Chapter 14
A Hierarchical Filtering Approach for Online Damage Detection Using Parametric Reduced-Order Models

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Abstract This chapter presents a hierarchical Bayesian framework for the system parameter identification of vibrating systems using spatially incomplete and noisy output-only response measurements. The parameters to be identified are treated as random variables, whose distributions are approximated by a finite number of evolving particles. For each realization of the parameters, an output-only Bayesian filter is employed for the unknown input and state estimation, creating thus a bank of filters that are recursively weighted, upon assimilation of the measurement information, and subsequently updated in order to narrow down the range of system parameters and converge to the target values.

Keywords Sequential Bayesian inference (SBI) · Input–state–parameter estimation · Hierarchical particle filter · Evolution strategy · Crack detection

14.1 Introduction

The effective localization and quantification of cracks in structural and mechanical systems remains an active research topic in the engineering community. In contrast to crack or damage detection, for which many alternative methods of a certain degree of functionality have already been established, crack localization and quantification is considerably more complicated and, in most cases, requires the availability of extensive spatial information in combination with physics-based models. The problem becomes even more challenging under the constraint of real-time performance on in-service systems, calling thus for computationally efficient models [1] and recursive methodologies [2] able to operate in online fashion.

Within this context, parametric projection-based reduced-order models for cracked structures are developed, with the parameters corresponding to geometric properties of the crack [3]. These models are used in the context of sequential Bayesian inference problems for the online parameter estimation using limited output measurements. By so doing, the identification of crack features is seen as an input, state, and parameter estimation problem, which is decomposed into two levels: one dealing with the estimation of the state and unknown input and a second one for estimating the parameters.

14.2 Methodology

The problem of online crack detection using a limited number of output-only vibration measurements is tailored to a Bayesian context, which implies the availability of (1) actual vibration response measurements and (2) a physics-based representation of the system under consideration. As such, the problem can be seen as an input, state, and parameter estimation problem, which is formulated in terms of the following parametrized state-space model:

$$\mathbf{x}_{k+1} = \mathbf{A}_{\mathrm{d}}(\boldsymbol{\mu})\,\mathbf{x}_k + \mathbf{B}_{\mathrm{d}}(\boldsymbol{\mu})\,\mathbf{p}_k + \mathbf{v}_k \tag{14.1a}$$

$$\mathbf{y}_k = \mathbf{G}_{\mathrm{d}}(\boldsymbol{\mu})\,\mathbf{x}_k + \mathbf{J}_{\mathrm{d}}(\boldsymbol{\mu})\,\mathbf{p}_k + \mathbf{w}_k \tag{14.1b}$$

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where $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector, $\boldsymbol{\mu} \in \mathbb{R}^{n_\mu}$ denotes the parameter vector, $\mathbf{p}_k \in \mathbb{R}^{n_p}$ is the input vector, and $\mathbf{y}_k$ represents the vector of system observations. Lastly, the terms $\mathbf{v}_k \in \mathbb{R}^n$ and $\mathbf{w}_k \in \mathbb{R}^{n_y}$ denote the mutually uncorrelated, zero-mean, and white process and measurement noise signals.

Instead of the state augmentation technique, in which the unknown system parameters are appended to the state vector, this chapter is based on the use of a bank of filters that adapt themselves to the sought after system parameters. As such, the unknown parameter vector is treated as a random variable, and the problem is decomposed as follows:

$$\begin{aligned}\mathbb{E}\left[\mathbf{x}_k|\mathbf{Y}_k\right] &= \int \mathbf{x}_k \int p\left(\mathbf{x}_k, \boldsymbol{\mu}\,|\mathbf{Y}_k\right)\, d\boldsymbol{\mu}\, d\mathbf{x}_k \\ &= \int \mathbf{x}_k \int p\left(\mathbf{x}_k|\boldsymbol{\mu}, \mathbf{Y}_k\right) p\left(\boldsymbol{\mu}\,|\mathbf{Y}_k\right)\, d\boldsymbol{\mu}\, d\mathbf{x}_k \\ &= \int p\left(\boldsymbol{\mu}\,|\mathbf{Y}_k\right) \underbrace{\left[\int \mathbf{x}_k\, p\left(\mathbf{x}_k|\boldsymbol{\mu}, \mathbf{Y}_k\right)\, d\mathbf{x}_k\right]}_{\hat{\mathbf{x}}_k(\boldsymbol{\mu})} d\boldsymbol{\mu}\end{aligned}$$

where the term in the square brackets denotes the state estimate for a fixed parameter value. The above integral can be further simplified upon discretization of the parameter space, and the unknown probability density function can be obtained by means of the Bayes rule

$$p\left(\boldsymbol{\mu}_j \middle| \mathbf{y}_k\right) = \frac{p\left(\mathbf{y}_k \middle| \boldsymbol{\mu}_j\right) p\left(\boldsymbol{\mu}_j \middle| \mathbf{y}_{k-1}\right)}{\sum_{i=1}^{n_\mu} p\left(\mathbf{y}_k \middle| \boldsymbol{\mu}_i\right) p\left(\boldsymbol{\mu}_i \middle| \mathbf{y}_{k-1}\right)} = \tilde{w}_k^j \tag{14.2}$$

which essentially generates the weights assigned to each parameter sample j for $j = 1, \ldots, n_\mu$. The parameter dynamics are not dictated by a random walk model but are governed by the Covariance Matrix Adaptation—Evolution Strategy (CMA-ES) [4], which is invoked upon exceedance of the particle degeneracy threshold.

14.3 Results

The considered case study for the demonstration of the proposed algorithm consists of the wing-box panel shown in Fig. 14.1. The panel consists of a 3mm thick plate which is supported by two C-channel ribs at the two edges running in x direction and further stiffened by two angle section stringers, whose position is indicated by the dashed lines. The material properties

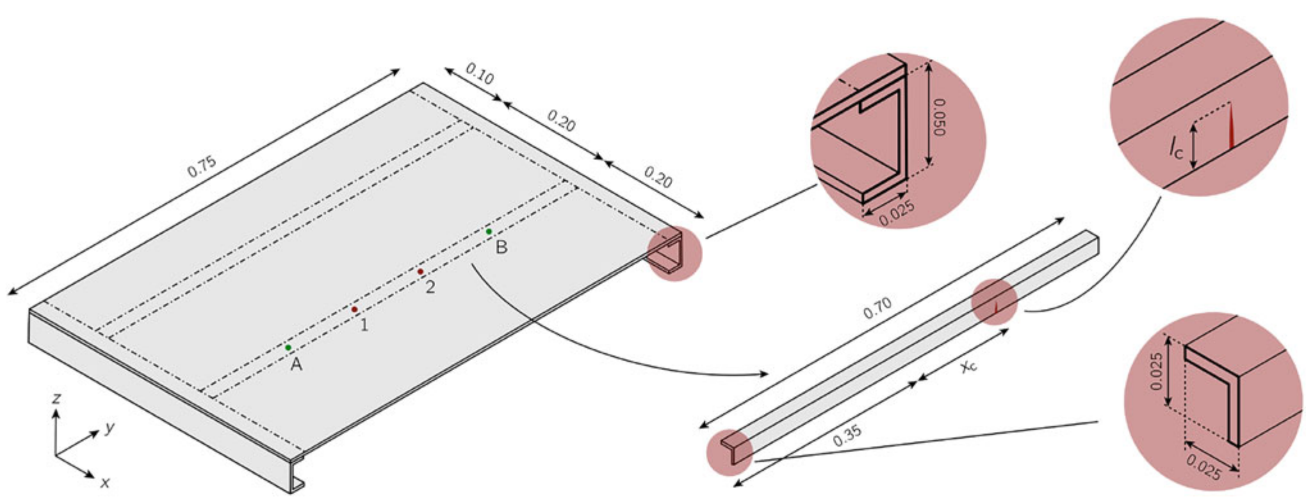

Fig. 14.1 Schematic representation of the wing-box panel geometry and crack parameter definitions; the sensing points are indicated by the red marks

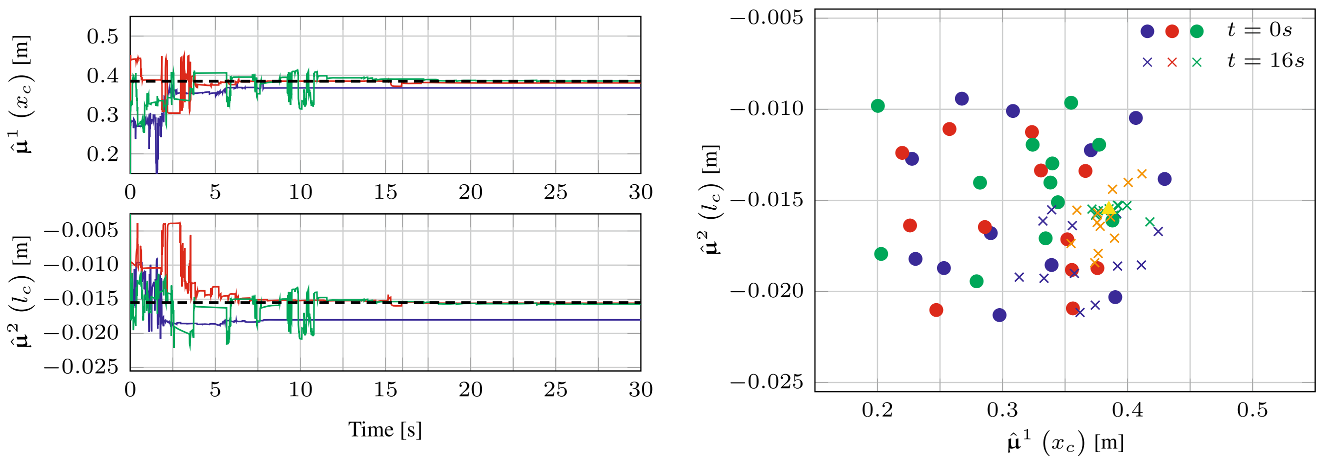

Fig. 14.2 Parameter estimates from three different runs depicted in red, blue, and green; the target values are designated by (left) black dashed lines and (right) a yellow triangular mark

are assumed to be linear elastic with $E = 73$ GPa, ν =0.3, and density ρ =2700 kg/m^3. A 2% proportional damping is further introduced to the first and fourth vibration modes, and the vibration response is measured at the red sensing points. In order to enable the online performance of estimation, the inverse problem is solved using a parametrized Reduced-Order Model (pROM), which is constructed according to the work presented in [5], and consists of 15 basis vectors. The crack, which is placed in the first stringer, is parametrized with respect to the position x_c from the middle point of the stringer in y direction, which ranges between −0.2 and 0.2 m from the midpoint of the stringer or between 0.15 and 0.55 in the cartesian coordinate y, as well as with respect to the length $l_c \in [0.0025, 0.0225]$. The vibration response of the panel is tested in operational conditions, in which the dynamics are assumed to be driven by a uniform pressure, whose magnitude is a Gaussian white noise process, applied on the plate in the direction of z axis.

The estimated time histories of the sought after parameters for three different runs of the algorithm are shown in Fig. 14.2, not only in terms of the time histories, for a simulation period of 30 seconds, but also in terms of the parameter samples between different simulation instances. It is observed, from both types of figures, that after 5 seconds of the simulation time, the parameter estimates have moved quite close to the target values and remain almost completely unchanged after the first 15 seconds.

14.4 Conclusions

A hierarchical technique for the joint estimation of state, input, and unknown system parameters of dynamic systems is presented in this chapter. The proposed scheme is based on the decomposition of the input–state–parameter estimation problem into two levels in which the input and state are jointly estimated, while the parameters, whose dynamics are governed by an evolution strategy, are estimated in a second level. The proposed approach avoids the augmentation of the state and features a decaying in time parameter variance, which is a fundamental convergence requirement for kernel-based methods.

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