Altered Chord Alternatives

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Abstract. Motivated by the empirically pleasing sound of E♭7sus4 as a substitution for G7 in the II–V–I progression Dm7–G7–CM7, we advocate for the use of this chord and four other uncommon four-note chord substitutions in jazz, taking Schönberg's *schwebend* as justification and explaining the details of our MATLAB code for analyzing a four-note chord's "diatonic citizenship."

Keywords: Altered scale · Schönberg · *Schwebend* · MATLAB · Jazz

1 Introduction

Jazz musicians often speak of chord extensions and substitutions as lending different "colors" to their playing. This paper takes a small step towards articulating these "colors" in a mathematically precise way. We hypothesize that perceived "color" is a function of external diatonic context ("In what scales does this chord live?") and internal intervallic structure ("What intervals does this chord contain?"). Our computer-generated chord diagrams elucidate both of those features. For each chord, a 12-tone ring illustrates its intervals; and underneath the 12-tone ring, we list the scales where the chord "lives." We refer to this collection of scales as the chord's *diatonic citizenship*.

We propose extending the practice of altered chords in jazz to what we call *alternative altered chords*. Section 2 summarizes the use of altered chords in jazz and concludes with our definition of alternative altered chords. Section 3 provides theoretical justification for the use of these alternative altered chords from Schönberg's work [5]. In Sect. 4, we describe our MATLAB code for determining a four-note chord's diatonic citizenship. Section 5 lists the alternative altered chords and their diatonic citizenship, and Sect. 6 outlines future directions.

2 Motivation

Here is a II–V–I progression, ubiquitous in jazz, in C major:

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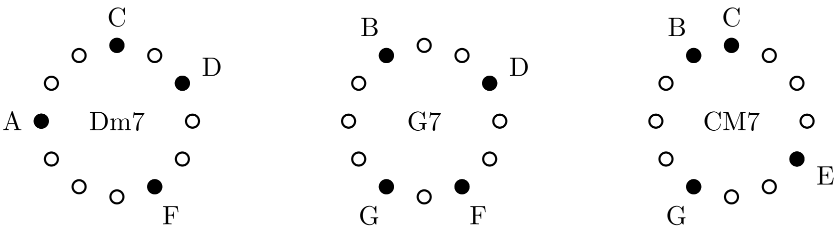
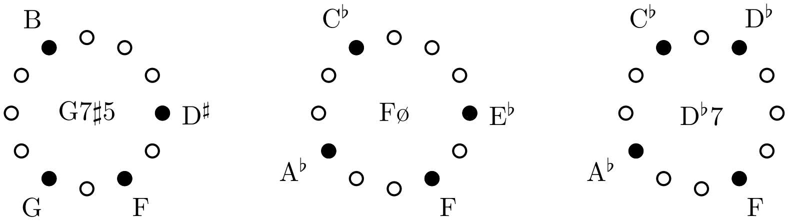


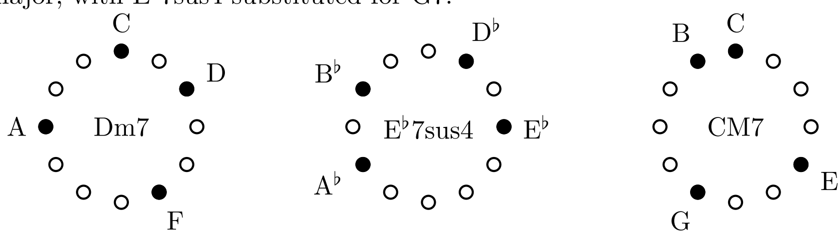
Comping and improvising over a II–V–I progression are among the first skills an aspiring jazz musician learns. After gaining familiarity with this progression, the musician graduates to playing the G altered scale over the G7 chord, rather than playing the G mixolydian scale. G mixolydian is the fifth mode of the C major scale (G, A, B, C, D, E, F), and G altered is the seventh mode of the A$^\flat$ melodic minor scale (G, A$^\flat$, A$^\sharp$, B, D$^\flat$, D$^\sharp$, F). In jazz, the *essential chord tones* of a seventh chord are the 1, 3, and 7. An *altered* G7 chord is created by altering any non-essential chord tone in G7, *i.e.* adding combinations of the $\flat 9$, $\sharp 9$, $\flat 5$, or $\sharp 5$ to combinations of the 1, 3, or 7 of G7. Below are common examples of altered G7 chords.



An altered G7 chord represents a compromise between

(i) creating chromatic voice-leading opportunities between Dm7 and CM7, which is accomplished by plugging the semitonal gaps in the C major scale;
(ii) preserving the dominant function of G7 by retaining its essential chord tones G, B, and F; and
(iii) maintaining at least one common tone between each pair of chords (*harmonisches Band*, [5]).

Let us turn to an example lacking (ii) and (iii). Consider the following II–V–I in C major, with E$^\flat$7sus4 substituted for G7.



$E^\flat$7sus4 consists of exactly the ♭9, ♯9, ♭5, and ♯5 of G7, with none of the essential G7 chord tones. Notice how $E^\flat$7sus4 plugs four out of the five semitonal gaps left by the C major scale, which form a $G^\flat$ major pentatonic scale ($G^\flat$, $A^\flat$, $B^\flat$, $D^\flat$, and $E^\flat$). There are five four-note chords available to us from this scale: $E^\flat$7sus4, and four others that lie outside the G altered scale—that is, outside the usual source for G altered chords.

Definition 1. *Let the tonic be C major. Then an* ***alternative altered chord*** *is one of the five four-note chords drawn from the notes $G^\flat$, $A^\flat$, $B^\flat$, $D^\flat$, and $E^\flat$.*

(Each alternative altered chord has no intersection with the C major scale. Note that only one of the five chords, $E^\flat$7sus4, could be considered a traditional altered chord, since it lives in the G altered scale.) This definition can be transposed. For example, if the tonic is G major, then to find the source of alternative altered chords to substitute for D7, begin a major pentatonic scale a tritone away from G: $D^\flat$, $E^\flat$, F, $A^\flat$, and $B^\flat$.

3 Theoretical Framework

The empirically pleasing sound of substituting $E^\flat$7sus4 for G7 in the context of a C major II–V–I progression suggests that our arsenal of altered chords could be extended to include those chords which *only* satisfy (i), and neither (ii) nor (iii), from the list of compromises in Sect. 2. As Schönberg writes in [5], pg. 133: "Quite certainly there are harmonic means, which at present have just not been theoretically determined, whose capacity for forming cadences or, far more, for admitting them, is just as great as that of IV, II, V and I."

If we were to use only triads, the V in a II–V–I would be necessary to solidify the key. We don't hear the B of C major without the V triad: II (D-F-A), V (G-B-D), I (C-E-G). But since jazz employs seventh chords, our key is already determined by the notes in the II (D-F-A-C) and the I (C-E-G-B). Thus we can dispense with key determination, connecting the two chords II and I via one chord that lies completely outside the key, using this external chord as "chromatic glue." Here is what we mean by the claim that $E^\flat$7sus4 ($E^\flat$-$A^\flat$-$B^\flat$-$D^\flat$) serves as "chromatic glue" between Dm7 and CM7: $E^\flat$7sus4 links D to C via $D^\flat$, and A to G via $A^\flat$; and as the movements C to B and F to E are already chromatic, the $B^\flat$ and $E^\flat$ serve to add momentary diversions.

On the subject of V substitutions, Schönberg brings up the III and the VII. He writes in [5], pg. 134: "First of all, looking for a substitute for V, we shall consider the suitability of III." Then he offers pros and cons of this substitution: It shares two common tones with I, a drawback; but it contains the leading tone, and it creates a nice root progression. "Yet, it is not commonly used; hence, we shall not use it much either, but shall remember why we do not: chiefly because it is not commonly used. That means, it could be used." He also considers substituting VII for V: "It does indeed determine the key, it does lead to the closing chord; but it, too, is not in common practice today, and so we shall disregard it." While his discussion only concerns diatonic substitutions, we include it as part of the theoretical justifcation for using alternative altered chords, because it demonstrates Schönberg's acute awareness of the idiom *rules are made to be broken*: There is nothing inherently wrong with using an uncommon substitution. Indeed, the narrative arc of [5] involves establishing rules and discarding them.

As final justification for the alternative altered chords, which will be enumerated in Sect. 5, we quote Schönberg at length: "A piece can also be intelligible to us when the relationship to the fundamental is not treated as basic; it can be intelligible even when the tonality is kept, so to speak, flexible, fluctuating (*schwebend*). Many examples give evidence that nothing is lost from the impression of completeness if the tonality is merely hinted at, yes, even if it is erased. And [...] the analogy with infinity could hardly be made more vivid than through a fluctuating, so to speak, unending harmony, through *a harmony that does not always carry with it a certificate of domicile and passport carefully indicating country of origin and destination*" ([5], pp. 128–129, emphasis added).

So, our five alternative altered chords, the five four-note chords lying entirely outside C major, contribute this *schwebend* quality in a way that standard G7 altered chords (drawn from the G altered scale, containing at least one essential chord tone of G7, hence overlapping with the C major scale) cannot. We propose the V in a II–V–I jazz progression be replaced by a chord that merely serves as "chromatic glue," departing from the tonic entirely, losing any trace of dominant chord function, and maintaining no common tones between chords.

4 Methodology

Restrictions. We impose the following limitations on this investigation.

(A) We assume our II–V–I progression is in the key of C major. So, we are looking for G7 substitutions beyond those residing in the G altered scale (or the $A^\flat$ melodic minor scale). The reader can transpose our C major results to II–V–I progressions in other major keys.
(B) For choice of scale, we restrict our attention to major, melodic minor, harmonic minor, and harmonic major, since these are the scales most commonly associated with the jazz tradition, and since each scale in this four-scale cycle can be changed to the next by altering just one semitone (the third or the sixth note in the scale).
(C) We will be concerned with four-note chords, also known as tetrachords, for the simple reason that these are more easily playable on guitar (the author's instrument) than extensions of higher cardinality.

MATLAB Code Overview. We modifed the MATLAB code [3], which was described briefly in [2] and in greater detail in [4] (in the context of a random walk on a graph that generates modulation excercises for guitar). First, we build a large matrix, `chordsModesKeysScalesRoots`, containing all four-note combinations in 12-TET major, melodic minor, harmonic minor, and harmonic major.

```
M=[2 2 1 2 2 2 1];    % major intervals
m=[2 1 2 2 2 2 1];    % melodic minor intervals
```

```
hm=[2 1 2 2 1 3 1];  % harmonic minor intervals
hM=[2 2 1 2 1 3 1];  % harmonic Major intervals
scaleMatrix=[M; m; hm; hM];
scaleMatrix2=cat(2,scaleMatrix,scaleMatrix);
n=0;
chordsModesKeysScalesRoots=[];
for K=0:11   % goes through keys
 for s=1:4   % goes through scales
  for i=1:7   % goes through modes
   for j=(i+1):i+6
    for k=(j+1):i+6
     for l=(k+1):i+6
      n=n+1;
      int1=sum(scaleMatrix2(s,i:j-1));  %------------
      int2=sum(scaleMatrix2(s,j:k-1));  % four intervals
      int3=sum(scaleMatrix2(s,k:l-1));  %
      int4=sum(scaleMatrix2(s,l:i+6));  %------------
      note1=mod(K+sum(scaleMatrix2(s,1:i))- ...
       scaleMatrix2(s,i),12);          %------------
      note2=mod(note1+int1,12);        % four notes
      note3=mod(note2+int2,12);        %
      note4=mod(note3+int3,12);        %------------
      % now write this data into the matrix:
      chordsModesKeysScalesRoots(end+1,:)=[int1,
       int2, int3, int4,  % four intervals
       i,     % mode
       K,     % key
       s,     % scale
       note1, note2, note3, note4]; % four notes
end   end   end   end   end   end
```

The matrix `chordsModesKeysScalesRoots` turns out to have dimensions 6720×11. Each row contains a chord built from the scale interval vectors `M`, `m`, `hm`, and `hM`. For example, row 222 of `chordsModesKeysScalesRoots` is

`[2 2 3 5 5 0 2 7 9 11 2]`

where the numbers `[2 2 3 5]` describe the intervals in the chord, the numbers `[5 0 2]` describe a mode (5), key (C), and scale (melodic minor) where this chord can be found, and the numbers `[7 9 11 2]` describe the notes: G, A, B, D. As another example, row 4437 of `chordsModesKeysScalesRoots` is

`[5 2 2 3 5 7 4 2 7 9 11]`

where the numbers `[5 2 2 3]` describe the intervals in the chord, the numbers `[5 7 4]` describe a mode (5), key (G), and scale (harmonic major) where this chord can be found, and the numbers `[2 7 9 11]` describe the notes: D, G, A, B. These examples show there are duplicate chord entries—as there should be, since the same note combinations can arise in multiple modes, keys, and scales. Next we build a smaller matrix grouping together the chords that are equal up to permutation. (The only permutations appearing are circular shifts, due to the nature of the construction of the matrix `chordsModesKeysScalesRoots`.)

```
sameChordNotes=[];
m=0;
for i=1:size(chordsModesKeysScalesRoots,1)
% ^runs down rows of chordsModesKeysScalesRoots
 if ismember(i,sameChordNotes)==0
 % ^checks if row number has already been assigned
  n=0; % resets sameChordNotes column to 0
  m=m+1; % increments sameChordNotes row
  for j=i:size(chordsModesKeysScalesRoots,1)
  % ^checks all chords after current index
   for k=0:3
    if isequal(chordsModesKeysScalesRoots(j,[8:11]), ...
     circshift(chordsModesKeysScalesRoots(i,[8:11]),k))==1
     % ^checks if the notes are the same up to shift
     n=n+1;
     sameChordNotes(m,n)=j;
end   end   end   end   end
```

The matrix `sameChordNotes` turns out to have dimensions 393×32, meaning that there are 393 possible four-note chords in all keys of 12-TET drawn from the major, melodic minor, harmonic minor, and harmonic major scales (and that the same chord appears in at most 32 rows in `chordsModesKeysScalesRoots`). Each row of `sameChordNotes` lists the rows of `chordsModesKeysScalesRoots` that contain the same chord. For example, row 30 of `sameChordNotes` is:

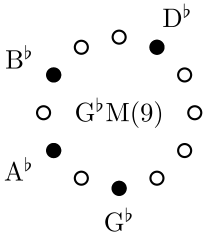
```
[37 82 107 136 177 222 247 276 1137 1182 1207 1236 1277 1322 ...
1347 1376 3922 3947 3976 4017 4342 4367 4396 4437 0 0 0 0 0 0 ]
```

This is the row of `sameChordNotes` dedicated to (permutations of) the chord G-A-B-D, which is why it lists the rows 222 and 4437 of `chordsModesKeysScalesRoots` that we displayed earlier.

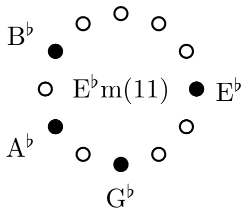
To create the lists of "diatonic citizenship" in Sect. 5, we wrote additional code that takes four notes as input, looks up their information in `sameChordNotes` and `chordsModesKeysScalesRoots`, and returns all the keys and scales where the four-note chord lives, formatted for LaTeX . We also wrote code that turns MATLAB output into vector graphics by way of PSTricks. Due to space limitations, we must omit this code. It is available upon request.

5 Results

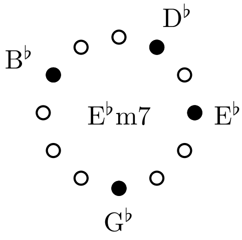
Let us look at all five tetrachords that completely plug the semitonal gaps in the C major scale. (There is more than one possible name for each chord. Names below were assigned for convenience.) Beneath each chord, we list its diatonic citizenship, adopting italicized abbreviations for scale types: *M* for the major scale, *m* for the melodic minor scale, *hm* for the harmonic minor scale, and *hM* for the harmonic major scale. For chord types, we use "M" for major triads and "m" for minor triads. Diatonic citizenship was determined according to the MATLAB code and scale restrictions in Sect. 4.



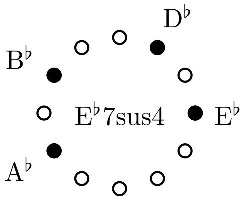
$\mathrm{C}^\sharp/\mathrm{D}^\flat{}_M$ $\mathrm{C}^\sharp/\mathrm{D}^\flat{}_m$
$\mathrm{F}^\sharp/\mathrm{G}^\flat{}_M$ $\mathrm{F}^\sharp/\mathrm{G}^\flat{}_{hM}$
B_M B_m



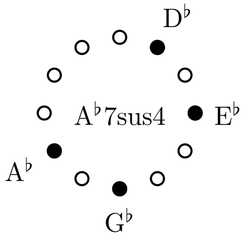
$\mathrm{C}^\sharp/\mathrm{D}^\flat{}_M$ $\mathrm{C}^\sharp/\mathrm{D}^\flat{}_m$
$\mathrm{D}^\sharp/\mathrm{E}^\flat{}_m$ $\mathrm{D}^\sharp/\mathrm{E}^\flat{}_{hm}$
$\mathrm{F}^\sharp/\mathrm{G}^\flat{}_M$
B_M



$\mathrm{C}^\sharp/\mathrm{D}^\flat{}_M$ $\mathrm{C}^\sharp/\mathrm{D}^\flat{}_m$
$\mathrm{F}^\sharp/\mathrm{G}^\flat{}_M$
$\mathrm{A}^\sharp/\mathrm{B}^\flat{}_{hm}$
B_M B_{hM}



$\mathrm{C}^\sharp/\mathrm{D}^\flat{}_M$ $\mathrm{C}^\sharp/\mathrm{D}^\flat{}_m$
$\mathrm{F}^\sharp/\mathrm{G}^\flat{}_M$
$\mathrm{G}^\sharp/\mathrm{A}^\flat{}_M$ $\mathrm{G}^\sharp/\mathrm{A}^\flat{}_m$ $\mathrm{G}^\sharp/\mathrm{A}^\flat{}_{hm}$ $\mathrm{G}^\sharp/\mathrm{A}^\flat{}_{hM}$
B_M



$\mathrm{C}^\sharp/\mathrm{D}^\flat{}_M$ $\mathrm{C}^\sharp/\mathrm{D}^\flat{}_m$ $\mathrm{C}^\sharp/\mathrm{D}^\flat{}_{hm}$ $\mathrm{C}^\sharp/\mathrm{D}^\flat{}_{hM}$
E_M
$\mathrm{F}^\sharp/\mathrm{G}^\flat{}_M$ $\mathrm{F}^\sharp/\mathrm{G}^\flat{}_m$
B_M

Recall that we are interested in widening the palette of available "colors" in altered chords, where the vague notion of "color" could refer to a chord's internal intervallic structure or to its external diatonic context. First, regarding internal intervallic structure: Note that all of these alternative altered chords lack the tritone interval present in a V7 chord; and the seconds and fourths in these chords generally outnumber the major and minor thirds.

Second, regarding external diatonic context: Observe that the use of any of these alternative altered chords could be framed as temporary modulation from the key of C major into C♯/D♭ major, C♯/D♭ melodic minor, B major, or F♯/G♭ major—keys that are adjacent to C major (C♯/D♭, B), or directly across the circle of fifths (F♯/G♭). Therefore we may say that substituting alternative altered chords for G7 in the II–V–I progression Dm7–G7–CM7, as opposed to substituting the usual altered chords drawn from the G altered scale, maximizes Schönberg's *schwebend*, in the sense that C♯/D♭ major, C♯/D♭ melodic minor, B major, and F♯/G♭ major share fewer notes with C major than the G altered scale (which is G♯/A♭ melodic minor) shares with C major.

In conclusion, refering to the itemized list of compromises in Sect. 2, we advocate for V7 substitutions in jazz that let (i), chromatic voice leading, take precedence over (ii), preservation of the V7's dominant function, as well as over (iii), maintaining common tones between two chords. As Schönberg writes in [5], "harmonic usage is often created by coincidences of voice leading" (pg. 115).

6 Future Directions

Composition. By losing the essential chord tones of G7, the substitutions we propose effectively erase the V in the II–V–I cadence in C major. How would jazz sound if every V were replaced with these alternative altered chords? Do we need to preserve the essential chord tones of G7? (Recall that the essential chord tones are the 1, 3, and 7—so, the G, B, and F.) In other words, do we really need the V in a II–V–I, or do we just need a chord that adds tension by temporarily transporting us from the tonic while also serving as the chromatic glue between the II and the I? We will rewrite a few jazz standards using our alternative altered chords instead of dominant seventh chords, and we will disseminate the recordings online. Our code could be easily modified to investigate additional scales (*e.g.* octatonic), or to map out four-note chords in 31-TET and other non-standard tuning systems. We look forward to collaborating with microtonal guitarists on this front.

Analysis. This work is part of our long-term goal of applying the rich ideas in [1] and [6] to the jazz tradition. Specifically, we hope to explore the voice-leading dance of nearly even tetrachords—instead of nearly even triads—in and around (perfectly even) diminished seventh chords—instead of augmented triads—by analyzing examples from well-known jazz recordings, quantifying notions of "color" in chromatic harmony along the way.

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