A Critical Look at the Use of Wavelets in Damage Detection

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Abstract. The use of wavelets is a popular tool in vibration-based damage detection due to their ability to identify the changes in the instantaneous frequency of structures. Despite their widespread use in damage detection, few studies systematically investigate the efficacy of different waveforms in identifying variations in instantaneous frequency and damage detection. This article aims to address this gap in the literature. Wavelet properties, like center frequency and vanishing moment, are of importance for the wavelet selection process. For this, first, the moving load analysis was performed on the numerical model of the undamaged and damaged bridge with a low level of damage, and acceleration time histories were obtained at different observation points. The continuous wavelet transform (CWT) using different wavelet functions including Symlets, Daubechies, Coiflets, Gaussian, were applied on the acceleration time histories. Then, the absolute difference of wavelet coefficients between damaged and undamaged acceleration signals were used in order to evaluate the success of each wavelet in damage detection. At the end, different resolutions were observed in high and low scales by using different vanishing moment, and by use of a wavelet coefficient line plot, Gaus2 showed the more sensitivity to detect the damage location compared to the others.

Keywords: Wavelet transform · Damage detection · Moving load · Bridge · Accelerations · Vibrations

1 Introduction

Vibration-based monitoring of structures to detect damage has been a popular application in the field of structural health monitoring (SHM). Within this context, use of a powerful signal processing tool can be instrumental distinguish the subtle changes in the recorded vibration signals and detect the damage in structures. One of the oldest and widely used method for signal processing is Fast Fourier transform (FFT) [1, 2]. Despite its power in providing valuable information about the global response of the structure, FFT has certain limitations. For example, it is limited to stationary signals and cannot capture the nonlinear properties of the signals. In addition, time information is missing using FFT [3, 4]. To overcome these limitations, an alternative called short time Fourier transform (STFT) was introduced by Gabor [5]. STFT is capable to provide time and

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frequency localization and analyze the non-stationary signals. STFT divides the signal into a number of equally long windows and apply FFT on each window [6]. However, using the same size window limits the capabilities of STFT to analyze both low and high frequency components of the signal concurrently [7]. To overcome this limitation windowing technique was generated by Morlet, a geophysical engineer, named wavelet at the end of 1970s. Wavelets use of short-time windows to capture the high frequency content of the signal and long-time windows to analyze the low frequency information [8].

Wavelet transformation (WT) has been applied successfully on different studies to detect the damage in the bridge structures [8–11]. Since the different wavelet forms can affect the results of the wavelet transform, a result comparison of the different wavelets can be beneficial to provide a deep understanding of that. As such, this paper aims to investigate the efficacy of different wavelets functions to detect damage from acceleration signals of a simply supported bridge generated by a single moving load. The acceleration responses of the damaged and undamaged bridge FE model at 3 different observation points were analyzed by applying continuous wavelet transform (CWT) using four types of wavelet with different vanishing moments to quantify the efficacy of each wavelet in identifying the difference between damaged and undamaged responses and, therefore, detecting the damage.

2 Continuous Wavelet Transform (CWT)

The Continuous wavelet transform (CWT) is a robust time-frequency tool that decomposes the signals into a set of coefficients in two dimensions, scale and shift, using Eq. (1). The basis wavelet function is stretched (scaled) and translated (shifted) and compared against the original signal. The amplitude of wavelet coefficient, $|W_x(a,b)|$, plotted against the scale coefficient, a, shift coefficient, b, constitute the wavelet coefficient map [8, 12].

$$CWT\{x(t)\} = W_x(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\psi^*\left(\frac{t-b}{a}\right)dt \quad \text{for a} > 0 \tag{1}$$

where, ψ(t) is the mother wavelet and ψ^*(t) its complex conjugate. The mother wavelet is scaled and translated, using a and b respectively. There is no consensus in the structural health monitoring about which mother wavelet maximizes the chance of damage detection. The aim to maximize the damage feature utilized to detect the damage is usually what drives the choice of the wavelet. With this in mind, in this study, four wavelet types are chosen with different vanishing moment (m) including Daubechies (db) (m = 2,4,6,8), Coiflet (coif) (m = 1,3,5), Gaussian (gaus) (m = 2,4,6,8) and Symlet (sym) (2,4,6,8), which are mostly used in damage detection studies [10, 11, 13, 14], to compare the performance of the different wavelets in detecting damage. Figure 1 illustrates some of this mother wavelets.

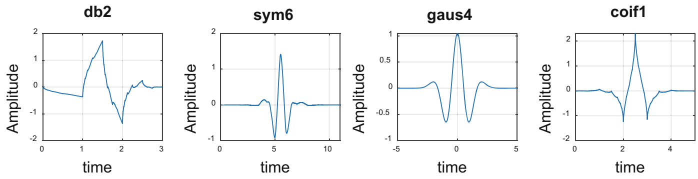

Fig. 1. Example of some mother wavelets used in this study

Although it is common to work in frequency domain in engineering practice, the wavelet coefficients are generally presented in terms of scale, which is inversely proportional to frequency. Therefore, a low scale implies high frequency and conversely a high scale, that is a stretched wavelet, implies a low frequency. Equation (2) provides the relationship between frequency and scale [8, 10]:

$$F_s = \frac{F_c}{a\Delta} \tag{2}$$

where F_s is the pseudo-frequency in Hz corresponding to scale a, F_c is the center frequency of the wavelet in Hz, a and Δ are the scale of the wavelet and sampling rate of the record, respectively.

3 Numerical Analysis of the Bridge Using a Single Moving Load

The numerical model used in the study is a simply-supported concrete bridge modeled in SAP2000 (Fig. 2), with the elasticity modulus of concrete 32 GPa. The span length is 50 m and the bridge is discretized at every 1 m. The moving load analysis was performed using a 400 KN constant load crossing the bridge at a speed of 50 km/h (13.8 m/s). The damage has been simulated by reducing the flexural stiffness (EI) of the 2 elements (*i.e.,* 2 m length) by 5% at the mid-span, and the acceleration time histories were extracted at 3 observation nodes (nodes13, 26, 40), as shown in Fig. 2.

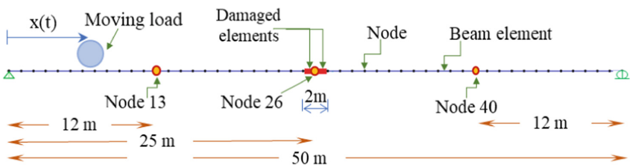

Fig. 2. The sketch of the bridge subject to damage and moving load.

Figure 3 shows the relative insensitivity of the acceleration time history recorded at the mid-span. The change in the natural frequencies of the bridge due to the simulated damage was less than 0.1%.

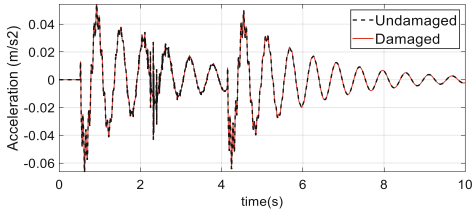

Fig. 3. Mid-span acceleration of the bridge crossed by the moving load

4 Damage Detection Using CWT

In this article, the acceleration responses without noise effect were used and the difference in the wavelet coefficient map between undamaged and damaged signals used for damage detection. Figure 4 shows the wavelet coefficient map of the acceleration signal in node 26 shown in Fig. 3 for both undamaged and damaged cases using the Coiflets 1 (coif1). Also presented in Fig. 4 are the predominant frequencies of the bridge in the vertical direction and the corresponding scales computed using Eq. (2). For both signals, the wavelet coefficient map reveals high energy content near the fundamental frequency of the bridge, which is in line with the expectation that the single-span bridge behavior is dominated by its first structural mode. In addition to this, there is a series of rather high coefficients at 0.5 s and 4.12 s in the low-frequency region (i.e. high scale) as the result of moving load entering and leaving the bridge.

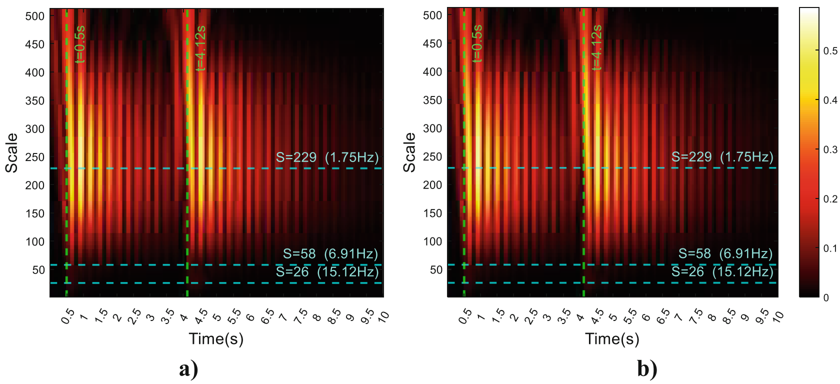

Fig. 4. CWT of acceleration signals shown in Fig. 3, a) Undamaged and b) Damaged

Although the wavelet coefficient maps for the damaged and undamaged cases presented in Fig. 4 seem very similar to each other, the absolute difference between the two maps plotted in Fig. 5 shows that the simulated damage leads to a shift in the wavelet coefficients. Figure 5a and Fig. 5b, represent the absolute difference in the wavelet coefficient maps up to a scale of 512 and 1024, respectively. In both figures there is a high difference close to the natural frequency of the bridge (i.e., f = 1.75 Hz; scale 229) implying the presence of the damage in the bridge albeit without providing any information regarding the location of the damage. On the other hand, in the low frequency (high scale) region, a stark difference between the wavelet coefficients from the damaged and undamaged cases at t = 2.239 s and 2.384 s can be observed. This instant is the period when the moving load crossed over the 2 m long damaged portion of the bridge indicating that the absolute difference of the wavelet coefficient maps of damaged and undamaged cases using the Coiflet 1 wavelet not only detect presence of the damage but also can provide information about the location of the damage. The location of the damage can be identified better if even lower frequencies (higher scales) are used in constructing the wavelet coefficient maps as shown in Fig. 5a and Fig. 5b. Here, it should be noted that acceleration time histories used in Fig. 5 are recorded at the damage location.

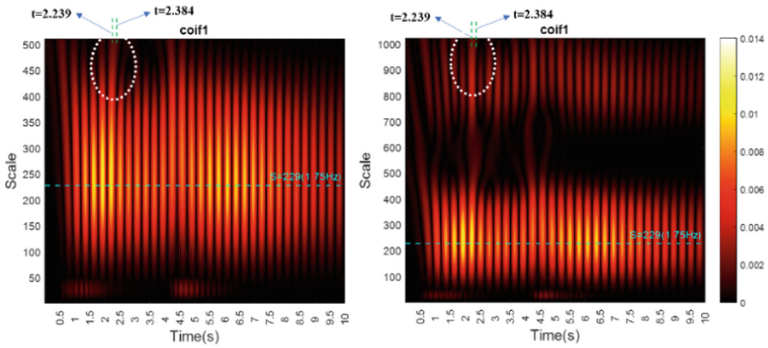

Fig. 5. The absolute coefficient difference of CWT between damaged and undamaged acceleration signal at node 26: a) scale 1–512 and b) scale 1–1024

4.1 Performance of Different Wavelet Bases in Damage Detection

In this section, the performance of different wavelet bases with different vanishing moments in identifying and locating damage is investigated. Figure 6 shows the absolute difference between the wavelet coefficient maps of acceleration signals recorded at the midspan of the damaged and undamaged bridges computed using four wavelet functions with different vanishing moment. For all wavelet bases, the damage clearly changes the energy content of the acceleration signal close to the fundamental frequency of the bridge. For the Gaussian wavelet base, this region is the only one that seems to be affected by the damage. Recalling that the lower frequency region provides more information regarding the location of the damage, it could be stated from Fig. 6, that Gaussian wavelet base can be mainly used detecting damage as it only provides useful information in the high frequency region and not the low frequency region.

A similar observation can be made for Daubecies, Symlet and Coiflet wavelet bases with higher vanishing moments. Figure 6a, 6b and 6d show that the absolute difference in the wavelet coefficient maps in the high scale (low frequency) region between the damaged and undamaged cases decreases significantly with an increase in the vanishing moment of the Daubecies, Symlet and Coiflet wavelets. Accordingly, db2, sym2, and coif1 wavelets provide the best information regarding damage when used in a CWT application to plot the absolute difference between the wavelet coefficient maps of the damaged and undamaged cases.

In Fig. 6 the results were presented for accelerations recorded at the damage location. To evaluate the efficacy of the wavelets in detecting and locating damage from accelerations that are far from the damage location, absolute difference in wavelet coefficient maps between damaged and undamaged cases obtained from accelerations recorded 12m left (node 13 in Fig. 2) were created and plotted in Fig. 7. In Fig. 7, only the wavelets with the lowest vanishing moments for each wavelet base was plotted for brevity.

The similarities between Fig. 6 and Fig. 7 suggest that any potential damage can be detected and located from accelerations recorded both close to the and far away from the damage location. The instant the moving load crossed the observation node13 (t = 1.36 s) and the damage location (between t = 2.239 s and 2.384 s) is also plotted in Fig. 7.

To study the variation of the absolute difference in coefficients between damaged and undamaged cases in the low frequency region, a section plot of Fig. 7 is created at a scale of 900 and plotted in Fig. 8. The absolute coefficient difference of CWT between damaged and undamaged acceleration signal at node 13 for a scale of 900. Here it should be noted that the scale value of 900 is selected randomly at the high scale range. An interesting observation can be made from Fig. 8. Although Daubecies, Symlet and Coiflet wavelets provides much higher coefficients, and arguably more information, in the lower frequency region when the entire coefficient map (Fig. 7) is investigated, the Gaussian wavelet provides a much clearer picture of the damage location. All four wavelets investigated produce the highest coefficients at the instant the load crosses the damage location. However, the value of the highest coefficient is very close to several other peaks observed for the Daubecies, Symlet and Coiflet making it difficult to differentiate the damage location. On the contrary, the peak obtained using the Gaussian wavelet at the damage location distinguishes clearly from the other peaks in the same graph leaving little doubt about the location of the damage. In other words, focusing only on a certain scale rather than the entire coefficient map leads to a different conclusion made earlier using Fig. 6: The Gaussian wavelet provides not only an opportunity to detect the damage but also identify the location of the damage.

To generalize this observation, the absolute difference in coefficients obtained from CWT analysis of the acceleration signals recorded 12 m right of the damage location (node 40 in Fig. 2) at the scale of 900 was plotted in Fig. 9. The load crosses this observation point at t = 3.25 s. The observations made for the signals recorded at node 13 depicted in Fig. 8 and summarized above is equally applicable to the signals recoded at node 40.

Finally, the absolute difference of wavelet coefficients at a very low scale (high frequency) region was investigated. For this, the absolute difference of wavelet coefficients between damaged and undamaged cases computed using Gaussian 2 (gaus2) wavelet for a scale of 1 was plotted (Fig. 10). The results indicate that, the only information that is provided by the wavelet coefficients in the high frequency region is the entrance and exit of the load on the bridge. Although, the coefficients are somewhat higher in the damage

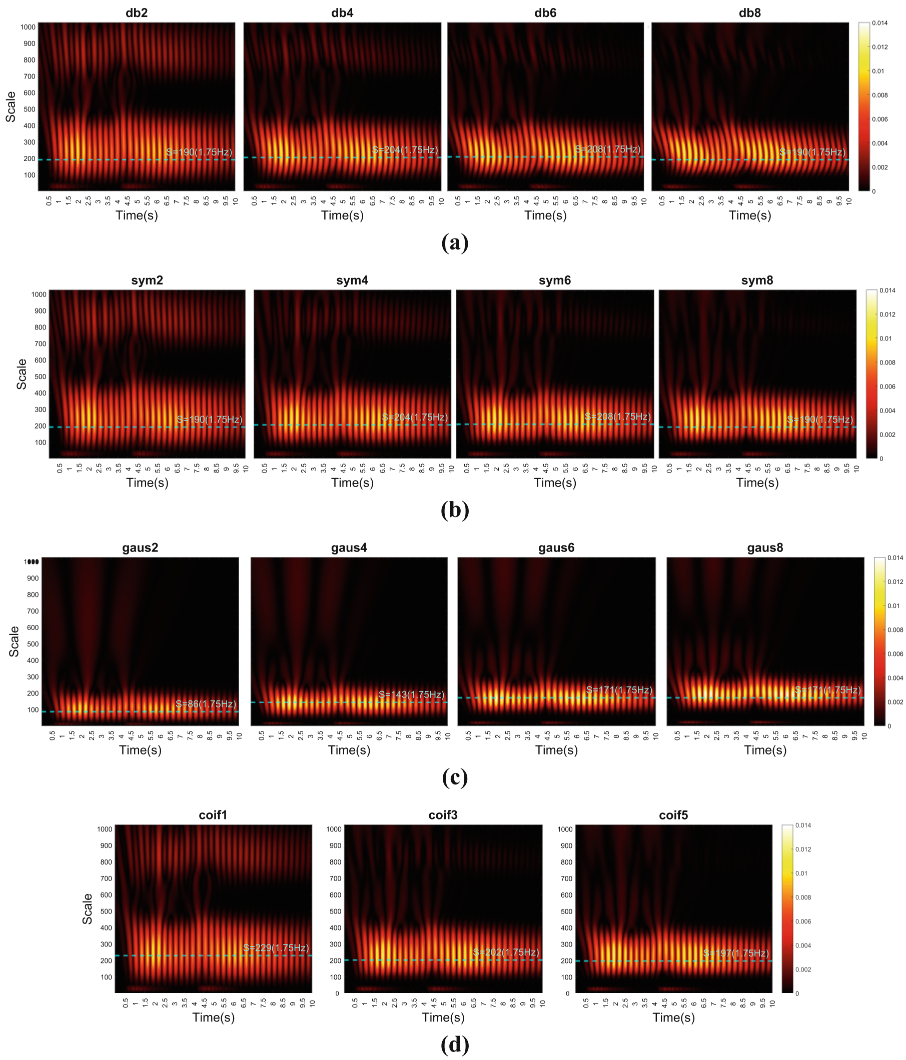

Fig. 6. The absolute coefficient difference of CWT between damaged and undamaged acceleration signal at node 26: a) Daubechies wavelet, b) Symlet wavelet, c) Gaussian wavelet d) Coiflet wavelet.

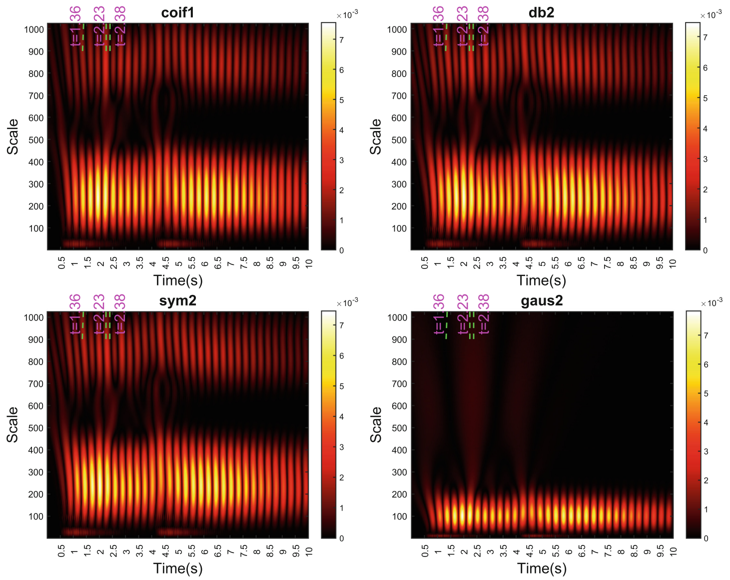

Fig. 7. The absolute coefficient difference of CWT between damaged and undamaged acceleration signal at node 13

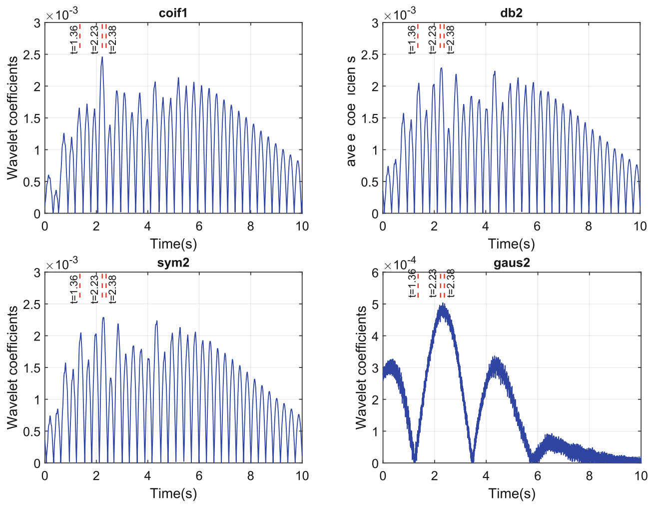

Fig. 8. The absolute coefficient difference of CWT between damaged and undamaged acceleration signal at node 13 for a scale of 900

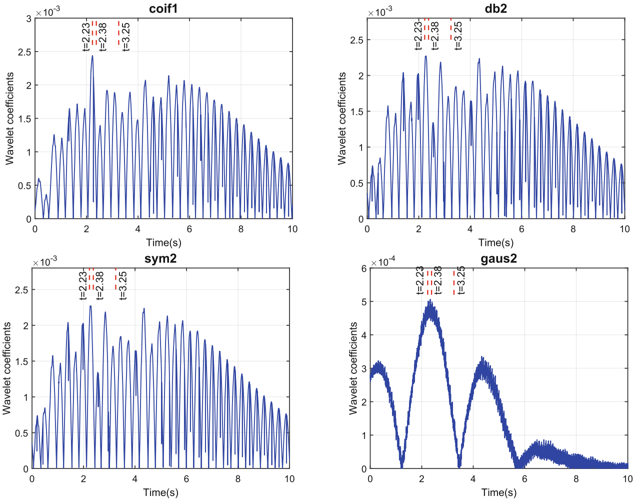

Fig. 9. The absolute coefficient difference of CWT between damaged and undamaged acceleration signal at node 40 for a scale of 900

region, the values are much lower compared to those observed at the exit and entrance of the load.

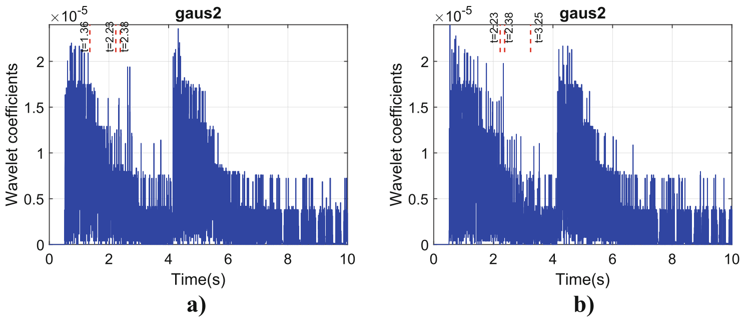

Fig. 10. The absolute coefficient difference of CWT between damaged and undamaged acceleration signal for a scale of 1 using Gaussian 2 wavelet: a) node 13 and b) node 40

5 Concluding Remarks

The article investigates the efficacy of different wavelets for damage detection and location from the accelerations recorded under a single moving load. Damage detection is based on the absolute difference in the wavelet coefficient maps in damaged and undamaged states. This method looks promising as the damage can be detected clearly, particularly at the fundamental frequency of the bridge. In addition, the damage location could also be located successfully from the accelerations recorded at different locations of the damage using the information at the low frequency range of the wavelet coefficient map.

Of the four wavelets bases used in the study, all of them could successfully detect the damage, particularly from the information at the fundamental frequency. When the entire wavelet coefficient map was used, the Gaussian wavelet provided the least information in the low frequency region as the coefficients for this wavelet in this region are much smaller compared to those at the fundamental frequency. However, when the coefficients at a given scale in the low frequency region was examined, the Gaussian wavelet gave the best results in terms of damage location. For all wavelet bases, the variants with lower vanishing points provided better information in the low frequency region. The study needs to be expanded to consider measurement noise and different damage locations to establish a framework for damage detection based on wavelet coefficient maps.

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