Passive and Active Suspension Systems Analysis and Design

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Abstract Active suspension systems allow one to improve the performance of a vehicle, to reduce the fuel (energy) consumption and exhaust emissions. This in turn allows one to improve the transport traffic and well-being in cities. The analysis and design of several types of semi-active and active suspension systems is provided in this paper.

Keywords Automotive suspensions systems · Active suspension systems · Modelling and simulation · Vehicle performance

1 Introduction

Increased competition on the automotive market has forced companies to research into alternative strategies to classical passive suspension systems [1, 2]. To improve handling and comfort performance, instead of a conventional static spring and damper system, semi-active and active systems are being developed [5, 6]. A semi-active suspension system involves the use of a dampers or spring with variable gain [3, 4]. Such systems can only operate on three fixed positions: soft, medium and hard damping or stiffness. Additionally, a semi-active system can only absorb the energy from the motion of a car body.

Alternatively, an active suspension system possesses the ability to reduce acceleration of sprung mass continuously as well as to minimise suspension deflection, which results in improvement of tyre grip with the road surface, thus, brake, traction control and vehicle maneuverability can be considerably improved.

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2 Passive Suspension System

A one-degree-of-freedom (1-DOF) of a passive suspension system is given in Fig. 1. (We do not take into account the mass and stiffness of a wheel in this example).

A mathematical model of a passive suspension system can be obtained from the Newton's second law according to the free-body diagram, given in Fig. 2, as

$$m\ddot{x}_1(t) = -k(x_1(t) - x_0(t)) - b(\dot{x}_1(t) - \dot{x}_0(t)) \tag{1}$$

where

- m is the ¼ car body mass,
- k is the suspension spring coefficient,
- b is the suspension damping coefficient,
- x_0 is the road vertical disturbance (input signal),
- x_1 is the vertical displacement of the sprung mass (output signal).

Re-arrange Eq. (1) as follows:

$$m\ddot{x}_1(t) + b(\dot{x}_1(t) - \dot{x}_0(t)) + k(x_1(t) - x_0(t)) = 0 \tag{2}$$

Equation (2) can be represented in the standard form as:

$$\ddot{x}_1(t) + \frac{b}{m}(\dot{x}_1(t) - \dot{x}_0(t)) + \frac{k}{m}(x_1(t) - x_0(t)) = 0 \tag{3}$$

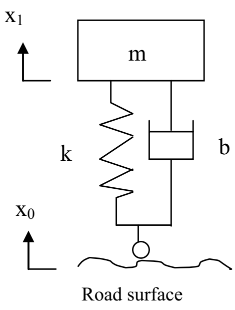

Fig. 1 A 1-DOF passive suspension system

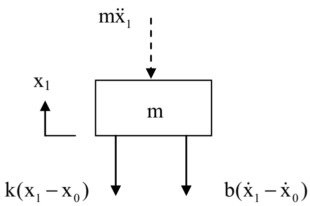

Fig. 2 The free-body diagram

Represent Eq. (3) in the Laplace form

$$s^2X_1(s) + s\frac{b}{m}(X_1(s) - X_0(s)) + \frac{k}{m}(X_1(s) - X_0(s)) = 0 \tag{4}$$

or

$$s^2X_1(s) + s\frac{b}{m}X_1(s) + \frac{k}{m}X_1(s) = s\frac{b}{m}X_0(s) + \frac{k}{m}X_0(s) \tag{5}$$

Equation (5) can be represented as

$$X_1(s)\left[s^2 + \frac{b}{m}s + \frac{k}{m}\right] = X_0(s)\left[\frac{b}{m}s + \frac{k}{m}\right] \tag{6}$$

The transfer function between the vertical displacement of the sprung mass and the road vertical disturbance can be obtained from (6) as

$$\frac{X_1(s)}{X_0(s)} = \frac{\frac{b}{m}s + \frac{k}{m}}{s^2 + \frac{b}{m}s + \frac{k}{m}} \tag{7}$$

The transfer function (7) can be represented in the form of polynomials

$$G(s) = \frac{X_1(s)}{X_0(s)} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \tag{8}$$

where

$$b_0 = \frac{k}{m}, \quad b_1 = \frac{b}{m}, \quad a_0 = \frac{k}{m}, \quad a_1 = \frac{b}{m}$$

The standard form of a second order system in the transfer function representation is given as

$$G(s) = \frac{k_{ss}\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2} \tag{9}$$

where

k_{ss} is the steady-state gain,
ς is the damping ratio,
ω_n is the natural frequency.

Compare Eqs. (7) and (9) we can obtain the damping ratio ς and natural frequency ω_n of the passive suspension system as

$$\omega_n = \sqrt{\frac{k}{m}}, \quad \varsigma = \frac{\frac{b}{m}}{2\omega_n}. \tag{10}$$

The simulation of the passive suspension system has been performed using the MATLAB package.

The following parameters are used:

$$\begin{aligned} &1/4 \text{ body mass}, \ m = 530.6 \text{ kg} \\ &\text{suspension spring coefficient}, k = 22{,}750 \text{ N/m}, \\ &\text{suspension damping coefficient}, \ b = 700 \text{ Ns/m}. \end{aligned} \tag{11}$$

The transfer function of the passive suspension system (7) is obtained in the form:

$$G_p(s) = \frac{1.3193\,s + 42.876}{s^2 + 1.3193\,s + 42.876} \tag{12}$$

The frequency response characteristics of the mass vertical displacement $x_1(t)$ versus the road vertical disturbance $x_0(t)$ is given in Fig. 3:

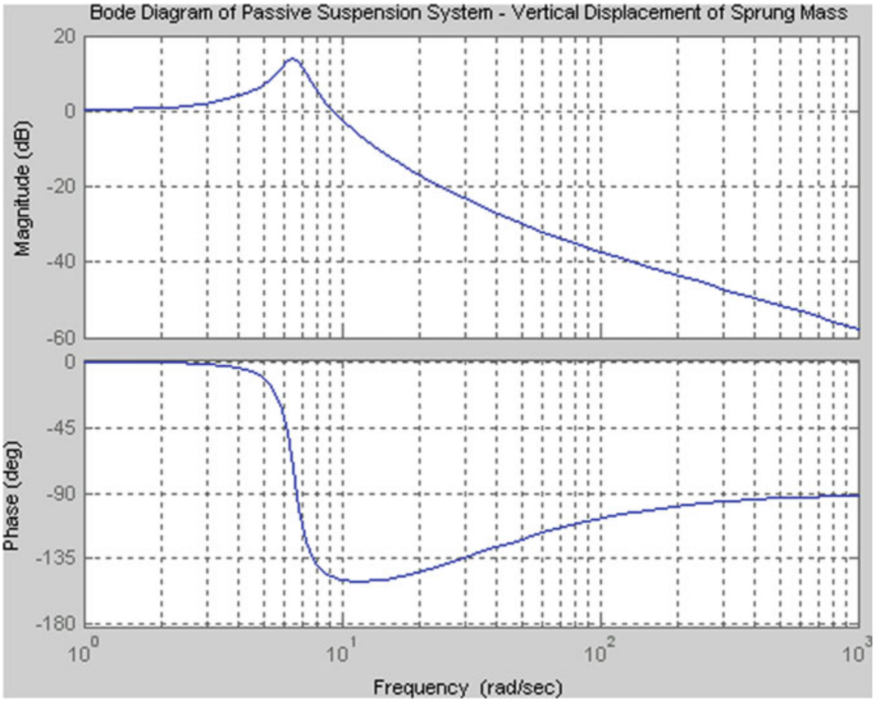

Fig. 3 Frequency response characteristics of the mass vertical displacement versus the road vertical disturbance of the passive suspension system

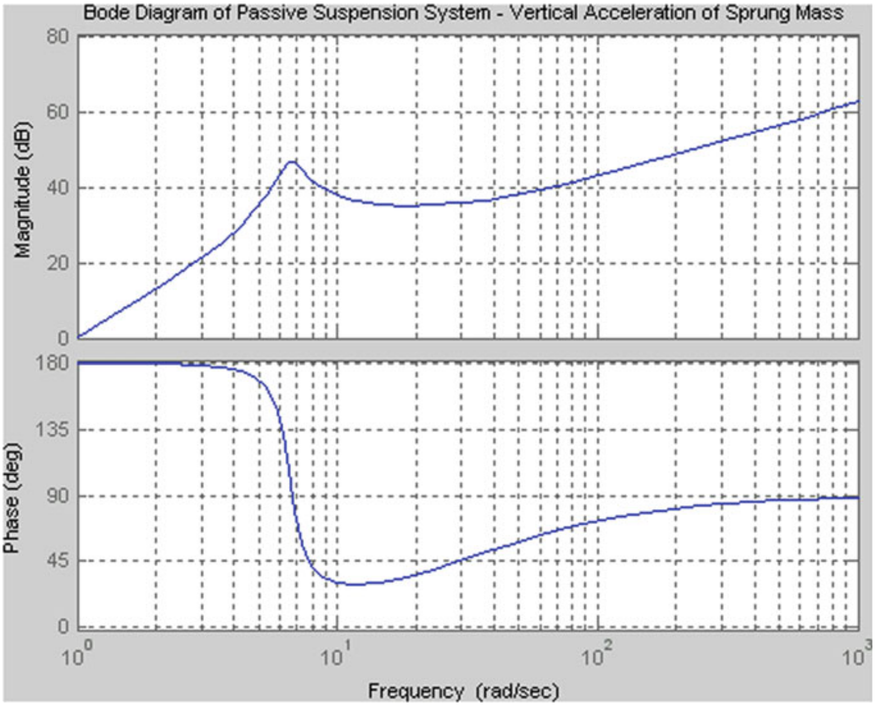

Fig. 4 Frequency response characteristics of the mass vertical acceleration versus the road vertical disturbance of the passive suspension system

The frequency response characteristics of the mass vertical acceleration $\ddot{x}_1(t)$ versus the road vertical disturbance $x_0(t)$ is given in Fig. 4.

Represent the system (8) in the block-diagram form in order to obtain the behaviour of the passive suspension in the time domain. The input-output relationship can be obtained from (8) as

$$X_1(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} X_0(s) \tag{13}$$

If we define:

$$Q(s) = \left[\frac{1}{s^2 + a_1 s + a_0} \right] X_0(s) \tag{14}$$

then, we can re-write (13) as

$$X_1(s) = (b_1 s + b_0) Q(s) \tag{15}$$

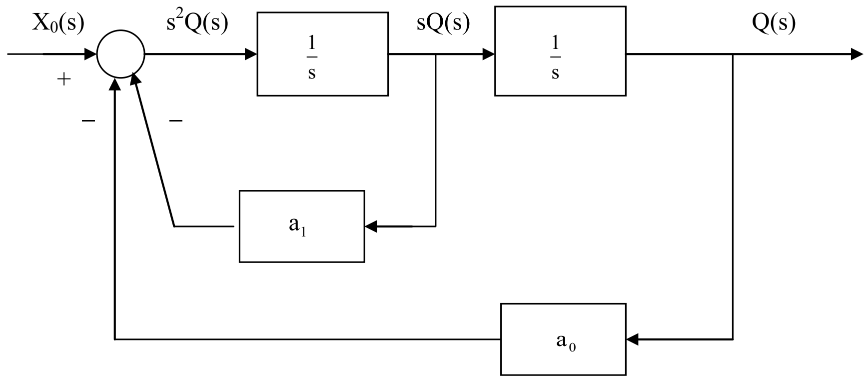

Fig. 5 Representation of the system (8) in the block-diagram form

Therefore,

$$X_1(s) = b_1 sQ(s) + b_0 Q(s) \tag{16}$$

From (14) we can obtain the following:

$$s^2Q(s) + a_1 sQ(s) + a_0 Q(s) = X_0(s) \tag{17}$$

Re-arrange Eq. (17) in the following form:

$$s^2Q(s) = -a_1 sQ(s) - a_0 Q(s) + X_0(s) \tag{18}$$

Represent (18) in the block-diagram form (Fig. 5):

Now, we can represent the complete system using Eq. (16) in the following block diagram form (Fig. 6):

Denote:

$$\begin{aligned} Q_1(s) &= Q(s) \\ Q_2(s) &= sQ_1(s) \end{aligned} \tag{19}$$

Then, Eq. (18) can be represented in the following form:

$$sQ_2(s) = -a_1 Q_2(s) - a_0 Q_1(s) + X_0(s) \tag{20}$$

The output can be obtained from Eq. (16) as

$$X_1(s) = b_1 Q_2(s) + b_0 Q_1(s) \tag{21}$$

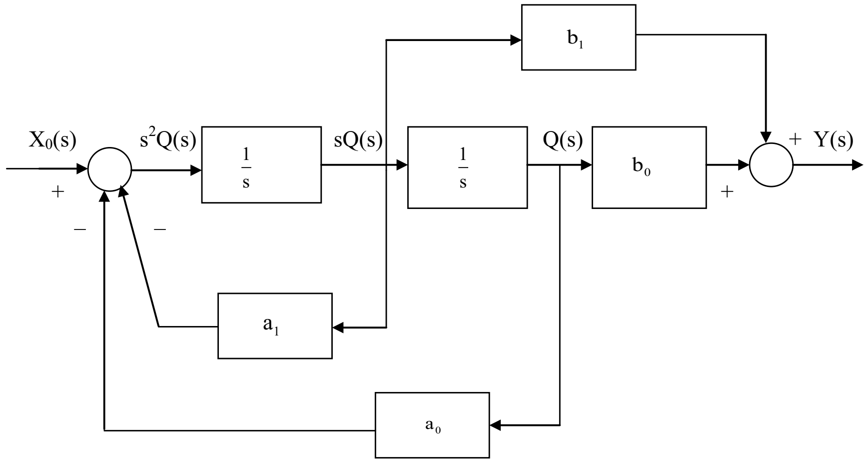

Fig. 6 Representation of the equation (18) in the block-diagram form

Assuming that the initial conditions are zero, the state-variable model in the time-domain can be represented in the following form:

$$\begin{aligned} \dot{q}_1(t) &= q_2(t) \\ \dot{q}_2(t) &= -a_1 q_2(t) - a_0 q_1(t) + x_0(t) \\ x_1(t) &= b_1 q_2(t) + b_0 q_1(t) \end{aligned} \tag{22}$$

The system is designed using standard blocks from the Simulink library. This is given in Fig. 7.

The impulse response of the passive suspension system is given in Fig. 8.

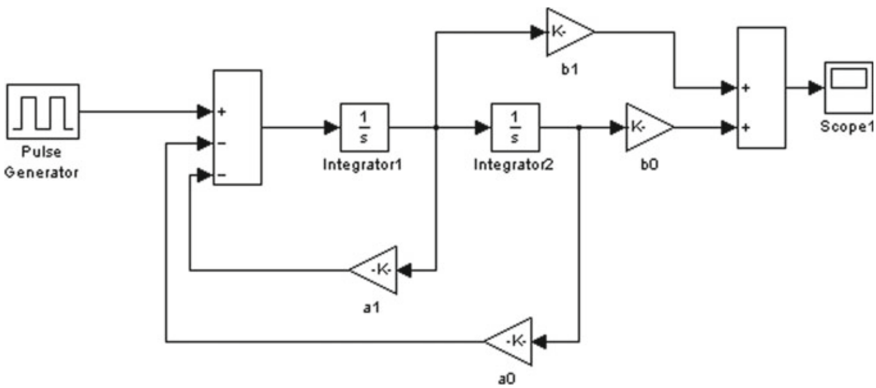

Fig. 7 Representation of the complete system in the Simulink form

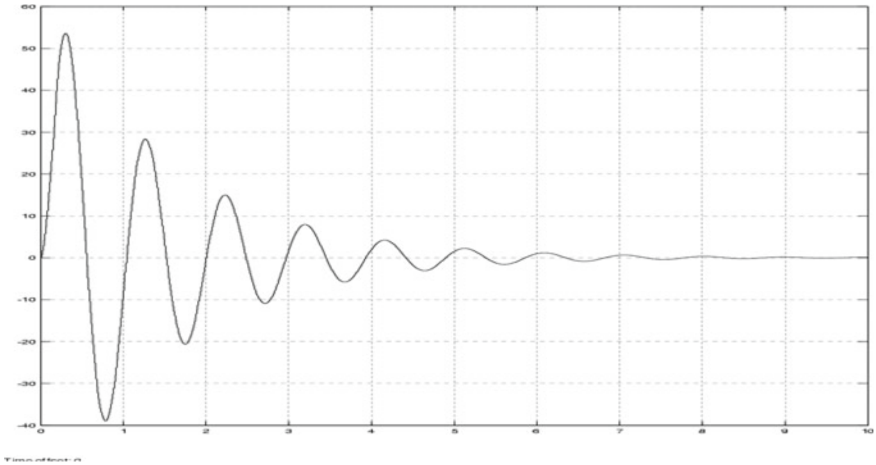

Fig. 8 The impulse response of the passive suspension system.

3 Active Suspension System

Sky-hook damper control system (Fig. 9)

It is well known that sky-hook control is effective in suppression of sprung mass vibrations. Thus, implementation of this system can dramatically improve comfort of driving. A theory of operation of a sky-hook damper system is given in Fig. 10.

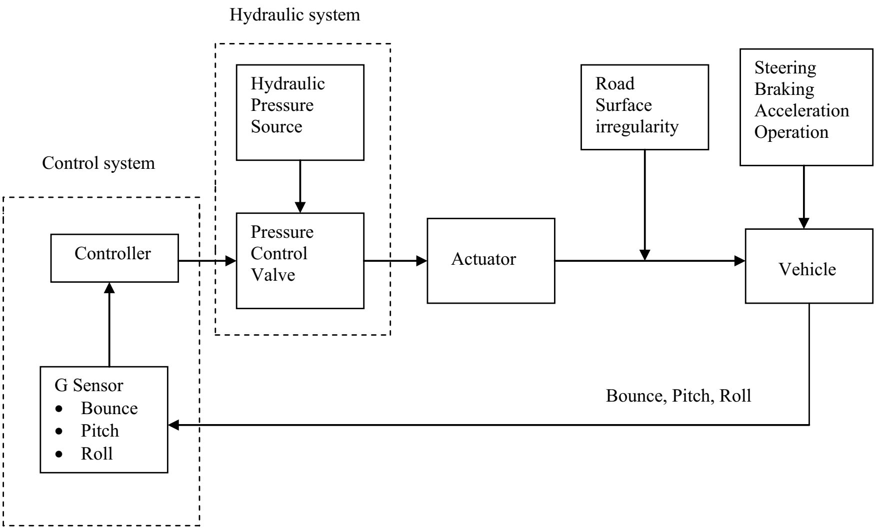

Fig. 9 Hydraulic and control systems for an active suspension

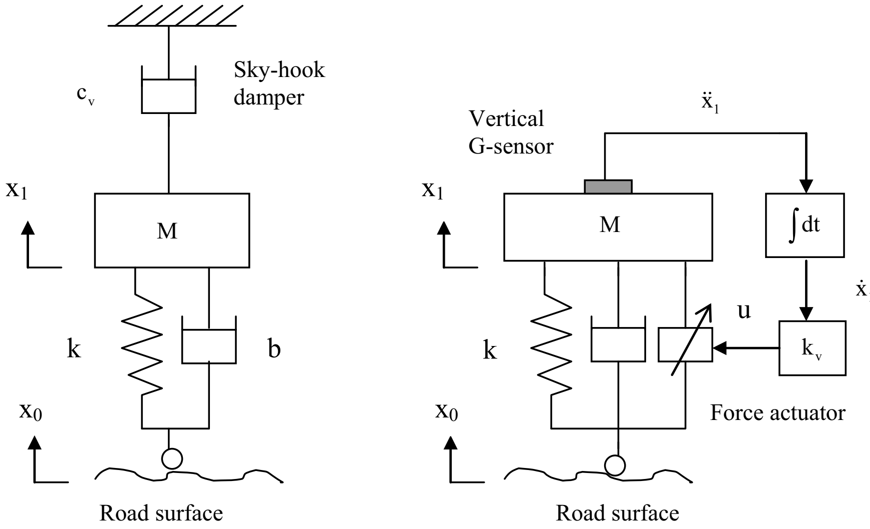

Fig. 10 The idea of sky-hook damper control for a 1-DOF active suspension system

A sky-hook damper is a virtual damper and control signal $\boldsymbol{u(t)}$ is calculated using absolute velocity of a car body as follows:

$$u(t) = -c_v \dot{x}_1(t) \tag{23}$$

where

$\dot{x}_1(t)$ is the absolute velocity of vertical sprung mass motion,
k_v is the coefficient of the sky-hook damper.

A mathematical model of an active suspension system can be represented in the following form:

$$m\ddot{x}_1(t) + b(\dot{x}_1(t) - \dot{x}_0(t)) + k(x_1(t) - x_0(t)) = u(t) \tag{24}$$

Substitute (23) into (24):

$$m\ddot{x}_1(t) + b(\dot{x}_1(t) - \dot{x}_0(t)) + k(x_1(t) - x_0(t)) + k_v\dot{x}_1 = 0 \tag{25}$$

Represent the Eq. (25) in the Laplace form as

$$ms^2X_1 + bsX_1 - bsX_0 + kX_1 - kX_0 + k_vsX_1 = 0 \tag{26}$$

Re-write Eq. (26) in the following form

$$X_1(ms^2 + bs + k + k_vs) = X_0(bs + k) \tag{27}$$

The transfer function between the mass vertical displacement and the road disturbance can be obtained from Eq. (27) as:

$$\frac{X_1(s)}{X_0(s)} = \frac{\frac{b}{m}s + \frac{k}{m}}{s^2 + \frac{b+k_v}{m}s + \frac{k}{m}} \tag{28}$$

The simulation of the active suspension system (28) has been performed on MATLAB with the parameters given in (11) and $k_v = 650$ Ns/m.

The transfer function of the active suspension system (28) is obtained in the form:

$$G_a(s) = \frac{1.3193\,s + 42.876}{s^2 + 2.827\,s + 42.876} \tag{29}$$

The frequency response characteristic of the mass vertical displacement $x_1(t)$ versus the road disturbance $x_0(t)$ is given in Fig. 11.

The frequency response characteristic of the mass vertical acceleration $\ddot{x}_1(t)$ versus the road disturbance $x_0(t)$ is given in Fig. 12.

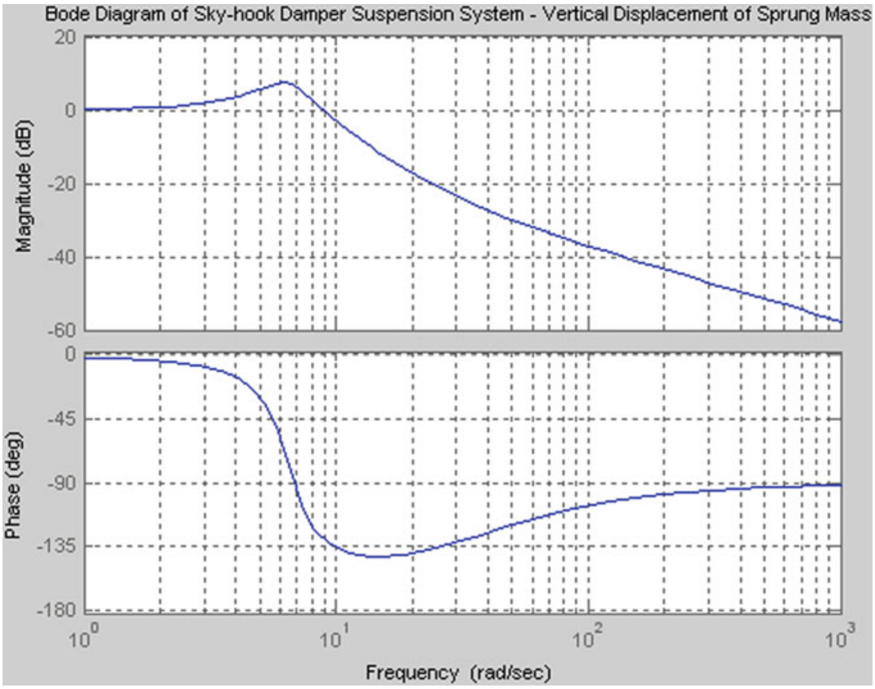

Fig. 11 Frequency response characteristics of the mass vertical displacement versus the road vertical disturbance of the active suspension system

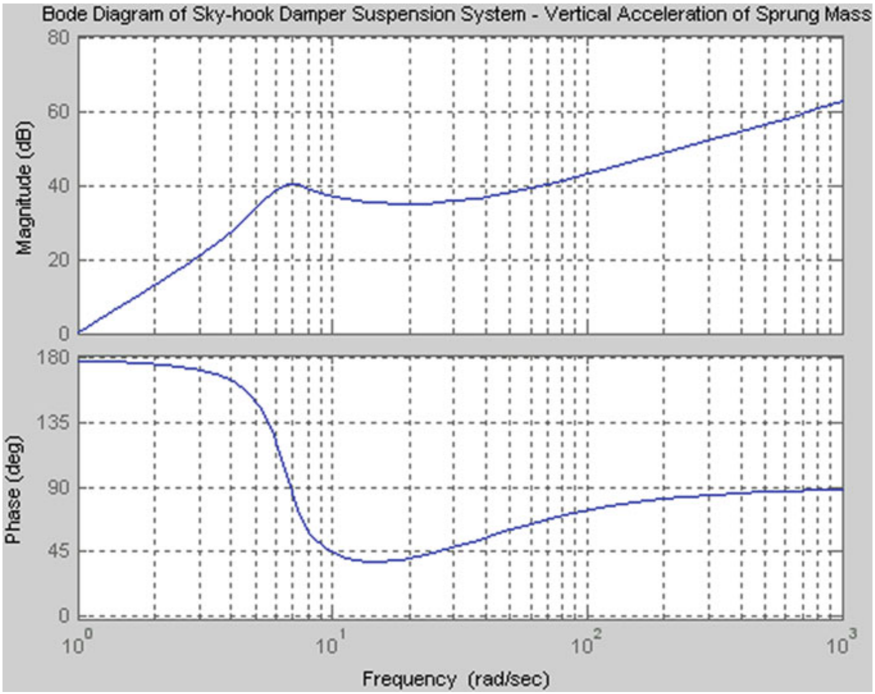

Fig. 12 Frequency response characteristics of the mass vertical acceleration versus the road vertical disturbance of the active suspension system

It can be seen from Figs. 3 and 11 that the magnitude of the peak (on the natural frequency) of mass vertical displacement of the active suspension is lower than that of the passive suspension. It follows from Figs. 4 and 12 that the peak of mass vertical acceleration has also been suppressed on the active suspension.

The impulse response of the active suspension system is given in Fig. 13.

It can be seen from Figs. 8 and 13 that the transition response has been reduced from 9 s (on the passive suspension) to 3 s (on the active suspension).

Thus, the given above MATLAB/Simulink results prove the advantages of the active suspension system when compare with the passive suspension.

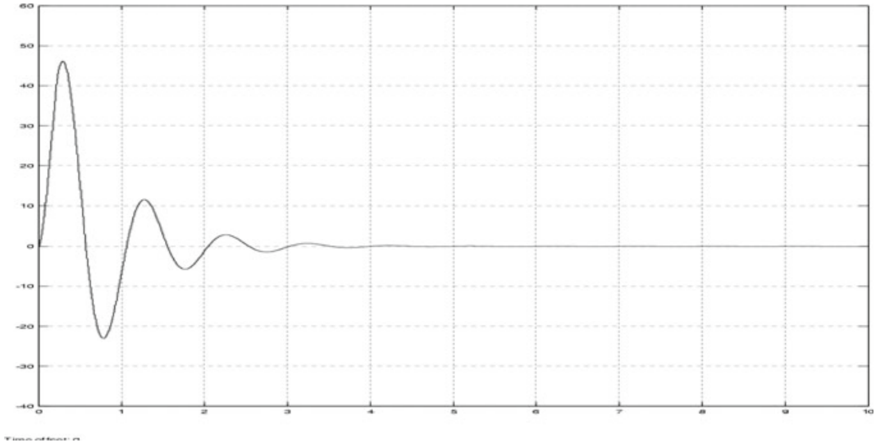

Fig. 13 The impulse response of the active suspension system

4 Conclusions

The analysis of several types of semi-active and active suspension systems is given in this paper. It has been demonstrated that active suspension systems provide the better performance in the frequency domain. The time response of an active suspension system on the applied input is faster as well. The signal from an accelerometer is used for the control system. Three-axes accelerometers are usually available on vehicles. Such accelerometers are specifically designed for the applications on vehicles. The accelerometers are not expensive and robust devices. They work in the acceptable range of driving and environmental conditions. Therefore, accelerometers are the preferable alternative to liner potentiometers and velocity sensors to measure the vertical displacement and velocity of the displacement of a car-body. The numerical integration of the signal from an accelerometer can be implemented on the automotive ECU (Electronic Control Unit).

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