Chapter 7
Tooth Relieving of Worm Hobs for Cutting Novikov Gears with Double Lines of Action

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7.1 Introduction

The peculiarity of the formation of screw and relieved surfaces by disk grinding wheels lies in the inevitable deviation of the profile of the ground surface from the profile of the wheel, which is called an organic error. For a straight basic rack, this organic error is minimized as much as possible, or, at the attained degree of minimization, an obtained profile is taken as the basic rack [1–3]. For screw and relieved surfaces with a profile of substantial and variable curvature, this organic error must be eliminated to the maximum degree.

A well-known example of a solution to this problem is the method of forming a profile of an unruled worm of the ZT2 type, proposed by F.L. Litvin in 1961 (later, in the monograph by Litvin and Fuentes [4], such worms were considered as worms of the ZF-II type). In this example, the axial profile of a grinding wheel was set in the form of a circular arc. Then, based on the condition that this arc is the contact line of the wheel and the ground surface, the angle of installation of the grinding wheel axis and the axial profile of the worm thread were sought. Obviously, to obtain a worm hob with the relieved surfaces of the teeth close in profile to the found worm threads, a solution for the inverse problem is required.

In the studies by Sandler et al. [2] and Sandler and Lagutin [5], the authors proposed a solution for a similar problem with respect to the convex thread profile of the worm and the hob for cutting the teeth of gears with liquid friction. They specified a substantially curvilinear axial profile of the working worm (or the hob's generating worm) and also took the profile of the grinding wheel in the axial section of the worm. The setting angle of the grinding wheel axis is determined on the basis of the condition that this axis intersects two normal lines to the thread axial profile and lies on a plane parallel to the axis of the worm. The proposed method

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ensures minimization of the organic error in the profiling of the worm thread and, taking into account a similar organic error in tool profiling, the necessary identity of the profile of the hob's generating worm.

In the work by Sandler and Lagutin [6], the authors made the first attempt to apply the general principles of this method to the study of the relieving process of worm hobs for cutting Novikov gears. In this case, two significant factors were taken into account. First, the curvature of the profile of the generating rack of these worm hobs is a variable in terms of not only its radius of curvature but also its sign. Second, the possibilities of the relieving machine allow setting the axis of the grinding wheel on a plane not parallel to the axis of the ground product that improves the grinding conditions for a number of parameters.

In this work, this research is developed and continued. This has solved the main issues of the relieving technique, which are functionally oriented to solving the following problems: providing the rear angles of the teeth necessary for the wear resistance of the cutting edges of the hob, minimizing the organic error of the profile of the generating rack, and determining the main parameters for setting up the relieving machine and the profile of the grinding wheel.

7.2 Novikov Gearing: Parameters of the Basic Rack

For the first time, helical gears with an initial point tangency of the circular-helical surfaces of the teeth were proposed by the eminent American inventor E. Wildhaber in 1926 with a US patent no. 1,601,750.[1] However, at that time, this invention went unnoticed, and such gears underwent rapid development only after the Soviet engineer M.L. Novikov formulated the general principle of their formation and showed that they will have an increased load capacity, primarily in contact endurance [7]. The fundamental novelty of Novikov's invention was analyzed in the works by S.P. Radzevich [8, 9].

Initially, M.L. Novikov proposed a version of helical overcentrode gears, in which tooth profiles, convex on the pinion and concave on the wheel, were delineated with the circular arcs in the face section of the gears. Later on, V.N. Kudryavtsev showed that the same effect can be achieved in gears synthesized on the basis of two incongruent basic racks with the circular-arc profiles and cut by two worm hobs: one for the pinion with convex teeth and the other for the wheel with concave ones [10].

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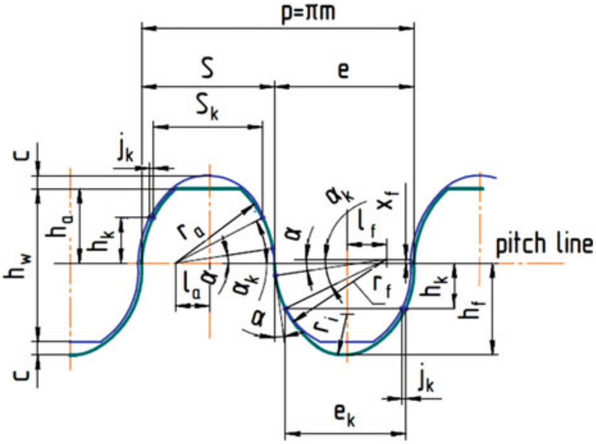
[1] A comment from the editor: It is proven [8] that the invention by E. Wildhaber (1926) is not workable at all, as the fundamental laws of gearing are violated in this invention. It is also proven [8] that the fundamental laws of gearing are fulfilled in Novikov gearing, and, therefore, Novikov gearing is workable. It is a wrong practice to combine Novikov gearing with the helical gearing by E. Wildhaber in a common term – the terms "Wildhaber–Novikov gearing" and "W-N gearing" are meaningless by nature.

Fig. 7.1 Basic rack for Novikov gears according to GOST 15023-76



However, at present, basic racks are standardized and widely used in production with a full tooth profile, which provides, in gears, two lines of action and allows for cutting the pinion and the wheel by one worm hob [11]. In particular, according to the Russian Standard GOST 15023-76, a basic rack consists of a convex addendum, a concave dedendum, and a short straight section between them. The geometry of Novikov gears with two lines of action is calculated according to the Russian Standard GOST 17744-72. Figure 7.1 shows the main parameters of a basic rack, the values of which depend on a range of modules.

The pressure angle α_k at the predetermined contact points on the tooth addendum and dedendum is equal to 27° for any case. These points are also characterized by parameters such as the distance h_k from the pitch line, the hob tooth width e_k at its addendum, and the space width S_k at its dedendum; the difference $j_k = e_k - S_k$ provides a backlash between the teeth of the wheels to be cut. The pressure angle α in the straight section of the tooth depends on the module and is equal to $\approx 8°$.

The hob tooth in the normal section is formed according to the dimensions of the space of the basic rack. Similarly, the space between the hob teeth forms the tooth of the wheel to be cut.

The convex profile of the hob tooth is outlined by the radius r_f, the center of curvature of which is located at a distance l_f from the axis of symmetry of the head and is shifted relative to the pitch line to the hob axis by x_f. The center of the concave arc radius r_a of the hob tooth space is located on the pitch line, at a distance l_a from the axis of symmetry of the space. Dimensions along the pitch line are the thickness e of the hob tooth and the width S of the hob space. The height h_f of the hob tooth head is equal to the dedendum of the gear to be cut. The depth of the space is equal to the sum of the addendum h_a of the gear and the radial clearance c between the gear and the hob when cutting. Moreover, in Fig. 7.1, the height h_w of the tooth active profile in the gear and the radii r_i of the arc transition curves on the head and in the space of the hob tooth are indicated.

7.3 Worm Hob Parameters

The main parameters of any worm hob are the axial module m, the radius r_F of the pitch cylinder, the number of treads z_0, the screw parameter $p = 0.5m\ z_0$ of the generating surface, and the number of teeth z_f in the face section. From these data, the lead angle γ_1 of the helical surface of the cutting edges on the hob pitch cylinder is determined from the expression:

$$\tan \gamma_1 = p/r_F \tag{7.1}$$

The worm hob design profile (a generating rack) is considered in the normal section of the generating worm, which is tangent to the screw front surface. For single-thread hobs, this section and the front surface practically coincides in the profiling zone.

When cutting the teeth on a gear hobbing machine, the plane of the generating rack is installed perpendicular to the direction of the teeth of the cut wheel, while the hob axis is set, taking into account the lead angle of the hob front surface.

When the teeth of the worm hob are relieved, its axis is installed in the centers of the grinding-relieving machine, that is, in the horizontal plane. For a reliable reproduction of the profile of the generating rack in the normal section of the hob, with the adopted profiling method, it is necessary to recalculate the parameters of the profile of the generating rack into the axial section of the tooth surface to be relieved.

First of all, it is necessary to determine the axial profile of the helical surface, on which the cutting edges of the hob teeth are located. To do this, the prescribed angle α of the normal profile of the straight section should be replaced by an angle α_1 of the axial profile according to the formula:

$$\tan \alpha_1 = \tan \alpha / \cos \gamma_1 \tag{7.2}$$

The curvature radii r_a and r_f of the two parts of the active profile in the normal section are replaced by the corresponding radii r_{a0} and r_{f0} in the axial section. For single-thread hobs, these radii with a sufficient approximation are determined by the Meusnier theorem from the expression:

$$r_{a0,f0} = r_{a,f} / \cos \gamma_1 \tag{7.3}$$

The arc radius r_i on the head of the hob tooth, which is a processed fillet of the wheel tooth, in the normal section is replaced with the corresponding radius r_0 in the axial section of the hob. According to Euler’s formula, these radii are related by the expression:

$$r_0 = r_i / \cos^2 \gamma_1 \tag{7.4}$$

The thread thickness b in the axial section should be determined on the minimum ground radius $r_{\min}$ of the hob, proceeding from the thickness b_n of the normal section at the end point of machining:

$$b = b_n / \cos \gamma_{(r_{\min})} \tag{7.5}$$

where $\gamma_{(r_{\min})} = \operatorname{atan}(p/r_{\min})$ is the lead angle on the hob dedendum cylinder.

The calculated coordinates of the axial profile of the helical surface of the cutting edges are related to the coordinates of the generating rack profile by the dependencies:

- For point A_1 on the tooth dedendum:

$$x_{01} = x_{n1}; z_{01} = z_{n1} / \cos \gamma_{x01}$$
$$\tan \alpha_{01} = \tan \alpha_{n1} / \cos \gamma_{x01},$$

where:

$$z_{n1} = 0{,}5\pi m_n + l_a - (r_F - x_{n1}) \cot \alpha_{n1}$$
$$\gamma_{x01} = \operatorname{atan}(p/x_{01}),$$

- For point A_2 on the tooth addendum

$$x_{02} = x_{n2}$$
$$z_{02} = z_{n2} / \cos \gamma_{x02}$$
$$\tan \alpha_{02} = \tan \alpha_{n2} / \cos \gamma_{x02},$$

where:

$$z_{n2} = (x_{n2} - r_F + x_f) \cot \alpha_{n2} - l_f$$
$$\gamma_{x02} = \operatorname{atan}(p/x_{02})$$

The found parameters of the axial section of the helical surface of the cutting edges of the hob teeth are also parameters of the axial section of the relieved surfaces of the teeth.

For clarity, let us consider Fig. 7.2, which shows an example of the basic rack for gears with the module $m_n = 12$ mm.

For full-profile Novikov gears, the pitch line of the basic rack divides the tooth height in half. The active parts of the hob tooth profile are limited by the dimensions 10.2 and 10.32 mm from the pitch line. At the same points, there is a joint of the radius sections of the profile. The tooth thickness in the normal section at the minimum machining diameter is $b_n = 31.7$ mm.

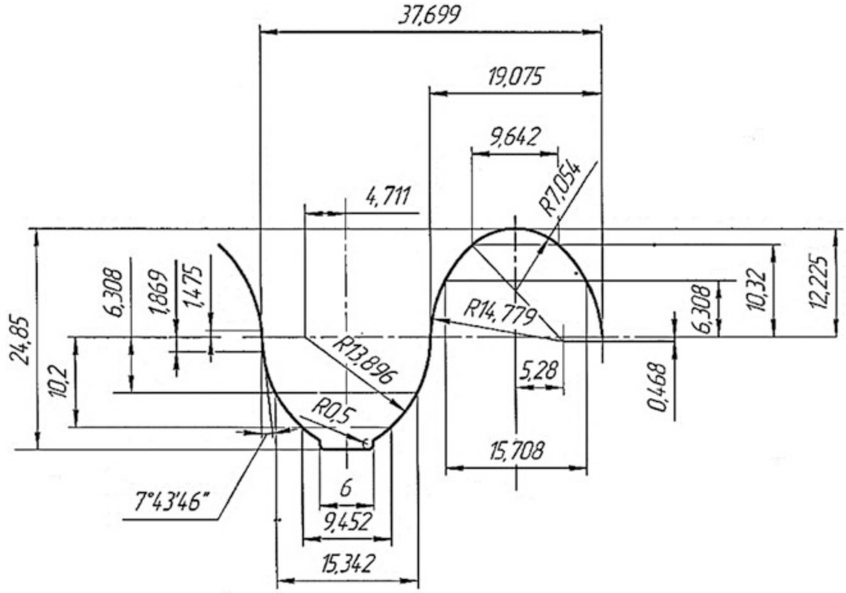


Fig. 7.2 An example of the basic rack of the hob for gears with the module $m_n = 12$ mm

The "predesigned parameters" of the worm cutter are as follows: the outer diameter is 180 mm; the number of hob threads $z_0 = 1$; the number of teeth in the face section $z_f = 10$; the pitch diameter is 155.55 or its radius $r_F = 77.775$ mm; and the fall of the relieving cam on the outer cylinder $K = 10$ mm.

The "calculated parameters" are as follows: the axial module $m = 12.038$; the screw parameter of the generating worm $p = 6.019$; the lead angle on the pitch cylinder $\gamma_F = 4.425°$; the parameter of relieving $k = K z_f/2\pi = 15.915$; the minimum radius of grinding on the cylinder of hob spaces $r_{\min} = 66.215$; the lead angle on this cylinder is 5.194°; and the maximum tooth thickness $b = 31.83$.

The parameters of the axial profile of the hob generating surface are as follows: the minimum pressure angle on the straight section $\alpha_{1\ \min} = 7.7523°$; the concave arc radius on the tooth head $r_{0f} = 14.823$; the convex arc radius in the space $r_{0a} = 13.938$; and the radii of transitive curves on the tooth head $r_{i\ f} = 7.096$ and in the space $r_{i\ a} = 7.002$. The pressure angles at special points of the work area are as follows:

- At a point of conjugation of the radii r_{0f} and r_{if} on the head of the tooth $\alpha_{0f} = 46.95°$
- At the contact point on the head of the tooth $\alpha_{01} = 27.35°$
- At a point of conjugation of the radii r_{0a} and r_{ia} in the space $\alpha_{0a} = 47.34°$
- At the contact point in the space $\alpha_{02} = 27.08°$

7.4 Parameters of Radial–Axial Relieving

The hobs for cutting the involute gears have an almost straight tooth profile with the pressure angle $\alpha_0 \approx 20°$. The relieving of such hobs is usually carried out by the radial method.

The tooth of hobs for cutting Novikov gears has a substantially curved profile with a large difference in pressure angles. The pressure angle in the straight section of the tooth profile near the line is $\alpha_0 \approx 8°$, and, therefore, the rear angles in this section are minimal, which, during cutting, significantly reduces the wear resistance of the cutters as a whole, since it is precisely in this section that they are intensively formed wear sites [12]. In addition to the arc sections with large pressure angles, a large shift of the profile relief surface relative to the pitch line will occur, which will appear at re-sharpening of the hob. So, such hobs cannot be relieved with only a purely radial method, and, therefore, radial–axial relieving must be used.

In the general case of radial–axial relieving of the teeth of the worm hob [1–3], the direction of relieving motion is perpendicular to the hob axis an angle φ_c, which ensures obtaining a necessary rear angle $\lambda_б$ near the cutting edges of the hob teeth.

The grinding wheel axis O_w–O_w is turned (in the projection to the horizontal plane) at an angle φ_0 to the hob axis O_1–O_1 (see Fig. 7.5) and is simultaneously inclined to this plane by the angle β_w. Such a method of setting the axis of the grinding wheel eliminates the large differences in the curvature of the grinding surface of the wheel, improves the grinding conditions, and increases the life of the grinding wheel.

During the relieving, the hob rotates around its axis O_1–O_1 with the angular velocity ω_1, the grinding wheel moves along the hob axis at a speed $p\omega_1$ and performs reciprocating motion with a speed $k\omega_1$, where $k = K\,z_f/2\pi$ is the relieving parameter and K is the recession of the Archimedean spiral of the cam of the relieving mechanism on the angular pitch of the hob teeth. The relieving carriage slides are turned perpendicular to the hob axis by an angle φ_c, which allows increasing the rear angle λ_b on the lateral surface of the tooth.

The current radius r_w of the grinding wheel is determined at the point of its contact with the tooth of the hob on its current cylinder. For the initial position of the wheel, it is advisable to take the tangency of its maximum radius r_{wm} with the minimum radius of the work piece surface $r_{i\ \min}$.

With radial–axial relieving, the lead angle γ_0 of the relieved surface at each of the two selected points ($i_{1,2}$) is determined from the expression [1, 2]:

$$\tan\gamma_{0i} = [p \pm k\,\sin\,(\alpha_i + \varphi_c)/\cos\,\alpha_i]/x_{0i} \tag{7.6}$$

where the radius x_{0i} of the location of the selected point on the ground surface is determined from the drawing of the product.

The sign "+" in square brackets refers to the teeth side where the lead angle of the relieved surface is greater than the lead angle of the generating surface of the hob, sign "–", respectively, to the opposite side of the teeth.

From formula Eq. (7.6), it follows that the values of the angles γ_{0i} depend on the angle φ_c of the installation of the relieving support, which is determined by the required rear angle near the cutting edge of the tooth. For hobs with variable profile curvatures, this angle is determined in the area with the smallest value of the angle α_{0i}, that is, in the considered example for a straight-line segment near the pitch line, where $\alpha_{0\ \min} = 7.7523°$.

7.5 Calculation of the Angle φ_c Installation of the Relieving Support of the Machine

For worm hobs intended for finish cutting of involute gears with module 12 mm of the thermally hardened steel, the Russian State Standard 9324-80 recommends parameters of radial relieving providing a normal relief angle of ≈4° on a pitch cylinder. Based on this recommendation, for the considered type of hob, we establish the following dependence, which ensures the determination of φ_c with sufficient accuracy for practical calculations:

$$(\tan \gamma_{0\mathrm{F}} - \tan \gamma_{x\mathrm{F}}) \cos \alpha_{0\min} = \tan 4° \tag{7.7}$$

Expanding the meanings of $\tan \gamma_{0\mathrm{F}}$ from Eq. (7.6) and $\tan \gamma_{x\mathrm{F}} = p/r_\mathrm{F}$, we obtain the following expression for determining the value of φ_c:

$$\sin \left(\alpha_{0\min} + \varphi_{c\mathrm{R,L}}\right) = 0.44\, r_\mathrm{F}/\left(K z_\Phi\right) \tag{7.8}$$

The subscript "$_{\mathrm{R,L}}$" relates to the right and left sides of the hob tooth. For the example considered above with $\alpha_{0\ \min} = 7.7523°$, we obtain $\varphi_{c\mathrm{R,L}} = 12.229°$, taking into account the price of the rotation angle scale that we set according to the rounding rules. $\varphi_{c\mathrm{R,L}} = 12°$.

Table 7.1 shows the calculated parameters of the profile of the hob tooth and the lead angles of the relieved surface of the teeth at seven key points of the profile $i = 1 \ldots 7$:

1. The boundary of the active section of the profile on the tooth leg
2. The point of contact on the leg of the hob tooth
3. The conjugation of the arc concave section with a rectilinear section
4. The point on the hob pitch cylinder
5. The conjugation of the rectilinear and arc sections on the tooth head
6. The point of contact on the head of the hob tooth
7. The boundary of the active section of the profile on the tooth head

In Figs. 7.3 and 7.4, the graphs of the dependence of the lead angles γ_{0i} of the relieved surface on the radius $x_{0i} = r_i$ of the profile point, taking into account the pressure angle α_{0i} at $\varphi_c = 12°$, are shown for both sides of the tooth.

Table 7.1 Parameters of hob tooth profile in key points (for the given example)

Numbers of profile points	Radius of point, mm $x_i = r_i$	Pressure angle in axial section, α_{0i}°	Lead angle of screw surface of cutting edges, γ_i°	Lead angles of relieved surface on the tooth sides:	
				Right, γ_{0iR}°	Left, γ_{0iL}°
1	67.575	47.338	5.09	21.207	−11.853
2	71.467	27.079	4.814	13.598	−4.200
3	75.906	7.752	4.587	8.576	0.446
4	77.775	7.752	4.425	8.373	0.435
5	79.25	7.752	4.343	8.219	0.427
6	84.083	27.35	4.094	11.679	−3.635
7	88.095	46.951	3.909	16.439	−9.001

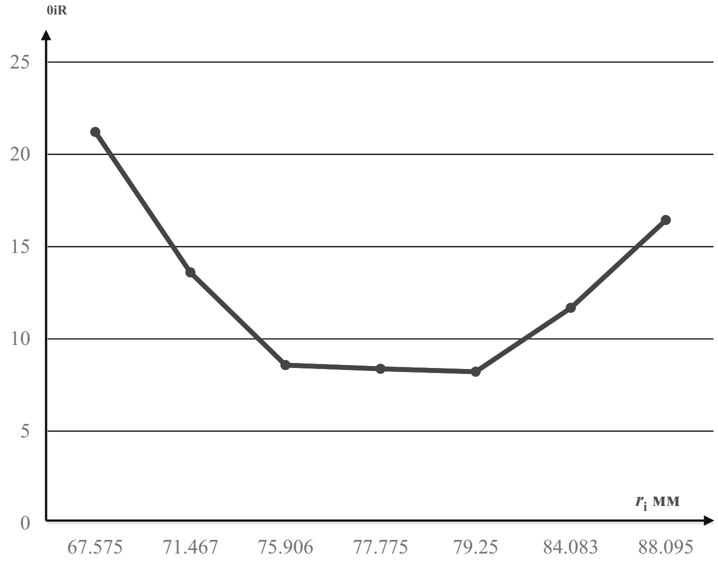


Fig. 7.3 Graph of the function $\gamma_{0iR} = f(r_i)$ – right tooth side

Analysis of the calculated data shows that the lateral surfaces of the teeth of such a hob are organically characterized by the presence of paired points with equal lead angles of the relieved surface, but the values of these angles differ significantly from the lead angle of the helical generating surface. In this regard, still one task when choosing the relieving parameters is to exclude pruning of the cutting edges of the teeth.

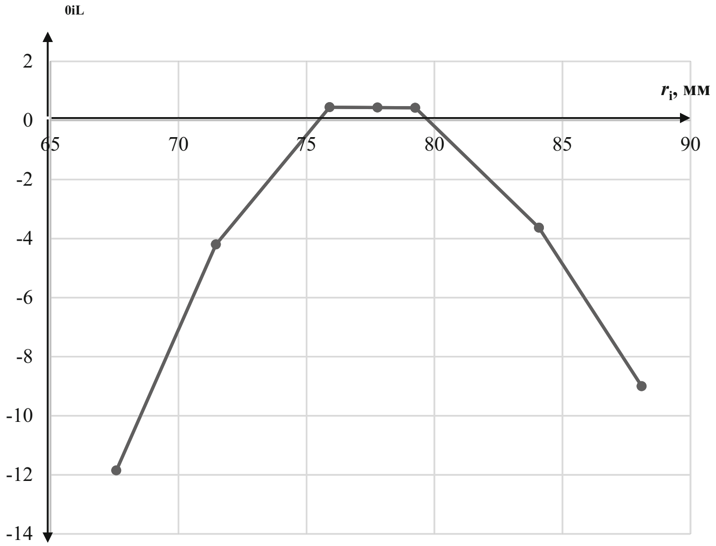


Fig. 7.4 Graph of the function $\gamma_{0i\mathrm{R}} = f(r_i)$ – left tooth side

7.6 Choice of the Designed Points to Construct Designed Normals

The choice of two points of the axial profile of the relieved surface with the parameters $A_1(x_{01}, z_{01}, \alpha_{01})$ and $A_2(x_{02}, z_{02}, \alpha_{02})$ must satisfy two conditions. First, as much as possible, the part between the selected points should cover the active part of the tooth profile, including contact points ($\alpha_0 \approx 27°$). Second, undercutting of the tooth cutting edges should be excluded.

To do this, on the right side of the teeth of the hob with right-hand threads, the angle β_w of setting the axis of the grinding wheel in a plane parallel to the hob axis should not significantly exceed the maximum lead angle of the generating surface. When relieving of the opposite side of the tooth, the angle β_w should not be substantially less than the lead angle of the helical surface of the cutting edges on the tooth head.

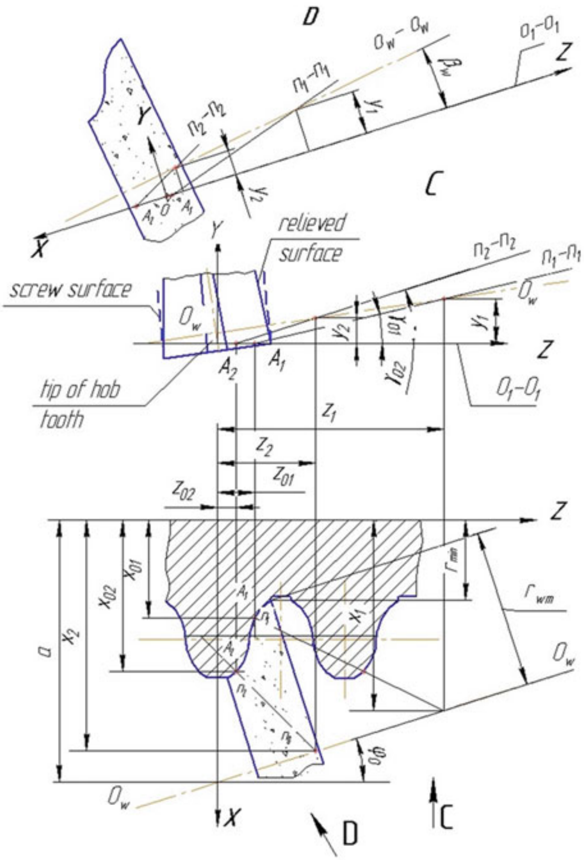
Point A_1 should be selected on the tooth leg of the hob and point A_2 on its head. A study conducted with concrete examples (see the calculation example below) showed that these conditions are most fully satisfied if, as point A_1, we select the design contact point in the tooth space on a concave arc at $\alpha_{01} \approx 27°$ (point 2 in the Table 7.1). As point A_2, we should choose the conjugation of the convex arc section with the transition curve on the tooth head at $\alpha_{02} \approx 47°$ (point 7 in Table 7.1). In this case, the angle β_w of the inclination of the wheel axis is less than the smaller of the values of the angle γ_{0i} at the selected design points.

7.7 Determination of the Angle β_w of Inclination of the Grinding Wheel Axis

The design scheme for determining the angle β_w of setting the axis of the grinding wheel for radial–axial relieving of surfaces with a variable curvature profile is shown in Fig. 7.5.

In the coordinate system *XYZ* (see Fig. 7.5), the *Z*-axis is directed along the axis of the hob, the *X*-axis is aligned with the axis of symmetry of the thread, *XZ* is the plane of the axial section of the hob, and the *Y*-axis is perpendicular to this section. The plane of the axis of the grinding wheel is parallel to the *Y*-axis and intersects the *X*-axis at a distance *a* to the *Z*-axis under an angle $\varphi_0 \neq 0$.

Fig. 7.5 The design scheme for determining the angle β_w of the installation of the axis of the grinding wheel for radially axial occultation of surfaces with a profile of variable curvature



As shown in the lower projection of Fig. 7.5, the value of distance a is determined as:

$$a = r_{i\ \mathrm{min}} + 0.5\mathrm{b} \tan \varphi_0 + r_{\text{им}} / \cos\ \varphi_0 \tag{7.9}$$

where b is the thickness of the worm thread being ground with a minimum radius $r_{i\ \mathrm{min}}$.

The equation of the plane containing the axis of the grinding wheel is written in the form:

$$x(z) = a – z\ \tan \varphi_0 \tag{7.10}$$

The coordinate z_i of the intersection of normals restored from points A_1 and A_2 with the plane of the axis of the grinding wheel is:

$$z_{i\,(1,\,2)} = (a – x_{0i} + z_{0i} \tan \alpha_i) / (\tan \alpha_i + \tan \varphi_0) \tag{7.11}$$

The two other coordinates $x_{i(1.2)}$ and $y_{i(1.2)}$ of each of the intersection points of the normals with this plane are determined through z_i. When determining the coordinate $y_{i(1.2)}$ we should consider the frontal projection (view C):

$$x_{i\,(1.2)} = a – z_i \tan \varphi_0; y_{i\,(1.2)} = (z_i – z_{0i})\ \tan \gamma_{0i} \tag{7.12}$$

The angle β of the inclination of the axis of the grinding wheel is determined in the tangent function (from top view on the plane of the axis of the grinding wheel) by the formula:

$$\tan \mathrm{\beta_w} = \frac{y_1 - y_2}{\sqrt{(x_1 - x_2)^2 + (z_1 - z_2)^2}} \tag{7.13}$$

7.8 Choice of the Angle φ_0 for Relieving the Tooth Flanks

As mentioned above, turning the hob axis by the angle φ_0 improves grinding conditions by increasing the curvature of the surface of the wheel at the points of contact with a processed surface and reduces the difference between the angle β_w of the inclination of the wheel axis and the lead angles of the helical line of the cutting edges of the hob teeth. The choice of the angle φ_0 is determined by two restrictions. On the one hand, it is necessary to minimize the mentioned difference of angles β_w and γ_{0i}. On the other hand, the value φ_0 should not lead to pruning of the cutting edges of the teeth on the opposite side of the thread of the generating surface, i.e., the grinding wheel should enter the space without damaging the adjacent tooth.

Now, we consider the issue of pruning prevention in more detail. The minimum distance between the back (flat) side of the grinding wheel and the opposite side of the thread of the generating surface takes place opposite to the most protruding point on the head of the hob tooth. Such a point is a contact point on the tooth head with a pressure angle $\alpha_{0i} \approx 27°$.

Due to the difference between the angle β_{w} of the wheel inclination and the lead angle γ_i of the generating surfaces, the dangerous zone of approach of the surfaces is shifted from the horizontal plane (down when processing the right side of the tooth). The dangerous approach reaches its maximum value at the extreme point of intersection of the face sections of the outer wheel cylinder (radius r_{wm}) and the cylinder of the hob radius ($r_{\mathrm{F}} + h_{\mathrm{k}}$) at the contact point on the tooth head. For simple geometric reasons, the distance from this point to the horizontal plane is determined by the formula:

$$l_{\max} = r_{\mathrm{wm}} \sin \psi, \tag{7.14}$$

where:

$$\psi = a\cos\left(\frac{r_{\mathrm{wm}}^2 + (r_{\mathrm{wm}} + r_{\min})^2 - (r_{\mathrm{F}} + h_{\mathrm{k}})^2}{2r_{\mathrm{wm}}(r_{\mathrm{wm}} + r_{\min})}\right) \tag{7.15}$$

The linear displacement Δ of the back surface of the grinding wheel to the middle of the space at a length $l_{\max}$ in a screw projection onto the axial plane of the cutter to both tooth sides will be:

$$\Delta_{\mathrm{R,L}} \approx \pm l_{\max}\left[\tan \beta_{\mathrm{wR,L}} \cos \varphi_0 - p/(r_{\mathrm{F}} + h_{\mathrm{k}})\right] \tag{7.16}$$

where the sign "+" relates to the right side of the right-hand hob and the sign "−" to its left side.

In the angular dimension, this displacement is determined from the expression:

$$\tan \varphi_\beta = \Delta/(r_{\mathrm{F}} + h_{\mathrm{k}}) \tag{7.17}$$

The pruning of the cutting edges of the opposite side of the thread is guaranteed to be excluded to satisfy the condition $\varphi_0 + \varphi_\beta \leq \alpha_{0i}$, where the value α_{0i} is accepted for the key point 6 from Table 7.1. For a preliminary calculation of the relieving parameters, the angle φ_0 should be considered as $\varphi_0 = 18°$. Then, having determined the angle φ_β by formula Eq. (7.17), it is possible to correct the angle φ_0 by increasing it to the maximum permissible value:

$$\varphi_{0\max} = \alpha_{0i} - \varphi_\beta. \tag{7.18}$$

Given that increasing φ_0 reduces the desired value of the angle β, such an adjustment is advisable.

As an example, we will calculate the relieving parameters β_w and φ_0 as applied to the hob considered above and study the features of the resulting side surfaces.

As the initial points for constructing the designed normals, we take the points: on the leg A_1 at the contact point with the pressure angle $\alpha_{01} \approx 27°$ (key point 2 from Table 7.1) and, on head A_2 at the conjugation between the active profile and the transition curve with the angle $\alpha_{02} \approx 47°$ (key point 7). For the coordinates of these points and the exact values of the pressure angles, see Table 7.1.

The maximum radius of the grinding wheel is assumed to be $r_\mathrm{wm} = 60$ mm, the minimum radius of the relieved surface on the space $r_\mathrm{min} = 66.215$ mm, and the height of the grinding profile of the hob tooth $H_1 = 23.785$ mm, for both sides of the teeth of the manufacturing surface. The value of the turning angle of the relieving support to provide a rear angle in the middle section of the profile $\varphi_\mathrm{c} = 12°$ for both sides of the teeth. The angle φ_0 is preliminarily considered to be equal to 18°.

The lead angles γ_{0i} of the relieved surface at points A_1 and A_2 are shown in Table 7.1 for each side of the tooth of the right-hand hob. On the right side of the tooth, at which the lead angle of the relieved surface is greater than that of the generating worm, they are $\gamma_{01} = 13{,}598°$ and $\gamma_{02} = 6439°$. On the opposite left tooth side, correspondingly, we have $\gamma_{01} = -4200°$ and $\gamma_{02} = -9001°$.

The negative values of the lead angles γ_{0i} of the relieved surface at the points of the arc sections of the profile indicate its transformation in these sections into an analogue of the left helical surface, which is a characteristic for single-thread hobs with a standard lateral rear angle of 4° or more, with the number of modules in the pitch diameter of 12 or more.

Table 7.2 summarizes the main stages of the calculations with a mention of the needed formulas.

For the right side, the difference between the values of the initial φ_0 and the adjusted $\varphi_{0\max}$ angles in the preliminary calculation is less than 1°, and, therefore, a change in φ_0 is not required. For the left side, the final calculation is carried out by increasing the angle φ_0 to 19° and recalculating β_w, which has changed very slightly and, in practice, in both cases, is rounded to the value $-0.1°$.

In the studies by Lagutin and Sandler [1] and Sandler et al. [2], it was shown that the discrepancy between the profiles of the grinding wheel surface and the hob tooth flank to be ground will be minimal if in the vicinity of the calculated contact point, the line of their contact will not cross the axial section of the hob but touch it.

This condition is satisfied in the case when the axis of the grinding wheel is inclined to the axial plane at an angle β_w, which is determined depending on the pressure angle α_0, the lead angle γ_{0i}, and the radius r_i of the relieved surface, taking into account the installation angle φ_0 of the wheel axis in the projection onto the horizontal plane, by the expression:

$$\tan\beta_\mathrm{w} = \tan\gamma_{0i}\cos\alpha_0 / \cos\left(\alpha_0 + \varphi_0\right) \qquad (7.19)$$

Table 7.2 Calculation of the parameters β_w and φ_0 of the hob teeth

Data	Formula number	Estimated values		
		Preliminary		Final
		Right	Left tooth side	
$\varphi_0°$	Entering values	18.0	18.0	19.0
a	(9)	134.47	134.47	135.15
x_1	(12)	107.76	107.76	107.21
z_1	(11)	82.21	82.21	81.14
y_1	(12)	17.17	−5.17	−5.13
x_2	(12)	122.47	122.47	122.44
z_2	(11)	36.94	36.94	36.91
y_2	(12)	9.47	−5.09	−5.08
$\beta_w°$	(13)	9.19	−0.095	−0.064
$\psi°$	(15)	34.64	34.64	34.64
l_{max}	(14)	34.11	34.11	34.11
Δ	(16)	2.81	2.49	2.47
$\varphi_\beta°$	(17)	8.93	7.95	7.89
$\varphi_{0max}°$	(18)	18.42	19.40	19.46

Table 7.3 Values of the angle β_w calculated for the different sides of the hob tooth

Point numbers	1	2	3	4	5	6	7
x_{0i}, mm	67.575	71.467	75.906	77.775	79.25	84.083	88.095
$\alpha_{0i}°$	47.34	27.08	7.75	7.75	7.75	27.35	46.95
$\gamma_{0i}°$ (right side)	21.207	13.598	8.576	8.373	8.219	11.679	16.439
$\beta_w°$ (right side)	32.22	16.96	9.42	9.198	9.029	16.641	25.441
$\gamma_{0i}°$ (left side)	−11.853	−4.20	0.446	0.435	0.427	−3.625	−9.001
$\beta_w°$ (left side)	−18.882	−5.29	0.491	0.479	0.470	−4.590	−14.325

Table 7.3 shows the values of the angle β_w calculated according to this dependence for seven key points of the axial section of the right and left sides of the hob teeth:

In Fig. 7.6 (the upper part), a graphical interpretation of the dependence Eq. (7.19) of the angle β_w on the hob radius r_1 is displayed for the right side of the teeth. Let us draw two boundary straight lines: one through the starting points A_1 and A_2 of the designed normals and the second horizontal line at a minimum value $\beta_w = 9.19°$ (from Table 7.3) until they mutually intersect at some point C.

The sections of the profile that fell into the sector between the boundary lines have practically no organic error, since any straight line drawn through the intersection point C inside the sector also passes through two points of the axial profile of the relieved surface. Outside the obtained sector, the relieved profile has a small organic deviation from the given profile of the grinding wheel.

In the lower part of Fig. 7.6, a similar dependence of the angle β_w on the hob radius r_1 is shown for the left side of the teeth. We also draw two boundary straight lines. One of them passes through the points of the graph corresponding to the values

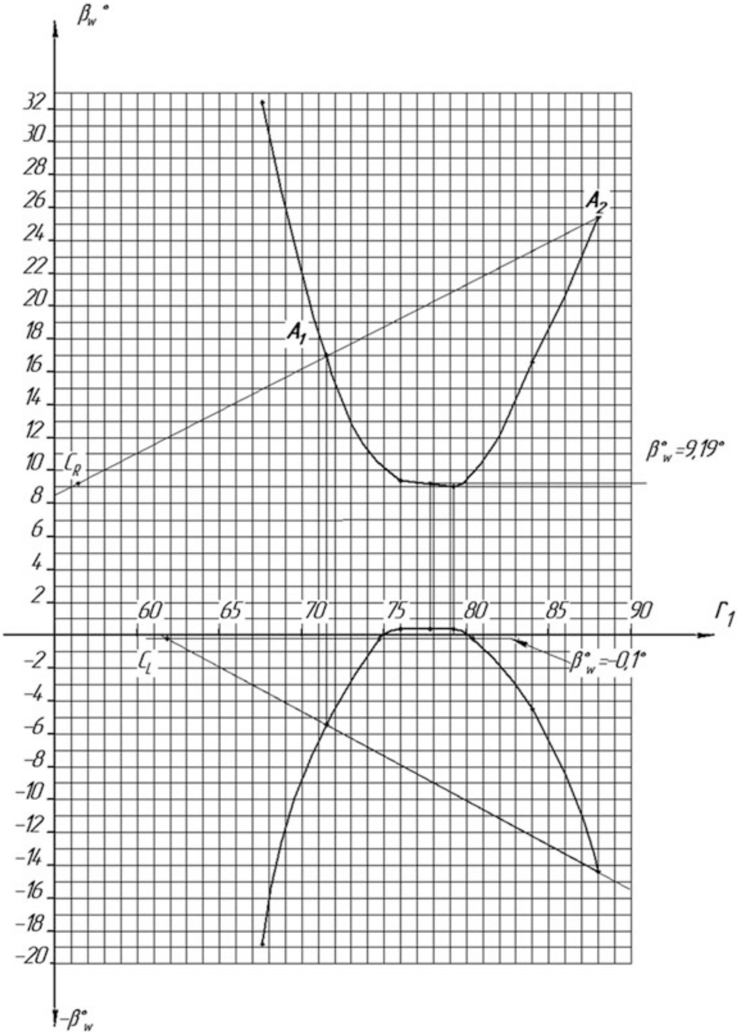


Fig. 7.6 The dependence of the angles β_w from the current radius r_1 of the hob

of the radius at points A_1 and A_2. The second line runs horizontally, touching the graph at its upper point with a value of $\beta_w = -0.1°$. The intersection of these lines forms a sector in which there is practically no organic deviation of the relieved profile.

7.9 Profiling of Grinding Wheels

Significant difficulties in the manufacture of worms and worm hobs with a profile of substantial and variable curvature are due to the fact that the wheel dressing copiers included in the delivery of grinding machines are not suitable for reproducing such a profile [1, 2]. Therefore, it is necessary to design a special knurling roller or create a special ruling device. At present, with the development of computer numerical control (CNC), ruling devices with diamond rollers and programmed control of their movements relative to the working surface of the grinding wheel have gained a certain distribution.

The characteristic of the surface of the grinding wheel, i.e., the line of its contact with the surface to be ground, is oriented along the axial section of the latter. Therefore, profiling of the wheel in its axial section with a sufficient degree of approximation reproduces the section of the surface being ground that is normal to a helical line with an inclination angle equal to β_{w}. That is, the axial profile of the grinding wheel for single-thread hobs practically repeats the profile of the basic rack. In this case, the value of the angle α_{w} of the wheel profile in a rectilinear section is determined from the expression:

$$\tan \alpha_{\mathrm{w}} = \tan \left(\alpha_{0\ \min} + \varphi_0\right) \cos \beta_{\mathrm{w}} \tag{7.20}$$

In general, the coordinates of the axial profile of the grinding wheel, based on the coordinates of the axial section of the relieved surface of the hob teeth, the lead angles of this surface, and the found angle of inclination of the wheel axis, are determined by the following expressions:

$$\begin{aligned} z_{\mathrm{w}} &= z_{0i} / \left(\cos \beta_{\mathrm{w}} + \tan \gamma_{0i} \sin \beta_{\mathrm{w}}\right) \\ x_{\mathrm{w}} &= x_{0i} + z_{\mathrm{n}}^2 \sin 2\beta_{\mathrm{w}} \tan \gamma_{0i} / \left(4x_{0i}\right) \end{aligned} \tag{7.21}$$

7.10 Sensitivity to the Hob Regrinding

If the relieving of surfaces with a variable curvature profile is performed by the radial–axial method, then the axial component of the relieving parameter $k_{\mathrm{x}} = k \sin (\alpha_{0i} + \varphi_{\mathrm{c}})/\cos\alpha_{0i}$ included in formula Eq. (7.6) is variable along the profile. In this case, strictly speaking, the profile distortion that occurs during regrinding of the hob teeth on the front surface is inevitable. However, the considered profile has points with identical pressure angles α_{0i} (for example, at the conjugate points of large and small radii on the tooth’s head and leg) and the lead angles γ_{0i} of the relieved surface, and, therefore, at these points and in the areas between them, the profile deviations during regrinding are minimal.

7.11 Conclusion

1. The technology of relieving of the tooth flanks of the worm hobs for cutting gears with Novikov gearing should provide the necessary rear angles of the hob teeth and the quality of the implementation of the basic rack of gears by the generating worm of the hob.
2. Changing the lead angle of the relieved surface of the tooth hobs with a basic rack for Novikov gears with two lines of action has a parabolic dependence on the radius and pressure angle of the tooth profile, and the relieved tooth surfaces of such hobs are organically characterized by the presence of paired profile points with equal lead angles.
3. A technique has been developed for calculating the parameters of installation, movement, and profile of the grinding disk wheel when grinding the tooth flanks, which allows obtaining at least four contact points between the wheel and the grinding surface in the axial section of the hob. Thus, the organic error of grinding when profiling the grinding wheel is minimized, as close as possible to the basic rack profile.
4. It was determined that the minimum rear angle of the hob teeth takes place near the hob pitch cylinder and that the installation angle φ_c of the relieving support of the grinding-relieving machine is necessary to calculate based on the minimum value of the rear angle of 4°. The technique for calculating this angle is provided.
5. It was revealed that when grinding both the right and left sides of the teeth, it is preferable to assign the angle φ_0 of the installation of the axis of the grinding wheel to 18° and clarify it from the condition to prevent pruning of the cutting edges on the opposite side of the space between the teeth. The technique for refining this angle is provided.
6. When relieving the left side of the teeth of the right-hand cutter, the directions of the lead angles of the grinding surface and the helical surface of the cutting edges have different signs. Accordingly, the angle β_w of inclination of the axis of the grinding wheel to the horizon can also have a sign opposite to the lead angle γ_1 of the generating surface of the hob.
7. The above technique is also proposed for use in the development of control programs for setting up a grinding-relieving machine and the dressing mechanism of a grinding wheel in the manufacture of worm hobs on relieving machines with numerical program control.

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