A Set-Theoretic Model of Meter and Metric Dissonance

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Abstract. A set-theoretic of model of musical meter is formalized, building up from time points to pulses to meters to metric relations. The model of metric relations formalizes work on metric dissonance, and refines the displacement/grouping taxonomy under current usage.

Keywords: Musical meter · Set theory · Metric dissonance · Syncopation · Hemiola · Polymeter

1 Introduction

During the 1970's, mathematical set theory emerged in North America as the predominant method for exploring and communicating the structure of the chromatic 12-note universe, emphasizing the properties of its chordal and scalar subsets, and the relations in which they participate [1, 2]. The paradigm was productively transferred onto the diatonic 7-note universe [3], and eventually onto patterns of musical time, with a focus on cyclic rhythms in universes of eight or more beats [4–7]. These cycles are implicitly metric, in the sense that they project at least a unit pulse, and a slower pulse that marks points of cyclic orientation or renewal. But the metric models are minimal, in the sense that they track only those two pulses. In the European tradition, meter is deep, projecting at least three pulses of different speeds, and sometimes as many as seven [8]. The predilection in Asia and Africa, as well as in globally circulating popular and electronic dance music, for cycle-lengths that are powers of 2 or multiples of 6, together with the propensity to bodily entrain (dance) at a rate that is faster than unit and slower than cycle, suggests that deep meter is a broader phenomenon [9–11].

Maury Yeston laid the groundwork for a set-theoretic model of deep meter by defining meter as an inclusion relation between pulses; the transitivity of inclusion invites a recursive application [12]. Lerdahl and Jackendoff further prepared the terrain by defining pulses as sets of time points, and by representing meters as dot arrays with a depth dimension [13]. Using these formulations as a foundation, the first half of this paper proposes a set-theoretic model of deep meter, integrating the perspective of recent studies in the psychology of metric induction.

The second half of the paper builds a model of relations between distinct deep meters. Metric relations, as defined here, are equivalent to what music theorists commonly refer

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to as metric *dissonance*: the superposition or juxtaposition of distinct, partly incommensurable meters, which have the capacity to scramble patterns of neural activation [14, 15]. Since the 14th century, metric relations have been associated in Europe with psychological or semantic states such as difficulty, disorientation, conflict, mental instability, and yearning for the unattainable [16–19]. Metric relations are the basis of metric modulations, which are essentially local processes of metric change, and metric form, patterns of metric change across relatively long stretches of musical time. They thus are of central concern to musical analysts who recognize rhythm and meter as dynamic elements of musical structure and experience.

2 A Set Theoretic Model of Meter

The model of metric relations is built up in three stages, each of which converts sets of the previous stage into elements of the subsequent one. Time points are elements of pulses, which are elements of meters, which are elements of metric relations.

2.1 Time point and span

The axiomatic elements are *time points*, which have no properties other than their temporal addresses [20, 21]. Pairs of distinct elements $x < y$, where $<$ represents temporal precedence, give rise to *time spans*, evaluated as $(y - x) > 0$.

Time points are distinct from the musical events whose onsets mark them [22]. Events and their onsets are *res extensae* that exist in musical sound. Time points are *res cogitantia* that exist in the mind. The correspondence of time points to musical events is not 1:1. There are musical events, such as grace notes, that do not mark time points [13]. Conversely, there are *virtual* time points that are unmarked by musical events [23]. The onsets themselves are smeared across spans, or bins [24], and reduced to points through the mental operation of *quantization* [25].

Time-span sizes are mentally assigned rather than prosthetically measured, and give rise to comparative rather than absolute values. Augustine of Hippo wrote in the 4th century that "I confidently answer—insofar as a trained ear can be trusted—that this syllable is single and this double.... It is in you, oh mind of mine, that I measure the periods of time" [26].[1] If two adjacent time spans are brief, our mind spontaneously determines whether they are equal. If unequal but integrally proportioned, we subitize the number of concatenated shorter spans that fit the longer span, so long as that number is small.

2.2 Pulse

Definition 1. A pulse P is an ordered set of time points whose adjacent elements are separated by a constant value, $\tau(P)$, the pulse's period.

[1] Book 11, Chapter 28, paragraph 34.

The constant-value constraint is often referred to as the *isochrony* property. The property applies to time points, not to the musical events that mark them, which may be literally isochronous (if machine-generated), or notionally isochronous (if human-generated, like a drumbeat), or neither, if the series includes virtual time points (as do most songs and instrumental compositions). It is important to keep in mind this last possibility, since it is easy to default to a prototypical conception of a pulse as a parade of isochronous onsets emitted from a uniform auditory source. Often pulses have gaps, as when a series of alternating half and quarter notes induces a quarter-note pulse, or their isochronous onsets are split into multiple streams, as when a quarter-note pulse is induced by anti-phased half-note pulses in the prototypical rock drummer's alternation of bass and snare.

Pulse periods, equivalent to slow frequencies, can be represented by absolute durations, with Herz or milliseconds, or by relative duration, with standard symbols, or counts of beats or measures.

2.3 Meter

A meter is a set of pulses related by inclusion. It is useful to divide the study of meter into minimal meters, with exactly two pulses, and deep meters, with three or more pulses. Minimal meters are to deep meters as intervals are to chords.

2.3.1 Minimal Meter

Definition 2. A *minimal meter* M is a pair of pulses (P_1, P_2) such that $P_1 \subset P_2$. The definition implies $\tau(P_1) > \tau(P_2)$, and thus that P_1 is *slower* and P_2 *faster*.

The ordering of pulses from slowest to fastest will be preserved as the model develops.

A minimal meter is classified by a function $\beta(\mathrm{M}) = \frac{\tau(P_1)}{\tau(P_2)}$, which evaluates the ratio of its constituent pulses. By Definition 1, both pulses are periodic, and by Definition 2, they are related by inclusion. Thus the time points of P_1 are periodic selections of the time points of P_2, and the range of $\beta(\mathrm{M})$ is the positive numbers >1.

Two classes of minimal meters merit special attention:

Definition 3. A minimal meter M is *duple* if $\beta(\mathrm{M}) = 2$, and *triple* if $\beta(\mathrm{M}) = 3$.

Definition 4. A minimal meter is *normal* if it is either duple or triple.

"Normal" designates a class of meters that have historical significance in Europe and the Americas, and perhaps elsewhere. I intend it as a technical term that implies no judgement of value.

2.3.2 Deep Meter

Definition 5. A *deep meter* M is a set of three or more distinct pulses (P_1, P_2, …, P_k), ordered from slowest to fastest, such that every pair of pulses forms a minimal meter.

A deep meter M is classified by a function, $\beta(\mathrm{M}) = \frac{\tau(P_1)}{\tau(P_2)}, \frac{\tau(P_2)}{\tau(P_3)}, \ldots, \frac{\tau(P_{k-1})}{\tau(P_k)}$ whose range consists of multisets of numbers, each of which classifies an adjacent minimal meter, ordered from slowest to fastest. Consider for example M' = (4.5 s, 1.5 s, .75 s, .375 s, .125 s), which also might be notated as (dotted breve, whole, half, quarter, tripleted eighth) or as (36, 12, 6, 3, 1). This meter has five distinct pulses, hence four adjacent minimal meters, whose classification is β(M') = (3 2 2 3).

18th-century European compositional theory stipulates that a meter is defined not as a list of pulses, but rather as a selection of two orienting pulses: a tactus, or counting pulse, and a downbeat pulse [27]. In this view, M' (as defined above) is not yet a meter. It is, rather, a genus that gives rise to a multitude of specific meters. For example, depending on which durational values are selected and which pulses are prioritized, M' could be represented using, among others, the following 18th-century meter signatures: $\frac{3}{1}, \frac{6}{2}, \frac{2}{4}, \frac{2}{2}, \frac{4}{4}, \frac{3}{8}, \frac{6}{8}$, and $\frac{12}{8}$, representing eight distinct meters. That historical tradition has left a strong residue in modern textbooks, which classify "meters" according to their signatures, and more subtly in theoretical and perceptual research, which often nominates a single pulse as the tactus [13, 19, 28], implying that a change of tactus, all else invariant, is a change of meter. The view taken here is that reference pulses are external to a model of meter. Meter is a system of relations, which need not be directed or oriented. A change of reference pulse is not a change of meter, and two listeners inducing the same pulses are hearing the same meter, even if their awareness or bodily response is oriented to different speeds.

2.3.3 Properties of Deep Meters

Among the properties of deep meters M = (P_1, P_2, ..., P_k) are the following:

Definition 6. The slowest pulse in M, P_1, is its *span pulse.*

Definition 7. The fastest pulse in M, P_k, is its *unit pulse.*

Definition 8. The *cardinality* of M is *k*.

Definition 9. The *size* of M is the ratio of its unit and span pulses, $\frac{\tau(P_1)}{\tau(P_k)}$, which is equal to the product of the elements in its ordered set β(M), $\prod_{n=1}^{k-1} \frac{\tau(P_n)}{\tau(P_{n+1})}$.

Definition 10. A meter M is *saturated* if all elements of β(M) are prime numbers.

If a meter M is saturated, then it has no interior gaps in M which could be filled by additional pulses. Saturated meters are particularly significant because of our propensity to fill gaps in the pulse spectrum through spontaneous processes of subjective metricization [28].

Definition 11. A deep meter is *normal* if each of its adjacent minimal meters is normal; equivalently, it is classified a multiset of 2's and 3's exclusively.

2.4 Metric Relations

Definition 12. Two saturated meters are *related* if (1) their unit pulses are identical, $x_k = y_k$, and (2) their span pulses have equal periods, $\tau(x_1) = \tau(y_1)$.

Since both conditions are based on equality, the defined relation inherits the properties of symmetry, reflexivity, and transitivity proper to equality relations.

The first condition insures that X and Y are drawing from a common universe of time points. Recall that not all time points of the unit pulse need be onset-marked. A unit pulse might be a proper superset of two onset-marked pulses in different streams, or presented at different times.

Since $x_1 = y_1$ implies $\tau(x_1) = \tau(y_1)$, the two conditions together entail that X and Y are of equal size. The saturation condition insures that X and Y also have equal cardinality k, permitting disjoint pairing by shared subscript, and thus by identical or similar speed.

Definition 13. Given two saturated meters $P = (P_1, P_2, \ldots, P_k)$ and $Q = (Q_1, Q_2, \ldots, Q_k)$ that are related as in Definition 11, pulses p_n and q_n are *associated*, for $1 \leq n \leq k$.

This term will be central to the work carried out in the next section.

3 Kinds of Metric Relations

In recent North-American scholarship, when two distinct meters of equal cardinality are combined simultaneously or successively, they are said to create **dissonances**, which are partitioned into two principal classes, **displacement** and **grouping** [19, 29]. Displacement covers situations often referred to as syncopation, anti-phasing, turning the beat around, and shadow meter. Grouping covers hemiolas and polymeters. I will substitute other terms for "grouping" and "displacement," because both are rooted in properties and conceptions specific to early-modern Europe,[2] and thus introduce impertinent implications for other repertories. Nevertheless, the following classification of metric relations draws the boundary at the same location. Grouping dissonances are further distinguished as simple and complex [31]. Recent work [32–34] identifies and models a third class that combines aspects of displacement and grouping dissonance; the taxonomy developed here defines a hybrid genus that is consistent with that work.

I propose classifying metric relations by a procedure whose components are summarized here, and detailed in the remainder of this paper. First, associated pairs of individual pulses are classified as identical, co-periodic or anti-periodic. Second, metric relations

[2] When pulses of the same period but different phases are combined, a displacement model involves determining (a) which of the two pulses is the source, and (b) whether the copy is displaced in the positive or negative direction. The determination is sometimes arbitrary for music that lacks pitch, or whose pitch-combinations don't adhere to historical European principles of dissonance regulation. Grouping is avoided because it is not a property of meters, whose pulses consist of time points that cannot be grouped into "larger time points." The conception of slow pulses "grouping" faster ones is based on a malformation that has likely origins in ancient Greek theories of poetic meter. I elaborate this point in [30].

are classified as ordered sets of associated pulse-pair classes. Finally, the classes of metric relations are mapped onto three genera, which are equivalent to the three classes identified in recent literature on metric dissonance: displacement, grouping and hybrid.

3.1 Pulse-Association Classes

Associated pulse pairs (x_n, y_n) are mapped to one of three classes by a function $\varphi(x_n, y_n)$.

Definition 14. If $x_n = y_n$, then the pulses are *identical*, and $\varphi(x_n, y_n) = \text{IDENT}$.

Definition 15. If x_n and y_n are not identical, but their periods are equal, then they are *properly co-periodic*, and $\varphi(x_n, y_n) = \text{CO-P}$.

"Properly co-periodic" excludes the trivial case where the pulses are identical; as with proper inclusion, this licenses one to cut locutionary corners by dropping the adverb. Pulses that are properly co-periodic share no time points.

Definition 16. If x_n and y_n have unequal periods, then they are *anti-periodic*, and $\varphi(x_n, y_n) = \text{ANTI-P}$.

3.1.1 Constraints on Associate-Pulse Classes

Definitions 13–15 suggest a reformulation of Definition 11: two saturated meters (X, Y) are related if $\varphi(x_1, y_1) \neq \text{ANTI-P}$ and $\varphi(x_k, y_k) = \text{IDENT}$. The first constraint is motivated by the observation that when $\varphi(x_n, y_n) = \text{ANTI-P}$, then their intersection set $x_n \bigcap y_n$ is a slower pulse which is an element of X, Y, or both.

An additional constraint governs associated pulses of intermediate speed: if some associated pulse-pair is co-periodic, then all slower associated pairs, up to and including the span pulse, are co-periodic as well. Consider some associate pair $\{x_n, y_n\} \mid 1 < n < k$, and a slower associate pair, $\{x_m, y_m\} \mid m < n$. By definition of meters X and Y, $x_m \subset x_n$ and $y_m \subset y_n$. Assume now that $\varphi(x_n, y_n) = \text{CO-P}$. Then $x_n \bigcap y_n = \emptyset$. Accordingly, $x_m \bigcap y_m = \emptyset$, and thus $\varphi(x_m, y_m) = \text{CO-P}$.

3.2 Metric-Relation Classes

The classification system for associated pulse pairs (x_n, y_n) serves as the basis for classifying the relation between the meters (X, Y) of which they are elements.

Definition 17. Let (X,Y) be meters related as in Definition 11. Then the relation X R Y is classified by a function $\varphi(X, Y) = (\varphi(x_1, y_1), \varphi(x_2, y_2), \ldots, \varphi(x_k, y_k))$, whose image is a multiset, or *string*, of pulse-association classes.

If the classification of associated pulses were unconstrained, we would quickly suffer a combinatorial explosion of metric-relation classes. Fortunately, the constraints already adopted, plus one additional one proposed below, filter out most combinations. To review:

1) All strings end with IDENT;
2) No string begins with ANTI-P;
3) CO-P is preceded only by CO-P.

A fourth constraint is adopted to eliminate redundancy caused by adjacent identity relations. Since consecutive identity-pairs (IDENT, IDENT) = $(\text{IDENT})^2$ are structurally no different than a single identity-pair IDENT, they do not profit from independent investigation. This motivates the mapping in (4), which reduces the cardinality to one that was already inventoried at a previous level of k.

(4) $(\text{IDENT})^n \rightarrow \text{IDENT}$, for $n > 1$.

The strings that survive these filters, up to a metric depth of five pulses, are listed in Table 1, where they are labelled from (a) to (m). Superscripts count consecutive repetitions of a term. The comments in the final column serve as the basis for assigning the strings to genera of metric relations in the next sub-section.

Table 1. Metric-Relation Classes $\varphi(X, Y)$ for meters up to a depth of five pulses

k =	Label	$\varphi(X, Y) =$	Comments
1	(a)	IDENT	Universal root
2	(b)	(CO-P, IDENT)	Root of co-periodic genus
3	(c)	$((\text{CO-P})^2$, IDENT)	Left-extension of (b)
	(d)	(IDENT, ANTI-P, IDENT)	Root of anti-periodic genus
	(e)	(CO-P, ANTI-P, IDENT)	Root of hybrid genus
4	(f)	$((\text{CO-P})^3$, IDENT)	Left-extension of (b)
	(g)	(IDENT, $(\text{ANTI-P})^2$, IDENT)	Internal expansion of (d)
	(h)	(CO-P, $(\text{ANTI-P})^2$, IDENT)	Internal expansion of (e)
	(i)	$((\text{CO-P})^2$, ANTI-P, IDENT	Left extension of (e)
5	(j)	$((\text{CO-P})^4$, IDENT)	Left-extension of (b)
	(k)	(IDENT, $(\text{ANTI-P})^3$, IDENT)	Internal expansion of (e)
	(l)	$((\text{CO-P})^2$, $(\text{ANTI-P})^2$, IDENT)	Left extension of (h); internal expansion of (i)
	(m)	(IDENT, ANTI-P, IDENT, ANTI-P, IDENT)	Elided concatenation of (d)

3.3 Three Genera of Metric Relations

Through a procedure to be described in this section, the thirteen classes listed in Table 1 reduce to three genera, one of which comes in two species.

Definition 18. The *co-periodic genus* consists of strings of the form $((\text{CO-P})^a, (\text{IDENT})^b)$, for $a, b \geq 1$.

The co-periodic genus corresponds to what is characteristically referred to as extended syncopation, anti-phasing, shadow meter, turning the beat around, or displacement dissonance.

Definition 19. The *anti-periodic genus* consists of strings of the form $((\text{IDENT})^a, (\text{ANTI-P})^b, (\text{IDENT})^c)$, for $a, b, c \geq 1$.

The anti-periodic genus corresponds to what is referred to as hemiola, polymeter, and grouping dissonance. If $b = 1$, as is the usual case, the relation is simple. This is the hemiola that is familiar from Baroque cadences in triple meter, from 3-over-2 polymeters, and from the alternation of ternary and binary subdivisions of half-measures, which arise characteristically, for example, in Korean *pung'mul* and Spanish *flamenco* [8, 35]. If $b > 1$, the hemiola is complex [31, 36]. This formation involves other co-prime polymeters, including 4-over-3, 9-over-2, 9-over-8, and so forth.

Definition 20. The *hybrid genus* consists of strings of the form $((\text{CO-P})^a, (\text{ANTI-P})^b, (\text{IDENT})^c)$, for $a, b, c \geq 1$.

These are equivalent to the hybrid forms identified in [32–34]. Most of the examples analyzed in those writings are composed by Johannes Brahms, suggesting that an alternative label might be the *Brahms Genus*.

Leaving aside the universal root (a), the only entry in the table that does not fit this taxonomy is the final one, (m), where a pulse of intermediate speed functions simultaneously as the span pulse of a fast simple hemiola and as the unit pulse of a slow simple hemiola. As this is an elided replication of an existing genus, I am reluctant to establish a new genus to contain it. Otherwise, I conjecture that the three-fold taxonomy introduced here exhaustively covers metric relations of yet greater depth ($k > 5$), which are (in any case) of diminishing frequency since they approach the limit of the number of pulses that can be simultaneously tracked or entrained [28].

4 Extensions

This paper has adopted several limitations which could be loosened in future work. First, saturated meters need not be normal; higher primes could be substituted for 2's and 3's without affecting other aspects of the model [12]. The simplest case would be anti-metric 5-over-2, as in Holst's Mars movement from "The Planets," and the finale of Ravel's string quartet. Second, relations among three or more meters, which have been noted in the analytical literature, could be explored. Figure 1 sketches some of the simplest possibilities:

(a) Three meters, each pair of which is simply anti-periodic [12].
(b) Three meters, each pair of which is co-periodic. For an example from Schumann, see [18].

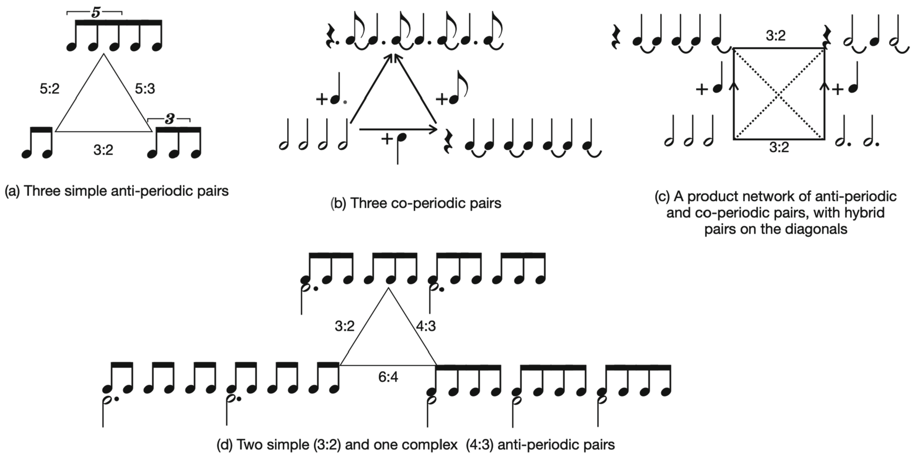

Fig. 1. Four portraits of multiple metric relations.

(c) Four meters in a product network [4], with anti-periodic pairs on one axis, co-periodic pairs on the other, and hybrid pairs on both diagonals.
(d) Three meters, two pairs of which are simply anti-periodic in a 3:2 ratio, the third pair of which is complexly anti-periodic in 4:3 ratio. For examples from Dvorak and Brahms, see [31].

There may be some motivation to regard (c) and (d) as underlying two of the identified genera. For any pairing of meters at opposite vertices of (c), one or both of the meters at the complementary vertices may be implicitly present, even if not explicitly articulated. Thus, any binary hybrid metric relation may be viewed as the product of a co-periodic relation and an anti-periodic one, a quaternary design whose intermediate pulses may be concealed or under-articulated. Similarly, any binary complex anti-periodic relation between two pulses may be viewed as the elision of *n* simple relations, a ternary design with the *n – 1* intermediate terms (*Vermittlungen*) elided out [36, 37]. Underlying these structural proposals is a *Gestalt* hypothesis about cognition: that explicit gaps in a well-defined structure are imagined to be notionally present. This same hypothesis is invoked at earlier levels of the model, where it was posited that gapped sets of onset-marked time points are completed by virtual time points, and that gapped sets of pulses on the speed continuum are filled (saturated) by processes of subjective metricization.

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