



Modeling of the Behavior of Concrete Specimens Under Uniaxial Tensile Stresses Through the Use of a 3D Probabilistic Semi-explicit Model

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Abstract. This paper presents the numerical modeling of plain concrete specimens subjected to uniaxial tensile stresses. The simulations are performed using a three-dimensional macroscopic probabilistic model for semi-explicit concrete cracking. As it is well known, concrete structures are largely sensible to the scale effects that can be attributed, among other reasons, to the heterogeneous nature of the material. The model used herein, which is developed in the framework of the finite element method, considers the material heterogeneity through the assumption that each finite element represents a volume of heterogeneous material, with mechanical properties of tensile strength and fracture energy being randomly distributed over the mesh according to the Weibull and lognormal distributions, respectively. The cracks are created with different energy dissipation according to an isotropic damage law. The results are obtained through Monte Carlo simulations using a parallelization strategy with OpenMp to allow feasible 3D simulations of real structures in a viable computational time. With the purpose of modeling the uniaxial tensile test and verifying the prediction of the scale effects, simulations of three prismatic plain concrete specimens with different sizes are performed.

Keywords: Numerical modeling · Probabilistic cracking model · Scale effects · Uniaxial tensile test · Finite element method

1 Introduction

Concrete is a heterogeneous composite material whose mechanical behaviour is subjected to the so-called scale effects. This phenomenon is related to the dependence of the global response of a given concrete structure to its size or volume. Some reasons can be pointed out to explain this phenomenon: the heterogeneous

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nature of concrete; the physical and chemical changing during its production and hardening; the micro-cracking due to drying; and the porosity resulting from the presence of water [8, 10].

As stated by [7], the size effect is related to two levels: (1) the material level that determines intrinsic constitutive relations for concrete cracking, and (2) the structural level to account for size effect in design methods and finite element analysis. The material heterogeneity and size effect are strictly correlated aspects and should be taken into account when dealing with concrete structures modeling.

In this work, a 3D macroscopic probabilistic model for semi-explicit concrete cracking is applied to numerical simulations of the behavior of plain concrete specimens under tensile stresses. The purpose of these simulations is to investigate the model's capability to reproduce the phenomenon of scale effect on concrete tensile strength. The model considers the heterogeneity of the material through a probabilistic approach, and a Monte Carlo (MC) procedure is used to ensure the accuracy of the results. The MC procedure is implemented using a parallelization strategy, reducing the computational time.

2 Semi-explicit Probabilistic Model for Concrete Cracking

The semi-explicit probabilistic model is developed in the context of the finite element method; its main principle is to incorporate the concrete heterogeneity in its formulation. For this, it is assumed that each finite element represents a volume of heterogeneous material, with its heterogeneity degree (r_e) evaluated by the following ratio: finite element volume (V_e) divided by coarsest aggregate volume (V_a). Therefore, to describe the material heterogeneity, the tensile strength (f_t) and fracture energy (G_c) are randomly distributed for each finite element according to its respective r_e .

In this modeling, it is considered that the creation and propagation of one crack within the element itself induces some local dissipation of energy. The element is considered damaged when the total amount of energy that it can consume is reached. The evolution of this dissipative process is mathematically represented through a probabilistic isotropic damage law [2]. At a macroscopic level, the creation and propagation of a crack is the consequence of the elementary failure of successive elements that randomly appear and can coalesce to form the macroscopic cracks. In that context, the model does not deal with crack propagation laws in the sense of fracture mechanics [3, 10].

The stress-strain relation of the material in a stage of damage can be expressed in terms of the undamaged stress-strain relation, as described in Eq. (1), where $\tilde{\mathbf{E}}$ and \mathbf{E}_0 are, respectively, the elastic modulus of the damaged and undamaged material and D is the damage variable. The damage evolution can be given by Eq. (2), where $\tilde{\varepsilon}_0$ represents the damage initialization strain; $\tilde{\varepsilon}_{f_i}$ represents the maximum critical strain and $\tilde{\varepsilon}^k$ stands for the equivalent strain.

$$\sigma = \tilde{\mathbf{E}}\varepsilon, \quad \tilde{\mathbf{E}} = \mathbf{E}_0(1 - D), \quad (0 \leq D \leq 1) \quad (1)$$

$$D = 1 - \frac{\tilde{\varepsilon}_0}{\tilde{\varepsilon}^k} \left[1 - \frac{(\tilde{\varepsilon}^k - \tilde{\varepsilon}_0)}{(\tilde{\varepsilon}_{fi} - \tilde{\varepsilon}_0)} \right] \tag{2}$$

This constitutive law is completely defined by the tensile strength and volumetric density of dissipated energy (g_c). This latter can be evaluated considering the use of an energetic regularization technique [1], taking into account the material fracture energy, as follows: $g_c = G_c/l_e$; where l_e represents the elementary characteristic length and is evaluated as: $l_e = (V_e)^{1/3}$.

2.1 Random Distribution of the Material Properties

The tensile strength of the material is distributed according to the Weibull distribution [11,12]. Its probability density function for a random variable x is described in Eq. (3), where $b > 0$ is the shape parameter and $c > 0$ is the scale parameter of the distribution.

$$f(x, b, c) = \frac{b}{c} \left(\frac{x}{c}\right)^{b-1} e^{(-\frac{x}{c})^b} \tag{3}$$

The mean μ and the variance σ^2 of the distribution can be seen in Eq. (4) and Eq. (5), where Γ is the Gamma function given by $\Gamma(\eta) = \int_0^\infty x^{\eta-1} e^{-x} dx$. If η is a positive integer then $\Gamma(n + 1) = n!$ what means that $\Gamma(n) = (n - 1)!$.

$$\mu = c\Gamma\left(1 + \frac{1}{b}\right) \tag{4}$$

$$\sigma^2 = c^2\Gamma\left(1 + \frac{2}{b}\right) - \mu^2 \tag{5}$$

The fracture energy of the material is distributed according the lognormal distribution. Its probability density function is defined by $f(x, b, c) : x \in (0, \infty] \rightarrow \mathcal{R}$, as can be seen in Eq. (6), where, μ is the mean and σ the standard deviation of the variable's natural logarithm.

$$f(x, \mu, \sigma) = \frac{1}{\mu\sigma\sqrt{2\pi}} e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}} \tag{6}$$

The expected value $E(X)$ and variance $Var(X)$ are given by (Eq. (7)) and (Eq. (8)). The standard deviation is considered as the dispersion measure of the distribution and is defined as $d_{log} = \sqrt{Var(X)}$.

$$E(X) = e^{\mu + \frac{\sigma^2}{2}} \tag{7}$$

$$Var(x) = \left(e^{\sigma^2} - 1\right) e^{2\mu + \sigma^2} \tag{8}$$

2.2 Parameters Estimation

The Weibull distribution parameters are estimated through an iterative numerical procedure developed to solve a non-linear system of equations. The system combines the equations of the mean and standard deviation of the distribution (Eq. (4) and Eq. (5)) with the analytical scale law proposed by [9]. This formulation comes from an experimental investigation correlating concrete heterogeneity and scale effect. This law, applied here to the elementary level, estimates the expected values of the mean and standard deviation of a given volume of concrete. With the procedure, a pair of (b, c) is obtained for each element as a function of its volume, maximum aggregate size, and compressive strength (f_c). More details of the analytical expressions as well as the description of the iterative procedure implementation can be found in [5].

The lognormal distribution has two parameters; however, as it is assumed that the fracture energy is an intrinsic material property with a constant mean value, the only parameter that must be determined is its standard deviation d_{log} . The fracture energy mean value can be taken equal to the experimental value obtained by [6] ($G_c = 1.3141 \times 10^{-4}$ MN/m). Thus, an inverse analysis procedure was carried out for estimating d_{log} , based on several simulations of a macrocrack propagation test on a very large double cantilever beam specimen (DCB), modeling the experimental test performed by [6]. From this procedure, a function to define the value of the parameter for each mesh element related to its heterogeneity degree (Eq. 9) was proposed. The full description of the inverse analysis procedure is beyond the scope of this work, but more details can be found in [4].

$$d_{log}(r_e) = (A \ln(r_e) + B) \times G_c, \quad r_e \in [1, 3000] \quad (9)$$

where, $A = -8.538$ and $B = 70.88$.

3 Modeling of the Uniaxial Tensile Test

Simulations using different volumes of plain concrete are performed to verify the presence of scale effects on the direct tensile strength of concrete in cubic and prismatic specimens. Since the model is macroscopic and its main objective is to treat the macrocrack propagation and not macrocrack initialization, a difference was expected between the numerical and experimental results concerning the tensile strength values. Therefore, the following simulations are performed:

1. Simulations of four concrete cubes with different volumes to verify the accuracy of the model and the set of estimated parameters.
2. Simulations of the four concrete cubes after calibrating the evaluation of mean and standard deviation of tensile strength, proposing an adjustment function to be used in the cases of simulation of concrete specimens under uniaxial tensile stresses.
3. Simulations of three prismatic plain concrete specimens with the purpose of validation of the proposed adjustment function.

The concrete properties are the following: Young's Modulus $E = 36$ GPa; Poisson's ratio $\nu = 0.2$ and volume of the largest aggregate $V_a \approx 9 \times 10^{-4} \text{ dm}^3$. The meshes are composed of linear tetrahedrons. The number of elements was fixed to ensure that the elements' failure has the same impact on the global response of the problem; this means that the ratio V_t/V_e is kept the same in all cases. The boundary and loading conditions of the cubes and prisms are consistent with the simulated direct tensile test, with incremental displacements applied in the longitudinal direction and increments corresponding to $\delta = 0.1 \times 10^{-4} \text{ dm}$. The measures of the specimens are in decimeters. For each Monte Carlo simulation, 400 samples were run to ensure a statistically consistent result.

3.1 Simulation of Cubic Specimens - First Evaluation

The simulations will be carried out on four cubes with different volumes whose geometry and mesh characteristics can be seen in Fig. 1. The four meshes are composed of 96 elements. More details about the cubic specimens are reported in Table 1, where the following information is presented: the simulation reference (REF), the specimen height and cross-section, the total volume of the cubes (V_T), the ratio V_t/V_a , the heterogeneity degree (V_e/V_a) and the empirical (theoretical) values of mean and standard deviation of tensile strength related to each analyzed specimen.

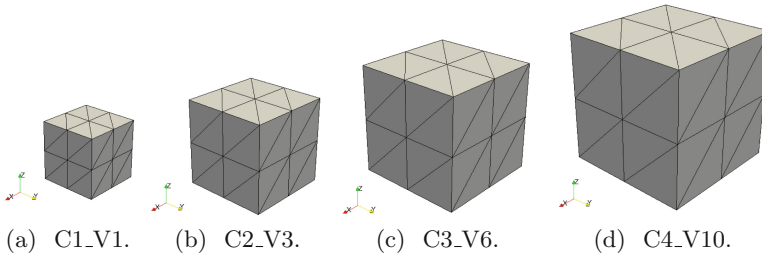


Fig. 1. Geometry and meshes characteristics of cubic specimens.

The results of these simulations are displayed in Table 2, where Δ_{f_t} and Δ_{SD} are the comparison between numerical results and experiments, obtained by the respective division of numerical by empirical values of f_t^{mean} and SD (standard deviation). Notice that as the volume of the cubes increases, the difference between the expected and numerical tensile strength values increase. A reason for this behavior is the model's characteristics and purpose, i.e., it is a macroscopic model responsible for providing fine information about localized macrocrack propagation at a structural level. Thus, the model is not formulated to reproduce the crack creation (initialization) as its principal feature. Therefore, the obtained difference between the results is understandable.

Table 1. Detailed information about the cubic specimens.

REF	Height	Cross-section	V_T	V_i/V_a	V_e/V_a	Empirical	
						f_t^{mean} (MPa)	SD(MPa)
C1_V1	1.0000	1.0000	1	1111	11.52	3.17	0.23
C2_V3	1.4423	2.0801	3	3333	34.57	2.83	0.16
C3_V6	1.8171	3.3019	6	6666	69.15	2.64	0.13
C4_V10	2.1544	4.6416	10	11111	115.16	2.50	0.11

Table 2. Results of the first simulation on the cubes

Reference	Numerical		Comparison	
	f_t^{mean} (MPa)	SD(MPa)	Δ_{f_t}	Δ_{SD}
C1_V1	3.454	0.1849	1.09	0.80
C2_V3	3.273	0.1505	1.16	0.94
C3_V6	3.157	0.1335	1.20	1.03
C4_V10	3.155	0.1468	1.26	1.33

3.2 Simulation of Cubic Specimens - Second Evaluation

The simulations presented in this section aim to minimize the difference between numerical and expected results. For this purpose, a parameters calibration is proposed employing a proportional decrease in the elementary values of the mean and standard deviation of the tensile strength, provided by the analytical expressions describing the scale effect. Thus, the coefficient of variation of tensile strength theoretically obtained remains unchanged through this strategy. The adjustment is based on the difference between numerical and empirical results measured by the ratio Δ_{f_t} (see Table 2). The proportional decrease percentage, defined as $df_t(\%)$, is given in Table 3. This table also presents the numerical results of the second group of simulations and their comparison with the empirical data.

Table 3. Results of the second simulation on the cubes

Reference	$df_t(\%)$	Numerical		Comparison	
		f_t^{mean} (MPa)	SD(MPa)	Δ_{f_t}	Δ_{SD}
C1_V1	-10	3.097	0.1665	0.98	0.70
C2_V3	-13	2.854	0.1329	1.01	0.83
C3_V6	-17	2.618	0.1097	0.99	0.84
C4_V10	-20	2.526	0.1119	1.01	1.02

As can be seen, the mean value of the tensile strength is precisely achieved, whereas its standard deviation became more accurate only for the case of C4_V10. A level of discrepancy related to standard deviation was already expected and can be justified considering the following aspects: a) the decrease is proposed concerning Δ_{f_t} ; b) it is a complex task to reproduce coefficients of dispersion of probabilistic parameters [9]; c) there is a substantial difference between the number of MC samples (400 analyzes) and the number of tests (around 13 experiments per concrete type and specimen size).

From these results, a function is proposed to estimate the percentage of decrease of the mean and standard deviation of tensile strength related to the mesh heterogeneity degree. The objective is to apply this function in the specific cases of direct tensile tests simulations and use the calibrated values for estimating the (b, c) parameters of the Weibull distribution. This function is defined as $df_t\left(\frac{V_e}{V_a}\right)$, and is described in Eq. 10.

$$df_t\left(\frac{V_e}{V_a}\right) = -4.8\left(\frac{V_e}{V_a}\right)^{0.3} \quad (10)$$

3.3 Simulations of Prismatic Specimens - Validation

For validation purposes, numerical analyzes on three prismatic specimens with different sizes, defined as P1, P2, and P3, are performed. Its geometry and mesh characteristics can be seen in Fig. 2. The number elements is equal to 837. More details about the prismatic specimens are reported in Table 4. The results are presented in Table 5 showing that there is a difference between numerical and analytical values of f_t^{mean} . However, the maximum discrepancy is around 14% for the case of P3, and the scale effect phenomenon is verified. Therefore, the results can be considered satisfactory. Besides, it is essential to highlight the mesh size distinction regarding to the cubic specimens; in this case, the number of elements is almost ten times increased.

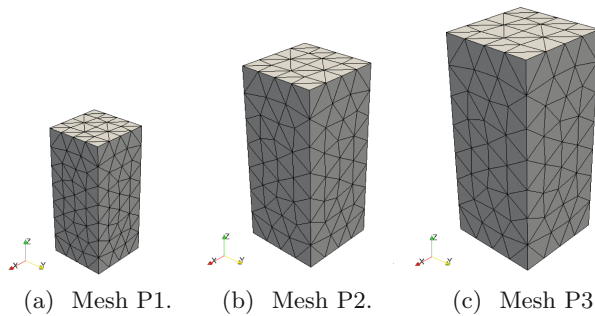


Fig. 2. Geometry and meshes characteristics of prismatic specimens.

Table 4. Detailed information about the prismatic specimens.

REF	Height	Cross-section	V_T	V_t/V_a	V_e/V_a	Empirical	
						f_t^{mean} (MPa)	SD(MPa)
P1	2.20	0.9506	2.10	2333	2.78	2.94	0.1795
P2	3.00	1.7689	5.32	5911	7.06	2.67	0.1324
P3	3.52	2.4336	8.58	9533	11.38	2.54	0.1132

Table 5. Results of the prismatic specimens simulations.

REF	Numerical		Comparison	
	f_t^{mean} (MPa)	SD(MPa)	Δ_{f_t}	Δ_{SD}
P1	3.196	0.1144	1.09	0.64
P2	3.004	0.1176	1.12	0.89
P3	2.907	0.1006	1.14	0.89

A general overview of the global mechanical behavior of the performed simulations is presented through the load-displacements curves of each MC simulation, displayed in Fig. 4(a–c). Only twenty samples are reported in the graphics to allow clear visualization of the typical profile of $P - \delta$ curves. A comparison in terms of $(\sigma \times \varepsilon)$ curves is presented in Fig. 4(d). A more significant distinction between P1 and P2 mean curves are observed due to their volumes’ larger difference. Moreover, the maximum discrepancy between numerical and empirical values of f_t^{mean} is related to P3. The typical cracking pattern of the analyzed prismatic specimens is illustrated in Fig. 3, where is presented the results of the damage variable at the final stage of the simulations for one of the MC samples of each specimen. In these figures, the light grayish blue elements represent the damaged elements. The dark gray represents the undamaged ones. The others with intermediate colors represent elements in the damaging process.

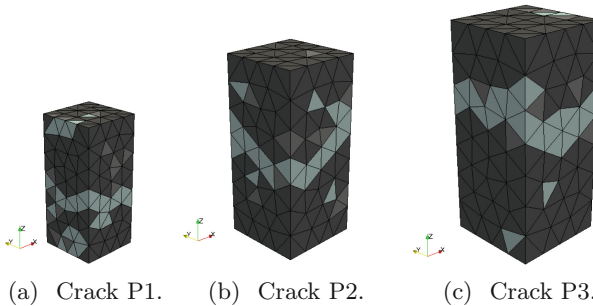


Fig. 3. Example of typical cracking pattern at the final stage of the simulations.

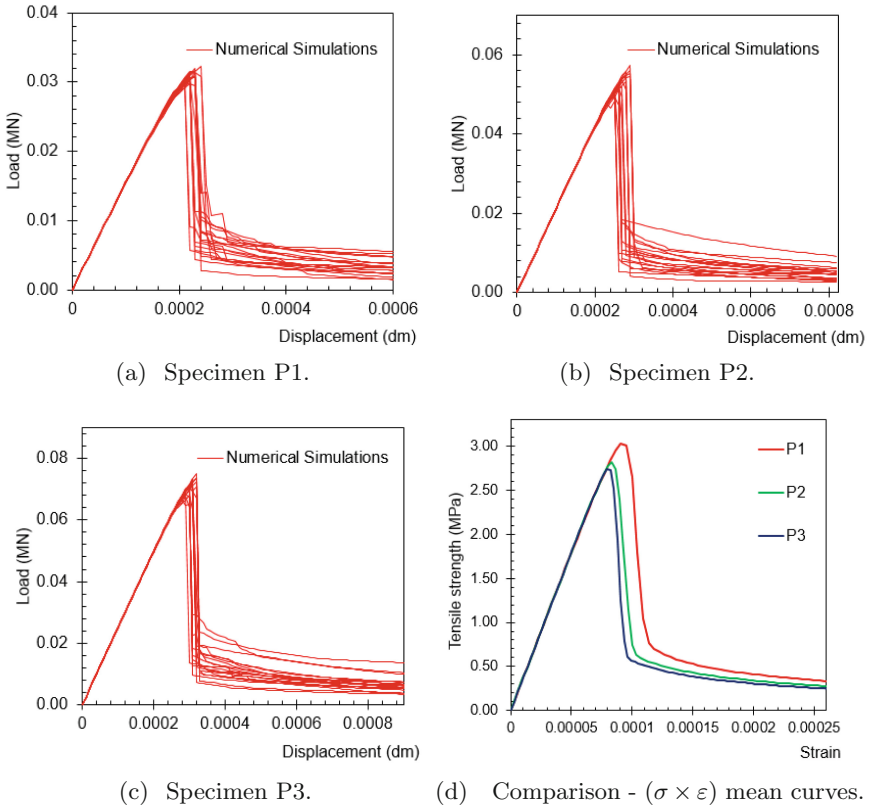


Fig. 4. Global response of Monte Carlo simulations of the prismatic specimens.

4 Conclusions

In this paper, a 3D probabilistic semi-explicit cracking model is applied to simulate the behavior of concrete specimens under tensile stress in order to reproduce scale effects. After calibrating the parameters, the mean value of tensile strength was precisely achieved in the cubic specimens simulations, whereas its standard deviation became less accurate. However, the results are promising since the main objective was to achieve its mean value. Through these results, an adjustment function to be used in this cases of uniaxial tensile stress simulations was proposed to calibrate the parameters, and validation simulations were performed. The scale effect was verified in the prismatic specimens, although a slight difference between numerical and analytical values was observed. However, as the maximum discrepancy is around 14% and, considering that the results are concerning a very complex phenomenon, they can be considered satisfactory.

References

1. Bazant, Z., Oh, B.: Crack band theory for fracture of concrete. *Mater. Struct.* **3**, 155–177 (1983)
2. Rastiello, G.: Influence de la fissuration sur le transfert de fluides dans les structures en béton. Stratégies de modélisation probabiliste et étude expérimentale. Ph.D. thesis, Université Paris-Est, IFSTTAR, Paris, France (2013)
3. Rastiello, G., Tailhan, J.L., Rossi, P., Dal Pont, S.: Macroscopic probabilistic cracking approach for the numerical modelling of fluid leakage in concrete. *Ann. Solid Struct. Mech.* **7**, 1–16 (2015)
4. Rita, M.R.: Implementation of a 3D macroscopic probabilistic model for semi-explicit concrete cracking. Ph.D. thesis, Universidade Federal do Rio de Janeiro, UFRJ, Rio de Janeiro, Brazil (2022)
5. Rita, M.R., et al.: Simulação numérica de viga de concreto protendido em duplo balanço utilizando um modelo probabilístico. In: Proceedings of the XLI Ibero-Latin-American Congress on Computational Methods in Engineering. ABMEC, Foz do Iguaçu, Brasil (2020)
6. Rossi, P.: Fissuration du béton: du matériau à la structure-application de la mécanique lineaire de la rupture. Ph.D. thesis, L'Ecole Nationale des Ponts et Chaussées, Paris, France (1988)
7. Rossi, P.: Size effects in cracking of concrete: Physical explanations and design consequences. *Proc. IA-FRAMCOS* **2**(3), 1805–1810 (1995)
8. Rossi, P., Richer, S.: Numerical modelling of concrete based on a stochastic approach. *Mater. Struct.* **20**, 334–337 (1987)
9. Rossi, P., Wu, X., Le Maou, F., Belloc, A.: Scale effect on concrete in tension. *Mater. Struct.* **27**, 437–444 (1994)
10. Tailhan, J.L., Dal Pont, S., Rossi, P.: From local to global probabilistic modeling of concrete cracking. *Ann. Solid Struct. Mech.* **1**, 103–115 (2010)
11. Weibull, W.: A statistical theory of the strength of materials. In: Proceedings of the Royal Swedish Institute for Engineering Research (1939)
12. Weibull, W.: A statistical distribution function of wide applicability. *J. Appl. Mech.* **18**, 293–297 (1951)