Robust Control of Perishable Inventory with Uncertain Lead Time Using Neural Networks and Genetic Algorithm

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Abstract. The expansion of modern supply chains constantly triggers the need of maintaining resilience and agility for higher profit. There is a need to change the standard methods of inventory control to new approaches that are highly adaptable to uncertainties that emerged as a result of supply chains globalization. In this paper, a novel approach based on neural network, state-space control and robust optimization is proposed to support the perishable inventory replenishment decisions subject to uncertain lead times. We develop an approach based on the Wald criterion to compute optimal robust (i.e. "best of the worst" case) controller parameters. We incorporate lead-time specific perturbations through plausible scenarios using several lead times sets. Based on extensive numerical experiments, the obtained solutions highlight that the approach provides stable and robust solutions even for high lead times.

Keywords: Inventory control · Simulation · Optimization · Uncertain lead time · Neural networks · Genetic algorithm

1 Introduction

Over the last decade, the inventory systems have expanded significantly. Nowadays, they are exposed to the highly changeable environment. Not only the uncertainty of market demand can contribute to rising costs, but also the uncertainty of perishability process, variable lead-time, delays. Nowadays, one of the utmost important goals of modern supply chains with growing uncertainty is to build and maintain agility [1]. Fullfilling orders can be challenging tasks in case of variable environment where customers expect more flexibility than ever. Increasing the efficiency of order management systems in terms of automating many steps that requires manual involvement can enhance the goods flow, increase profitability and prevent shortages.

There is a lot of work with optimal inventory policies dedicated to the system with demand uncertainty while including no uncertainties connected to the production and distribution processes instabilities [2]. For example in [3], the effect of time value of money and inflation on optimal ordering policy is investigated but in the case of zero lead-time. Many policies have been proposed for inventory problems under stochastic

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demand and constant lead time, for example, the basestock policy (also called "order-up-to" policy) and is widely implemented in industry, but the existed methods are not sufficient to keep the modern supply chain at optimal levels because of constant lead-time assumption. For example, in [4] the optimal basestock levels are calculated in the subject of uncertain demand. Therefore, there is a need to develop methods that cope also with uncertainty in the supply chains in terms of lead times. Lead time in inventory management is the time between placing an order to replenish inventory and order receiving. Lead-time uncertainty is usually concerned with unexpected shipment (or production) delays [2]. Lead time is one of the utmost important factors that affect the stock level at any point in time. The areas which are affected by this kind of uncertainty are the agri-food, electrical, medicine (e.g. blood supply chains) and many more. With a view to the above matters, a lot of practitioners and researchers are active in this area of study. In [5] a model to minimize the total cost of an integrated vendor-buyer supply chain when the lead time is stochastic is proposed with constant demand rate assumption. Another example is study [6] in which an inventory model with the randomly variable lead time is developed also under constant demand assumption. In real supply chains constant demand is not often encountered, hence some more advanced methods based on robust optimization started to be implemented in industry.

Robust optimization is considered as a promising approach to deal with uncertainties [7]. The robust optimization has been widely studied in supply chains problems showing promising computational results for problems under demand uncertainty (e.g., see [4, 8–11]).

In the above papers involving demand uncertainty, the supply-side is assumed to be deterministic and order lead times are assumed to be either zero or fixed. There are a few papers that deal with supply and demand uncertainty. An inventory control model under demand and lead time uncertainty is studied in [12] where the tri-level optimization-based approach is used, but without considering the perishable products. Furthermore, there is a work that includes lead-time uncertainty and uses a robust optimization approach [13] – there is an approach based on Benders' decomposition to calculate optimal robust policy parameters. The work proposes the approach for robust optimization and applies it to the basestock problem. We want to extend this study to the case with perishable products and developing also a controller based on neural networks in the proposed approach, not only an optimization method.

In this paper, we proposed a method to reduce the influence of lead-time uncertainty on the robustness of the inventory system with perishable products. The presented approach for inventory control uses the combination of artificial neural networks and genetic algorithm optimization. For method validation, the nonlinear, discrete-time model of inventory system is implemented in Matlab environment together with neural network and applied to the problem of control the perishable goods flow. The proposed method is tested with the use of the set of initial conditions, different lead times, a variety of lead-times uncertainties, and two fixed shelf-lives. For developed controllers, the testing errors are calculated and the analysis of lead-time uncertainty on testing error, stock level and order quantity is presented.

2 Problem Definition and Assumptions

In this paper, we focus on the problem of inventory system control with lead-time uncertainty and perishable products. The problem considers the calculation of order quantity while balancing two conflicting goals: deliver a sufficient number of products on time and keep inventory levels down. The purpose of such an inventory control system is to determine when and how much to order. This decision should be based on the current inventory state, the expected demand, the lead-time, possible delays, and other cost factors. In this paper, we propose the approach, which includes the solution steps for the problem of inventory system optimization in case of uncertain lead-time. For the offline testing of the control approach the nonlinear, discrete-time perishable inventory with fixed lifetime products, proposed in [14], is implemented in Matlab environment. The considered class of inventory system assumes that stored products have a fixed shelf-life. The following assumptions are used for formulating the model and the investigated problem:

1. The review period is constant and equals one day.
2. The products are sold according to FIFO policy.
3. The inventory contains a single type of product.
4. Lead-time s may be uncertain.
5. Shortages are allowed but are not backlogged. Excess demand is lost.
6. There is one stocking point in each period.
7. Demand is a time-varying function.
8. Deterioration occurs as soon as the items are received into inventory.
9. The shelf-life l is fixed and known a priori. After l days all items from the same batch are expired and became unsellable waste. Lost units are not replaced.

The applied notation of applied inventory model is presented in the Table 1.

Table 1. The model parameters and variables – applied notation

Symbol	Definition
N	Length of the simulation horizon
$k \in \{1,2,\ldots,N\}$	Discrete-time
l	The shelf-life of a product
$i \in \{1,2,\ldots,l\}$	Index of state variables
s	Lead time
s_Δ	Uncertain lead time
s_0	Nominal lead time
Δ	Lead time perturbation
$d_{\max}$	The maximum demand in one period k

(*continued*)

Table 1. (*continued*)

Symbol	Definition
$\mathbf{x}(k)$	Vector of state variables
$y(k)$	Inventory level (on-hand stock)
$u(k)$	Order quantity
$d(k)$	Aggregated demand
$d_i(k)$	Demand for products of age i
$h(k)$	Aggregated amount of sold products
$h_i(k)$	Sold products of age i
n	Number of neurons in the hidden layer
$\mathbf{v}$	The vector of network weights
a_j	Activation function in the first layer
e_j	Activation function in the second layer
c_j	Transformation function in the second layer

In the applied inventory model, the demand is modelled as an unknown a priori, bounded function of discrete-time: $0 \leq h(k) \leq d(k) \leq d_{max}$. There is full demand satisfaction when the number of sold products: $h(k) \in \mathbb{R}_{\geq 0}$ is equal to the current demand $d(k) \in \mathbb{R}_{\geq 0}$, $h(k) = d(k)$. The maximum value of imposed demand for products per k period is constrained by $d_{max} \in \mathbb{R}_{>0}$. The orders are calculated in regular intervals on the basis of the expected demand $d(k)$ and the inventory state $x_i(k) \in \mathbb{R}_{\geq 0}$. The inventory state can be divided into two parts: (a) the on-hand stock per age i $x_{s+1}(k), x_{s+2}(k), \ldots, x_l(k)$; (b) work-in-progress deliveries $x_1(k), x_2(k), \ldots, x_s(k)$. In this model, i represents the age of products, e.g. $d_{s+1}(k)$ is the demand for the freshest products available in the inventory. The total amount of the sold products is given by: $h(k) = \sum_{i=1}^{l} h_i(k)$, where: $h_i(k) \in \mathbb{R}_{\geq 0}$ – sold products of age i.

In general we assume that lead-time s may be not known exactly. In such case the uncertain lead-time is denoted as s_Δ, and takes the following additive form:

$$s_\Delta = s_0 + \Delta \tag{1}$$

where: s_0 is a nominal value of lead-time and Δ is unknown, but bounded perturbation such that $|\Delta| \leq \delta$.

As inventory systems become more complex, representing them with differential equations or state-space models becomes highly advanced. Considering that, for efficient implementation in Matlab, the model is formulated using a state-space approach. State-space representation of this system is given by l equations:

$$\begin{cases} x_1(k+1) = u(k) \\ x_2(k+1) = x_1(k) - h_1(k) \\ \vdots \\ x_l(k+1) = x_{l-1}(k) - h_{l-1}(k) \end{cases} \tag{2}$$

State variable $x_i(k) \in \mathbb{R}_{\geq 0}$ keeps the information about products quantity of age i. Order quantity $u(k)$ is a nonnegative and real number. A more profound explanation of inventory model fundamentals is presented in [14].

3 Proposed Approach

The main purpose of the proposed approach is to calculate order quantities and their frequency for the inventory system under lead-time uncertainty while obtaining optimal performance in terms of shortage and holding costs minimization. The proposed approach uses two main tools: (a) genetic algorithm (GA) which is used for the learning stage; (b) neural network (NN) which is designed for the goods flow control in the perishable inventory system with lead-time uncertainty. In Table 2, there are main parameters that are assumed in the proposed approach.

We adopted the artificial neural network as a controller to control the flow of perishable products in case of lead-time uncertainty. Furthermore, the proposed approach includes also genetic algorithm application for the learning process of neural networks. A genetic algorithm is used as an optimization tool for calculating neural network weights. The proposed approach can be represented by diagram in Fig. 1.

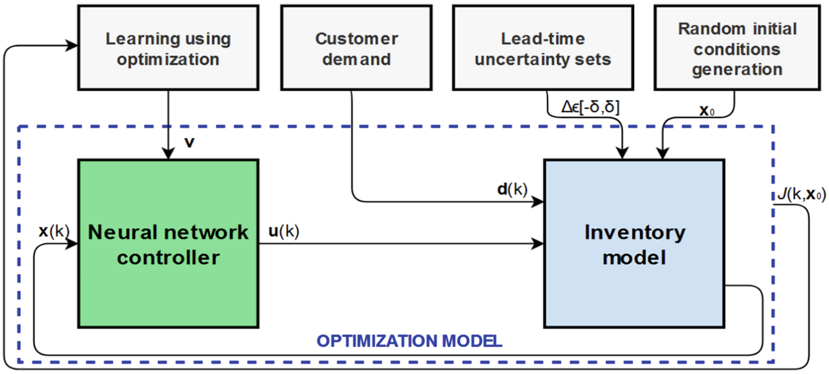

Fig. 1. The diagram of proposed approach.

The proposed approach can be explained as follows: the first step is to generate the random initial conditions of inventory state in range (0, 2). Next step is the optimization process. The goal of the optimization process is to tune the weights of neural networks that minimize the quality cost for the worst-case scenario of lead-time uncertainty. The quality cost is represented as a weighted sum of lost sales and holding cost with the assumption that the cost of lost sales is 3 times higher than the cost of holding cost. Finally, testing process of the obtained results is performed using a set of different initial conditions of inventory and different lead-times uncertainties (from the selected uncertainty range).

Table 2. Parameters of GA and NN.

Approach part	Parameter	Value
GA	The number of variables	$n(l+2)$
	The maximum number of generation	4000
	Population size	2000
	Parallel computing	True
NN	ANN model	Multi-layered perceptron
	The number of neurons in hidden layer	3
	The number of input node	l
	The number of output node	1
	The number of hidden layer	1
	The number of hidden node	3
	Activation function on the hidden layer	Satlin
	Activation function on the output layer	Poslin

The developed neural network controller consists of three layers: input, hidden and output layer. The applied structure of the neural network is depicted in Fig. 2. The input of the neural network controller is the state vector $\mathbf{x}(k) \in \mathbb{R}_{\geq 0}$ which is the number of products on every shelf – shelf represents the age of the product. The products are picked from the shelf to fullfill the current orders. The output of the neural network is the control $u(k) \in \mathbb{R}_{\geq 0}$ which is the order quantity generated in order to satisfy the demand $\mathbf{d}(k) \in \mathbb{R}_{\geq 0}$. The applied structure is a feed-forward network, in which the activation functions: saturating linear transfer function a_{j}, positive linear transfer function e_{j} and transformations c_{j} and $\breve{u}$ occur. The controller on the basis of current stock age and work-in-progress deliveries is able to generate the optimal order quantity for each day k. The weights are the elements of vector $\mathbf{v}$.

The learning process is formulated as an optimization problem of a perishable inventory system with uncertainty with the use of the genetic algorithm. The objective of the considered optimization problem is to establish weights of the neural network (Fig. 2) so that the inventory system may satisfy the customers' needs (3) and minimize the holding cost (4) at the same time. The first criterion is describing the number of lost sales due to stock shortages:

$$J_h = \sum_{k=s+1}^{N} (d(k) - h(k)) \tag{3}$$

As a second criterion for optimization, the surplus of stock over demand is considered:

$$J_y = \sum_{k=s+1}^{N} m(k) \tag{4}$$

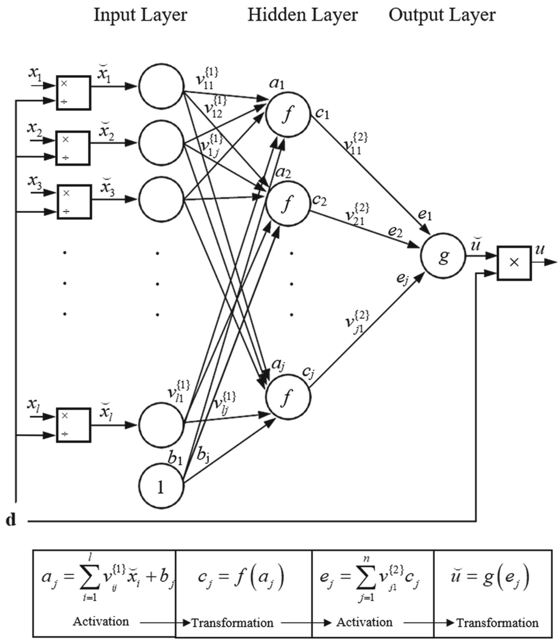

Fig. 2. The applied structure of the neural network controller.

where:

$$m(k) = \begin{cases} y(k) - d(k) & \text{for } y(k) \geq d(k) \wedge \hat{y}(k) \leq d(k) \\ 0 & \text{for} \qquad \text{otherwise} \end{cases} \tag{5}$$

The inequalities in above relationship (5) eliminate the penalty for the stock which results only from the initial conditions $\mathbf{x}_0$, where $\hat{y}(k)$ is free response of the system.

Formulated criteria can be written as the weighted cost function:

$$J = 3 \cdot J_h + J_y \tag{6}$$

Formally, the optimization problem may be stated as follows:

$$\begin{aligned} &\min_{\mathbf{v}} \max_{\Delta} J(\mathbf{v}, \Delta) \\ &\text{s.t.} \quad -\delta \leq \Delta \leq \delta \end{aligned} \tag{7}$$

The optimization is performed for assumed set of initial inventory states $\mathbf{x}_0$. As a result of the optimization process, the vector of weights $\mathbf{v}$ is obtained. In this way, the inventory controller can be optimized with a view to uncertain demand, perishability and the state vector $\mathbf{x}(k)$. This approach provides flexibility and resilience, making the inventory system being more robust for uncertain changes of lead-time. In the further part of the work, the proposed approach is called robust neural network controller (in short: RNN controller).

4 Simulation Results

In this section, we apply the proposed approach to control the perishable inventory system with uncertain delay. The simulation research is divided into five parts. The first one is focused on analyzing the performance of the proposed controllers in terms of lead-time influence on testing error. In the second part, the effect of the lead-time perturbation on stock level and cost function is investigated. In order to present the performance of proposed approach, we prepare the numerical example with the data extracted from one of the retail outlet [15]. The data contains the daily demand for milk in one month. In the next part, we extend the simulation research by applying larger lead-time uncertainty. In order to show the numerical example of the performance of RNN controllers, the fourth part contains the time responses of the perishable inventory control system. The fifth part of the simulation study is devoted to the analysis of testing error using different sizes of test sets in case of different lead-time perturbations.

For simulation purposes, the learning set contains 180 different inventory states. The initial conditions of the state vector are generated using random numbers in the range: (0, 2). Single inventory state represents a different level of initial stock level of product of different shelf-life. The general simulation parameters take the following values: the review period is one day, simulation horizon equals month (31 days), shelf-life l is fixed, adopted issuing policy: FIFO. The parameters of the main parts of the approach are listed in Table 2 (previous section).

4.1 Lead-Time Influence on Testing Error

In this subsection, the results of the testing process for the following nominal lead-times $s_0 \in \{2, 3, 4, 5\}$, lead-time perturbation of one day and shelf-life of 9 days are presented. The size of the test set is 1000 different initial inventory states. The obtained results are listed in Table 3.

Table 3. Cost function value and testing error for different lead-times.

s_0 (days)	J	Testing error (%)
2	1.4686	1.69%
3	1.3408	2.23%
4	1.4584	1.98%
5	2.4153	2.66%

The results show that testing error is the smallest for $s_0 = 2$ among considered cases and the biggest for the highest considered lead-time $s_0 = 5$. Nevertheless, the testing error is not exceeded about 3% in all considered cases. The cost function value J takes the highest value for the highest considered nominal lead-time.

4.2 Lead-Time Perturbation Influence on the Learning Process

This subsection is devoted to the investigation of lead-time perturbation and its influence on the learning process. In Table 4 the results of the learning process for the lead-time perturbation bounds: $\delta \in \{0, 1, 2, 3\}$, the nominal lead-time value of 5 days and products with the shelf-life of 12 days are presented. The lowest value corresponds to the no perturbation scenario and the highest to the high lead time uncertainty scenario. The estimated learning time increase with the perturbation bound used for the controller tuning. Learning time was approximately in the range 50–100 min on computer with Ryzen 5950X CPU.

Table 4. Cost function for worst-case scenario for different lead-time perturbation bounds.

δ (days)	J	Cost increase
0	0.8287	0
1	1.2011	0.37
2	1.7234	0.89
3	2.4317	1.60

In the analyzed case, a threefold increase in lead-time perturbation bound leads to about 2 times higher costs in terms of holding space and lost sales. In the assumed scenario, the inventory system without uncertainty in the lead-time is able to generate about 31% less cost J in comparison to the smallest assumed lead-time perturbation bound ($\delta = 1$).

4.3 Robustness of Proposed Approach

For the purpose of robustness analysis the simulation with different lead-time uncertainty is performed. The simulation scenario is prepared as follows: the demand for milk product is extracted from the retail outlet; the simulation scenario starts with a sufficient level of stock in the inventory – it means that inventory initial states are adopted to the lead-time in the analyzed case; the assumed expiration date equals 12 days; the weights of the design RNN are optimized for different perturbation bounds $\delta \in \{0, 1, 2, 3\}$ and the nominal lead-time of 5 days, whereas the simulation is conducted for the perturbated lead-times ranging from 1 to 9 days. In Fig. 3, the surface of the cost function is presented.

Figure 3 visualizes the cost function values of optimized robust neural controllers for the different variants of lead-time uncertainty. It can be seen that the smallest cost function is achieved for perturbations Δ smaller than 0. The most interesting situation is for the $\Delta > 0$. As it can be seen in Fig. 3 the controller, which does not include the uncertainty during the learning process $\delta = 0$, obtains high cost function values for $\Delta > 0$. This is because the controller was not able to be prepared for unknown uncertainties and it causes a lot of shortages in the inventory. On the other hand, the most robust behaviour for the highest lead-time is achieved by the controller of perturbation bound $\delta = 3$. In this case, other controllers ($\delta < 3$) obtain worse control quality.

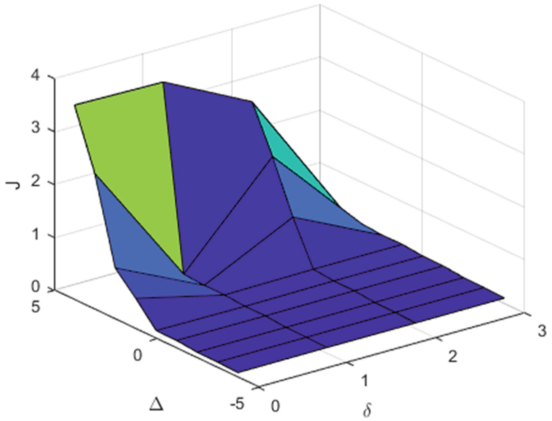

Fig. 3. Cost function values for controllers optimized for different values of perturbation bound δ and simulated using different lead-time perturbations Δ.

Moreover, in order to analyze the effect of lead-time influence on the level of stock the surface with the stock level is also generated (see Fig. 4).

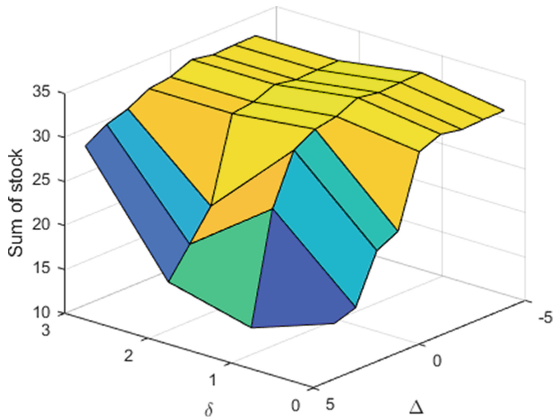

Fig. 4. Sum of stock for controllers optimized for different values of perturbation bound δ and simulated using different lead-time perturbations Δ.

It can be seen that for $\Delta \leq 0$ the stock level is similar for all optimized controllers. The change starts to be visible for $\Delta > 0$ where the stock level decreases. It can be observed that for $\Delta = 4$ the following relationship is satisfied: the higher δ the more stock is stored in the inventory. It means that controller optimized using the highest perturbation bound ($\delta = 3$) is more accurate in determining a sufficient amount of stock to minimize shortages.

4.4 Time Responses of the Obtained NNC Controllers for Long-Lead-Time Scenario

In this subsection, the time responses are investigated. The case with the high lead-time is selected ($s_\Delta = 8$) and the same parameters of the simulation are assumed as in point 4.3 with the only difference in the initial inventory state. In this subsection, we assumed zero initial inventory state, which means that inventory is completely empty at the beginning of the simulation. To start with, the monthly demand is plotted in Fig. 5 and lost sales are illustrated in Fig. 6.

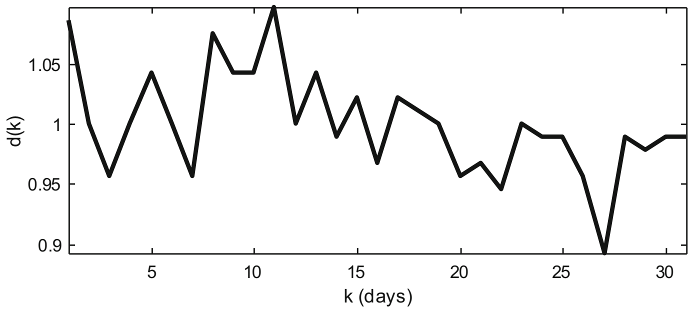

Fig. 5. Demand scenario for milk products.

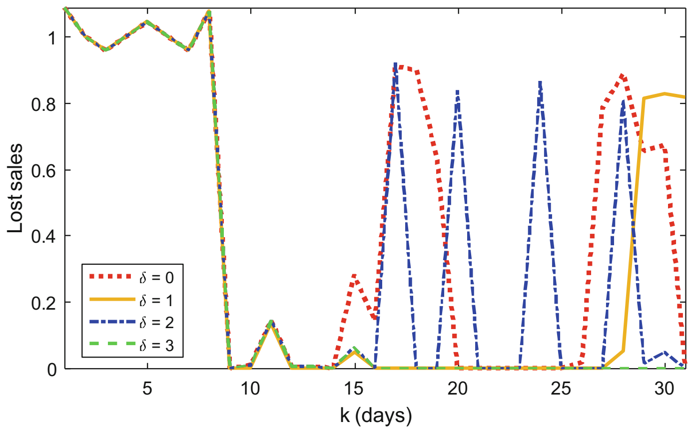

Fig. 6. Lost sales for controllers optimized using different perturbation bounds δ.

It is clearly visible that least lost sales are for $\delta = 3$. This observation implies that variable demand is satisfied with the highest level of service for $\delta = 3$ among considered controllers. It is important to highlight that the significant shortages characterize the non-robust controller ($\delta = 0$). The next time response is the order (Fig. 7).

In Fig. 7, it can be seen that the controller tuned for the perturbation $\delta = 3$ calculates the orders that follows the changes in demand without oscillations and overshoots. The other controllers generate the highly oscillating order quantities which result in higher shortages which can be seen in Fig. 6.

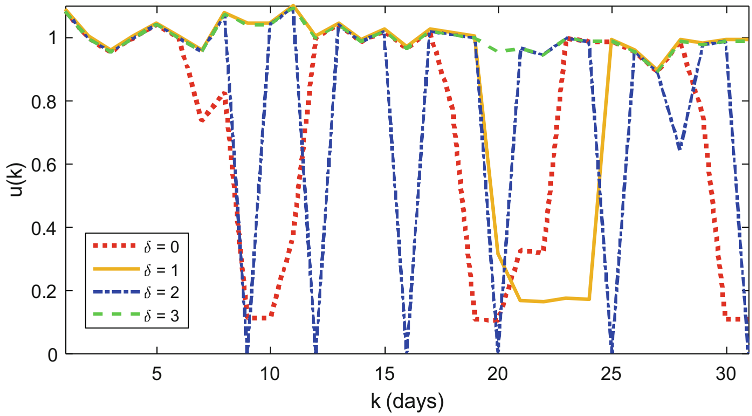

Fig. 7. Orders for controllers optimized using different perturbation bounds δ.

4.5 Test Size Influence on Testing Error

The next section is focused on the analysis of investigating the test size influence on the testing error. Figure 8 illustrates the obtained testing errors during the testing phase of RNN controllers.

Fig. 8. Learning error for different lead-time perturbation bounds and test set sizes.

In Fig. 8 can be observed that testing error, for the highest perturbation bound for all test set sizes, is the smallest among the considered cases. On the contrary, the highest learning errors occur for the controller that controls the inventory system without considering any uncertainty. It can be noted that the testing error for all considered cases is in the range from 2.3% to 4.5%.

5 Conclusions

In brief, we developed a robust neural network controller to manage the perishable items in case of uncertain lead times. In order to optimize the developed model, we adopted the robust optimization approach based on Wald criterion. Simulation research was conducted to illustrate the proposed approach performance with the use of a real demand. Our numerical results demonstrate that controllers learned using greater uncertainty bounds are more prone to outperform the controllers learned using smaller perturbation bounds in case of high lead-time. It is evident that neglecting the uncertain nature of the lead-time has serious consequences. For example, for controllers which are learned using smaller perturbation bound, the inventory level dropped below the sufficient minimum of full demand satisfaction in case of high lead-time values. What is more, learning using an evolutionary algorithm in the case of a perishable inventory system with uncertainty provides testing error not greater than 3.8%. On the basis of conducted research it can be noted that the RNN controllers are able to order the proper amount of products in an exact time for a given uncertainty set. The order quantity calculated by the controllers is nonnegative and bounded which is of utmost importance in the case of practical implementation goals. Moreover, the stock level smoothly follows the reference demand value and do not cause any unnecessary overstocks. All these advantages are achieved in the environment of uncertain lead-time. Looking also at the limitations, our proposed approach can be extended in considering demand uncertainty and lead-time uncertainty at the same time. It is one of the main topics for our further researches.

References

1. Lau, H.C.W., Wong, C.W.Y.: Infrastructure of an agile supply chain system: a virtual agent modeling approach. In: Sloot, P.M.A., Abramson, D., Bogdanov, A.V., Gorbachev, Y.E., Dongarra, J.J., Zomaya, A.Y. (eds.) ICCS 2003. LNCS, vol. 2659, pp. 432–441. Springer, Heidelberg (2003). https://doi.org/10.1007/3-540-44863-2_43
2. Chu, J., Huang, K., Thiele, A.: A robust optimization approach to model supply and demand uncertainties in inventory systems. J. Oper. Res. Soc. **70**, 1885–1899 (2019). https://doi.org/10.1080/01605682.2018.1507424
3. Taleizadeh, A.A., Nematollahi, M.: An inventory control problem for deteriorating items with back-ordering and financial considerations. Appl. Math. Model. **38**, 93–109 (2014). https://doi.org/10.1016/j.apm.2013.05.065
4. Bienstock, D., Özbay, N.: Computing robust basestock levels. Discret. Optim. **5**, 389–414 (2008). https://doi.org/10.1016/j.disopt.2006.12.002
5. Sajadieh, M.S., Jokar, M.R.A., Modarres, M.: Developing a coordinated vendor-buyer model in two-stage supply chains with stochastic lead-times. Comput. Oper. Res. **36**, 2484–2489 (2009). https://doi.org/10.1016/j.cor.2008.10.001
6. Maiti, A.K., Maiti, M.K., Maiti, M.: Inventory model with stochastic lead-time and price dependent demand incorporating advance payment. Appl. Math. Model. **33**, 2433–2443 (2009). https://doi.org/10.1016/j.apm.2008.07.024
7. Gholizadeh, H., Fazlollahtabar, H.: Robust optimization and modified genetic algorithm for a closed loop green supply chain under uncertainty: case study in melting industry. Comput. Ind. Eng. **147**, 106653 (2020). https://doi.org/10.1016/j.cie.2020.106653
8. de Ruiter, F.J., Ben-Tal, A., Brekelmans, R.C., den Hertog, D.: Robust optimization of uncertain multistage inventory systems with inexact data in decision rules. Comput. Manag. Sci. **14**, 45–66 (2017)
9. Ben-Tal, A., Golany, B., Nemirovski, A., Vial, J.P.: Retailer-supplier flexible commitments contracts: a robust optimization approach. Manuf. Serv. Oper. Manag. **7**, 248–271 (2005). https://doi.org/10.1287/msom.1050.0081

10. Bertsimas, D., Thiele, A.: A robust optimization approach to inventory theory. Oper. Res. **54**, 150–168 (2006). https://doi.org/10.1287/opre.1050.0238
11. Kim, J., Do, C.B., Kang, Y., Jeong, B.: Robust optimization model for closed-loop supply chain planning under reverse logistics flow and demand uncertainty. J. Clean. Prod. **196**, 1314–1328 (2018). https://doi.org/10.1016/j.jclepro.2018.06.157
12. Rahdar, M., Wang, L., Hu, G.: A tri-level optimization model for inventory control with uncertain demand and lead time. Int. J. Prod. Econ. **195**, 96–105 (2018). https://doi.org/10.1016/j.ijpe.2017.10.011
13. Thorsen, A., Yao, T.: Robust inventory control under demand and lead time uncertainty. Ann. Oper. Res. **257**(1–2), 207–236 (2015). https://doi.org/10.1007/s10479-015-2084-1
14. Chołodowicz, E., Orłowski, P.: Development of new hybrid discrete-time perishable inventory model based on Weibull distribution with time-varying demand using system dynamics approach. Comput. Ind. Eng. **154**, 107151 (2021). https://doi.org/10.1016/j.cie.2021.107151
15. Thakkar, H., Ravalji, J.: Demand management of perishable products using supply chain management concept. Int. Conf. Ind. Eng. (2013)