Assessing 3D Concrete Structures at ULS with Robust Numerical Methods

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Abstract. After a rapid state of the art, an innovative numerical method dedicated to reinforced concrete is presented. Constructed upon a strong mathematical basis, this method gives qualitative results on real-life projects. It illustrates the contribution dual analysis can have to the civil engineering world, bringing together robustness, readable results, and error bracketing.

Keywords: Concrete structures · ULS · Numerical methods · Optimization

1 Introduction

Assessing the real state of a concrete structure at ULS is often challenging. By definition, the ULS imposes to consider nonlinear law of material, and concrete is known to have complex heterogeneous behavior. Not only the reinforcements must obviously be taken into account, but the concrete itself can show scattered values of limit strength, especially in tension. Eurocodes conservatively recommend to consider a null tensile limit for the concrete, and to apply a safety factor to the compressive strength. This point of view is only one-dimensional, and some adaptations must be made when dealing with multi-axis stressing like confined concrete.

Hence the engineers find themselves with two possibilities for analyzing concrete structures at ultimate states:

– Applying the codes, generally by boiling down complex problems to easier ones. For instance, transversal sections of slender structures are classically calculated thanks to 2D tools, providing the ULS longitudinal reinforcement steel ratio. For more complex geometries, engineers can apply the strut-and-tie method (an illustration of limit analysis) to find compressive and tensile parts of concrete and then find the design of the needed reinforcements. Although this method is still used nowadays, it requires a certain level of expertise.

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– When the above methods cannot be easily applied, like on complex massive 3D structures, as a last resort, engineers turn to full 3D finite elements nonlinear elastoplastic analysis. This solution, which can be very time-consuming, is not so easy to put into practice, and often leads to more questions than answers: which software to use? What kinds of finite elements? What kind of 3D nonlinear law to use for concrete? How to take into account the reinforced bars (which are one-dimensional) into a 3D model? And even so, once the analysis has been made, how to efficiently post-process the 3D results?

After a rapid state of the art, we present hereafter a new numerical method to assess the ULS of reinforced concrete. The presentation is illustrated all along with real project results, showing the ability of the method to cope with existing structures' complexity without oversimplifying them.

2 Current Methods and Their Limitations

2.1 When Standards Can't Solve Your Problem

In a civil engineering office, standards are always the first call. If by any chance, your problem complies with the standard framework, the study can be straightforward and, furthermore, you have the pleasant feeling to enforce an absolute law. Unfortunately, the situation is rarely as simple, mainly because the standard framework is narrow and its universality also has to be questioned.

As explained in [4] for design codes [5–9], regarding flexural slender beams the plan section hypothesis is universally accepted and codes have almost all the same predictions (less than 10% of standard deviation). This reassuring result is not the same for reinforced concrete beams subjected to shear for instance, where very large discrepancies can appear among standards (factor of more than 2). This particular example shows the difficulty to establish sound simplified computational methods as soon as there is no agreed basis for a rational theory. The present comparison, over different national codes, makes clear the limitations of standards universality as much regarding different countries at the time being as regarding standard evolution for the existing structures. On this subject, let's point out that the paradigm shift, from new constructions to aging existing structures, will increase the need for timeless/worldwide prescriptions.

On the other hand, beyond this non-universality, complex 3D RC structures suffer from a lack of completely general rules of design prescribed by standards. Thus, when dealing with complex problems, the strut-and-ties method arise to be the most usual way to bring out a reasonable design.

2.2 Strut-and-Ties Responses

When it comes to assessing ULS in reinforced concrete complex geometry, strut-and-ties method is one of the most widely used. This method stems from the Ritter [1] and Mörsh [2] work at the end of the nineteenth century, which introduced the truss analogy and became practical thanks to Schlaich and Shäfer simplifications presented in [3].

Roughly, in this method, an RC structure is seen as a truss made from the rebars in traction and the concrete in compression. Thus, a stress diagram must be designed, which can be done by hand or sometimes automatically. Although a yield capacity is sought, this stress diagram is often provided thanks to a load path resulting from an elastic stress distribution (Fig. 1).

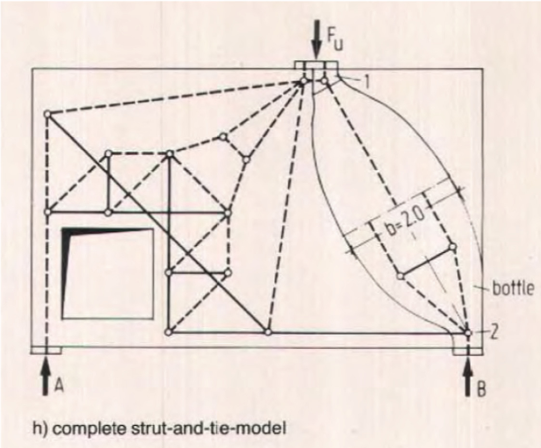

Fig. 1. Complete strut-and-tie-model of a deep beam with large hole [3]

Let's notice that the strut-and-ties method imposes compliance to the standard limit values on the traction stress in reinforcements and on the compression stress in concrete. As explained in [3], those limitations imply that the structure is designed according to the theory of plasticity lower bound theorem, and therefore, is in the spirit of the limit analysis formalism used in the presented work.

Dealing with simple structures and loadings, those diagrams can be straightforward, but they become really complex as soon as the problem is more unusual. In such cases, only experimented engineers can bring out a solution thanks to a deep knowledge of the theory. And what about an existing structure? When reinforcements are already laid out in the structure, the engineer is not free anymore to find a load path but has to trace back the design history and justify its viability. This is not the spirit of strut-and-ties that iterates on design to establish the verification.

The presented numerical method doesn't seek to replace strut-and-ties as a design tool, and neither does the understanding it gives at the first stage. However, in its application of the lower bound theorem in a more exhaustive way, it seems to be much more efficient when it comes to justifying, especially a posteriori, any kind of 3D reinforced structures.

2.3 3D Elastoplastic Analysis as the Last Solution

Let's come back to the standards discrepancies described in Sect. 2.1. A unified concept is often based on a strong physical basis. However, physical bases also mean laborious and time-consuming computations and standards can't prescribe unreachable computations to the operational world. Therefore, simplifications are made even if they imply unevaluated security margins and sometimes inconsistencies.

This fear of physical computation is largely caused by the difficulty to complete a relevant elastoplastic analysis on a 3D mesh, and then to bring out from them a readable result. While very general software, like Abacus or Ansys, offer a large number of different types of mesh and constitutive law, using them appears to be really hard for everyday life civil engineers. Indeed, those software tools are general mechanical software and don't offer proper tools for civil engineers. Moreover, uncontrolled convergence of non-linear models can prevent computation to succeed while hard work has already been done on the modelization part.

The presented below numerical method intends to solve those limitations regarding, at the same time, a suitable software tool and a robust numerical method. Let's add that several existing software already attempt to offer operational tools without over simplifying the problem, like for instance ATENA or DIANA (see [13] and [14]). However, to our knowledge, the dual analysis presented hereafter doesn't exist in any other commercial piece of software.

3 A General Optimization Formalism to Assess ULS for Reinforced Concrete Structures

Let's look into the following RC structure, a member of an externally and internally prestressed box girder subjected to cracking (Fig. 2):

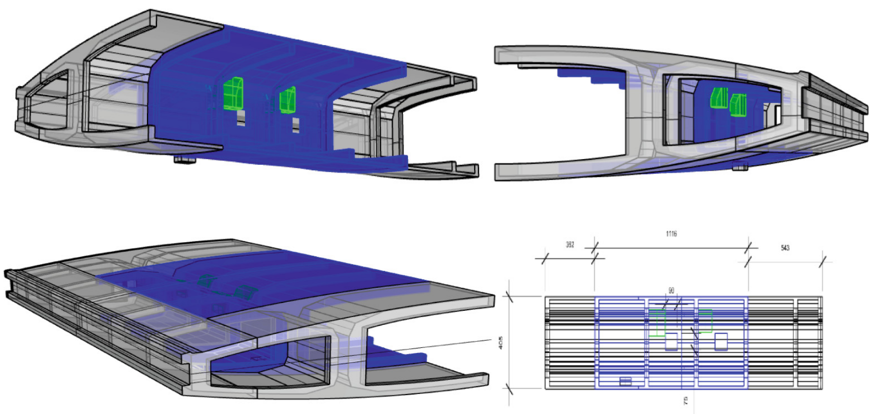

Fig. 2. CAD model of the prestressed box girder subjected to cracking. Half of the box girder is modeled due to longitudinal vertical plan symmetry. Only a few panels in the vicinity of supports are computed. External prestressing anchorage are in green.

Then, consider a civil engineer responsible for answering the following question: “Do we have to strengthen the structure or can we let it this way?”.

For this example, standards are poor help once the material limit values are determined. Moreover, since it concerns an existing structure, they have evolved since the first design whose story is hard to trace back. Therefore, a 3D finite element model is needed, either to help the strut-and-ties stress diagram, or to justify the structure full part.

3.1 An Inhouse IPM[1] Solver

Non-linear mechanical problems generally mean, mathematically speaking, constrained optimization problems (which are much more difficult than unconstrained ones). Therefore, reliable computation tools for civil engineering are necessarily based on strong mathematical foundations. An answer to this reliability need is the primal-dual interior point method, which offers great robustness as much as important calculation speed. In a simple way, this method, adopted in the present work, consists in applying Newton’s method to a set of perturbed non-linear equations.

There exists a large number of available optimization solvers that can handle a wide variety of problems and one can find a good review of them in [18]. However Strains’s will to exploit at most the mechanical insight lead the company to develop its own optimization software based on the primal-dual interior point method. Therefore, it must be pointed out that the strength of the whole presented numerical method results widely from this inhouse solver development and, in the same way, the quality of the presented results largely benefit from this demanding work.

3.2 The Static Approach or How to Find the Best Equilibrium State

As explained before, the strut-and-ties method would have tried to find an equilibrium state according to an elastic load path and elastoplastic materials strength limitations. This equilibrium is chosen among an infinite number of possibilities and is entirely dependent on the engineer’s ability to intuit the best one. Moreover, to find the struct-and-tie diagram the engineer often needs to work out an elastic 3D computation which leads to a time-consuming study, even if the 3D model is only an intermediate step.

The presented method proposes to take advantage of the 3D model, in order to find the best equilibrium state numerically. The only additional work is to model each reinforcement, as shown below (Fig. 3):

[1] Interior Point Method.

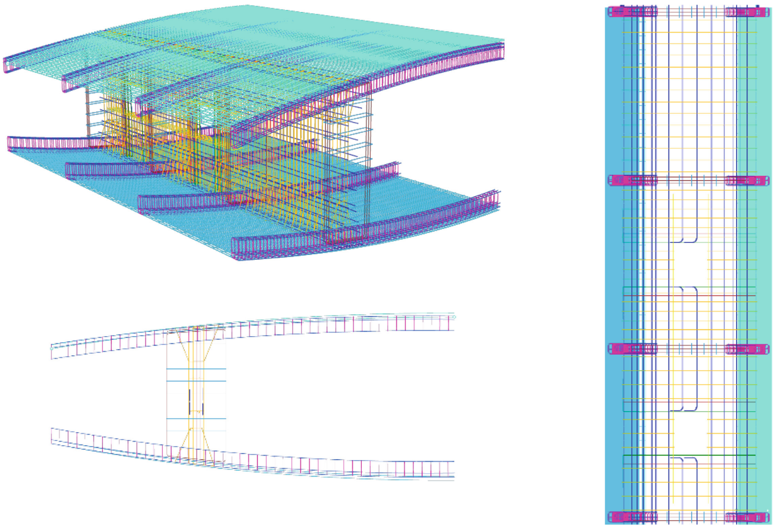

Fig. 3. CAD Model of the reinforcements embedded in concrete needed for the nonlinear analysis

A founded remark could be that drawing all the reinforcements geometry in a CAD model can be very time-consuming and that the everyday life engineer can't cope with this task. And, indeed, it would have been the case if the last decades' great improvements concerning CAD Software would have not occurred. Thanks to those powerful tools, this work only takes nowadays a few hours (once the data are collected) which doesn't seem prohibitive in front of the whole study time span.

Once the CAD model is performed, a 3D mesh is generated automatically, as well as the intersection of all the volumic mesh's tetras with the reinforcement cad lines. Then the algorithm tries to maximize the complementary energy, which only depends on the stress field, among stress values complying with the materials limit values. Different kinds of behavior can be prescribed: elastic, elastoplastic, or rigid-plastic (each with frictional contact if needed).

First of all, the used variational formulation is innovative, in the way it links concrete to reinforcement dealing only with stress variables, which can't be explained in the present paper. Let's only precise that this formulation is based on the Hybrid Equilibrium Element introduced in [16] and extended in [17], and which inherits from the work of [15]. Thus the general energy functional to be minimized is of the form:

$$\Pi_c(\sigma, \lambda) = U_c(\sigma) - V_c(\sigma) \pm \oint_{\Gamma_e \setminus \Gamma_u} \lambda^T (\mathcal{N}^T \sigma - \bar{t}) d\Gamma \tag{1}$$

The first term $U_c(\sigma) - V_c(\sigma)$ is the classical complementary energy. Imposing strongly the equilibrium on the whole structure and minimizing this first term one can retrieve an equilibrium stress field solution of the approximated mechanical problem. However, the stress field approximation used in the hybrid equilibrium formulation ensures only the equilibrium inside each finite element and does not consider equilibrium between elements. This fact explains the need to enforce *a posteriori*, and so in a weak form, the equilibrium on each finite element boundary. This is the reason of the more unusual second term $\oint_{\Gamma_e \backslash \Gamma_u} \lambda^T (\mathcal{N}^T \sigma - \bar{t}) d\Gamma$, where the Lagrange multipliers λ can be seen as generalized displacements. Under some conditions on the λ's discretization, which are respected in our case, a strong equilibrium is retrieved making the stress field codiffusive.

The second major advance is the mathematical formulation of the problem. It inherits from the work of [10] and [11] which brought out the interest of conic constraints in limit analysis. This formulation gives access to a robust solver (see Sect. 3.1) and allows to deal with all kinds of analysis (elastic, elastoplastic, limit analysis) in the same extended framework. This numerical robustness is enhanced by the ability to lead limit analysis. Indeed, limit-analysis is on its own a very direct and robust approach to assess a ULS state. While it is commonly used in geotechnical studies, structural studies hardly take advantage of this method, mainly for traditional reasons. For a civil engineer confronted with a complex RC structure, limit analysis can be seen as a very efficient tool giving access to the largest scaling value accepted for a particular loading. This scaling value is all the more interesting as the associated stress state is provided, illustrating the equilibrium scheme. Thus, static limit analysis can be seen as the optimal strut-and-ties scheme according to an automatically found safety factor (Fig. 4).

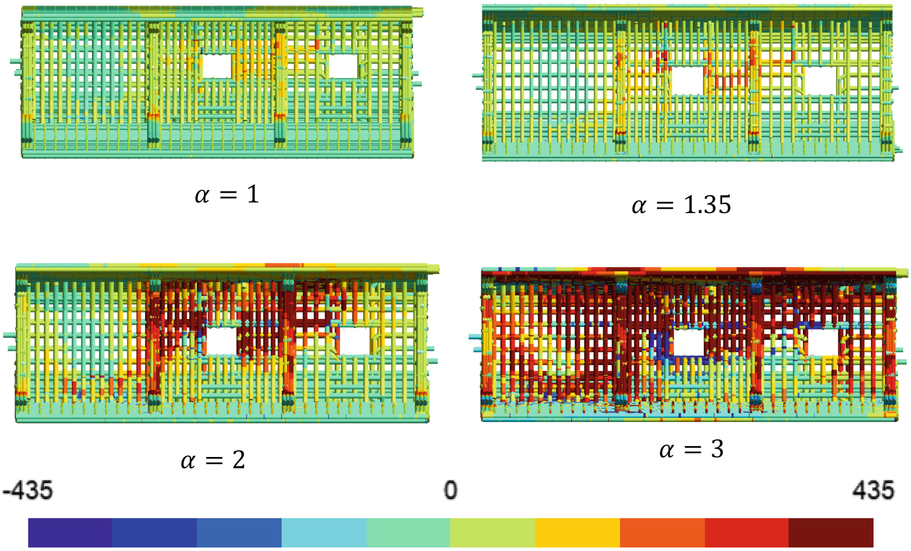

Fig. 4. Stress reinforcement evolution according to α, for the loading α(PP + P_ext + P_int). Stress value in MPA, positive values are traction.

3.3 The Kinematic Approach or How to Find the Worst Mechanism

The former static approach answers the question: what is the best equilibrium stress field according to some stress limitations? To find out the answer it extends numerically strut-and-tie rational and generalized it to any kind of structure. It must be pointed out that using a stress field as the only variable to find out a mechanical problem solution is rather uncommon. Indeed, discretization of the displacements, instead of stress, are known to give smaller size problems as explained in [12]. For instance, a much more classical method is a displacements formulation together with a Newton Raphson scheme that uses a given strain increment to find out the stress and the plastic strain. Thus, in displacements formulations, stress values are only retrieved “at the end” thanks to the constitutive law applied to a displacement field solution.

Regarding the presented method, a kinematic approach is also implemented. It consists of the search of a displacement field minimizing the potential energy plus the dissipation. Constrains on this minimization are only about boundary conditions, friction, and some slack variables.

The kinematic information completes advantageously the information already given by the equilibrium stress field, knowing that its computation cost is much less important. If the true solution is well bracketed, the retrieved displacement field gives access to a failure mechanism in adequation with the static approach stress field (Fig. 5).

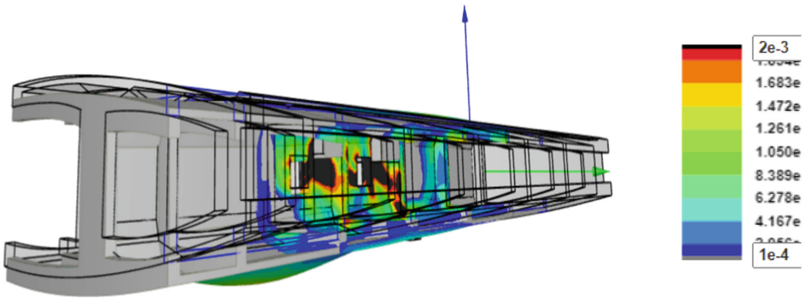

Fig. 5. Plastic strains on deformed geometry given by the kinematic approach (scaling factor 20).

3.4 At the End: Quantify the Error

Dual approach comparisons show that the choice to lead a static or a kinematic approach is far from having no impact. Indeed, with access to a kinematic result and a static result at the same time, it becomes possible to compute the distance between the two solutions. As the dual approach complies with all the mechanical equations except the constitutive law, this distance can be seen as an error in constitutive law (as shown in [19]).

It could seem expensive to lead two computations, meaning a static one and a kinematic one, with the only goal to assess the error. Indeed, numerous error estimators, post-treating a kinematic approach, give access to this error evaluation. This subject has been widely discussed in [12], and it appears that performing a dual approach is a very reliable way to assess the error and can be as expensive as some other error post-treatments. Moreover, it gives access to an equilibrium stress field which is of significant value regarding the understanding of the structure (as strut-an-ties are). Eventually, the lower bound theorem assures that the static approach under-estimates the resisting value, making the computation conservative (Table 1).

Table 1. Equations' verification (strong or weak) according to each approach

Equations verification	Static	Kinematic
Equilibrium	strong (co-diffusive)	weak
Compatibility	weak	strong
Constitutive Law	Only verified when true solution is found	Only verified when true solution is found

4 Conclusion

To quote the fib Model Code for Concrete Structures 2010 [20]: "Non-linear methods of analysis may be used for both ULS and SLS, provided that equilibrium and compatibility are satisfied and adequate non-linear behavior for materials is assumed". In spite of such recommendations, only the compatibility of displacements is widely used in modern software, whereas equilibrium seems much more difficult to achieve.

In this article we described an already operational numerical method, allowing civil engineers to comply with the quoted standard prescription. The advantages of this dual analysis based on strong mathematical bases were discussed, offering a powerful and reliable tool to cope with all the complexity of an existing or new 3D RC structure.

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