Optimization for Total Hip Arthroplasty Applications

Abstract Optimization in total hip arthroplasty (THA) is the ultimate intend of surgeries at all phases of the treatment including during, pre-, and post-surgery stages. There are many factors that play key roles in THA optimization that relate to surgeon, patient, implant, and method of surgery. In this work, we present studies that are related to optimization of THA from a variety of perspectives that are categorized into experimental and mathematical optimizations. Improvements for THA operations are recommended in the conclusion section.

1 Introduction

Optimization in total hip arthroplasty (THA) is a common objective of many researchers, and it has been the goal of several articles with different objectives of THA. Different optimization approaches have been followed mainly in numerical and somewhat using theoretical and mathematical optimization. In the next section, we will cover work completed with regard to THA of computational optimization techniques with their aims and objectives. What follows would be the theoretical optimization methods that are very rare to see in THA applications. The conclusion section is devoted to potential work that can be accomplished in optimization improvement opportunities that could take place in THA applications from both computational and theoretical perspectives.

2 Experimental Optimization for THA Success

Experimental optimization described in this section aims to cover the THA research done with data collected and optimal values attained based on the research outcomes. For instance, a review of the studies on biomechanics of experimental and computational periprosthetic femoral fracture (PFF) fixation methods up to 2017 is described and compared in [1]. This summary concluded the validation of computational

E. Tokgöz, *Total Hip Arthroplasty*, https://doi.org/10.1007/978-3-031-08927-5_10

results with experimental data's completion to be essential. Computational studies are observed to be useful in studying fixation methods or conditions (such as bone healing) that are difficult to study in vivo or in vitro with some issues. The need of optimality for PFF fixation is the consensus of the reviewed studies.

In [2] optimization of periarticular injection techniques is investigated by identifying and mapping the periarticular neural anatomy of the hip as a part of THA. Taking into account of both gross and microscopic neural anatomy of the human hip joint with particular key areas of hip mechanoreceptors and free nerve endings, the hip joint is supplied by the femoral, obturator, sciatic, and superior gluteal nerves, as well as the nerves in the quadratus femoris. It is noted that the maximum concentration of sensory nerve endings and mechanoreceptors is found at the anterior hip capsule, especially superiorly. The labrum innervation is maximized 120 degrees from 10 o'clock position clockwise. Hence, after the cup and liner are placed, periarticular injections should be infiltrated toward the remnant labrum 120 degrees from 10 o'clock position clockwise. Another optimization technique is to construct an optimal solution during the surgery. A single intraoperative AP pelvis x-ray is found to be a quick, reliable, and inexpensive means of determining acetabular abduction, acetabular medialization, leg length, and femoral alignment in [3]. Plain x-ray can be easily used to assess screw position and cup seating. This approach allows the surgeon to make a decision on changing the position of the prosthesis if needed before the conclusion of the case. In cases where changes are needed, reliable correction was produced in 97% of cases. In some instances, however, intraoperative considerations are such that these considerations will supersede generic alignment and leg length targets.

The use of computer-assisted surgery for improving prostheses implantation for computer-aided surgery has been proven reliable with accuracy and reproducibility in the positioning of the implants. Several navigation devices are used, and the anterior pelvic plane (APP) is used as the reference for characterizing the patient's orientation in 3D. In an attempt to generalize for optimization purposes, the "plane of Lewinnek" or "safe zone" is introduced in [4] based on superficial anatomical landmarks with pubic symphysis and both superior anterior iliac crests. The sophistication of interrelated human body network has a weight distribution of impact; pathological issues that exist in locations such as spine can have more impact than other locations. Lumbar pathological issues such as abnormal spinopelvic motion can increase the risk of hip dislocation even with the Lewinnek et al.'s (1978) "safe zone" placement of acetabular component that assumes anteversion $15° \pm 10°$ and inclination $40° \pm 10°$ angles [5]; it is noted that the "safe zone" is recently questioned in several studies and may not accurately predict THA instability based on the anteversion and inclination measurements [6]. The dislocation rate after THA is observed to be as high as 92% and as low as 75% in the spinopelvic abnormality cases [5]. In an attempt of investigating the relationships between APP and sacral slope in [7] upon measuring 328 patients' lateral radiographs of the pelvis in standing position, poor correlation between APP and sacral slope suggests using the reference to the APP for the per-operative orientation in the 3D space while individually adjusting the preoperative planning to the sacral slope. The importance of preoperative planning is emphasized in [8] as the first step in adult reconstructive surgery of the hip. Proper execution helps to minimize intraoperative problems and avert complications. It is also observed to reduce surgical trial and error that eventually reduced operative time. Clinical results can be ultimately improved in addition to possible operational planning shortening for a new implant system and technical skill improvement for performing THA. Figure 1 displays a sleeve that is placed to maximize stability and ingrowth of a previously detected osteotomy that distorted the proximal femoral anatomy.

It is observed to be natural having leg length changes after THAs, and biomechanical reasoning is investigated in the research literature. The design and placement of the implant are main factors with interlocking of medullary canal and implant. Flexible structure of the bone and the strength of the material used for the implant eventually causes the sinking of the implant into the bone and causing the imbalance. The design of the femoral implant can be targeting to achieve fixation of the medullary canal and implant in the mediolateral dimension or anteroposterior engagement of the bone. It is concluded by the authors that osseointegration of the cementless THA and postoperative leg length discrepancies are impacted by the femoral challenges; optimal intra- and extramedullary geometry fitting and offset restoration of cementless femoral stems are major common challenges among the implants used, and they cannot necessarily offer optimal fit or offset restoration [9].

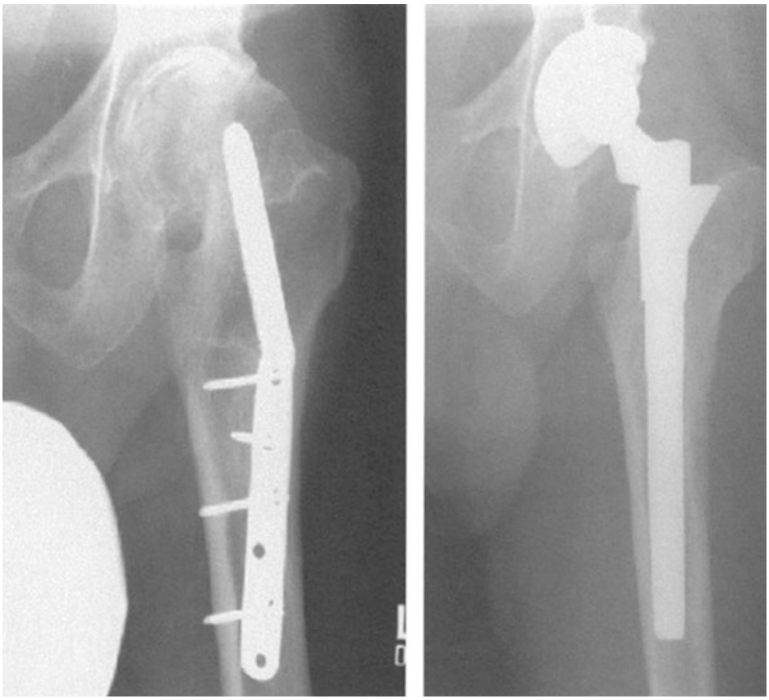

Fig. 1 A sleeve is placed to maximize stability and ingrowth of a previously detected osteotomy that distorted the proximal femoral anatomy [73]

Biomechanical data of THA patients that underwent anterolateral surgery is collected during the post-surgery gait training by using crutches and a mobile robot that assisted them during walking in a clinical setting [10]. In comparison to the control group that had no use of robotics for gait training, the analysis revealed the robotic-assisted group to have significantly higher absolute walking speed, higher relative walking speed (0.2 vs. 0.16 m/s, $p = 0.043$), or shorter relative cycle time. This particularly impacts the time that the patients spend in the clinic.

Finite element analysis (FEA) is one of the methods used for PFF fixation upon THA by altering the loading and boundary conditions along with the isometric and physiologic loadings [69]. FEA is useful for determining optimized hip implant 3D modeling to improve mechanical strength by optimizing stress and strain distribution. The major issue in the use of FEA on PFF is the standardization of the technique and methods used [11]. The increase in the overall rigidity of the construct eventually determined to increase the stability of the fracture, as a result of the motion across the fracture or the overall stiffness of the instrumented femur [1]. The increase in rigidity is due to the following:

A. Changing the rigidity of the connectors based on one or more of the following:
   (a) By using screws instead of cables (see, e.g., [12]).
   (b) By using cables instead of wires (see, e.g., [13]).
   (c) By using double-wrapped wires compared to single wrapped (see, e.g., [14]).
B. Plate and strut modifications changing stiffness (see, e.g., [15]).
C. Using longer revision stems (see, e.g., [16]).

Due to variability of clinical techniques used, unclarity of how a designed experiment would perform at an optimal level needs to be explained clearly. Construct stiffness appears to be the focus of majority of the work in progress, but it may be misleading due the possibility of a highly stiff plate causing stress shielding in the underlying bone. In this case, either the fracture heals or the construct itself does not fail. More studies on biomechanical quantification of such outcomes for periprosthetic femoral fracture are needed [11, 70–72].

The biomechanical performance of different configurations of cables, wires, and screw positions is tested in [17] computationally. Four different screws are used in three fixation methods consisting of placement of three cable- screw combinations proximal to the fracture with the only difference between the methods being the positioning of the cable- screw pairs proximally. It is concluded that the choice of the location impacts the fixation strength and the option that yielded the best fixation strength has the potential to reduce the refracturing of the bone. This best option determined is expected to yield to the highest stiffness that may achieve the optimal mechanical stability; FEA agreed with the experimental results. Optimization of hip implant prosthesis' structure including stem, head neck, and acetabular cup with the objective of minimizing aseptic loosening, bone resorption, and stress shielding in the implant is investigated in [18] by utilizing the topology optimization tool of FEA. The authors reported that weight reduction and stress distribution behavior of the implant under varying conditions are better understood by using topological optimization of the implant. Optimal weight and mechanical characteristics of CoCrMo alloy made femoral hip stems are investigated in [19] using FEA. The final model attained not only satisfied the ISO design conditions for femoral stems but also reduced the weight by about 15% of the original model and fulfilled the mechanical expectations. Computational optimal values are attained for determining the best hip neck design for reducing the mechanical stresses and avoid stress shielding of the prosthesis in [20] for a combination of neck cross sections including elliptic, circular, oval, and trapezoidal shapes with different side profiles including circular arcs, flat, and knife edges. Trapezoid cross section combined with flat and circular arc side profiles is determined to provide outstanding stress, strain, and deformation results. The goal in [21] is to reduce stresses and deformation developed in the prosthesis by using 3D models developed with fenestrations. Slot fenestration design is shown to have the minimum deformation upon applying FEA among big loop fenestration design, design without fenestration, slot fenestration design, and many loops fenestration design models. Variational neck diameters on five hip implants are tested in order to minimize stress levels and optimize neck thickness in [22] using FEA. The best design is attained for the hip prosthesis with 9 mm neck diameter that had the maximum reduced stress. Acetabular cup's corner shape variation is analyzed using FEA in [23] for circular arc, sharp, and spline shapes interfaced with ball head at four micro separations. Reduced risks of striping wear, fracture, and fatigue in the hip prosthesis are attained for the spline-shaped corners of the acetabular cup that resulted in reduced stresses, strains, and contact pressure on the ball head interface compared to others. To optimize geometry of the implant by using bio-inspired lattice structures, FEA investigation resulted to indicate Schwarz diamond lattice structure among the three options in [24]. This structure is shown to have the best functional performance that provided the minimum level of Von Mises stresses and maximum safety factor. Impact of outer shell is investigated with the use of titanium femoral stems in [25]. FEA is used on these stems with porosities (BCC structure) of 90, 77, 63, 47, 30, and 18% and with and without outer shells of thickness 0.5, 1.0, 1.5, and 2 mm. The stems without shells are determined to have frequent fatigue failure, while the shell with porosities showed an increase in stress shielding behavior up to 28% as a result of the stress distribution behavior and fatigue strength analysis.

Physiological and implant failure mode analysis and functional recovery after THA are analyzed for surgical success. Femoral offset (FO) changes following THA can help with analysis of hip muscle activities to observe functional physiological restoration noting that FO accuracy and functional recovery are correlated [26]. Analysis of 13 hip muscles' moment arms of 18 unilateral THA patients in vivo revealed a potential improvement of abductor and external rotator function upon 2–3 mm of FO restoration; an increased FO observed to reversely correlate with length of both flexor and adductor moment arms during the gait and stance phases, respectively. A decrease of both abductor and external rotator moment arms during the whole gait and a decrease in extensor moment arms during the stance phase are correlated with a decreased FO after THA [26]. This approach can particularly help with presurgical planning for functional restoration of the hip.

Reasons for THA implantation failure are complicated and can include patient-, material-, and non-patient-related factors (such as inadequate surgical technique). Femoral head-neck interface by fretting and corrosion damage is determined as another factor contributing to THA failure [27, 28]. Determination of the exact reasons of the degradation process appears to be not possible; however, the following factors are determined to impact the implant failure:

- Body mass index [29].
- Taper length [29].
- Time in situ [30].
- Mixing of alloys [31, 32].
- Femoral head size [33, 34].
- Flexural rigidity [35].
- Female taper angle [34].
- Taper-angle mismatch [36].
- Taper diameter [37].
- Stem surface roughness [16, 30, 38–42].

Effectiveness of cemented and uncemented fixation types on the liner wear risk is analyzed theoretically by using FEA in [43]. The key intraoperative factors playing roles in determining the wear risk during surgical planning that are used in modeling included head material, head size, liner thickness, cervical-diaphyseal angle, and center of rotation positioning. Biomechanical restoration analysis was based on two types of 3D liner models' simulation of ultrahigh molecular weight polyethylene. Liner thickness and acetabular fixation techniques are determined to be significantly related to wear risk. A proper prevention technique to the cause of polyethylene liner wear is observed to be the use of a cemented fixation with a thick liner in the right center of rotation.

THA dislocation rate's exposure to cup positioning and abductor mechanism's reconstruction are evaluated on cementless THA operations of 1318 patients on the data collected in a span of 20 years in [44]. The radiological assessment of 28 or a 32 mm femoral head-sized THAs cups is based on positioning and hip rotation center, and reconstruction of the abductor mechanism is conducted by measuring the lever arm distance and the height of the greater trochanter. It is concluded for the observed 38 dislocations by using multivariate regression analysis on the implant and physiological data that these dislocations are most associated with the following:

- A greater distance to the anatomic hip rotation center.
- Acetabular inclination and version angles for hips outside two safe windows of cup position.
- Lever arm distance and height of the greater trochanter abductor mechanism.
- Hip's abductor muscle weakness.

A fixation method utilized in [45–47] is differentiation of plates that depend on plate comparisons based on the following:

- Rigidity (flexible vs. rigid).
- Formation material (titanium vs. stainless steel).

- Thickness.
- Support (use of cables, different screw types, double plating, locking, multidirectional, etc.)
- Position of plating (anterior, lateral, etc.)
- Stem design and dimensioning (long vs. short, etc.)

Upon the failure of initial rigid fixation attempt by using a polyaxial femoral plating following THA, the refracture fixation attempt was by utilizing polyaxial femoral plating as a flexible fixation method in [45]. Biomechanical effects of fixation methods using FE modeling and Vancouver type C fracture on a clinical case are compared in the study. It is observed that short bridging used for PFF rigid fixation can defeat fracture movement and prevent healing to eventually cause failure. On the contrary, non-locking plate with a longer bridging utilizing flexible fixation promoted better healing. The length of bridging appeared as the most impactful parameter on fracture location and stiffness. The path to optimum fixation construct design is conjectured to be by using a computational approach as such in [45].

Micromotion is another concern in biomechanical analysis of designed implants. For instance, in the case of cementless fixation, micromotion at the bone-implant interface has been reported to affect bone ingrowth [48]. Low micromotion, typically below 28 μm, results in bone ingrowth, while excessive micromotion that can be assumed to be above 150 μm results in the growth of fibrous tissue inhibiting biological fixation [49]. Micromotion on bone-implant interface mainly depends on the implant primary stability that relates to several factors including implant macrogeometry, elastic modulus mismatch with the bone, fixation technique, and the bone tissue quality with its defects [50].

In an attempt of optimize the dose of topical tranexamic acid for primary THA, both 1 and 2 gr. tranexamic acid are found to significantly reduce postoperative drain blood loss in [51], therefore the use of topical tranexamic acid at the end of surgery is found to be effective and safe for reducing postoperative blood loss in primary THA. Topical tranexamic acid at a dose of 1 gr. may be sufficient and cost-effective, with fewer side effects than the higher dose.

For an optimal range of motion, it is important to obtain acceptable offset and anteversion, and for the most appropriate lever arm and muscle strength around the hip [52]. In an experimental study by using sawbones with four different angles of femoral anteversion (16, 34, 47, and 59 degrees), the effectiveness of a modular femoral neck system is tested to determine optimal outcomes. The femoral neck system consisted of two neutral and four types of retroverted necks for the correction of femoral anteversion and offset in THA. Reconstruction of the preoperative anteversion and offset in the normal femur were achieved with the neutral neck. The long neck with 15 degrees of reversion was effective for the mildly or moderately anteverted femur with insufficient correction for the severely anteverted femur. An optimal value of the medial component of femoral offset in femora with anteversion of less than 47 degrees is determined to be useful by using this modular neck system for correction. Patients that have greater anteversion may find femoral necks that have a greater degree of retroversion useful which is a feature rarely seen in the clinical situations [8].

Noting that there is a wide range of choices in the selection of femur implants along with the patient-based anatomical variations, it is natural to see that there is no one-fit for all implants that can satisfy the conditions of structural strength for longer durability. Von Mises stress calculations are used for measuring the life of an implant when continuous cyclic load associated with regular day-to-day activities is applied to the implant. In [53], the trapezoidal-shaped stem with three different cross sections is considered with the femoral head size, acetabular cup thickness, backing cup thickness, and trunnion geometry varied to arrive the best possible combination. Using ANSYS R-19, the acetabular cup, backing cup, and trunnion top surface radius and femoral head sizes are varied experimentally. Trunnion interface is determined to not play a significant role with respect to the structural strength of the implant.

Patients with acetabular dysplasia were operated by circumferential acetabular medial wall displacement osteotomy in [54] to reconstruct the acetabulum during total hip arthroplasty. All patients had cementless acetabular components implanted. The authors' early recommendation was that circumferential acetabular medial wall displacement osteotomy optimizes the reconstruction of the acetabulum in patients with hip dysplasia (Fig. 2).

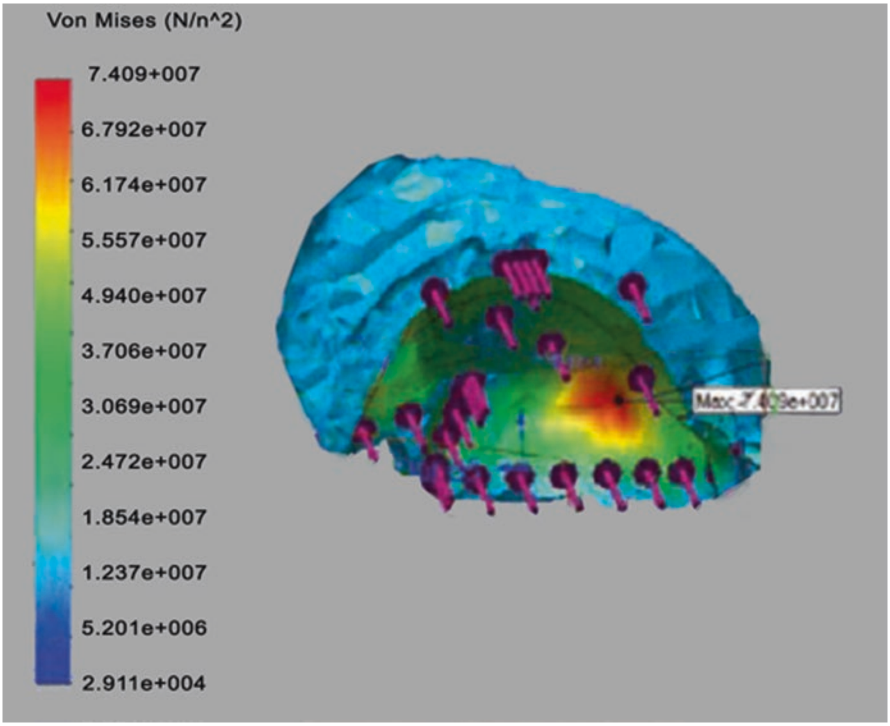

Fig. 2 An example of Von Mises stress calculations for a normal positioned cup [55]

Aseptic loosening in THA relates to optimal values used for cement mantle thickness and uniformity along with the optimal values of material mix that is patient dependent. The resultant progressive development of detrimental cement mantle defects is seen in patients upon non-optimal choices. The stresses act on cement mantle, and the associated interfaces also play primary roles in the long-term success of THA. The coverage of various surgical techniques that assist in creating an optimally thick, symmetric, and homogeneous cement mantle to control and minimize high cement stresses that initiate debonding and cement fracture is explained in [56]. Inherent properties of the cement for increased strain resistance and reduced microfractures include the following:

- Increased strength.
- Reduced brittleness.
- Improved interface adherence.

We refer to [56] for further details on how optimal application of the cementation can be achieved with the corresponding surgical details.

In an attempt to optimize risk for post-surgical operations, an evidence-based tool is used to address modifiable risk factors for adverse outcomes after primary hip surgeries in [57]. Identification, intervention, and mitigation of risk through evidence-based patient optimization are accomplished through nurses who screened patients preoperatively, identified and treated risk factors, and followed patients for 90 days postoperatively. Comparison of patients participating in the optimization program is compared to both historical and contemporary cohorts. Identification and optimization of the risk factors resulted in lower hospital length of stay and postoperative emergency department visits. Patients in the optimization cohort had a low mean value of length of stay and had significant decrease in 30- and 90-day emergency department visits. Additionally, the optimization cohort had a significant increase in the percentage of patients' discharge. There are also nonsignificant reductions in readmission rate and transfusion rate, and surgical site infections are observed.

Another evidence-based optimization attempt is made in [58] to determine the factors that play crucial roles in medical failure before THA to determine which modifiable risk factor is the most dangerous one. Factors such as smoking, abnormal body mass index (BMI), uncontrolled diabetes, and poor nutritional status are increasingly associated with complications after THA by researchers in the literature. Upon review of about 48 thousand primary THA procedures conducted in 2018, increased risk of postoperative infection, readmission, any complication, and mortality after primary THA is determined to be associated with the factors as follows:

- Low albumin.
- Elevated BMI.
- Use of tobacco.
- Diabetes.

Low albumin is determined to be the greatest risk factor among these factors. The importance of preoperative optimization and agreement of the patient and surgeon on the procedure to be applied is emphasized by the authors [58].

Empirical optimization of blood management in THA patients focusing on both hematopoiesis and hemostasis is conducted in [26]. In this large, single-center, retrospective study on 986 unilateral THA patients, the effectiveness and safety of an optimized blood management program in THA are analyzed. It is suggested by the authors that patients receiving primary unilateral THA should have multiple boluses of intravenous tranexamic acid combined with topical tranexamic acid, recombinant human erythropoietin, and iron supplements that can reduce the calculated total blood loss, hemoglobin drop, transfusion rate, and postoperative length of stay without increasing the incidence of venous thromboembolism or mortality.

Another method of optimization is through the utilization of simulation that is commonly used in wear testing and THA longevity. For instance, a hip simulator wear study was undertaken in [59] to investigate the contradiction of ex vivo studies failure to substantiate a relationship between roughness and the clinical wear factor. Five million cycles are applied to three explanted femoral heads on new acetabular liners with the simulator wear rate being five times the ex vivo value. A substantial difference is determined for the relationship of surface roughness and wear resulting from the simulation testing and unidirectional wear screening methods.

3 Mathematical Optimization for THA

Theoretical optimization by using mathematical formulation is a very rare application in THA. By theoretical optimization we mean THA-related mathematical optimization formulas generated by using the relevant constraints. For instance, topology optimization is used in [60] for having minimum compliance to ensure sufficient load-bearing capacity of the porous implant with very low thickness.

A hemispherical cup affixed to a superior flange that has an optimally graded porosity is introduced in [60] as an alternative concept for a 3D printed cage with a multifunctional fully porous layer. This model focused on 1877 computed tomography (CT) scan images of a 38-year-old male for (3D) modeling of the individual's pelvic bone. Assuming the relative density ρ as the design variable, the optimization problem can be stated as in Eq. 1 with elastic properties of the implant reducing the stiffness mismatch with the bone. This equation lowers the stress levels and micromotion at the bone-implant interface and ensures appropriate bone ingrowth while maintaining load-bearing capacity.

$$\min_{x} C(x) = \frac{1}{2}\sum_{n}^{i=1} u_i^T K_i u_i \tag{1}$$

such that

$$\sum_{n}^{i=1} v_i x_i \le V^*$$

$$U(x)K(x) = F$$

$$0 < x_{\min} \le x \le x_{\max} \le 1$$

where

- *C*: Implant compliance.
- *F*: Global force vector applied to the implant.
- *U*(*ρ*): Global nodal displacement vector.
- *K*: Global stiffness matrix of the implant.
- *P*: Vector of relative densities.
- *ρe*: Relative density of each element e.
- *V**: Prescribed volume fraction of solid material.
- *νe*: Volume of each element.
- *n*: Total number of elements.

Pelvic bone geometry and assignment of elastic properties are based on the CT scan image. Tetrahedron-based unit cell topology is used as the building block of the porous implant. This chosen topology offers load-bearing capabilities and enables bone ingrowth. Finite element analysis is used with topology optimization targeting minimum strain energy, to ensure the necessary load-bearing capacity [60]. The results are attained via analysis of the mechanics of materials with density-based topology optimization, additive manufacturing constraints, and bone ingrowth requirements integrated into the problem formulation. The micromotion is calculated as the relative sliding distance between the bone and the implant surfaces in [60]. A reduction in the maximum contact stress on the bone surface by 21.4% and a decrease in the bone-implant interface peak micromotion by 26% are attained upon numerical analysis. These numerical results indicate implant long-term stability and enhanced bone ingrowth. Even though a clinical loading case of one-legged standing is used for the analysis, the attained numerical results need to be further analyzed using different loading scenarios such as walking, running, and stair climbing.

The dislocation of the hip prosthesis is dependent of specific patient anatomy and artificial joint design. Selection of the stem model and size, the head diameter and its offset, and the acetabular cup orientation is designed at the time of preoperative planning for determining optimal geometry of the reconstructed hip. Various works had suggestions about the "optimal" acetabular cup position. In [61, 62], a larger head diameter is determined to transform a prosthetic impingement into a bone impingement. This assertion is confirmed in [63] by using a numerical model that incorporated multiple combinations of geometric examined factors including the head diameter, the acetabular cup anteversion, and its inclination. This multibody model of the hip joint implant allowed demonstrating the impact of the head size, acetabular cup inclination, and anteversion on the joint range of motion with freedom to parameterize the model to simulate other geometrical parameters of the reconstructed joint.

An optimal shape of the stem using multiple objective functions is necessary due to multiple factors causing the cement fracture. A genetic algorithm is used for multi-objective design optimization of the femoral stem of a cemented THA in [64] by focusing on failure factors of cement. The objective of the study was to determine a stem geometry considering multiple factors at the same time. Two objective functions are used for determining the largest maximum principal stress of proximal and distal sections in the cement mantle with each of the two models having boundary conditions of walking and stair climbing. A 3D finite element model of the proximal femur was developed from a composite femur. The minimization results of these four objective functions are attained by using the neighborhood cultivation genetic algorithm. Upon analysis, the geometry that leads to a decrease in the proximal cement stress and the geometry that leads to a decrease in the distal cement stress are found to be different while walking and stair climbing conditions matched. Among the five stem designs, one design is identified as the "better design" for all objective functions. In their article, the authors had shown the usefulness of multi-objective optimization through genetic algorithm use for shape optimization of the femoral stem in order to avoid cement fracture.

Inverse dual optimization is applied in [65] with an improved classical wear model and an original computational algorithm. The model was applied on titanium and cast Co-Cr alloy that are commonly used in THA applications. The classical mathematical model for wear optimization of hip implants reads,

$$W = K\frac{B \times D}{H}$$

where

- K: The wear constant specific for each material.
- B: Biomechanical load.
- D: Sliding distance of the acetabular semi-sphere of the implant (mm), measured as the number of rotations of the implant multiplied by half the distance of its circular-spherical length.
- W: Wear.
- H: The hardness of the implant material.

The mathematical optimization model for generating the inverse dual optimization model is designed as

$$W - K\frac{B \times D}{H}\ {}_{2} \cong \ 0$$

subject to

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} \le \begin{bmatrix} |K_j| \\ |B_j| \\ |D_j| \\ |H_j| \\ |W_j| \end{bmatrix} \le \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$$

Specific constraints are chosen in [65] to determine the solution for the corresponding model. Same mathematical modeling and nonlinear optimization are used in [66] on four commonly used ceramic materials for THA including ZTA Biolox, ZTA Biolox-Delta, Alumina (Al_3O_2), and Zirconium (ZrO_2). Acceptable numerical results are attained for dual optimization with low residuals. 2D graphical optimization and 3D interior multi-objective optimization are used to attain the results.

In [67], both analytical and numerical techniques are utilized to evaluate the use of a perforated, titanium funicular shell to support the proximal femoral cortex in THA. Modeling is accomplished by using beam on elastic foundations and 2D elasticity theory based on the principal interactions between the femoral cortex, the metal shell, the implant stem, and the acrylic bone cement. This model is translated into a nonlinear design optimization problem that helped to determine the dimensions of the implant and reinforcing shell that minimized an objective function by using a simplified material failure criterion. In this formulation, the five design variables used included the following:

- *L*: Implanted length of the prosthesis stem and reinforcing shell.
- *Rs*: Prosthesis stem radius.
- *Rhi*: Inner shell radii.
- *Rho*: Outer shell radii.
- *Rbi*: Reamed radius of the bone.

The effect of the design variables on dominant stresses is investigated by formulating an objective function that penalized stress concentrations. The design optimization study then consisted of minimizing this objective function for achieving the smoothest load transfer between components. The nonlinear programming problem for determining the optimum values of the design variables in the system is formulated as follows:

$$\max z = \left| \frac{d_i}{d_{yi}} \right| + w_1^{w_1} + w_2^{w_2} + w_3^{w_3} - 3$$

such that

$$w_i = 10 \times \left(\left| s_i \right| + s_i \right) + 1; i = 1,2,3$$

$$s_1 \equiv Rho - Rhi \geq 0.01$$

$$s_2 \equiv Rhi - Rs \geq 1$$

$$s_3 \equiv Rbi - Rho \geq 1$$

$$Rs \geq 1$$

$$50 \geq L \geq 300$$

$$10 \geq Rbi \geq 14$$

Upon determining a solution to the nonlinear optimization, an optimal design is determined between the two cases considered. This optimal design is found to be reasonable in terms of dimensions, and the boundary conditions being realistic since even perfect prosthesis-bone contact, as represented in the other case, would loosen postoperatively. Although the absolute stress levels are probably not accurate in this optimal design, the other case had doubtful boundary conditions since perfect prosthesis collar to bone contact both in tension and compression is not physically reasonable [67]. A linear programming problem is formulated as the linearization of the nonlinear programming problem above to determine linear approximation through sequential (i.e., iterative) methodology by determining repetitive linear solutions to the sequential problems. As a part of problem formulation, power series expansion of the nonlinear function can be used, and simplex algorithm can be a well suit for determining a solution.

Mathematical modeling for optimization of THA planning by incorporating joint functionalities is designed in [68]. Maximum posterior estimation is used for optimal planning to ensure the best balance of joint functionalities and bone-implant spatial relations. The training set is designed by an experienced surgeon, and a statistical model is derived from the training data sets. The mathematical model is formulated as a stochastic optimization problem by using Gaussian distribution. The solution to the optimal problem indicated two of the functionalities' improvements, while four of the functionalities to be the same as the surgeon determined planning values.

4 Conclusions and Possible Improvements

In this article we covered both computational and mathematical optimization concepts from both theoretical and practical perspectives with their relevant factors. As it can be realized from the practical standpoint, there are many factors that are difficult to incorporate into a single model with several characteristics unless it is a personalized design. There are several variables that can be incorporated into an optimization model including but not limited to the following:

- Surgeon.
- Surgery type.
- Implant modularity.
- Body mass index.
- Bone quality.
- Cement vs. cementless design.
- Cement material mix.
- Cement thickness.
- Taper length.
- Length of surgery.
- Mixing of alloys for implant design.
- Femoral head size.
- Flexural rigidity.
- Taper angle.
- Taper-angle mismatch.
- Taper diameter.
- Stem surface roughness.
- Stem surface topology design.
- Implanted length of the prosthesis stem and reinforcing shell.
- Prosthesis stem radius.
- Inner shell radii.
- Outer shell radii.
- Micromotion.
- Reamed radius of the bone.
- Bad habits (such as tobacco use, etc.)
- Additive manufacturing considerations (if applicable).
- Changing the rigidity of the connectors based on one of the following:
  - By using screws instead of cables.
  - By using cables instead of wires.
  - By using double-wrapped wires compared to single wrapped.
- Plate and strut modifications changing stiffness.
- Using longer revision stems.
- The wear constant specific for each material.
- Biomechanical load considerations.
- Sliding distance of the acetabular semi-sphere of the implant (mm), measured as the number of rotations of the implant multiplied by half the distance of its circular-spherical length.
- Wear factors.
- The hardness of the implant material.
- Implant volume.

- Interface adherence.
- Rigidity of plating (flexible vs. rigid).
- Formation material of plating (titanium vs. stainless steel).
- Thickness of plate.
- Plate support (use of cables, different screw types, double plating, locking, multidirectional).
- Position of plating (anterior, lateral, etc.)
- Stem design and dimensioning (long vs. short, etc.)
- Muscle strength.
- Nervous system conditions.
- Dose of medicine used pre-, during, and post-surgery.
- Preoperative planning methodology.

One of the challenges for using the optimization results attained for THA is the variation of the methods and analyzed outcomes. It is possible that the differentiation in the methods used for attaining measurable outcomes may not make it possible for comparative outcomes except for methods such as finite element analysis. A practical way of learning best practices from each other would be a shared platform of detailed measurements and outcomes attained. Such data along with machine/deep learning applications can help attaining predictive outcomes for varying patient conditions. This can reduce preoperative surgeon's planning time and possibly increase the implant design and surgical operation method's choice. Another method on attaining optimal results can be incorporating more factors into mathematical formulation and determining key parameters in the mathematical model through patient-specific data. This approach can be similar to the mathematical models covered in this article. Given the variability of the above-mentioned factors altogether per patient needs makes it a challenge even in the case of applying advanced techniques that take place in THA.

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