# Evolution of the R-value and Its Determination Based on Reverse Fitting for Sheet Metal

**Jun Zhao, Zhenkai Mu, Qingdang Meng, and Haoran Wang**

**Abstract** The plastic anisotropy R-value is an important material parameter for constitutive models, and the fact that the R-value varies with deformation has been recognized. However, there is not a unified method so far to determine the variable R-value. Therefore, based on the initial definition of the R-value, this paper proposes a reverse fitting model to determine the evolution of the R-value for sheet metal. Meanwhile, the limitations of the existing calculation methods for the R-value have been compared and analyzed in detail. The results show that although the longitudinal plastic strain–transverse plastic strain curve is close to a straight line, the R-value changes greatly. With the increase in the uniaxial tensile plastic strain, the evolution of the R-value is generally decreased and the curves' form can be divided into two types: up convex and down concave. In addition, the reason why there are significant differences in the R-value calculated by different methods is clarified. The proposed reverse fitting model can greatly simplify the secondary development of constitutive model considering the evolution of anisotropy and enhance the accuracy of numerical simulation.

**Keywords** Anisotropic coefficient · Tensile test · Reverse fitting · Sheet metal · R-value

## Introduction

The mechanical properties of sheet metals generally show obvious anisotropy when they undergo severe plastic deformation during manufacturing processes such as cold rolling. The macroscopic characteristics of anisotropy mainly reflect the variation of the yield stress and R-value with direction. Various phenomenological yield models have been proposed to describe the initial anisotropy for sheet metals [1–6]. In fact,

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during the plastic deformation process of the anisotropic sheet metal, the hardening as well as the deformation behavior is actually evolving. The hardening-induced anisotropy strongly affects the plastic deformation behavior, which has been observed experimentally [7], especially in the use of high-strength steel and aluminum alloy.

The R-value is an important material parameter for constitutive model, and it is defined as the ratio of the transverse plastic strain-increments to thickness plastic strain-increments in uniaxial tensile tests [8]. The evolution of texture will lead to the variation of the R-value in plastic deformation [9]. Consequently, the yield surface and flow potential need to be updated with the development of plastic strain [10]. However, the R-value is usually calculated based on measured strains in a certain strain range and is assumed to be a constant. Most anisotropic plasticity models also still use the initial yield stress and invariable R-value to determine the parameters of the yield function, only a few models consider the evaluation of the R-value [11–17]. In order to characterize the change law of the R-value during the plastic deformation, it is usually described as a function of the longitudinal plastic strain. Choi et al. [18] proposed an experimental method for the anisotropic evolution based on DIC and analyzed the evolution of the R-value in the deformation process. Rossi et al. [19] studied the planar anisotropy of sheet metals at large strains based on DIC. Safaei et al. [20] presented a phenomenological approach to describe the evolution of anisotropy during the plastic deformation based on the non-associated flow rule, and the curves of the R-value varying with longitudinal strain were obtained by fitting the longitudinal plastic strain–transverse plastic strain curves with a linear function and a third-order polynomial function, respectively. Yoshida et al. [21] made an interpolation between two yield surfaces at two different levels of the equivalent plastic strain to describe the evolution of the yield surface and gave the changes of the R-value for 3003-O aluminum sheet based on interpolation functions. Lian et al. [22] described the variation of the R-value with longitudinal strain for AISI 439 material. Based on the principle of work equivalence, the relationship between the R-value and equivalent strain was deduced, and the anisotropic behavior was predicted with the non-correlated flow criterion. However, it is not reasonable to regard the rolling direction strain as the equivalent strain.

Although some methods have been proposed to calculate the variable R-value, however, the application conditions and problems of these methods are not been analyzed in detail. In addition, there is no unified method so far to determine the relationship between the R-value and longitudinal plastic strain during deformation. Therefore, based on the initial definition of the R-value, this paper presents a reverse fitting method to characterize the evolution of the R-value with deformation. The proposed reverse fitting model can greatly simplify the secondary development of the phenomenological yield model considering the evolution of anisotropy and enhance the accuracy of FEM.

## Experiments

The experimental materials are DC01, DC04, and DC06. Specimens for the standard uniaxial tensile tests are cut along the rolling direction. The size of a specimen is shown in Fig. 1. The standard uniaxial tensile tests are carried out with an Inspekt 100 kN electronic universal material testing machine imported from Germany. The gauge elongation and width reduction of the specimen are measured by the axial extensometer and transverse extensometer, respectively. Then, the transverse–longitudinal plastic strain curves during the uniform deformation stage can be obtained, as shown in Fig. 2. The load–gauge elongation curves are shown in Fig. 3.

**Fig. 1** Size of the specimen

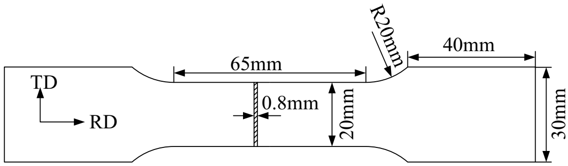



**Fig. 2** Longitudinal plastic strain–transverse plastic strain curves in the RD

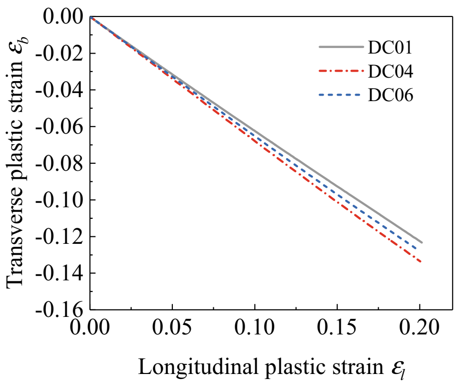
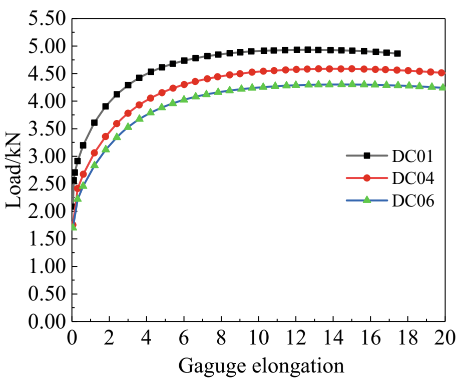



**Fig. 3** Load–gauge elongation curves



The plastic anisotropy R-value is defined as

$$R = d\varepsilon_b / d\varepsilon_t \tag{1}$$

where $d\varepsilon_b$ and $d\varepsilon_t$ are the transverse plastic strain-increment and thickness plastic strain-increment, respectively. It is difficult to measure the thickness of plastic strain. Therefore, Eq. (1) is usually translated to the following formula according to the incompressibility constraint.

$$R = -\frac{d\varepsilon_b}{d\varepsilon_l + d\varepsilon_b} \tag{2}$$

Equation (2) can be further transformed as

$$R = -\frac{d\varepsilon_b \big/ d\varepsilon_l}{1 + d\varepsilon_b \big/ d\varepsilon_l} = -\frac{k(\varepsilon_l)}{1 + k(\varepsilon_l)} \tag{3}$$

where $k(\varepsilon_l)$ is the slope of the transverse plastic strain–longitudinal plastic strain curve, and it can be assumed to be a function of the longitudinal plastic strain. According to Eq. (3), the solution of the R-value is essentially the determination of the slope of the plastic strain curve.

## Current Research Methods

### *Point Slope Method (PS Method)*

According to standard ISO 10113 [23], the total strain is usually used instead of the strain-increments in the calculation of R-value.

$$R = -\frac{\varepsilon_b}{\varepsilon_l + \varepsilon_b} \tag{4}$$

where $\varepsilon_l$ and $\varepsilon_b$ are the longitudinal plastic strain and transverse plastic strain at a certain time during the tensile process, respectively. They can be calculated by Eq. (5).

$$\begin{cases} \varepsilon_l = \ln\left(\dfrac{L_0 + \Delta L}{L_0}\right) - \varepsilon_s \\ \varepsilon_b = \ln\left(\dfrac{B_0 - \Delta B}{B_0}\right) + \nu\varepsilon_s \end{cases} \tag{5}$$

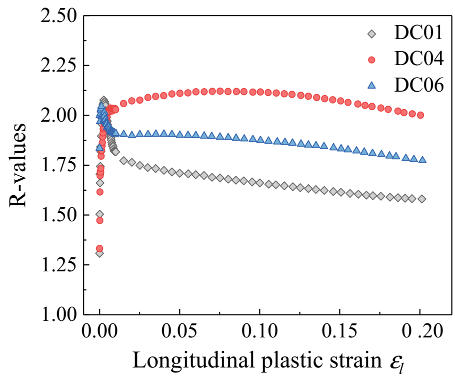



**Fig. 4** R-values obtained based on PS method

where $L_0$ and $B_0$ are the initial gauge length and width of the specimen, respectively. $\Delta L$ and $\Delta B$ are the gauge elongation and width reduction of the specimen, respectively. $\varepsilon_s$ is the initial yield strain and $\nu$ is Poisson's ratio. Then, the R-values during the uniform plastic deformation stage can be calculated according to Eqs. (4) and (5). In this paper, this method is referred to as Point Slope method (PS method).

In Fig. 4, the R-values significantly increase in the initial deformation stage and then tends to change smoothly. This is because the transition curve from elasticity to plasticity is approximately an arc, and the starting point of plasticity is approximately located in the middle of this arc curve. In fact, Eq. (4) is reasonable only when the deformation of the longitudinal and transverse of the specimen satisfies a certain proportion in the uniaxial tensile test. That is, the longitudinal plastic strain–transverse plastic strain curve is a straight line.

## *Piecewise Linear Regression Method (PLR Method)*

In accordance with standard ISO 10113, the linear regression of the transverse versus longitudinal plastic strain curve within a certain plastic strain is used to calculate the R-value. On this basis, in order to characterize, the R-value varies with the longitudinal strain. An et al. [24] proposed a new piecewise linear regression method. The transverse and longitudinal plastic strain discrete data are divided into $i$ partitions, as shown in Fig. 5. The slope $k_i$ of each interval can be fitted according to Eq. (6) and then the $R_i$-value of each interval can be calculated by Eq. (7). In this paper, this method is referred to as Piecewise Linear Regression method (PLR method). A comparison of the R-value calculated based on different partition sizes is in Fig. 6.

$$\varepsilon_b = k_i \varepsilon_l + c_i \tag{6}$$

$$R_i = -\frac{k_i}{1 + k_i} \tag{7}$$

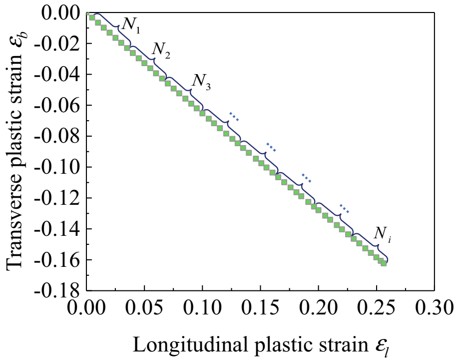



**Fig. 5** Schematic diagram of piecewise linear regression method

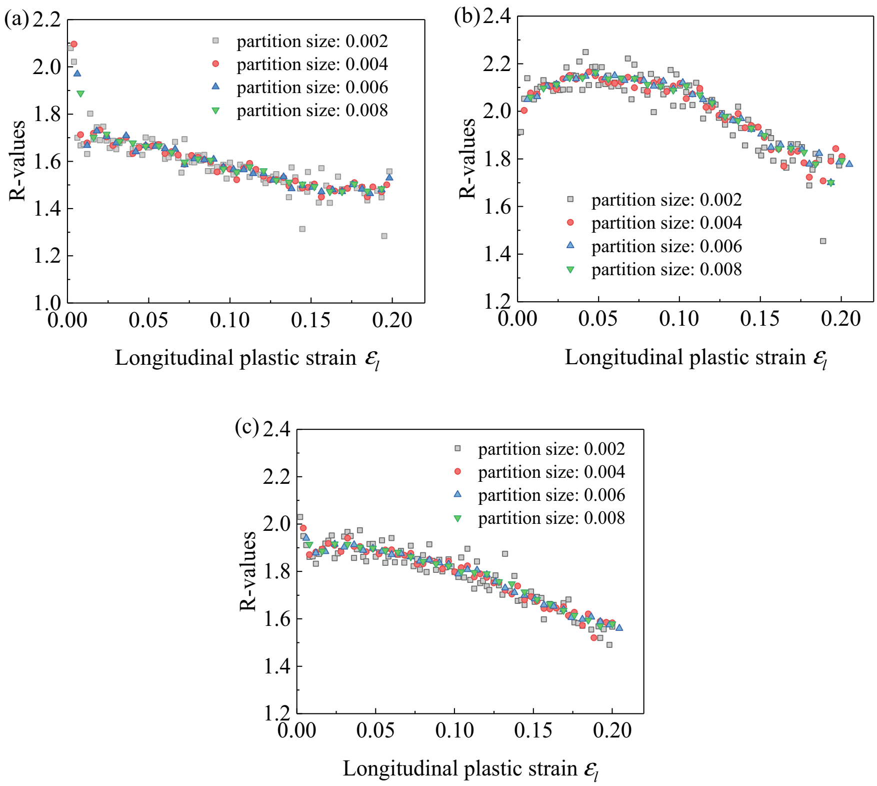



**Fig. 6** Comparison of the R-values obtained with different partition sizes according to PLR method: **a** DC01, **b** DC04, **c** DC06

The results show that PLR method is very sensitive to the partition size. The R-values are relatively dispersive when the partition size is too small, and the fitting distortion is easy to occur under this condition. Conversely, few discrete data points can be obtained when the interval size is too large, which cannot reflect the real change law of the R-value. Furthermore, only fitting these discrete points can get the relationship function between the R-value and longitudinal strain, but it will create a new fitting error. Nevertheless, to a certain extent, the results obtained with PLR method can reflect the change law of the R-value with the deformation when a suitable partition size is chosen. Therefore, it can be used to analyze the function form of the relationship between the R-value and longitudinal plastic strain.

### *Polynomial Fitting Method (Polynomial Method)*

Only the discrete data points of R-value can be obtained in the above method. For this reason, relevant scholars proposed to use polynomial function to fit the discrete data of the longitudinal plastic strain and transverse plastic strain [20]. It is assumed that the relationship between the transverse plastic strain and longitudinal plastic strain satisfies

$$\varepsilon_b = c_0 + c_1\varepsilon_l + c_1\varepsilon_l^2 + c_2\varepsilon_l^3 + \cdots + c_n\varepsilon_l^n \tag{8}$$

where $c_0$, $c_1$, …, $c_n$ are the undetermined coefficients. By deriving on both sides of Eq. (8), the following relationship can be obtained:

$$\frac{d\varepsilon_b}{d\varepsilon_l} = c_1 + 2c_2\varepsilon_l + 3c_3\varepsilon_l^2 + \cdots + nc_n\varepsilon_l^{n-1} \tag{9}$$

According to Eqs. (2) and (9), there is

$$R = -\frac{c_1 + 2c_2\varepsilon_l + 3c_3\varepsilon_l^2 + \cdots + nc_n\varepsilon_l^{n-1}}{1 + c_1 + 2c_2\varepsilon_l + 3c_3\varepsilon_l^2 + \cdots + nc_n\varepsilon_l^{n-1}} \tag{10}$$

In theory, the higher the polynomial order, the higher the fitting accuracy for the relationship between the transverse plastic strain and longitudinal plastic strain. However, the R-value varies with longitudinal strain will be more complex, even the change law is different as shown in Fig. 7. The function form of the R-value obtained with the polynomial method is also complicated, and there is no basis for choosing the polynomial order.

The principle of the above calculation methods is consistent. That is, fitting the data of the transverse plastic strain and longitudinal plastic strain by a function with some undetermined coefficients. Then, the slope of the plastic strain curve can be calculated by deriving these functions. In this paper, the methods based on this

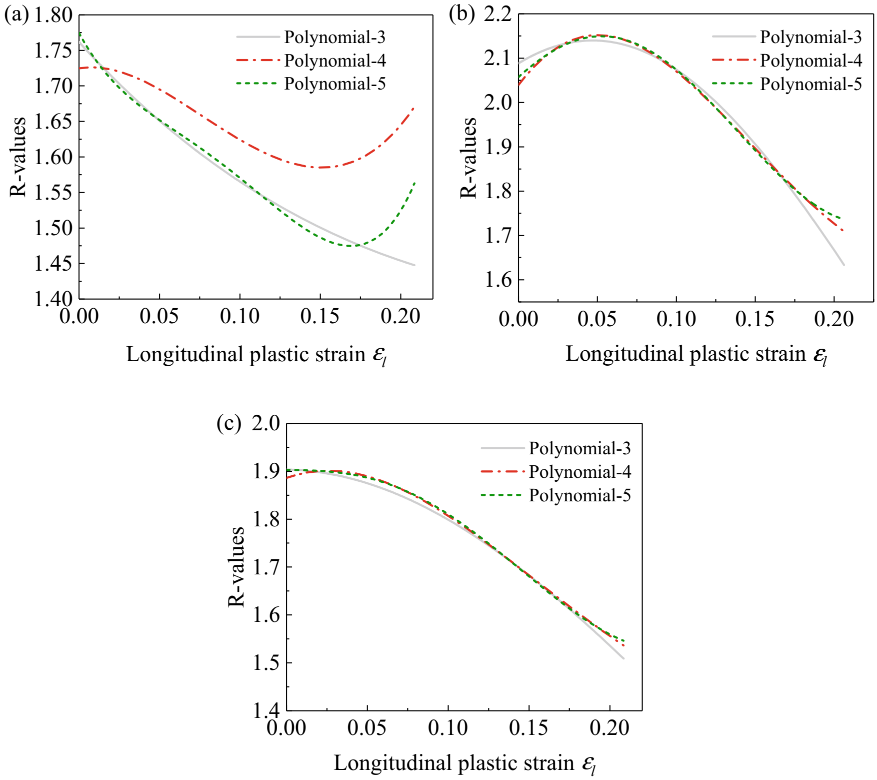



**Fig. 7** Comparison of the R-values obtained with different polynomial order: **a** DC01, **b** DC04, **c** DC06

principle are named the forward solution method. However, there is no basis for selecting a suitable fitting function, and the change law of R-value obtained with different fitting functions is vary greatly. In view of the shortcomings of the above calculation methods, a reverse fitting model of the R-value is proposed.

## Establishment of the Reverse Fitting Model

Assume that the relationship between the R-value and longitudinal plastic strain satisfies:

$$R = f(\varepsilon_l, a, b, c\ldots) \tag{11}$$

where $a$, $b$, and $c$ are the undetermined coefficients. According to Eq. (2), there is

$$-\frac{d\varepsilon_b}{d\varepsilon_l + d\varepsilon_b} = f(\varepsilon_l, a, b, c...) \tag{12}$$

Equation (12) can be further simplified as

$$\frac{d\varepsilon_b}{d\varepsilon_l} = -1 + \frac{1}{f(\varepsilon_l, a, b, c...) + 1} \tag{13}$$

Then, the relationship between the transverse plastic strain and longitudinal plastic strain can be obtained by integrating Eq. (13).

$$\varepsilon_b = -\varepsilon_l + \int \frac{1}{f(\varepsilon_l, a, b, c...) + 1} d\varepsilon_l + C \tag{14}$$

where $C$ is the integral constant which can be calculated according to the boundary condition $\varepsilon_b = \varepsilon_l = 0$ in the initial plastic deformation time. The undetermined coefficients can be determined by fitting the transverse plastic strain and longitudinal plastic strain data based on Eq. (14). Then, Eq. (11) can be determined.

According to the calculation results of the R-value based on PLR method in Sect. "Experiments", and the previous research results in literatures [20–22, 24], the curves of the R-value vary with longitudinal strain and can be approximately divided into two types: linear function and quadratic function. Linear function is a special form of quadratic function. It is assumed that the relationship function between the R-value and longitudinal plastic strain is a quadratic function:

$$R = a\varepsilon_l^2 + b\varepsilon_l + c \tag{15}$$

where $a$, $b$, and $c$ are the undetermined coefficients. According to Eqs. (12), (13), and (15), the following relationship can be derived:

$$\begin{cases} \varepsilon_b = -\varepsilon_l + \dfrac{2}{\sqrt{4a(1+c) - b^2}} \arctan\left(\dfrac{2a\varepsilon_l + b}{\sqrt{4a(1+c) - b^2}}\right) + C_1 & b^2 < 4a(1+c) \\ \varepsilon_b = -\varepsilon_l + \dfrac{1}{\sqrt{b^2 - 4a(1+c)}} \left[\ln\left|\dfrac{2a\varepsilon_l + b - \sqrt{b^2 - 4a(1+c)}}{2a\varepsilon_l + b + \sqrt{b^2 - 4a(1+c)}}\right|\right] + C_2 & b^2 > 4a(1+c) \end{cases} \tag{16}$$

where

$$\begin{cases} C_1 = -\dfrac{2}{\sqrt{4a(1+c) - b^2}} \arctan(\dfrac{b}{\sqrt{4a(1+c) - b^2}}) \\ C_2 = -\dfrac{1}{\sqrt{b^2 - 4a(1+c)}} \ln\left|\dfrac{b - \sqrt{b^2 - 4a(1+c)}}{b + \sqrt{b^2 - 4a(1+c)}}\right| \end{cases} \tag{17}$$

When the R-value varies with the longitudinal strain is approximately a straight line, the value of $a$ obtained by fitting will be approximately zero.

## Results and Discussion

According to uniaxial tensile tests, the specimens will undergo elastic deformation, uniform plastic deformation, diffuse instability deformation, located instability deformation, and fracture in turn. However, only the plastic strain data during the uniform plastic deformation stage can be obtained from the standard uniaxial tensile test. The installation position of the transverse extensometer is generally not on the narrowest cross-section. Therefore, the data closed to the necking should be removed to ensure the reliability of the plastic strain data. The longitudinal strain and transverse strain data within an interval of 0%–90% longitudinal strain during the uniform deformation stage are used to fit the undetermined coefficients. A comparison of the R-values based on different methods is shown in Fig. 8.

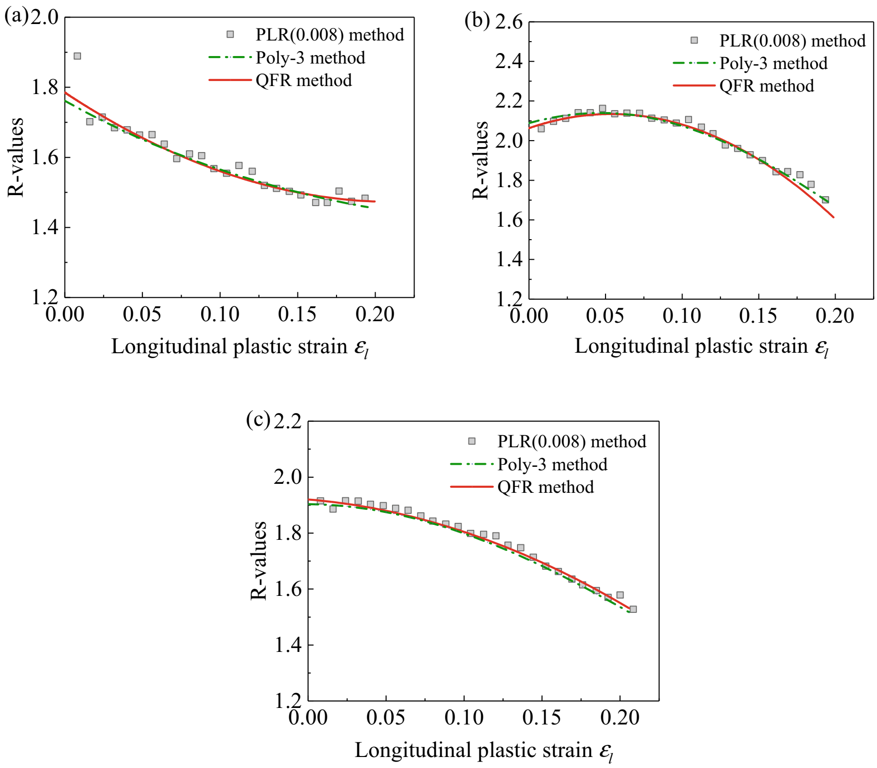



**Fig. 8** Comparison of the R-values based on different methods: **a** DC01, **b** DC04, **c** DC06

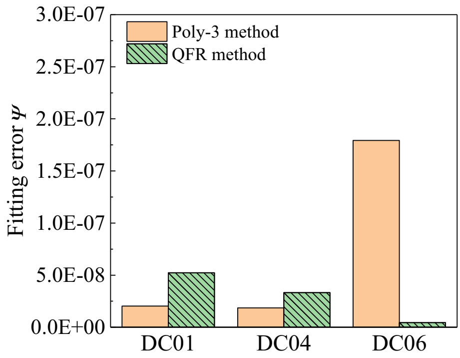



**Fig. 9** Comparison of the fitting error based on different methods

The prediction curves of the R-value based on reverse fitting model agree well with the discrete data points obtained with the PLR method. In order to evaluate the fitting accuracy of different methods, the following error evaluation formulas are established:

$$\psi = \frac{\Sigma\left(\varepsilon_b^{pre} - \varepsilon_b^{\exp}\right)^2}{\Sigma\left|\varepsilon_b^{\exp}\right|} \tag{18}$$

where $\varepsilon_b^{pre}$ is the predictive value of the transverse plastic strain and $\varepsilon_b^{\exp}$ is the experimental value of the transverse plastic strain. Figure 9 shows that the fitting error based on different methods. The reverse fitting model based on quadratic function has relatively high fitting accuracy, and the function form of the R-value obtained with QFR method is simpler than the polynomial method. It is easier to be applied in the secondary development of the constitutive model considering hardening anisotropy. In addition, the reverse fitting model proposed in this paper is not limited to the quadratic function. Therefore, the model has strong flexibility, which provides a valuable reference for the investigation of the R-value with complex variation law.

In order to clarify the effect of the strain curve slope on the calculation results of the R-value, according to Eq. (3), there is

$$dR = \frac{-1}{[1 + k(\varepsilon_l)]^2} dk \tag{19}$$

By Eqs. (3) and (19), the following relationship can be derived:

$$\frac{dR}{R} = \frac{1}{1 + k(\varepsilon_l)} \frac{dk}{k} \tag{20}$$

The above results show that the R-values approximately locate in $1 < R < 2.5$ for most deep drawing sheet metals. Then, the change range of the strain curve slope is $-0.5 < k(\varepsilon_l) < -0.71$. Therefore, according to Eq. (20), the calculation error of the slope will enlarge the prediction error of the R-value approximately by 2 ~ 3.5 times. Therefore, the significant differences in the R-value calculated by different methods are caused by the calculation error of the strain curve slope. In the future, the methods for determining the plastic strain curve slope should be studied to improve the accuracy of R-values.

## Conclusions

(1) According to R.Hill's orthotropic plasticity theory, the R-value is a function of the derivative of the transverse plastic strain to longitudinal plastic strain. The R-value is a constant only when the relationship of the above two strains is a linear function. Under this condition, the experimental results of the R-value can be obtained with the total strain of uniaxial tension.
(2) The relationship between the transverse strain and longitudinal strain is approximately a straight line, but the slight change in its slope will lead to a great change in the R-value. In order to obtain high-the precision anisotropic constitutive model for sheet metal, the evolution of R-value must be considered.
(3) The piecewise linear regression method can give the evolution trend of the R-value, but the difference in the partition size will result in the fluctuation of the R-value. The results obtained with this method are relatively discrete and the second fitting will introduce new fitting errors.
(4) The polynomial forward fitting method can ensure the fitting accuracy of the relationship between the transverse strain and longitudinal strain, but it cannot guarantee the accuracy of its derivative function. Because of the different orders of polynomial, the evolution of the R-value is diverse and the function form of the R-value is very complex.
(5) The proposed reverse fitting model of quadratic function with three undetermined coefficients can not only ensure the fitting accuracy of the uniaxial tensile strain relationship but also show a high degree of consistency in the evolution law of the R-value. Its function form is simple and the undetermined coefficients are few, which provides great convenience for the establishment of the high-precision anisotropic constitutive model for sheet metal.
(6) With the increase in the uniaxial tensile plastic strain, the evolution of the R-value is generally downward, and the curve form can be divided into two types: up convex and down concave.

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