



# Active Control Strategy of Pantograph Coupling with Rigid Catenary Considering the Time Delay in Sensor and Actuator

Wenyan Qi, Jiangwen Wang<sup>(✉)</sup>, Guiming Mei, and Weihua Zhang

State Key Laboratory of Traction Power, Southwest Jiao Tong University, No.111 First North  
Section of the Second Ring Road, Chengdu 610031, China  
wjw@swjtu.edu.cn

**Abstract.** The research on active control of the pantograph has been intensively studied, however, the control results in practice are not satisfactory as expected, for the existence of time delays coming from the input and output measurement. This paper focuses on the problem of time delays in the designing of the control strategy, including the sensor signal input and the action of the actuator. A multibody dynamic pantograph model is established based on the relative coordinate method, combined with the rigid catenary established by the modal superposition method. Firstly, the performance of the traditional PID controller with signal time delay is studied, including the separation or combination of sensor signal input delay and actuator signal output delay. Then, a novel fuzzy PID controller with an improved rule library is proposed, which further improves the control performance of the PID controller. Finally, to deal with the time delay of sensor and actuator well, a dynamic predictive fuzzy adaptive PID (DMC-Fuzzy-PID) controller is proposed by combining dynamic matrix control (DMC) with the improved fuzzy PID controller, which may support the engineering application of active control of pantograph.

**Keywords:** Active control · Time delay · Rigid catenary · Multibody dynamics · Fuzzy PID controller

## 1 Introduction

The pantograph catenary system (PCS) plays a vital role in the process of obtaining electric energy for the train, the continuous and stable contact state between the pantograph and the catenary is the key factor to ensure that the train obtains high quality electric energy [1, 2]. In the current collection quality evaluation process, the contact force is a very important indicator, excessive contact force will aggravate the mechanical wear of the pan-head, contact wire and other components; too small contact force will increase the contact resistance, cause electrical waste, and even cause the pan-head to go offline and arc burning. As the train speed increases, the complex coupled vibration between the pantograph and the catenary will become stronger, which is reflected in the greater fluctuations in the contact force and the sharp decline in current quality. With the

development of society and technology, especially in the Chinese transportation market, there is a great demand for the use of rigid contact catenary in main railway tunnels, such as the Sichuan-Tibet railway line with a maximum running speed of 200 km/h. The purpose of this paper is to use active control methods to make trains obtain better electric energy quality at 200 km/h.

However, signal measurement, transmission, and processing will cause time delays in the actual control process. These time delays often make the control result of the contact force of the PCS unsatisfactory. When the time delay increases, the control effect of the contact force becomes worse. In some cases, the control effect is worse than no control, which is very dangerous for the railway transportation system. Therefore, the engineering practicality of the active control of pantograph needs to design an active control strategy that considering the time delay.

In this paper, a traditional PID controller is applied to analyze the influence of time delay on the dynamic contact force, including sensor signal input and actuator signal output separate or combined. The schematic diagram of the signal delay control system is shown in Fig. 1. An improved fuzzy PID controller is proposed based on traditional PID control, which optimizes the PID controller’s control performance. Under the premise of considering the time delay, the DMC-Fuzzy-PID controller is proposed, which effectively improves the adverse effects of the signal delay of the sensor and actuator.

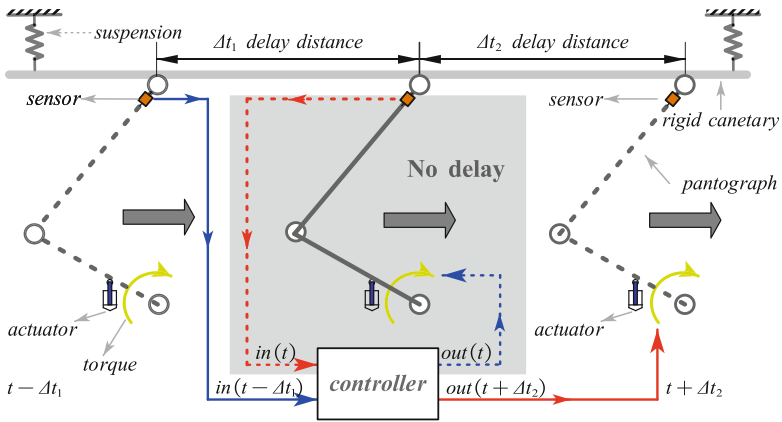


Fig. 1. The delay system for active control of pantograph.

## 2 PCS Model

### 2.1 Pantograph Model

Based on the relative coordinate system method in the field of multi-rigid body system dynamics, the R/W equation is used to establish the nonlinear dynamic equation of the

frame four-link structure, the dynamic differential equation of the pantograph model [3] is established as:

$$\begin{bmatrix} A \\ m_p \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \ddot{y}_p \end{Bmatrix} = \begin{Bmatrix} B + \Phi_q^T \lambda \\ b_p \end{Bmatrix} \quad (1)$$

### 2.2 Catenary Model

According to the static equilibrium condition and the theorem of kinetic energy and potential energy, the suspension structure of the rigid catenary is equivalent to a spring-mass system. The  $\pi$ -type structure and the contact line can be equivalent to a beam element [4] as shown in Fig. 2. where  $m_{eq}$  and  $k_{eq}$  are respectively the equivalent mass and equivalent stiffness of the suspension mechanism. The dynamic differential equation of rigid catenary is expressed as:

$$\sum_{j=1}^{NM} (m_{ij} \ddot{q}_j + k_{ij} q_j) = Q_i \quad (i = 1, 2, \dots, NM) \quad (2)$$

In the Eq. (2),  $m_{ij}$  and  $k_{ij}$  are the modal mass and modal stiffness respectively;  $Q_i$  is the generalized force,  $q_j$  is the generalized coordinate, and  $NM$  is the modal truncation order.

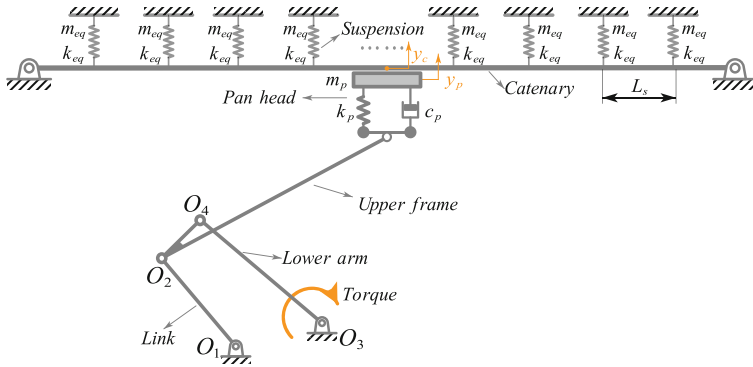


Fig. 2. Coupling model of PCS.

### 2.3 Contact Model

The pantograph and the catenary are coupled together by contact force, and the most commonly used contact force in the PCS is the penalty function method [5]. The contact force can be calculated by the Eq. (3).

$$F_c = \begin{cases} k_c(y_p - y_c) + c_c(\dot{y}_p - \dot{y}_c) & , y_p > y_c \\ 0 & , y_p \leq y_c \end{cases} \quad (3)$$

where  $k_c$  and  $c_c$  are contact stiffness and contact damping respectively. If  $y_p \leq y_c$ , the contact force becomes zero because the pantograph will no longer contact the catenary.

## 2.4 Control Objectives

The contact force of the PCS can be measured by the sensor, and the measured signal can be directly used as the feedback of the closed-loop system [6]. The goal of control is to make the contact force close to the target value, which can be expressed as:

$$e(t) = F_c(t) - f_0 \rightarrow 0 \quad (4)$$

in which normally,

$$f_0 \approx f_{mean} \quad (5)$$

where  $f_{mean}$  is the mean value of the contact force.

## 3 The Effect of Time Delay on PID Controller Performance

### 3.1 The Mathematical Model for PID Controller

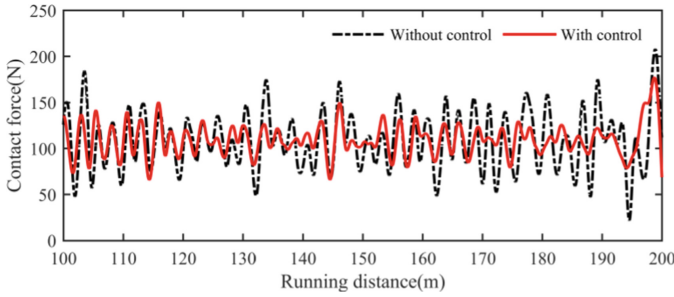
The PID controller is the most common controller in industry. PID stands for Proportional-Integral-Derivative [7, 8], the control signal  $u(t)$  is determined by the three terms  $K_p$ ,  $K_i$ ,  $K_d$  and the error signal  $e(t)$ , which can be shown as:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \dot{e}(t) \quad (6)$$

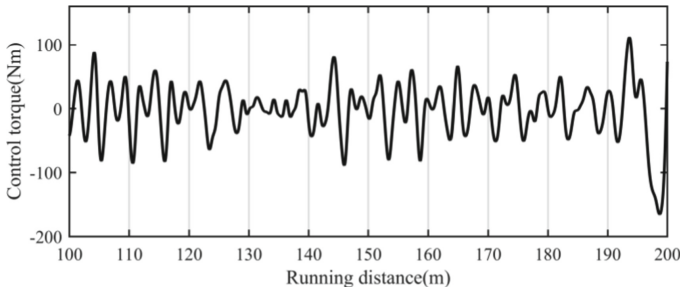
Equation (6) is the most general form and often reserved for the theoretical examination of the PID controller.

### 3.2 PID Control Performance of PCS

When the running speed of the pantograph is 200 km/h, the target value of the contact force is set to 108.8 N according to the formula  $f_0 = 70 + 0.00097v^2$ . The three parameters of the PID controller ( $K_p = 3.7$ ,  $K_i = 7.5$ ,  $K_d = 0.0049$ ) are selected, the control result of the PID controller for the contact force is shown in Fig. 3. It can be clearly seen that the contact force of the PCS is significantly reduced by the PID controller. Figure 4 shows the control torque output under the control parameters.



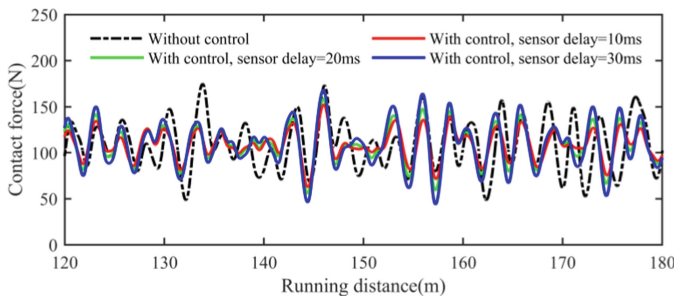
**Fig. 3.** The control result of the PID controller for the contact force.



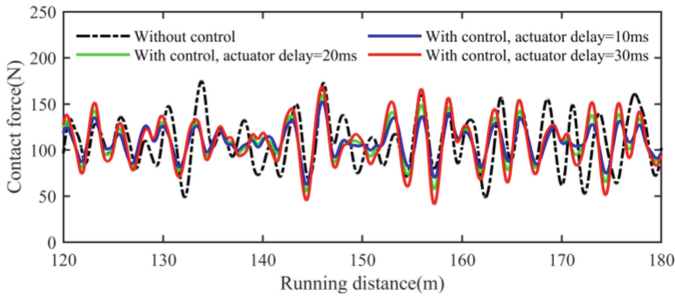
**Fig. 4.** Control torque output under control parameters.

### 3.3 The Effect of Time Delay on PID Controller Performance

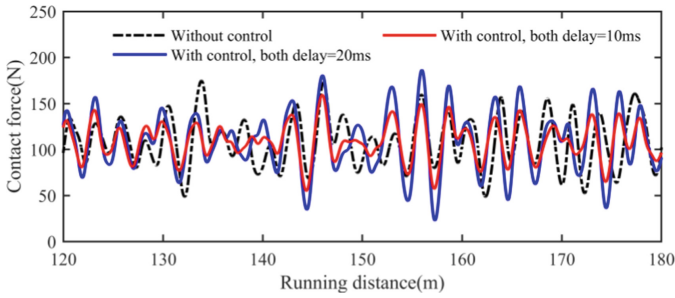
When considering the input time delay of the sensor, the output time delay of the actuator and the time delay of the two at the same time, the control effect of the contact force is shown in Fig. 5, 6, 7. It can be seen that the existence of time delay has a bad effect on the control effect of contact force. The greater the time delay, the worse the control effect of contact force. The standard deviation is selected as evaluating indicator of the current collection quality of PCS, through which controller performance is analyzed. Figure 8 shows the standard deviation of contact force with the time delays from 5 ms to 30 ms.



**Fig. 5.** Controller performance considering sensor time delay.

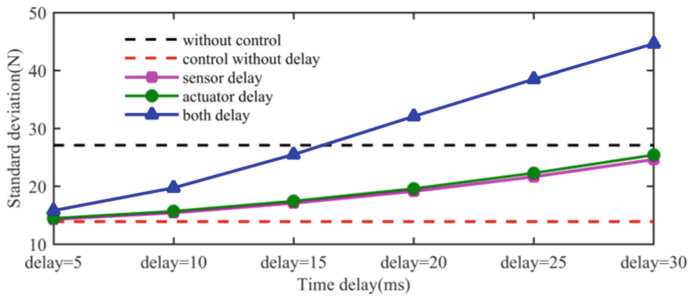


**Fig. 6.** Controller performance considering actuator time delay.



**Fig. 7.** Controller performance with time delay considering both sensor and actuator.

It can be seen from Fig. 5 to Fig. 8 that the PID controller still has the control performance that meets the requirements when there was very little delay. Further, when only the unilateral delay of the actuator or sensor is considered, the control performance will be significantly reduced if the delay time exceeds 10 ms. When the joint delay of the two is considered, the control performance deteriorates sharply as the delay increases. Therefore, on the one hand, the appropriate actuator and sensor with lower time delay should be chosen as far as possible. On the other hand, a more developed control method that can overcome large time delays should be applied.



**Fig. 8.** The standard deviation of contact force.

## 4 Controller Design Considering Time Delay

### 4.1 Improved Fuzzy PID Controller

Fuzzy PID control is based on PID control by adding fuzzy inference links, using fuzzy inference to adjust the three parameters of PID online, with strong anti-interference ability and self-adapting ability [9, 10]. the three parameters are determined by the Eq. (7).

$$\begin{cases} K_p = K_{p0} + K_{\Delta p} \cdot \Delta K_p \\ K_i = K_{i0} + K_{\Delta i} \cdot \Delta K_i \\ K_d = K_{d0} + K_{\Delta d} \cdot \Delta K_d \end{cases} \quad (7)$$

According to the expert experience of PID tuning and the compilation law of fuzzy control rule base, this paper improves the  $\Delta K_p$  rule base of traditional fuzzy PID and forms a new  $\Delta K_p$  rule base in Table 1.

**Table 1.** Improved Fuzzy rules

E	EC						
	NB	NM	NS	ZO	PS	PM	PB
NB	PB/NB/PS	PB/NB/NS	PB/NM/NB	PM/NM/NB	PB/NS/NB	PB/ZO/NM	PB/ZO/PS
NM	PB/NB/PS	PB/NB/NS	PM/NM/NB	PM/NS/NM	PM/NS/NM	PM/ZO/NS	PS/ZO/ZO
NS	PM/NB/ZO	PM/NM/NS	PM/NS/NM	PS/NS/NM	PS/ZO/NS	PS/PS/NS	PS/PS/ZO
ZO	PM/NM/ZO	PM/NM/NS	PS/NS/NS	ZO/ZO/NS	PS/PS/NS	PM/PM/NS	PM/PM/ZO
PS	PS/NM/ZO	PS/NS/ZO	ZO/ZO/ZO	PS/PS/ZO	PS/PS/ZO	PM/PM/ZO	PM/PB/ZO
PM	PM/ZO/PB	PM/ZO/PS	PS/PS/PS	PS/PS/PS	PM/PM/PS	PM/PB/PS	PB/PB/PB
PB	PB/ZO/PB	PB/ZO/PM	PB/PS/PM	PM/PM/PM	PM/PM/PS	PB/PB/PS	PB/PB/PB

Ideas for improving the  $\Delta K_p$  rule base:

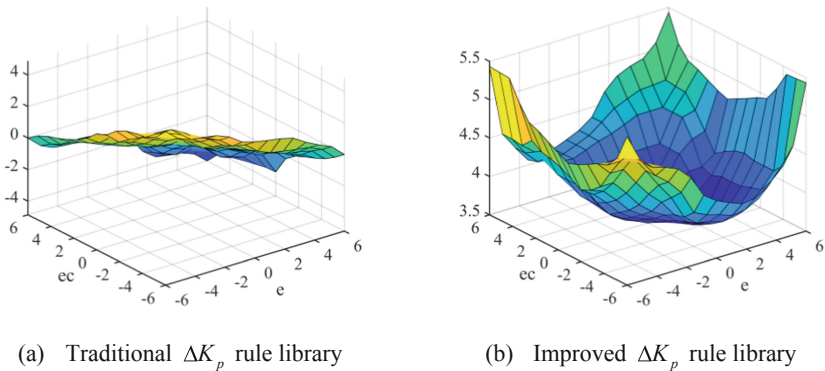
If  $e(t) \cdot ec(t) > 0$ , it indicates that the output of the system tends to diverge over time, in other words, the error  $|e(t)|$  will become larger and larger. At this time, to quickly reduce the error so that the system can reach the steady state again,  $K_p$  must be appropriately increased. From the formula  $K_p = K_{p0} + K_{\Delta p} \cdot \Delta K_p$  ( $K_{\Delta p} > 0$ ), the fuzzy control should output a positive  $\Delta K_p$  value, therefore, when the error  $|e|$  is large, to make the system have a fast response ability,  $\Delta K_p$  should take a large value, so when E and EC are both PB,  $\Delta K_p$  should be PB, but the  $\Delta K_p$  rule base of traditional fuzzy PID corresponds to NB (the  $\Delta K_p$  value of the fuzzy control output is negative), there is a contradiction, so this rule has been adjusted. According to this idea, the optimization and improvement of the rules with the same number of E and EC in the  $\Delta K_p$  rule library can be completed.

Similarly, if  $e(t) \cdot ec(t) < 0$ , it indicates that the output of the system tends to converge over time, in other words, the error  $|e(t)|$  will become smaller and smaller. At this time, to stabilize the output of the system in the vicinity of the steady-state value, it is only necessary to appropriately select  $\Delta K_p$  according to the magnitude of the deviation  $|e|$

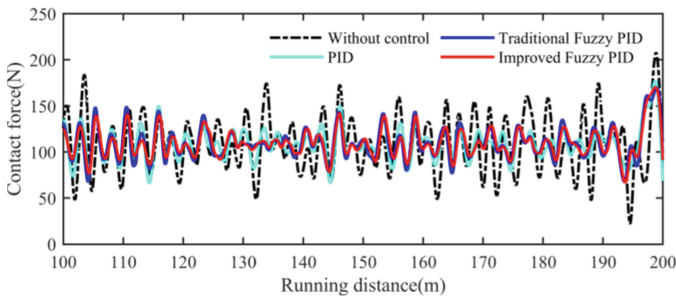
and the speed of the deviation change rate  $|ec|$ . according to this idea, the optimization and improvement of the E and EC rules in the  $\Delta K_p$  rule library can be completed.

The difference between the new  $\Delta K_p$  rule library of the improved fuzzy PID controller and the traditional  $\Delta K_p$  rule library can be intuitively reflected through the surface graph of the  $\Delta K_p$  rule library, as shown in Fig. 9.

Figure 10 shows the time domain curve of the dynamic contact force control performance of the improved fuzzy PID controller and other controllers when the time delay is not considered. Figure 11 shows the corresponding standard deviation. It can be seen that the three controllers have reduced the standard deviation of the dynamic contact force and improved the current collection quality of PCS. The improved fuzzy PID controller also shows better control performance than the PID controller and the traditional fuzzy PID controller.



**Fig. 9.** Comparison of surface maps of different  $\Delta K_p$  rule bases.



**Fig. 10.** Time domain curve of dynamic contact force with different controller.



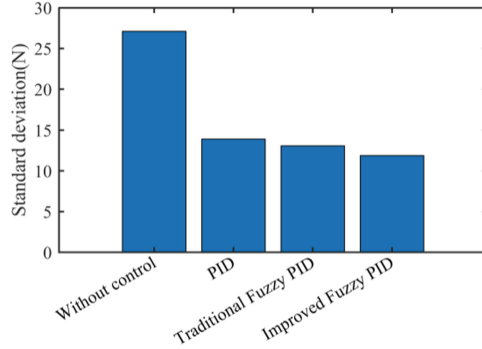


Fig. 11. The standard deviation of dynamic contact force.

### 4.2 Dynamic Predictive Fuzzy Adaptive PID Control (DMC-Fuzzy-PID)

Dynamic Matrix Control (DMC) is mainly composed of three links: predictive model, rolling optimization, and feedback correction [11, 12]. The role of the predictive model is to obtain the predictive output of the controlled object in the future. DMC approximately uses a finite set of step responses  $\{a_1, a_2, \dots, a_N\}$  as model parameters,  $N$  is the modeling time domain. The predicted output of the system under the action of  $M$  continuous control increments  $\Delta u(k), \Delta u(k + 1), \dots, \Delta u(k + M - 1)$  is

$$\tilde{y}_{PM}(k) = \tilde{y}_{P0}(k) + \mathbf{A}\Delta\mathbf{u}_M(k) \tag{8}$$

In the Eq. (8),  $\tilde{y}_{p0}(k) = [\tilde{y}_0(k + 1|k), \tilde{y}_0(k + 2|k), \dots, \tilde{y}_0(k + P|k)]^T$  is the initial predicted value of the controlled object,  $\tilde{y}_{pM}(k) = [\tilde{y}_M(k + 1|k), \tilde{y}_M(k + 2|k), \dots, \tilde{y}_M(k + P|k)]^T$  is the predicted value at a future moment,  $\Delta\mathbf{u}_M(k) = [\Delta u(k), \Delta u(k + 1), \dots, \Delta u(k + M - 1)]$  is the future  $M$  continuous control increments and  $\mathbf{A}$  is the dynamic matrix composed of the step response coefficients of the object, specifically

$$\mathbf{A} = \begin{bmatrix} a_1 & & & 0 \\ a_2 & a_1 & & \\ \dots & & & \\ a_M & a_{M-1} & \dots & a_1 \\ \dots & & & \\ a_P & a_{P-1} & \dots & a_{P-M+1} \end{bmatrix}_{P \times M} \tag{9}$$

The rolling optimization link is to find the control increment in Eq. (8). The optimal control law is determined by the quadratic performance index function.

$$\min \mathbf{J}(k) = \|\mathbf{w}_P(k) - \tilde{y}_{PM}(k)\|_Q^2 + \|\Delta\mathbf{u}(k)\|_R^2 \tag{10}$$

It is specified that  $M \leq P \leq N$ , for the time-delay system in this paper, to satisfy  $P > \tau/T$ , the length of the optimized time domain must be greater than the time-delay

interval, where  $\tau$  is the time delay. In the time delay interval, the predicted value cannot track the expected value, there is  $q = 0, i \leq \tau/T$ . In the actual controller design, we can generally choose

$$q_i = \begin{cases} 0, & i \leq \tau/T \\ 1 & \end{cases} \quad (11)$$

At this time,  $\mathbf{Q} = \text{diag}[q_1, q_2, \dots, q_P]$  constitutes the error weight matrix, and  $\mathbf{R} = \text{diag}[r_1, r_2, \dots, r_M]$  is the control weight matrix. Incorporate Eq. (8) into Eq. (10) to find the control increment that makes the objective function reach the minimum value. Obtained by calculation

$$\Delta \mathbf{u}_M(k) = (\mathbf{A}^T \mathbf{Q} \mathbf{A} + \mathbf{R})^{-1} \mathbf{A}^T \mathbf{Q} [\mathbf{w}_P(k) - \tilde{\mathbf{y}}_{P0}(k)] \quad (12)$$

The feedback correction link obtains the deviation by comparing the predicted output value of the prediction model with the actual output value of the object and uses the deviation to correct the prediction model.

$$\tilde{\mathbf{y}}_{cor}(k+1) = \tilde{\mathbf{y}}_{N1}(k) + \mathbf{h}e(k+1) \quad (13)$$

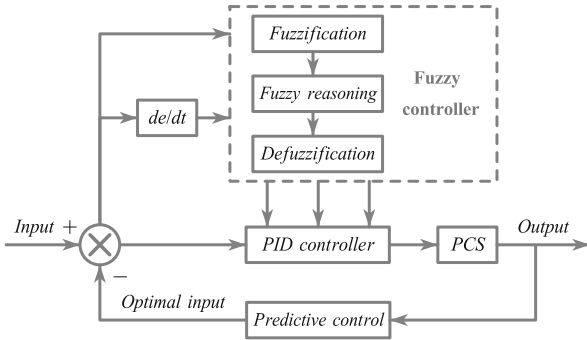
where  $\tilde{\mathbf{y}}_{N1}(k) = [\tilde{y}_1(k+1|k), \tilde{y}_1(k+2|k), \dots, \tilde{y}_1(k+N|k)]^T$ ,  $e(k+1) = y(k+1) - \tilde{y}_1(k+1|k)$ ,  $\mathbf{h}$  is the N-dimensional error weighted sequence. At this time, it is necessary to perform a transition operation on  $\tilde{\mathbf{y}}_{cor}(k+1)$  and finally obtain  $\tilde{\mathbf{y}}_{N0}(k+1)$  as the initial value of the model prediction at the next moment. The calculation expression is as follows:

$$\tilde{\mathbf{y}}_{N0}(k+1) = \mathbf{S} \tilde{\mathbf{y}}_{cor}(k+1) \quad (14)$$

The composition of the  $\mathbf{S}$  matrix is as follows:

$$\mathbf{S} = \begin{bmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \vdots \\ & & 0 & 1 \\ 0 & \dots & 0 & 1 \end{bmatrix}, \mathbf{S} \in \mathbb{R}^{N \times N}$$

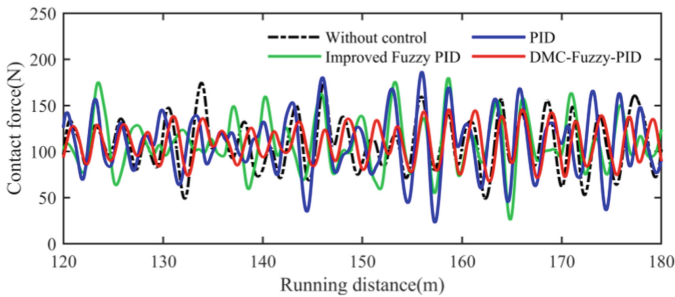
DMC-Fuzzy-PID controller is an organic combination of predictive control, Fuzzy control and PID control. The principle is to use the prediction controller to obtain the deviation between the optimal input value and the given input value and the deviation conversion rate as the input of the fuzzy PID controller. Then according to the principle of fuzzy control, the three parameters of the PID controller are adjusted online, and then the output is applied to the controlled object to determine the input value of the prediction controller. Figure 12 shows the structure of a control system using the DMC-Fuzzy-PID controller.



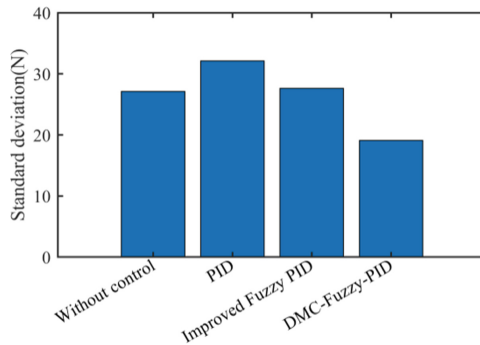
**Fig. 12.** The structure of DMC-Fuzzy-PID.

### 4.3 Simulation Verification of Controller Considering Time Delay

Figure 13 shows the dynamic contact force control performance of the traditional PID controller, the improved fuzzy PID controller, and the proposed DMC-Fuzzy-PID controller for the PCS when the sensor and actuator both have a delay of 20 ms. Figure 14 shows the corresponding standard deviation of the dynamic contact force.



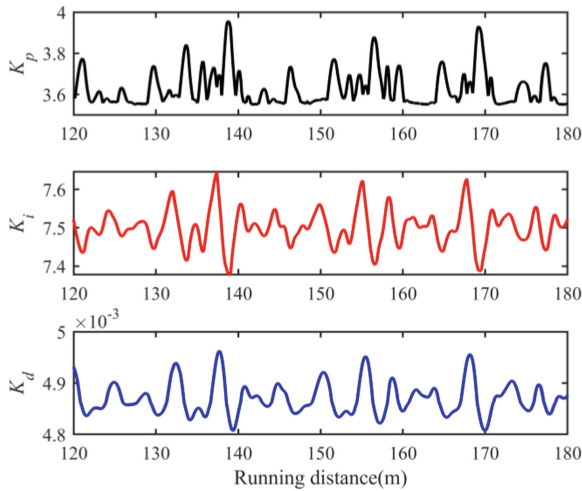
**Fig. 13.** Time domain curve of dynamic contact force with different controller, when both delay = 20 ms.



**Fig. 14.** The standard deviation for dynamic contact force, when both delay = 20 ms.

It can be seen from the Fig. 14 that although the improved fuzzy PID controller improves the standard deviation of the dynamic contact force to a certain extent, it is still greater than the standard deviation when the PCS is not controlled, and does not achieve a satisfactory control result. But the DMC-Fuzzy-PID obviously reduces the standard deviation of the dynamic contact force and improves the current collection quality of PCS. Figure 15 shows the adaptive adjustment of  $K_p$ ,  $K_i$ ,  $K_d$ .

In particular, the standard deviation of the dynamic contact force of different controllers under different time delays is extracted and shown in Table 2-4. It can be seen that when the time delay is 10 ms, regardless of whether the time delay of the sensor and the actuator is considered alone or both are considered, the three controllers all reduce the standard deviation of the dynamic contact force to a certain extent. When the time delay of both the sensor and the actuator is 20ms, the traditional PID controller and the improved fuzzy PID controller can not play a good control role, and the standard deviation of dynamic contact force increases by 18.42% and 1.84%, respectively. DMC-Fuzzy-PID controller reduces the standard deviation of dynamic contact force by 29.49% under the same conditions, which shows the superiority of the controller in the case of time delay.



**Fig. 15.** Adaptive adjustment of  $K_p$ ,  $K_i$ ,  $K_d$ .

**Table 2.** The control performance of different controllers, when considering the time delay of the sensor.

	Time delay = 10 ms	Time delay = 20 ms
PID	-43.10%	-29.40%
Improved Fuzzy PID	-49.22%	-42.90%
DMC-Fuzzy-PID	-55.89%	-52.54%

**Table 3.** The control performance of different controllers, when considering the time delay of the actuator.

	Time delay = 10 ms	Time delay = 20 ms
PID	-42.13%	-27.70%
Improved Fuzzy PID	-48.84%	-43.07%
DMC-Fuzzy-PID	-55.24%	-53.28%

**Table 4.** The control performance of different controllers, when considering the time delay of both the sensor and the actuator.

	Time delay = 10 ms	Time delay = 20 ms
PID	-27.16%	18.42%
Improved Fuzzy PID	-39.27%	1.84%
DMC-Fuzzy-PID	-51.19%	-29.49%

## 5 Conclusion

In this paper, a multi-rigid pantograph model based on the relative coordinate method and a rigid catenary model based on the modal superposition method are established. The related problems of sensor delay and actuator delay in the active control of pantograph are studied, and the following conclusions are drawn:

- (1) When the time delay of the sensor and the actuator are considered together, the current quality of the PCS will be worse than when considered separately.
- (2) Although the improved fuzzy adaptive PID controller improves the control performance of the PID controller to a certain extent, it is still not suitable for dealing with the time delay of sensor and actuator in the active control of pantograph.
- (3) The proposed DMC-fuzzy-PID controller has a satisfactory effect in dealing with the time delay problem in the active control of the pantograph. When the sensor and the actuator have a certain delay of 20 ms, the DMC-fuzzy-PID controller reduces the standard deviation of dynamic contact force by 29.49%.

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