Synergistic Effects in Queuing Systems and Related Statistical Problems

Gurami Tsitsiashvili(✉)

Institute for Applied Mathematics, Far Eastern Branch of Russian Academy Sciences, Radio Street 7, Vladivostok, Russia
guram@iam.dvo.ru

Abstract. In this paper, there is a approach to detect the results of elements interaction in multi-channel queuing system with large load and small queue. This method is extended to statistical estimates of characteristics of non-uniform Poisson flow, describing distribution of animals in some areas, a resolution of the most powerful decision rule for constructing of technical systems "friend – foe". Such approach gives possibility to expand applications area and to simplify using methods of research. These methods consists of structural analysis and construction of upper bounds of objective functions. It permits to shorten numerical calculations and to obtain explicit results.

Keywords: Multi-server queuing system · Almost deterministic one-server queuing system · Most powerful decision rule

1 RQ-Queuing Systems with a Large Number of Servers

Consider an RQ-system, i.e., a queuing system with orbit in which customer, which has not possibility to be served is directed to the orbit. When some server is released, the customer may be directed to the server in accordance with some protocol [1–3]. RQ-systems attract attention of specialists in queuing theory last years (see, for example, materials of Conference ITMM 2018 in Tomsk and 12th International Workshop on Retrial Queues and Related Topics (WRQ 2018). But calculations of RQ-systems with large number of servers are sufficiently complicated. To decrease a complexity of these calculations we use the theorem on the asymptotic behaviour of an n-server queuing system for $n \to \infty$. In this theorem, it is proved that at $T > 0$ for $n \to \infty$, the probability $P_n(T)$ of customers direction to the orbit during time interval $[0, T]$ tends to zero. So used theorem gives possibility to change objective functions of multi-channel RQ-system from its limit distribution to probability of customers direction to the orbit during time interval T.

1.1 Preliminaries

Consider n - server queuing systems with the parameter $n \to \infty$. Assume that an intensity of input flow is proportional to n and $e_n(t)$ is a number of input

A. Dudin et al. (Eds.): ITMM 2021, CCIS 1605, pp. 197–207, 2022.
https://doi.org/10.1007/978-3-031-09331-9_16

flow customers arriving until the moment t, $e_n(0) = 0$. Suppose that $q_n(t)$ is a number of working servers at the moment t, $q_n(0) = 0$, τ_j is the service time of j-th arriving customer and τ_j, $j \geq 1$, is a sequence of independent and identically distributed random variables (s.i.i.d.r.v.'s) with the distribution function (d.f.) $F(t)$ ($\overline{F} = 1 - F$). Here $F(t)$ has continuous density $f(t) \leq \bar{f}$, where $0 \leq \bar{f} < \infty$. This section is based on [4, Chapter II, § 1, Theorem 1]

Theorem 1. *Assume that the following conditions are true.*

(1) *For some* $a > 0$ *we have* $Ee_n(t) = nat$, $t \geq 0$.
(2) *There is* $B(n)$ *such that* $A(n) = \max(n^{1/2}, B(n))$ *satisfies the relation for* $n \to \infty$

$$\frac{B(n)}{A(n)} \to B \geq 0,\ \frac{\sqrt{n}}{A(n)} \to K \geq 0,\ \frac{n}{A(n)} \to \infty.$$

and $\max(B, K) = 1$*).*
(3) *Random processes* $x_n(t) = \dfrac{e_n(t) - Ee_n(t)}{B(n)}$ *C-converges to the centred Gaussian process* $z(t)$*, when* $n \to \infty$.
(4) *Random process* $\zeta(t) = \int_0^t \overline{F}(t-u)dz(u) + K\Theta(t)$, $0 \leq t \leq T$, *where* $\Theta(t)$ *is centred Gaussian process independent with* $z(t)$*, and its covariance function* $R(t, t+u) = \int_0^t \overline{F}(v+u)F(v)adv$ *and satisfies the formula* $P(\sup_{0\leq t\leq T} \zeta(t) > L) \to 0,\ L \to \infty$.
(5) *If* $\rho = aE\tau_j < 1$*, then for any* $T > 0$ *we have* $P\left(\sup_{0\leq t\leq T} q_n(t) \geq n\right) \to 0,\ n \to \infty$.

Designate $\mathcal{F}_1$ the space of deterministic functions on the segment $[0, T]$ with uniform metric ρ and denote $\mathcal{F}$ the set of bounded functional's f defined on $\mathcal{F}_1$ and continuous in the metric ρ : if $z = z(t), z_1 = z_1(t), z_2 = z_2(t), \ldots \in \mathcal{F}_1$ and $\rho(z, z_n) \to 0,\ n \to \infty$, then $f(z_n) \to f(z),\ n \to \infty$. Say that the sequence of random processes $z_n = z_n(t),\ n \geq 1$, C - converges to the random process $z = z(t)$ if for any functional $f \in \mathcal{F}$ we have that $Ef(z_n) \to Ef(z),\ n \to \infty$.

1.2 Main Results

In this subsection we used the following obvious inequality for RQ-systems

$$P_n(T) \leq P\left(\sup_{0\leq t\leq T} q_n(t) \geq n\right),\ n \geq 1.$$

Then from Theorem 1 it is possible to prove the relation

$$P\left(\sup_{0\leq t\leq T} q_n(t) \geq n\right) \to 0,\ n \to \infty \tag{1}$$

for n-channel RQ-systems with different input flows and so $P_n(T) \to 0,\ n \to \infty$..

Deterministic Input Flow of Customer Groups. Suppose that at the moments $1, 2, \ldots,$ groups of customers of the size $\eta_1 \geq 0, \eta_2 \geq 0, \ldots$ arrive in the n-channel RQ system. Here $\eta_1, \eta_2, \ldots$ are i.i.d.r.v.'s with integer values, $E\eta_1 = a,\ Var\ \beta_1 < \infty$. Define deterministic input flow as follows by the equality $e_n(t) = \sum_{k=1}^{[nt+\psi]} \eta_k,\ t \geq 0$, where ψ is independent of $\eta_k,\ k \geq 1,\ \tau_j,\ j \geq 1$, r.v. with uniform distribution on $[0, 1]$ and $[g]$ is the integer part of the real number g. For the n-channel RQ system with arbitrary protocol of customers direction to servers after their being in orbit the relation (1) is proved in [5].

Alternating Input Flow. This flow is defined by ON and OFF periods alternating with lengths $X_0 \geq 0,\ X_1 \geq 0, X_2 \geq 0, \ldots,$ and $Y_0 \geq 0,\ Y_1 \geq 0,\ Y_2 \geq 0, \ldots$ respectively. In [6,7] a continuous random flow with ON and OFF period is defined. Denote $F_1(t) = P(X_1 < t),\ F_2(t) = P(Y_1 < t),\ t \geq 0$, and suppose that

$$\overline{F}_1(t) = t^{-\alpha_1} L_1(t),\ \overline{F}_2(t) = t^{-\alpha_2} L_2(t),\ 1 < \alpha_1 < \alpha_2 < 2,$$

with $L_1(t) \to l_1 > 0,\ t \to \infty$, and $L_2(t)$ - slowly varying function and $b(t)$ is the inverse $1/\overline{F}_1(t):\ \ b(1/\overline{F}_1(t)) = t$.

Introduce i.r.v.'s $B,\ X,\ Y$, and r.v. Y_0 independent of $X_n,\ Y_n,\ n \geq 1$, so that $P(B{=}1) = \dfrac{\mu_1}{\mu},\ P(B{=}0) = \dfrac{\mu_2}{\mu},\ \mu{=}\mu_1 + \mu_2,\ \mu_1 = EX_1,\ \mu_2 = EY_1,$

$$P(X{\leq}x) = \frac{1}{\mu_1}\int_0^x \overline{F}_1(s)ds,\ P(Y{\leq}x) = \frac{1}{\mu_2}\int_0^x \overline{F}_2(s)ds.$$

Then random sequence $(X_k, Y_k),\ k \geq 0$ generates the ON–OFF process $W(t)$ as follows
$W(t) = BI_{[0,X)}(t) + \sum_{n=0}^{\infty} I_{[T_n, T_n + X_{n+1})}(t),\ t \geq 0$ where $T_0 = B(X + Y_0) + (1 - B)Y,\ T_n = T_0 + \sum_{i=1}^{n}(X_i + Y_i),\ n \geq 1$ and $I_A(t) = 1$ if $t \in A$ and $I_A(t) = 0$ else. The process $W(t)$ satisfies equalities $W(t) = 1$ if t is in ON-period, $W(t) = 0$ if t is in off-period, and stationary and $EW(t) = \mu_1/\mu = \alpha$.
Denote $A(t) = \int_0^t W(s)ds$, then $EA(t) = \alpha t,\ t \geq 0$. Let $n = n(N) = NM(N)$, $M = M(N) = [N^\gamma],\ \gamma > 0$, and assume that random functions $A_m(t),\ m = 1, ..., M,$ are independent copies of $A(t),\ e_n(t) = \left[\sum_{m=1}^{M} A_m(Nt) + \psi\right]$ For so defined alternating input fLow the formula (1) is proved in [5].

Erlang Input Flow. Assume that $E_n(t)$ is Poisson flow intensity $n\alpha$ and $e_n(t) = \left[\dfrac{E_n(t)}{r} + \psi\right],\ t \geq 0$, with random variable ψ independent of $\eta_k,\ k \geq 1,\ \tau_j,\ j \geq 1.$ and integer r. In [8] it is proved the formula (1) in condition $\alpha E\tau_j < 1$.

Consequently if the objective function of multi-channel RQ-system is $P_n(T)$, then it is possible to replace complicated calculations by known asymptotic Theorem 1.

2 Alternative Designs of High Load Queuing Systems with Small Queue

It is well known that queuing systems in high-load mode have long queues. A large number of publications are devoted to the study of asymptotic regimes in such systems (see, for example, [9]. Therefore, such modes of operation of these systems, that do not have large queues, are of great interest. These modes are convenient from an economic point of view, since the service device is almost fully loaded. On the other hand, this mode is also convenient for users which waiting times become small.

Multi-channel Queuing System $M|M|n|\infty$. Consider n – channel system with a Poisson input flow of intensity $n\lambda$ and the service time has an exponential distribution $1-\exp(-\mu_t)$. Such a system can be considered as an aggregation (Fig. 1, right) of n single-channel systems $M|M|1|\infty$ (Fig. 1, left) with Poisson input flows of λ intensity and a similar distribution of service times. Here, aggregation of n single-channel systems is understood as combining their input flows and combining service channels into a multi-channel system. Denote $\rho = \lambda/\mu$ load factor of the system $M|M|n|\infty$ and put A_n the stationary average waiting time, B_n the stationary average queue length.

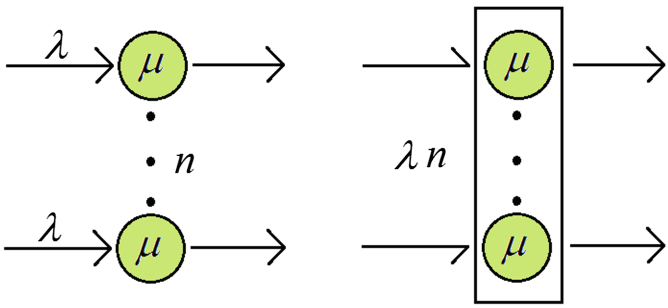

Fig. 1. Transformation of n single-channel systems $M|M|1|\infty$ into aggregated n - channel system $M|M|n|\infty$.

The following are obtained in [10].

Theorem 2. *1) If $\rho < 1$, then for some $c < \infty$, $q < 1$ the relation holds $A_n \leq c\, q^n$, $n \geq 1$, 2) If $\rho = 1 - n^{-\alpha}$, $0 < \alpha < \infty$, then for $n \to \infty$*

$$A_n \to \begin{cases} 0, & \alpha < 1, \\ 1/\mu, & \alpha = 1, \\ \infty, & \alpha > 1. \end{cases} \quad B_n \to \begin{cases} 0, & \alpha < 1/2, \\ \infty, & \alpha \geq 1/2. \end{cases}$$

This theorem develops and specifies the results of [11,12] in the direction of determining the changed structure of the queuing system.

It is clear that an alternative to the described mode of operation of a queuing system with a large load and a small queue can serve as an almost deterministic queuing system. Such a system operates on a specific schedule and its maintenance processes are almost cyclical [13]. The question arises as how to randomly perturb cyclic processes in order to keep a small queue in them along with a large load. Obviously, such perturbations will strongly depend on the distributions of random fluctuations.

Almost Deterministic Single-Channel Queuing System. Despite the importance of Theorem 2, such a queuing system design assumes its large size, which is not always convenient from an application point of view. It is clear that an alternative to the described mode of operation of a queuing system with a large load and a small queue can serve as an almost deterministic one channel queuing system (see, for example [13]).

Let's describe the single-channel queuing system $G|G|1|\infty$ by Lindley chain of waiting times for the service: $w_{i+1} = \max(0, w_i + \eta_i - \tau_i)$. Here τ_i is the interval between the arrival of i - th and $(i+1)$ - th customers, $M\tau_i = a$, and η_i – service time of i - th customer, $M\eta_i = b,\ 0 < a - b = \varepsilon$. Assume that random deviations from the distributions means are reduced as follows:

$$\eta_i^\varepsilon = b + \varepsilon^\alpha(\eta_i - b),\ \tau_i^\varepsilon = a + \varepsilon^\alpha(\tau_i - a)$$

and introduce Markov chain $w_i^\varepsilon,\ i \geq 0,\ w_0^\varepsilon = 0$, describing almost deterministic single-channel queuing system

$$w_{i+1}^\varepsilon = \max(0, w_i^\varepsilon + \eta_i^\varepsilon - \tau_i^\varepsilon) = \max(0, w_i^\varepsilon + \varepsilon^\alpha \delta_i).$$

Here $\delta_0, \delta_1, \dots,$ is a sequence of independent and identically distributed random variables, $\delta_i = \eta_i - \tau_i + \varepsilon,\ M\delta_i = 0$. In high load mode, when the load factor $\rho = \dfrac{b}{a}$ is close to one, the positive parameter $\varepsilon = (1-\rho)a$ is small: $\varepsilon \ll 1$. Value $\alpha > 0$ characterizes the rate of decreasing random perturbations with increasing loading.

Due to known results for a single-channel queuing system $G|G|1|\infty$ Markov chain $w_i^\varepsilon,\ i \geq 0$ has given for any $\varepsilon, \alpha:\ 0 < \varepsilon,\ 0 < \alpha$ the stationary distribution $\lim_{i\to\infty} \mathcal{P}\{w_i^\varepsilon > y\} = \mathcal{P}\{W_\alpha(\varepsilon) > y\},\ y \geq 0$. Using [15–25] it is possible to formulate following statements.

Statement 1. Let for some positive constants $\beta,\ c < \infty$ the inequality $M|\delta_1|^{2+\beta} \leq c$ takes place. Then for any $y \geq 0$ we have $\mathcal{P}\{\varepsilon W_0(\varepsilon) > y\} \to e^{-2y/d},\ \varepsilon \to 0$.

Statement 2. If for some fixed $\nu,\ 1 < \nu < 2;\ h_\nu > 0$, the following relations are true when $y \to \infty$ $P(\eta_1 > y) \sim h_\nu y^{-\nu};\ P(\tau_1 > y) = o(P(\eta_1 > y))$, or $P(\tau_1 > y) \sim h_\nu y^{-\nu};\ P(\eta_1 > y) = o(P(\tau_1 > y))$. Then there is a tail $R(y)$ of non - degenerate distribution function and $\Delta_\nu(\varepsilon) \sim c\varepsilon^{1/(\nu-1)},\ \varepsilon \to 0$, such that for any $y \geq 0$ we have $\mathcal{P}\{\Delta_\nu(\varepsilon) W_0(\varepsilon)/b > y\} \to R(y),\ \varepsilon \to 0$ or $\mathcal{P}\{\Delta_\nu(\varepsilon) W_0(\varepsilon)/a > y\} \to R(y),\ \varepsilon \to 0$.

Using Statement 1 it is possible to prove Theorem 3.

Theorem 3. *Assume that in a single-channel queuing system* $G|G|1|\infty$ *conditions of Statement 1 are true. Then the following limit relations are valid:* $W_\alpha(\varepsilon) \Rightarrow$ *(convergence in distribution)* $+\infty,\ 0 \leq \alpha < 1/2$; $W_\alpha(\varepsilon) \Rightarrow 0,\ 1/2 < \alpha$; $W_\alpha(\varepsilon) \Rightarrow \eta,\ \mathcal{P}\{\eta > y\} = e^{-2y/d},\ \alpha = 1/2$.

Using Statement 2 it is possible to prove Theorem 4.

Theorem 4. *Assume that in a single-channel queuing system* $G|G|1|\infty$ *conditions of Statement 2 are true. Then the following limit relations are valid:* $W_\alpha \Rightarrow +\infty,\ 0 \leq \alpha < 1/\nu$; $W_\alpha \Rightarrow 0,\ 1/\nu < \alpha$; $\varepsilon \to 0$.

The most simple variant of these theorems proves are based on following well known and elementary statement [14, Exercises 15-19 on pages 184-185].

Statement 3. Suppose that $X_n,\ n \geq 1$, is a sequence of positive real-valued random variables that converges in distribution to a non degenerate limit random variable X as $n \to \infty$. Then if a_n are positive real numbers with $a_n \to \infty$, then it follows that $a_n X_n \Rightarrow \infty$ and $X_n/a_n \Rightarrow 0$ as $n \to \infty$.

Thus, a parameter α, characterizing either the rate of convergence of the load factor to one in the system $M|M|n|\infty$, or a random fluctuation in the system $G|G|1|\infty$, allows to detect the convergence of the stationary waiting time to either zero or infinity.

3 Related Statistical Problems

In this section statistical estimates of characteristics of non-uniform Poisson flow, describing distribution of animals in some areas and resolution of the most powerful decision rule for constructing of technical systems discriminating "friend – foe". Main idea of this consideration is in a choice of convenient objective functions for next estimates. Such objective functions may be as relative errors of mean number of points of Poisson flow in some area so a calculation of the most powerful decision rule in a construction of technical system for discriminating "friend – foe". This results are based on the classification of statistical problems proposed in the monographs [32,33] and on the ideas of testing statistical hypotheses in the processing of physical and physico-technical observations [34,35].

Estimates of the Mean Number of Poisson Flow Points in Some Area. In geographical and geological investigations (see, for example [27]) there is a problem to estimate mean number of points in some area and to evaluate its quality. Let the study area is divided into m cells, and the number of points in the cell k is $n_k,\ k = 1, \dots, m$. As we deal with Poisson flow then the random variables $n_1, \dots, n_m$ are independent with Poisson distributions which have the parameters $\lambda_1, \dots, \lambda_m$. Consequently the random variable $N = \sum_{k=1}^{m} n_k$ has a Poisson distribution with the parameter $\Lambda = \sum_{k=1}^{m} \lambda_k$ and so $EN = \Lambda,\ VarN = \Lambda$.

Consider now random variable $\frac{N}{EN} = \frac{N}{\Lambda}$ and calculate its variance $Var\frac{N}{\Lambda} = \frac{1}{\Lambda}$. Consequently the following relation is true $\sqrt{Var\frac{N}{\Lambda}} = \frac{1}{\sqrt{\Lambda}}$. From Chebyshev-Bienome inequality we have $P\left(\left|\frac{N}{\Lambda} - 1\right| > \Lambda^{-1/3}\right) \leq \Lambda^{-1/3} \rightarrow 0,\ \Lambda \rightarrow \infty$. Therefore, the relative error of this estimate, constructed for non uniform Poisson flow decreases with the growth of total Λ.

Resolution of the Most Powerful Decision Rule. In the papers [28–31], a neural network converter 'Biometrics access code'is built on the basis of an electroencephalogram. The main indicator of the effectiveness of this converter is the probability of errors of the first α_1 kind when the probability of errors of the second kind α_2 is chosen by experts to distinguish between simple hypotheses "friend - foe". This distinction of hypotheses is made using the most powerful decision rule. A special role here is played by a set of sample characteristics, with the help of which these hypotheses are distinguished.

In this paper, we introduce a characteristic A of the resolution of the most powerful decision rule. The value of A is determined by the probability α_2, by the sample size n from independent and equally normally distributed random variables with variance σ^2 and the difference of the average $a_1 - a_2$ of these random variables when performing alternative hypotheses. It is established that the probability of errors of the first kind strongly (approximately as $\frac{\exp(-A^2)}{A\sqrt{2\pi}}$) depends on the resolution A of the most powerful solving rule.

This work is based on the classification of statistical problems proposed in the monographs [32,33], the Neumann-Pearson lemma and the well-known rule for finding the most powerful solving rule by the Bayesian solving rule. An important role here is played by the idea of testing statistical hypotheses when processing physical and physico-technical observations [34,35]. The main characteristic that determines the distinguishing ability of A in this statistical problem is the difference of the averages $a_1 - a_2$. This difference of parameters corresponding to the hypotheses "friend - foe" plays an important role in the design of the technical system that specifies the access code, thus $A = A(a_1 - a_2, \alpha_2, n, \sigma)$.

Consider a sample $x_1, \ldots, x_n$, consisting of independent random variables having a normal distribution with an average a and a known variance σ^2. From two hypotheses $H_1 = (a = a_1),\ H_2 = (a = a_2),\ a_1 > a_2$, the most likely hypothesis is selected. This choice is made under the assumption that the probability of an error of the second kind is $P(H_1/H_2) = \alpha_2$, where the value of α_2 is determined by experts (and in accordance with the requirements of GOST). In this assumption, we are looking for a decisive rule that minimizes the probability of a first-kind error $P(H_2/H_1)$. The search for the most powerful solving rule is based on the Neumann-Pearson lemma [32, chapter 3, § 1, 2] and is searched in the form

$$\frac{1}{n}\sum_{i=1}^{n} x_i > C \ \Rightarrow \ H_1,\ \frac{1}{n}\sum_{i=1}^{n} x_i \leq C \ \Rightarrow \ H_2. \tag{2}$$

The constant C is determined by the probability of an error of the second kind α_2 from the relations

$$\alpha_2 = P\left(\frac{1}{n}\sum_{i=1}^{n} x_i > C/H_2\right) = P\left(\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{\sqrt{(}x_i - a_2)}{\sigma} > \frac{\sqrt{n}(C-a_2)}{\sigma}/H_2\right)$$

Let's denote X a random variable with a normal distribution having zero mean and unit variance. Then from the above equalities we get

$$\alpha_2 = P\left(X > \frac{\sqrt{n}(C-a_2)}{\sigma}\right) = \int_{t(\alpha_2)}^{\infty}\frac{\exp(-u^2/2)}{\sqrt{2\pi}}du,\ t(\alpha_2) = \frac{\sqrt{n}(C-a_2)}{\sigma}. \tag{3}$$

It follows from the formula (2) that the constant C, defining the decisive rule (2), satisfies the equality

$$C = a_2 + \frac{t(\alpha_2)\sigma}{\sqrt{n}}. \tag{4}$$

Consequently we have

$$\alpha_1 = P\left(\frac{1}{n}\sum_{i=1}^{n} x_i \le C/H_1\right) = P\left(\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{\sqrt{(}x_i - a_1)}{\sigma} \le \frac{\sqrt{n}(C-a_1)}{\sigma}/H_1\right).$$

Hence the equality follows

$$\alpha_1 = P\left(X \le \frac{\sqrt{n}(C-a_1)}{\sigma}\right) = P\left(X \ge \frac{\sqrt{n}(a_1-C)}{\sigma}\right) =$$

$$= \int_{t(\alpha_1)}^{\infty}\frac{\exp(-u^2/2)}{\sqrt{2\pi}}du,\ t(\alpha_1) = \frac{\sqrt{n}(a_1-C)}{\sigma}. \tag{5}$$

Substituting the formula (4) into the formula (5), we get

$$t(\alpha_1) = A(a_1, a_2, \alpha_2, n, \sigma), \tag{6}$$

where the value

$$A(a_1, a_2, \alpha_2, n, \sigma) = \frac{\sqrt{n}}{\sigma}(a_1 - a_2) - t(\alpha_2) \tag{7}$$

defines the resolution of the most powerful decision rule (2).

Let us now consider how strong is the dependence of the probability of errors of the first kind on this value. To do this, we calculate for $t > 0$

$$J(t) = \int_{t}^{\infty}\frac{\exp(-u^2/2)}{\sqrt{2\pi}}du = \int_{t}^{\infty}\frac{\exp(-u^2/2)}{\sqrt{2\pi}u}d\frac{u^2}{2} \le \frac{\exp(-t^2/2)}{t\sqrt{2\pi}},$$

from here we get

$$J(t) = \int_{t}^{\infty}\frac{\exp(-u^2/2)}{\sqrt{2\pi}u}d\frac{u^2}{2} \ge \frac{\exp(-t^2/2)}{t\sqrt{2\pi}}\left(1 - \frac{1}{t^2}\right).$$

Combining the obtained inequalities, we find

$$\left(1 - \frac{1}{t^2}\right) \frac{\exp(-t^2/2)}{t\sqrt{2\pi}} \leq J(t) \leq \frac{\exp(-t^2/2)}{t\sqrt{2\pi}}, \ t > 0. \tag{8}$$

It follows that the function $J(t)$, determining the probabilities of errors of the first and second kind decreases very quickly with the growth of t.

Let's take $\alpha_2 = 10^{-9}, \ \alpha_1 = 7 \cdot 10^{-4}$ as a numerical example (these values are taken from [31]), then we can build an approximation $J(t) \approx \dfrac{\exp(-t^2/2)}{t\sqrt{2\pi}}$ and with an accuracy of 10^{-2}, get the values $t(\alpha_1) = 5.99781, \ t(\alpha_2) = 3.19465$. As a result, we come to equality

$$A(a_1, a_2, \alpha_2, n) = \frac{\sqrt{n}}{\sigma}(a_1 - a_2) - t(\alpha_2) = 9.19246.$$

Since $\alpha_1 = 7 \cdot 10^{-4}$, then combined with the formula (8) from the inequality

$$\alpha_1 \leq \frac{\exp(-A^2(a_1, a_2, \alpha_2, n)/2)}{\sqrt{A(a_1, a_2, \alpha_2, n)}}$$

it can be seen how much the resolution of $A(a_1, a_2, \alpha_2, n)$ affects the probability of an error of the first kind α_1, which is the main indicator in this statistical problem.

The formula (7), which determines the resolution of $A(a_1, a_2, \alpha_2, n)$, specifying the probability of an error of the first kind, despite its simplicity, contains a whole series of characteristics: the difference of the averages $a_1 - a_2$, variance σ^2, sample size n (and the probability of an error of the second kind α_2). Therefore, the choice of the characteristics of $a_1 - a_2, \ \sigma^2, \ n$ becomes a rather difficult task of designing the technical system described in [30]. Moreover, a special role here is played by the difference $a_1 - a_2 > 0$ of the average $a_1, \ a_2$, characterizing the distributions of samples describing the "friend - foe" states shared by the technical system.

4 Conclusion

The results presented in this paper go beyond the theory of probability and queuing. In these results, the main focus is not on proving probabilistic theorems of the greatest generality, but on obtaining explicit estimates of the comparison of queuing systems, statistical algorithms and programs before and after the transformation of their structure. The peculiarity of such results, and it is convenient to call them synergetic effects, is the strong dependence of the compared performance indicators when a certain parameter tends to zero or to infinity. However, this circumstance in no way reduces the requirements for the accuracy of the estimates obtained. According to the author, such estimates are most convenient to carry out during computational experiments. Another thing is that it is convenient to conduct such computational experiments working with complex systems if there are some analytical estimates of the marginal behavior of performance indicators.

References

1. Dudin, A., Nazarov, A.: On a tandem queue with retrials and losses and state dependent arrival, service and retrial rates. Int. J. Oper. Res. **29**, 170–182 (2017)
2. Artalejo, J., Gomez-Corral, A.: Retrial Queueing Systems. A Computational Approach. Springer, Heidelberg (2008). https://doi.org/10.1007/978-3-540-78725-9
3. Nazarov, A., Moiseeva, E.: Investigation of the RQ-system $MMPP|M|1$ by the method of asymptotic analysis in the condition of a large load. Izv. Tomsk. Polytech. Univ. **322**, 19–23 (2013). (in Russian)
4. Borovkov, A.: Asymptotic Methods in Queueing Theory. Wiley, New York (1984)
5. Tsitsiashvili, G., Osipova, M.: Synergetic effects for number of busy servers in multiserver queuing systems. Commun. Comput. Inf. Sci. Ser. **564**, 404–414 (2015)
6. Heath, D., Resnick, S., Samorodnitsky, G.: Heavy tails and long range dependence in on/off processes and associated fluid models. Math. Oper. Res. **23**, 145–165 (1998)
7. Mikosch, T., Resnick, S., Rootzen, H., Stegeman, A.: Is network traffic approximated by stable Levy motion or fractional Brownian motion? Ann. Appl. Probab. **12**, 23–68 (2002)
8. Tsitsiashvili, G., Markova, N.: Synergistic effects in a multi-channel queuing system with an Erlangian input flow. Bull. Pac. State Univ. **4**, 17–22 (2015). (in Russian)
9. Borovkov, A.A.: Asymptotic methods in queueing theory. Science, Moscow (1980). (in Russian)
10. Tsitsiashvili, G., Osipova, M.: Phase transitions in multiserver queuing systems. In: Dudin, A., Gortsev, A., Nazarov, A., Yakupov, R. (eds.) ITMM 2016. CCIS, vol. 638, pp. 341–353. Springer, Cham (2016). https://doi.org/10.1007/978-3-319-44615-8_30
11. Halfin, S., Whitt, W.: Heavy-traffic limits for queues with many exponential servers. Oper. Res. **29**(3), 567–588 (1981)
12. Jagerman, D.L.: Some properties of the erlang loss function. Bell Syst. Tech. J. **53**, 525–551 (1974)
13. Afanasyeva, L.G.: Queuing Systems with cyclic control processes. Cybern. Syst. Anal. **41**(1), 54–68 (2005). (in Russian)
14. Chung, K.L.: A Course in Probability Theory, 3rd edn. Stanford University, Academic Press (2001)
15. Borovkov, A.A.: Some limit theorems of the queueing theory. Probab. Theory Appl. **IX**(4) (1964), X (3) (1965). (in Russian)
16. Prokhorov, Y.V.: Transient phenomena in queuing processes. Litov. Math. Collection III **1**, 199–205 (1963). (in Russian)
17. Harrison, J.M.: The heavy traffic approximation for single server queues in series. J. Appl. Probab. **10**(3), 613–629 (1973)
18. Abate, J., et al.: Calculation of the Gl/G/l waiting time distribution and its cumulants from Pollaczek's formulas. Arch. Elektr. Uebertragung **47**(Pollaczek memorial volume), 311–321 (1993)
19. Beran, J., et al.: Long-range dependence in variable-bit-rate video. IEEE Trans. Commun. **43**, 1566–1579 (1995)
20. Bingham, N.H., et al.: Regular Variation. Cambridge University Press, Cambridge (1987)
21. Boxma, O.J.: Fluid queues and regular variation. Perform. Eval. **27**(28), 699–712 (1996)

22. Boxma, O.J., Cohen, J.W.: The M/G/l queue with heavy-tailed service time distribution. IEEE J. Sel. Areas Commun. **16**, 749–763 (1998)
23. Cohen, J.W.: Some results on regular variation for distributions in queueing and fluctuation theory. J. Appl. Probab. **10**, 343–353 (1973)
24. Cohen, J.W.: Superimposed renewal processes and storage with gradual input. Stochastic Process. Appl. **2**, 31–58 (1974)
25. Boxma, O.J., Cohen, J.W.: Heavy-traffic analysis for the GI/G/l queue with heavy-tailed distributions. Queueing Syst. **33**, 177–204 (1999)
26. Gnedenko, B.V., Korolev, V.Y.: Random Summation. CRC Press, Boca Raton (1996)
27. Stoyan, D., Kendall, W., Mecke, J.: Stochastic Geometry and Its Applications. Wiley, New York (1987)
28. Goncharov, S.M., Borshevnikov, A.E.: Construction of a neural network converter "Biometrics access code" based on the parameters of the visual evoked potential of an electroencephalogram. Reports of the Tomsk State University of Control Systems and Radioelectronics, vol. 2, no. 32, pp. 51–55 (2014). (in Russian)
29. Goncharov, S.M., Borshevnikov, A.E.: Prediction of the output parameters of the neural network converter "Biometrics-access code" based on an electroencephalogram. Inf. Secur. **21**(3), 302–307 (2018). (in Russian)
30. Borshevnikov, A.E., Goncharov, S.M., Dobrzhinsky, Yu.V.: On the requirements for the formation of synthetic images of an electroencephalogram for the tasks of highly reliable biometric authentication. Modern science: actual problems of theory and practice. Series: Nat. Tech. Sci. **2**, 27–29. (in Russian)
31. Borshevnikov, A.E.: Neural network model of a highly reliable biometric user authentication system based on an encephalogram. Oral report at the general Institute seminar of the IAM FEB RAS (2021)
32. Borovkov, A.A.: Mathematical statistics. Evaluation of parameters. Hypothesis testing. Moscow: Nauka. The main editorial office of the physical and mathematical literature (1984). (in Russian)
33. Borovkov, A.A.: Mathematical statistics. Additional chapters. Nauka, Moscow (1984). (in Russian)
34. Gortsev, A.M., Nezhelskaya, L.A., Shevchenko, T.I.: Estimation of the states of the MC-flow of events in the presence of measurement errors. Izv. Vuzov. Phys. **36**(12), 67–85 (1993). (in Russian)
35. Gortsev, A.M., Nezhelskaya, L.A.: Optimization of adapter parameters during observations of the MS flow. In Sat. Stochastic and deterministic models of complex systems, pp. 20–32. Publishing house of the Computing Center of the Siberian Branch of the USSR Academy of Sciences, Novosibirsk (1988). (in Russian)