of obtained stimulants, were compared in terms of discriminatory properties, and the results of linear ordering and grouping of objects obtained on their basis were compared. The differences identified in the results obtained were used to formulate a recommendation for a particular symmetric nominant transformation formula.

The data on four financial ratios (nominants) came from Notoria Serwis and characterised the financial situation of 24 companies from the *Machinery Industry* sector listed on the Warsaw Stock Exchange in 2019.

2 Applied Methods of Transformation of Symmetric Nominants with a Recommended Range of Values into Stimulants Normalised in the Range ⟨0, 1⟩

In the study, four methods were used to transform symmetric nominants with a recommended range of values into stimulants normalised in the range ⟨0, 1⟩ (the symbol of the method used in the resulting tables is given in brackets):

- transformation proposed by Kukuła (M1),
- transformation proposed by authors—this one assumes linear decrease of values of stimulant on both sides of recommended range of values of nominant (M2),
- transformation proposed by authors—this one assumes non-linear decrease of values of stimulant on both sides of recommended range of values of nominant and the situation when the decrease of values of the normalised stimulant close to the lower and upper limits of a recommended range of values is faster than for values close to minimum and maximum (convex functions) (M3),
- transformation proposed by authors—this one assumes non-linear decrease of values of stimulant on both sides of recommended range of values of nominant and the situation when the decrease of values of the normalised stimulant close to the lower and upper limits of a recommended range of values is slower than for values close to minimum and maximum (concave functions) (M4).

The formulas of above methods could be found in Batóg and Wawrzyniak (2021). The ideas of the above methods are presented in Figs. 1, 2, 3, 4.

It is worth to underline that in the case of methods M2, M3, and M4 the value of new minimum or new maximum is determined with the condition that the distance of the new minimum from the lower limit of the recommended range of values is the

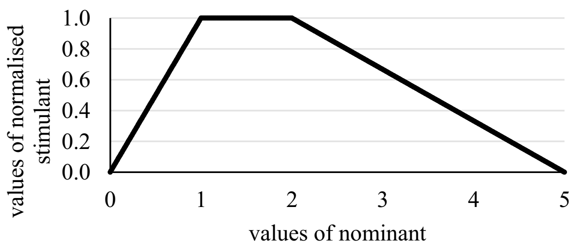

Fig. 1 Illustration of method M1

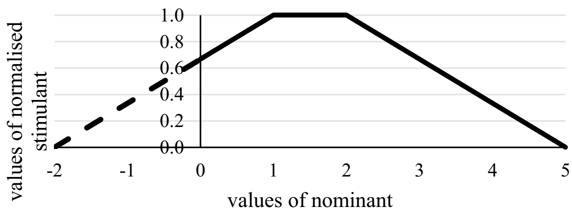

Fig. 2 Illustration of method M2

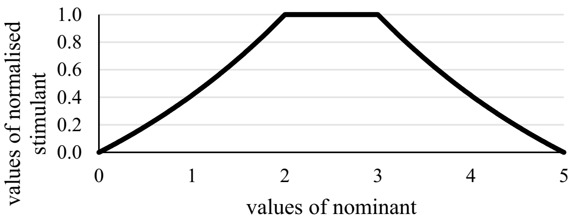

Fig. 3 Illustration of method M3

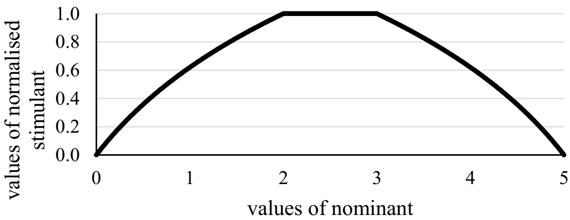

Fig. 4 Illustration of method M4

same as the distance of the new maximum from the upper limit of the recommended range of values.

3 Measures Used to Compare the Results Obtained

A synthetic measure used to rank the objects (companies) was defined as the arithmetic mean of the values of stimulants normalised in the range ⟨0,1⟩ based on the transformation methods discussed above—Eq. 1.

$$z_i = \frac{1}{K}\sum_{k=1}^{K} x_{ik} \tag{1}$$

where

x_{ik}—value of kth stimulant normalised in the range ⟨0,1⟩ for ith object,

z_i—value of the synthetic measure for ith object,

$i = 1, 2 \ldots, n$—number of object,

$k = 1, 2 \ldots, K$—number of stimulant normalised in the range ⟨0,1⟩.

The method of three averages was used to classify objects into groups (Nowak 1990). In this method, we obtain four groups, and the division points are: the overall average of all values, the average of values below the overall average and the average of values above the overall average.

The comparison of the obtained results started with an assessment of the discriminatory properties of the determined synthetic measures. To conduct this assessment, the G coefficient proposed by Sokołowski (Pociecha et al. 1988; Nowak 1990) was applied—Eq. 2.

$$G = 1 - \sum_{i=1}^{n-1} \min_i \left\{ \frac{z_i - z_{i+1}}{\max_i\{z_i\} - \min_i\{z_i\}}; \frac{1}{n-1} \right\} \tag{2}$$

where

$i = 1, 2 \ldots, n$—number of object,

z_i—value of synthetic measure for ith object.

The G coefficient takes values in the range $\langle 0, 1 - \frac{1}{n-1} \rangle$. The higher the value of G, the better discriminatory properties of the synthetic measure, that is, the greater ability of this measure to group objects.

The Pearson correlation coefficient (the relationship between the values of the synthetic measures) and Kendall's tau coefficient (the relationship between the rankings) were used to examine the consistency of the rankings (Salkind 2007). The values of these coefficients are in the range $\langle -1, 1 \rangle$, and the consistency of the rankings is greater when the values of the coefficients are close to one.

The following measures were used to test the consistency of the grouping of objects obtained by the method of three means on the basis of the values of the synthetic measures:

- simple matching coefficient (Everitt et al. 2011)—Eq. 3

$$C = \frac{1}{n} \sum_{i=1}^{n} c_i \cdot 100\% \tag{3}$$

where

$i = 1, 2 \ldots, n$—number of object,

$c_i = 1$ when ith object is in the same group in two classifications,

$c_i = 0$ when ith object is not in the same group in two classifications.

This measure allows to evaluate the percentage of objects which are in the same group in two classifications. The consistency of classifications is greater when this measure is close to 100%. This measure answers the question of whether for a given two synthetic measures we obtain the same grouping (the same belonging of objects to groups). It does not indicate which classification is better, but only which classification gives a grouping that is clearly different from the other classifications.

- Measure proposed by Nowak (Nowak 1985; Walesiak 2011)—Eq. 4

$$S = \frac{1}{v+u}\left(\sum_{s=1}^{u} \max_{r}\{k_{sr}\} + \sum_{r=1}^{v} \max_{s}\{k_{sr}\}\right) \quad (4)$$

$$k_{sr} = \frac{n_{sr}}{max\{n_{s\cdot}; n_{\cdot r}\}}$$

where

n_{sr}—number of objects, which belong at one time to the group $P_r^{(t)}$ and $P_s^{(q)}$ in partition $P^{(t)}$ and $P^{(q)}$,

$n_{s\cdot}$—number of objects in group $P_s^{(t)}$,

$n_{\cdot r}$—number of objects in group $P_r^{(q)}$,

$r = 1, 2, \ldots, v$, v—number of groups in partition $P^{(t)}$,

$s = 1, 2, \ldots, u$, u—number of groups in partition $P^{(q)}$.

Nowak's measure takes values in the range $\langle \frac{1}{n}, 1\rangle$. The average similarity of the most similar groups in two partitions is greater when the measure is close to one.

- Rand index (Rand 1971)—Eq. 5.

$$RI = \frac{1}{\frac{n(n-1)}{2}} \sum_{r,s;r<s} c_{rs} \quad (5)$$

where

r, s—number of object,

n—number of objects,

$c_{rs} = 1$ when objects r and s are in the same group in two classifications or objects r and s are not in the same group in two classifications,

$c_{rs} = 0$ when objects r and s are in the same group in one classification and are not in the same group in the second classification.

Rand index takes values in the range $\langle 0, 1\rangle$. The consistency of two classifications is greater when this measure is close to one. Rand index is different from other measures because it takes into account not only consistency of belonging of objects to the same group but also consistency of belonging of objects to the different groups.

4 Data and Results

The research objective was realised using data from Notoria Serwis on four indicator nominants characterising liquidity (current ratio), indebtedness (debt ratio) and efficiency of activity (receivables turnover and rotation commitments). The data concerned companies from *Machinery Industry* sector in 2019. The following recommended range of values for these indicator nominants can be found in the literature (Sierpińska and Jachna 2004; Hozer et al. 1997; Łuniewska and Tarczyński 2006; Gabrusewicz 2014):

- current ratio: ⟨1.2; 2⟩,
- debt ratio: ⟨0.56; 0.67⟩,
- receivables turnover: ⟨30 days; 60 days⟩,
- rotation commitments: ⟨30 days; 60 days⟩.

According to the procedure of linear ordering of objects, the values of indicator nominants were unified and transformed into stimulants normalised in the range ⟨0, 1⟩ according to methods 1, 2, 3 and 4, respectively, and then synthetic measures were determined on their basis and the companies were ordered according to the decreasing values of these measures—see Table 1.

The comparison of the obtained results began with the calculation of Sokolowski's coefficient, which allows the evaluation of the discriminatory properties of the determined synthetic measures. The value of this coefficient in this study takes values in the range ⟨0; 0.957⟩. Pearson correlation coefficients were then determined to examine the relationship between the values of the synthetic measures, and Kendall's tau coefficients were determined to examine the relationship between the rankings. Values of Sokolowski's coefficients, Pearson correlation coefficients and Kendall's tau coefficients are presented in Table 2.

Table 2 shows that the discriminatory properties of the obtained synthetic measures are similar, but slightly better discrimination of companies was obtained using the third transformation method. The differences between the results (values of the synthetic measure and the rankings of companies) obtained using this method of transformation of nominants and the results obtained using the other methods are confirmed by the values of both Pearson correlation coefficients and Kendall's tau coefficients—they are the lowest among all determined coefficients, that is, they prove the lowest consistency of companies orderings obtained by third method in relation to orderings obtained by first, second and fourth methods.

In the next stage of the research, the results of the grouping of companies made by the method of three averages obtained on the basis of the determined synthetic measures were compared.

The belongingness of companies to given groups is presented in Table 3, where group 1 includes the best companies from the point of view of the analysed ratios (after transformation of nominants into stimulants), and group 4—the worst companies.

Table 1 Values of the synthetic measures and the corresponding rankings of companies for each method of transformation of nominants into stimulants

Company	Values of the synthetic measures				Rankings of companies			
	M1	M2	M3	M4	M1	M2	M3	M4
Apator	0.943	0.979	0.962	0.986	5	3	3	3
Aplisens	0.360	0.544	0.331	0.593	24	24	24	24
Apsenerg	0.972	0.972	0.710	0.982	4	5	12	5
Bumech	0.889	0.889	0.750	0.918	9	13	8	13
Famur	0.750	0.750	0.750	0.750	18.5	21	8	21
Fasing	0.865	0.865	0.672	0.898	11	15	17	15
Feerum	1.000	1.000	1.000	1.000	1.5	1.5	1.5	1.5
Hydrotor	0.912	0.951	0.673	0.968	8	6	16	6
Izostal	0.750	0.892	0.750	0.921	18.5	11	8	12
Jwwinv	0.849	0.872	0.380	0.910	13	14	23	14
Lena	0.740	0.817	0.400	0.870	20	18	22	17
Mój	0.820	0.820	0.750	0.848	16	17	8	18
Newag	0.785	0.785	0.699	0.826	17	20	14	20
Patentus	0.940	0.940	0.681	0.960	6	7	15	7
Primetech	0.479	0.731	0.470	0.738	23	22	21	22
Rawlplug	0.852	0.852	0.750	0.882	12	16	8	16
Relpol	0.848	0.910	0.536	0.941	14	10	19	10
Secowar	1.000	1.000	1.000	1.000	1.5	1.5	1.5	1.5
Sonel	0.842	0.891	0.703	0.925	15	12	13	11
Sunex	0.649	0.792	0.500	0.848	21	19	20	19
Wielton	0.873	0.939	0.735	0.959	10	8	11	8
Zpue	0.975	0.975	0.755	0.983	3	4	5	4
Zrembch	0.939	0.939	0.920	0.958	7	9	4	9
Zuk	0.615	0.726	0.550	0.735	22	23	18	23

Source Own calculations

An analysis of the results in Table 3 shows that there were differences for methods of transformation both in the number of companies belonging to a given group and in the belongingness of a given company to a given group.

To confirm these observations, measures of consistency of groupings were calculated, the values of which are shown in Table 4.

Table 4 shows that the grouping results obtained on the basis of synthetic measures calculated with the use of different formulas of transformation of nominants into stimulants are quite different from each other, and this is particularly noticeable in the case of third method. The highest consistency of grouping of companies was obtained for second and fourth methods, i.e. when we assume linear (M2) and concave (M4) functions on both sides of the range of recommended values. In contrast, the

Table 2 Values of Sokolowski's coefficients, Pearson correlation coefficients and Kendall's tau coefficients for each method of transformation of nominants into stimulants

	M1	M2	M3	M4
Sokolowski's coefficients	0.446	0.453	0.478	0.466
Pearson correlation coefficients				
M1	1.000	0.938	0.724	0.930
M2		1.000	0.691	0.989
M3			1.000	0.630
M4				1.000
Kendall's tau coefficients				
M1	1.000	0.794	0.523	0.794
M2		1.000	0.470	0.985
M3			1.000	0.456
M4				1.000

Source Own calculations

lowest consistency of grouping of companies can be observed for second and third methods, i.e. when we assume linear (M2) and convex (M3) functions on both sides of the range of recommended values.

5 Conclusions

The research shows that for the companies examined and the selected indicator nominants:

1. the method of transformation of nominants into stimulants, assuming their symmetry, did not significantly affect the discriminatory properties of the synthetic measures obtained on their basis, although it was possible to indicate one method—the third one, which provided the greatest discrimination of the companies,
2. the greatest differences in ordering and grouping of companies were observed when comparing the results obtained with the third method (non-linear transformation assuming fast decrease of the values of the stimulants on both sides of the recommended range of values described by convex functions) with the results obtained with the other methods,
3. the smallest differences in ordering and grouping of companies were observed when comparing the results obtained with the second method (linear transformation) and the fourth method (non-linear transformation assuming slow decrease of the values of the stimulants on both sides of the recommended range of values described by concave functions).

Table 3 Belongingness of companies to given groups for each method of transformation of nominants into stimulants

Company	M1	M2	M3	M4
Apator	1	1	1	1
Apsenerg	1	1	2	1
Feerum	1	1	1	1
Hydrotor	1	1	3	1
Patentus	1	1	3	1
Secowar	1	1	1	1
Zpue	1	1	2	1
Zrembch	1	2	1	1
Bumech	2	2	2	2
Fasing	2	3	3	2
Jwwinv	2	2	4	2
Moj	2	3	2	3
Rawlplug	2	3	2	3
Relpol	2	2	3	2
Sonel	2	2	2	2
Wielton	2	1	2	1
Famur	3	4	2	4
Izostal	3	2	2	2
Lena	3	3	4	3
Newag	3	3	2	3
Sunex	3	3	4	3
Aplisens	4	4	4	4
Primetech	4	4	4	4
Zuk	4	4	3	4
Number of companies in groups				
Group 1	8	8	4	9
Group 2	8	6	10	6
Group 3	5	6	5	5
Group 4	3	4	5	4

Source Own calculations

On the basis of the above conclusions, it can be stated that in this particular study, the results obtained on the basis of the synthetic measure calculated with the use of the third method of transformation of nominants with a recommended range of values into stimulants normalised in the interval $\langle 0, 1\rangle$ differed the most from the other results of ordering and grouping of companies. For this transformation method, the synthetic measure had the best discriminatory properties.

Table 4 Measures of consistency of groupings

	M1	M2	M3	M4
Simple matching coefficients				
M1	100.0	70.8	45.8	79.2
M2		100.0	37.5	91.7
M3			100.0	37.5
M4				100.0
Nowak measures				
M1	1.000	0.656	0.431	0.716
M2		1.000	0.342	0.889
M3			1.000	0.382
M4				1.000
Rand indices				
M1	1.000	0.779	0.667	0.833
M2		1.000	0.620	0.917
M3			1.000	0.638
M4				1.000

Source Own calculations

Therefore, if the main objective of this study was to group companies as well as possible (to obtain groups that are most different from each other), it is this method of transformation of nominants that would be recommended to determine the synthetic measure.

At this stage of the research on the influence of nominant transformation methods on the results of object ordering and grouping, the authors do not formulate conclusions of a general nature, but wish to propose two solutions that may be useful in choosing the best transformation method:

- solution I—the choice of a particular transformation method is determined by meritorious reasons concerning the indicator nominant values outside the range of recommended values in a given sector of economy (expert solution),
- solution II—the choice of a particular transformation method is determined by preliminary research using the level of consistency of orderings of companies from the point of view of a single indicator nominant before and after the transformation—we choose that transformation method for a given indicator, which guarantees greater consistency (author's solution Batóg and Wawrzyniak 2020, 2021).

In order to verify the validity of the presented conclusions and solutions, the authors intend to compare the results based on data for a larger number of companies belonging to different sectors in future research.

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