# Damage Detection Based on Voltage Transfer Ratio Approach and Bayesian Classifier

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**Abstract.** Structural damage can result in observable changes of the signal acquired by network of PZT sensors, due to elastic wave interaction with damage. There are two approaches how to utilize PZT sensors for SHM purposes. One of the approaches follows closely classical ultrasonic testing. In that case, short-pulse excitation of PZT transducers is used, thus guided wave packets can be scattered on different elements of structure, eventually also on the damage itself. The disturbance of the scattered wavefield is the basis of damage detection and evaluation. In the different approach harmonic excitation of PZT is used, thus steady elastic waves are excited in the structure. The signal can be gathered in the pulse-echo scheme, i.e., when a single transducer is used both as an actuator and a receiver of waves, as well in the pitch-catch scheme, when a pair of transducers is used. An approach for damage detection with use of a network of PZT sensors excited with harmonic signals in the pitch-catch scheme will be presented and its properties and damage detection capabilities will be discussed. In addition method for data classification based on the Bayesian approach will be demonstrated and compared to other approaches to data classification.

**Keywords:** Damage detection · Structural Health Monitoring · PZT sensors applications · Bayesian classification

## 1 Introduction

Structural state in Structural Health Monitoring (SHM) systems is usually inferred based on collected set of numerical values, called the Damage Indices, being numerical characteristics of signals acquired from a network of sensors

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used for structure monitoring. In particular, for systems based on PZT networks structural state is inferred by means of elastic waves propagating between PZT actuator and PZT sensor. Elastic waves can be distorted during their transmission through the encountered damage or they can be reflected from it. Due to this phenomena, the signal acquired by the PZT sensor can be changed. As the result of elastic waves interaction with damage, the signal acquired by PZT network can be distorted, e.g. its amplitude or its phase can be changed.

The state of the structure is usually assessed by numerical signal characteristics, called the Damage Indices (DIs). The following examples of Damage Indices can be found in the literature as useful for SHM purposes [10]:

$$SL^1(g,s) = \left|1 - \frac{\int |f_{gs}|dt}{\int |f_{gs,b}|dt}\right|, \qquad SL^2(g,s) = \left|1 - \frac{\int (f_{gs})^2 dt}{\int (f_{gs,b})^2 dt}\right| \tag{1}$$
$$\mathrm{cor}(f_{gs}, f_{gs,b}), \qquad \mathrm{cor}(f_{gs}^{env}, f_{gs,b}^{env}).$$

where $f_{gs}$, $f_{gs,b}$ denote signal acquired for a given state of the structure and corresponding baseline and $f_{gs}^{env}$, $f_{gs,b}^{env}$ their envelopes. Similar examples can be formulated for systems of machine condition monitoring based on data acquired by vibration sensors.

Therefore for a system under continuous operation, a set of DIs

$$\{DI_j = DI(t_j), j = 1, \dots, N\} \tag{2}$$

is available, obtained with use of signals acquired by the system at $\{t_j, j = 1, \dots, N\}$ instants of time. The values $DI_j$ should depend on structural state, for properly configured SHM systems, but are also dependent on other factors of random nature, e.g. environmental conditions, measurement noise, etc. Therefore for a given state of the structure $M$, DI can be viewed as a realization of random variable $DI$ described by some probability density $p_M$, i.e.

$$P(a \leq DI \leq b) = \int_a^b p_M(x)dx, \tag{3}$$

where $P(a \leq DI \leq b)$ is the probability that $DI$ attain values in the interval $[a, b]$. For a series of $N$ independent measurements acquired for a given condition of the structure, it usually can be assumed that random variables $DI_j|M$, $j = 1, \dots, N$ are statistically independent and identically distributed (the so called *i.i.d.)*, that is:

$$\begin{aligned} &P(a_1, \leq DI_1 \leq b_1, \dots, a_N \leq DI_N \leq b_N) = \\ &P(a_1 \leq DI_1 \leq b_1) \cdot \ldots \cdot P(a_N \leq DI_N \leq b_N) \end{aligned} \tag{4}$$

where each of the probabilities $P(a_j \leq DI_j \leq b_j)$ are of the form given in (3).

## 2 Bayesian Decision Model

Structural damage or machine condition can be often quantified by a parameter $d$, e.g. the length of fatigue crack or area of delamination. In that case statistical regression models can be used for structural state assessment. In the Bayesian setting [6,7] it is assumed, there exist *a-priori* probability measure $\Pi$ on the space of possible structural states $\mathcal{D}$, such that:

$$P(d \in D) = \int_D d\Pi = \int_D \pi(y)dy$$

where $D$ is some set of $\mathcal{D}$ describing structural state of the interest, usually an interval, e.g. describing the crack length, and $\pi$ is *a-priori* probability density defined on $\mathcal{D}$. The *a-priori* distribution of possible structural states can be based on experts knowledge or can be estimated statistically with use of historical data about monitored structures (e.g. crack length occurrence for aircraft fleet). If neither is available, also the so called non informative *a-priori* distributions can be used to some extent. Every state of the structure $d$, defines a probability model $p_{\mathcal{M}|d}$ for Damage Indices values:

$$P(a \leq DI \leq b|d) = \int_a^b p_{M|d}(x)dx$$

where $P(a \leq DI \leq b|d)$ denotes the probability that $DI$ is in the interval $[a, b]$, provided the structural state is characterized by parameter value $d$.

*Crack Propagation Example.* An example could be crack quantification problem by a pair of PZT transducers. In that case, illustrated in Fig. 1, the crack propagate transversely to the sensing path, i.e. path connecting the sensors.

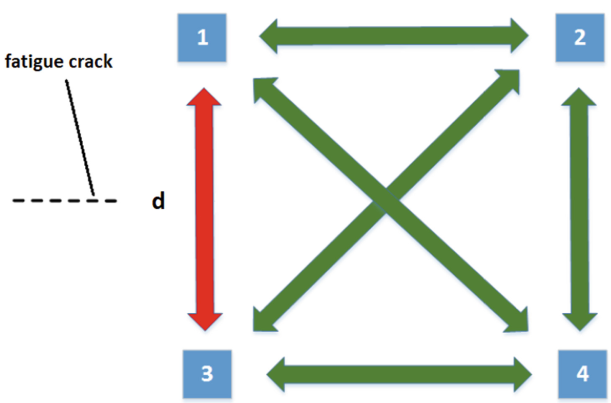



**Fig. 1.** Scheme of crack propagation within a PZT sensors network

Then one of the model for SHM system could be to assume:

$$DI|d \sim N(f(d), \sigma_0^2)$$

so $DI$ is normally (or log-normally) distributed with mean dependent on the distance $d$ of the crack tip from the sensing path and standard deviation $\sigma_0$ describing natural measurement noise (not damage dependent). The model can be extended then to subsequent sensing paths within the network. Since interaction of elastic wave with damage is a local phenomenon, the function $f$ should be bounded, i.e.:

$$c_1 \leq f(d), \text{ for } d \leq d_1, \qquad f(d) \leq c_2, \text{ for } d_2 \leq d,$$

so no information about $d$ can be extracted from $DI$ out of the interval $(d_1, d_2)$. Total probability of obtaining value of DI in the interval $(\delta_1, \delta_2)$ provided the crack tip is in the distance $(t_1, t_2)$ from sensing path is given by the formula:

$$\begin{aligned} P(\delta_1 \leq DI \leq \delta_2) &= \int_{t_1}^{t_2} \int_{\delta_1}^{\delta_2} p_{M|y}(x)\pi(y)dxdy \\ &= \frac{1}{\sqrt{2\pi}\sigma_0} \int_{t_1}^{t_2} \int_{\delta_1}^{\delta_2} e^{\frac{-(x-f(y))^2}{2\sigma_0^2}} \pi(y)dxdy. \end{aligned} \tag{5}$$

By Bayes formula the so called *a-posteriori* probability distribution of $d$, given $DI$ reads as follows:

$$p(d|DI = \delta) = \frac{p_{M|d}(\delta)\pi(d)}{p(DI = \delta)}, \tag{6}$$

where $p(DI = \delta)$ is the marginal density distribution of observing value $\delta$ of DI for a single measurement:

$$p(DI = \delta) = \int_{-\infty}^{+\infty} p_{M|y}(\delta)\pi(y)dy \tag{7}$$

For series of $i.i.d$ measurements for which $DI_j, j = 1, \ldots, N$ random variables are realized posterior distribution can be obtained similarly. Based on the posterior probability measure given measurement or series of measurements results, it is possible to improve initial knowledge about the structural state described by *a-priori* distribution $\Pi$.

From practical point of view, it can be often assumed that there are exist finite class $\mathcal{M}$ of structural states or machine conditions under interest. This can be due to structural design, e.g. only cracks of certain lengths need to be identifiable, or only damage indication is needed for maintenance action, but can be also due to limitation of the SHM method, e.g. DIs depend on damage size in discrete manner. In that case structural state assessment is the identification (a.k.a. classification) problem.

One of the classical application of Bayesian approach is decision rules optimization. In SHM setting, a decision $\delta$ can be viewed as a function from the space of measurement or series of measurements results, e.g. DIs to the space of all the defined states of the structure $\mathcal{M} = \{M_1, \ldots, M_K\}$:

$$\delta : DI \mapsto M_j \in \mathcal{M}. \tag{8}$$

In finite element case considered here, decision can be viewed as the outcome of SHM system - classification of the machine state, based on data acquired from network of sensors. In order to compare different decision rules (classifiers) a penalty rule (a.ka. *loss* function) for wrong classification is needed:

$$L : \mathcal{M} \times \mathcal{M} \to [0, +\infty) \tag{9}$$

which usually satisfies the condition:

$$L(M_j, M_j) = 0, \qquad j = 1, \ldots, K \tag{10}$$

so there is no penalty for good classification result. In the case of two state model $M_1$ can be assumed to represent no damage condition and $M_2$ can correspond to structural damage, then the loss function can be described by the table below (Table 1).

**Table 1.** General two state risk function

| | $M_1$ | $M_2$ |
|---|---|---|
| $M_1$ | 0 | $C_I$ |
| $M_2$ | $C_{II}$ | 0 |

In that case $C_I$ denotes the cost of *false calls* error (type I error), e.g. the cost of unplanned structure inspection and $C_{II}$ is the cost related to not recognized damage by the system (type II error), e.g. loss or overhaul cost of a monitored machine.

In the above considerations it was silently assumed, that the distribution of measurement outcome $p_M(DI)$ is known for a given state of the structure. This is rarely the case, unless there exist rich database of possible measurement outcomes for a given type of monitored element, which is yet not the case for SHM. Instead, it can be assumed there exist family of probability distributions, such that:

$$DI \sim p_\theta, \quad \theta \in \Theta_M. \tag{11}$$

Therefore in addition to finding an optimal classifier, also estimation of probability density for measurement outcome is necessary. Again, as for structural state, there might be defined *a-priori* distribution $\Lambda_M$ on the space of parameters $\Theta_M$ with density $\lambda_M$, such that average density distribution (a.k.a. marginal distribution)

$$\tilde{p}_M(DI) = \int_{\Theta_M} p_\theta(DI)\lambda_M(\theta)d\theta \tag{12}$$

can be used for measurement output modelling. While assumptions for $p_\theta$ can be quite general, proper definition of $\Lambda_M$ may require expert knowledge or training database.

Instead one can try to infer some properties of *a-priori* densities based on training dataset. One of the method for definition of the families $p_\theta$ based on training data for a given model $M$ is to use the so called maximum likelihood estimator (MLE) and its asymptotic properties [7]. This is the approach which is followed in this paper, the details of its definition and derivation can be found in [3].

## 3 Experiment Description and Experimental Data Evaluation

In the paper Voltage Transfer Ratio (VTR) approach was utilized for SHM with application of the voltage induced on PZT sensor as the response signal - proper for the structure assessment. This method, described in details in [3,4] is similar to Electromechanical Impedance (EMI) method [5], as it is based on harmonic PZT excitation signals, however voltage induced on PZT receiver is used as signal carrying information about the state of the structure instead of a single PZT sensor impedance as in EMI method. Due to Linear Time Invariant (LTI) systems theory, if a sinusoidal voltage $U_{in}$ is applied to PZT actuator, then the voltage signal $U_{out}$ on the receiver, induced by elastic waves, is also sinusoidal and has the same frequency as $U_{in}$, but can have different amplitude and can be phase shifted. The ratio, called the voltage transfer function:

$$TF = \frac{U_{out}}{U_{in}} \tag{13}$$

does not depend on time. It can be written in complex form as:

$$TF(\omega) = \frac{U_{out}}{U_{in}} = \frac{|U_{out}|e^{i(\omega t+\varphi(\omega))}}{|U_{in}|e^{i\omega t}} = |TF(\omega)|e^{i\varphi(\omega)} \tag{14}$$

where $|TF(\omega)|$, $\varphi(\omega)$ denote respectively - the amplitude ratio and the phase difference between output $U_{out}$ and input $U_{in}$ signals at a given frequency. Both components of TF carry the information about mechanical properties of the structure within the sensing range of a given pair of transducers, but can also depend on other factors, e.g. the properties of PZT sensors used or the distance between sensors.

Denoting as $TF(\omega)$ the transfer function for the actual state of the structure and as $TF_0(\omega)$ the components of the baseline transfer function, complex valued DIs used for structure assessment can be defined as follows:

$$DI(\omega) = \frac{TF(\omega)}{TF_0(\omega)} = \frac{|TF(\omega)|}{|TF_0(\omega)|}e^{i(\varphi(\omega)-\varphi_0(\omega))}. \tag{15}$$

For structure assessment, it is not necessary to use a single DI calculated at a given frequency. DIs behavior obtained for a range of frequencies can be better suited for damage detection and classification.

For the experiment a GFRP panel equipped with two networks containing 8 PZT sensors each was used (Fig. 2). Sensors localization was the same for both networks. Single layered PZT transducers produced by STEMINC (mod. SMD05T04R111WL), with diameter of 5 mm, thickness of 0.4 mm, made of SM111 material and of 450±10 kHz resonant frequency [9] were used in the experiment. The sensors were embedded into internal structure of the composite panel in its symmetry plane. For sensors excitation and signal acquisition, a dedicated system based on Analog Discovery 2 (AD2) module by DIGILENT [2] connected with 8 channels relay switch module designed in ITWL has been used. Signal generator was connected to A303 high voltage amplifier [1] in order to obtain 100 Vpp (symmetric) harmonic excitation signal in the frequency range of 200–350 kHz. In order to diminish the noise, measurements for a given excitation frequency were repeated 30 times and response signals were averaged.

Artificial damage was simulated for both networks by attaching small mass to the surface of the panel at 5 predefined locations with use of bitumen type paste (Fig. 2).

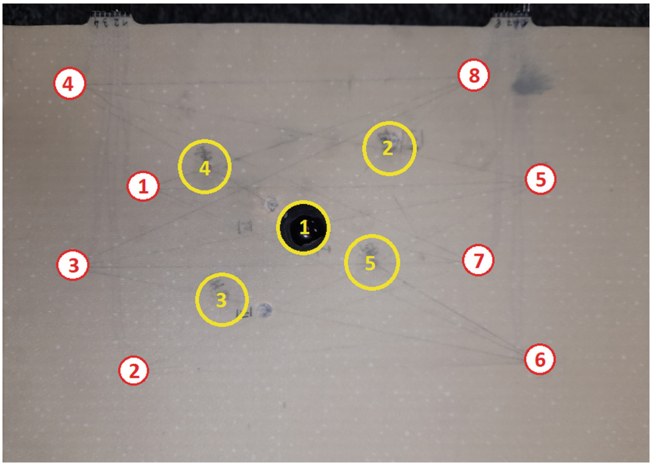



**Fig. 2.** View of the selected specimen used in the experiment with indication of PZT sensors and artificial damage localization

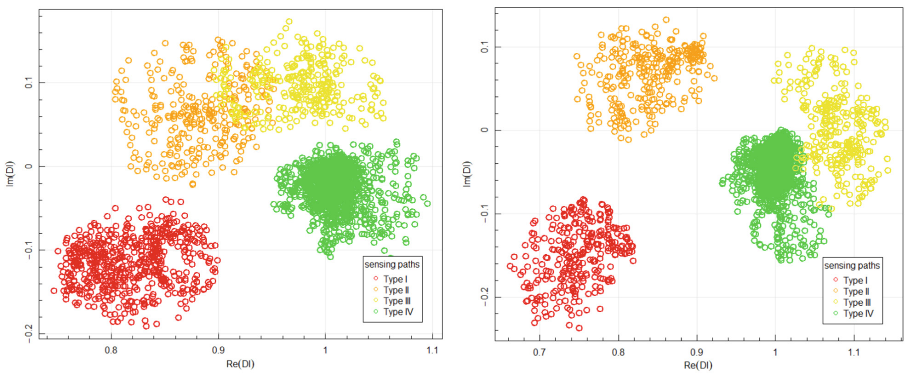
For both PZT networks, the following distinction of sensing paths with respect to introduced artificial damage has been introduced:

- Type I sensing paths which runs transversally through artificial damage and are sensitive to transmission mode of elastic waves interaction with damage;
- Type II sensing paths which are tangential to artificial damage or runs it its close proximity and can be affected by transmission mode (to some extent) and reflection mode of wave interaction with damage;
- Type III sensing paths which are separated from artificial damage but are affected by waves reflected from damage;
- Type IV sensing paths which are well separated from damage and are not influenced by its presence.

In Fig. 3 examples of Damage Indices obtained for two locations within the first PZT network, i.e. location no. 1 and location no. 3 (as shown in Fig. 2), are presented. The following classification of sensing paths has been made in those examples:

– Type I sensing paths are defined by the following pair of PZT transducers:
  - for location no. 1: 3–5, 1–7, 2–8, 4–6;
  - for location no. 3: 2–8, 3–6.
– Type II sensing paths are defined by the following pair of PZT transducers:
  - for location no. 1: 1–6, 4–7;
  - for location no. 3: 3–7, 2–5.
– Type III sensing paths are defined by the following pair of PZT transducers:
  - for location no. 1: 3–7, 1–5;
  - for location no. 3: 2–7, 3–5.

The rest of the sensing paths of the network were not significantly influenced by damage presence so these were classified as Type IV sensing paths. In similar manner the rest of data obtained for other artificial damage location for both PZT networks was labeled.




(a) for damage in location no. 1 (b) for damage in location no. 3

**Fig. 3.** Examples of Damage Indices obtained for the first PZT network

DIs obtained for a given state of the structure occupy similar area of the complex plane, irrespectively of the length and the direction of the sensing path being considered. This property is of significant importance for proper structure assessment based on Transfer Impedance method, since it opens a possibility for application of data classification methods, Bayessian classifier in particular. The data corresponding to Type I and Type II sensing paths, which were sensitive on the transmission mode of elastic waves interaction with artificial damage are well separated from DIs obtained for sensing paths not sensitive to damage (Type IV). For type III sensing paths, the obtained data sets can intersect both with data acquired for sensing paths influenced by damage (Type II) as in location no. 1 (Fig. 3a) as well as data corresponding to undamaged state of the structure (Type IV) as in location no. 3 (Fig. 3b), therefore it was observed that transmission mode of elastic wave interaction with damage has stronger effect on DIs than reflection mode of interaction.

Based on the obtained data, efficiency and properties of the proposed Bayessian classifier has been studied. For that purpose bootstrap data resampling method was applied [8] in accordance with the following procedure implemented using `boot` library available in R environment. Data acquired for network 1 corresponding to different groups and obtained for randomly selected sensing path and measurement series were used for Bayessian model definition. Then training data set was removed from the bootstrap sample and 100 data points of the remaining set were randomly selected and classified with use of the obtained model for the purpose of model validation. The percentage of correct and misclassified results was determined and the next step of boostrap resampling procedure was initiated. In similar way, nearest neighbor classifier performance was verified. For this algorithm, majority class of 20 nearest neighbor to a given data point was a basis for classification. In tables below confusion matrices of Bayessian and nearest neighbor classifiers are presented, which were calculated based on mean values of classification rates obtained for different groups of data after 300 steps of bootstrap procedure in both cases.

It can be noticed, that Bayessian classifier is characterized by very low percentage of false positive indications and reduced sensitivity to small disturbances of signal due to damage (which is the case for Type III sensing paths). For sensing paths not influenced by damage (Type IV), probability of DIs misclassification is less than 3% and probability of detection of Type III data is about 35% (sum of Type I, Type II and Type III classification probabilities). Sensitivity of nearest neighbor model to small damage is higher, however at the cost of significatnly higher false calls ratio. Reduced ratio of false positive indication can be of significant importance for particular applications, e.g. in the aerospace, where costs of unplanned maintenance procedures can be very high. In such cases reduction of false calls ratio can be more important to sensitivity to small damage which remains within damage tolerance bounds (Tables 2 and 3).

**Table 2.** Rate of classification of Bayessian model

| | | True class | | | |
|---|---|---|---|---|---|
| | | Type I | Type II | Type III | Type IV |
| Result class | Type I | 0.97 | 0.17 | 0 | 0 |
| | Type II | 0.03 | 0.70 | 0.12 | 0 |
| | Type III | 0 | 0.01 | 0.23 | 0.03 |
| | Type IV | 0 | 0.12 | 0.65 | 0.97 |

**Table 3.** Rate of classification of mearest neighbor model

| | | True class | | | |
|---|---|---|---|---|---|
| | | Type I | Type II | Type III | Type IV |
| Result class | Type I | 0.99 | 0.24 | 0 | 0 |
| | Type II | 0.01 | 0.58 | 0.2 | 0 |
| | Type III | 0 | 0.04 | 0.33 | 0.18 |
| | Type IV | 0 | 0.14 | 0.47 | 0.82 |

## 4 Summary

In the paper a new Bayesian approach to SHM data classification was proposed in the paper. The definition of approach is general and it can be applied to all type of data, not necessarily obtained with use of Voltage Transfer Ratio approach. Efficiency and validity of the classifier was confirmed based on the experimental data. Its damage detection rate was comparable with standard nearest neighbor classifier, but with significantly lower false calls ratio.

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