General Introduction to IOL Calculation

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Models

The law of refraction was defined in 1618 by Willebrord van Roijen Snell:

$$n_1 \cdot \sin \delta_1 = n_2 \cdot \sin \delta_2$$

where refractive indices = n_i and angle of incidence/reflection = δ_i. The application of Snell's law to the refractive surfaces enables, in principle, the calculation of any optical system (ray tracing). Unfortunately, this was not practical at the time, because the resulting equations could not be solved analytically without powerful computers due to the enormous computational power required. Carl Friedrich Gauss simplified this for paraxial space (cases of very small angular incidence) by replacing the sinusoidal functions with the angle itself. This made it much easier to calculate and introduces the familiar terms such as focal length, principal focal point, principal plane etc.

A further simplification is the approximation of real lenses by modelling them as infinitely thin lenses with a fictitious refractive index.

The Gullstrand Eye

The Gullstrand eye model was developed by the Swedish ophthalmologist and Nobel Prize winner Allvar Gullstrand and is a simplified eye model with standardized optical data (refractive indices, refractive values, spherical refractive surfaces, etc.) with which calculations can be made for optical systems/aids such as contact lenses and intraocular lenses [1]. All "classical" IOL formulas are based on the Gullstrand model.

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Liou and Brennan eye model

In contrast to the Gullstrand eye model, this newer model uses four aspherical surfaces and a gradient refractive index of the lens [2]. It offers a considerably more precise modelling ability but cannot be handled in its entirety with Gaussian optics. Computer-aided ray tracing is required for this purpose.

Historical development

With the advent of acoustic length measurement for the eye, formulae were developed to predict the refraction of the pseudophakic eye, before surgery. The first-generation formulae [3–6] were similar in that they all worked with Gaussian optics where lenses were assumed to be infinitely thin and the axial position of the IOL was assumed to be constant. These formulae were improved over several generations, but two input parameters, namely axial length and radius of curvature of the corneal anterior surface, must be taken into account in all formulae.

Sources of error

Axis length

The first generation of axial length measurements were made using ultrasound. Acoustic measurement with A-scan sonography, mostly used in the 10 MHz band and in the applanation mode, has an average tolerance of $\approx$ 0.2 mm. This corresponds to 0.5 dpt at eyeglass level for a normal eye. Significantly better results could be achieved by immersion A-scan because the indentation of the cornea as a source of error is eliminated. In small eyes (and in children in particular) the effect of measurement error is significantly bigger.

For most cases, acoustic axial length measurement has been surpassed by devices that use optical axial length measurement, such as the Zeiss IOL-Master introduced in 1999/2000. The measurement tolerance is reduced to 0.03–0.05 mm which corresponds to 0.15–0.2 dpt at eyeglass level. Newer methods such as OLCR (optical low coherence reflectometry) and ssOCT (swept source OCT) reduce the tolerances even further to $\approx$ 0.01 mm, increase the measurability on very turbid media and provide measurement data for all sections of the optical path, which is advantageous when newer algorithms are used.

Cornea

Normally, the corneal refractive power is estimated from paraxially measured local radii of the anterior surface. Some assumptions are hidden here, namely those of a sphere or minimal prolate asphere and a constant ratio of anterior and posterior surface radii. Measurement errors, asphericity and anterior or posterior surface ratio can all cause an error of $\approx$ 1/8 dpt [7]. The moisture of the patient's eye and fixation can also have dan effect on measurement accuracy. In device comparisons, corneal measurement accounts for more than 75% of the total refractive difference between the devices.

IOL position

The prediction of the axial position of the lens is the largest single source of error in IOL calculation [7]. Considerable improvements can be achieved by providing information on the partial distances of the optical path [8, 9]. In the near future, algorithms using ssOCT image data to determine the lens equator can be expected to provide further improvements, especially for short eyes.

Refraction

The postoperative refraction determination is a subjective procedure in which human factors of the examinee and examiner have an impact on the results. In addition, refraction can only be measured in discrete intervals. Distance, ambient light and other factors can also influence the results. Higher-order aberrations cause device dependence and overall less precisive values. The actual tolerance of refraction of pseudophakic patients ranges from 0.18 to 0.39 dpt, depending on the source and there is a strong dependence on visual acuity and type of IOL [9].

Formula error

The approximations of the Gaussian space and the infinitely thin lens are inaccurate for the human eye, because not all optical surfaces are rotationally symmetrical, large aperture angles and curvatures exist, and the lens thickness is not negligible. A major source of error is the corneal model. Using the Gullstrand model, the refractive power of the cornea is overestimated by ≈ 0.5 dpt, using the American keratometry indices, it is overestimated by more than 1.2 dpt. The estimation of the "effective lens position" (ELP, a fictitious value which should not be confused with the measured postoperative anterior chamber depth) is done by using the preoperatively measured parameters; axial length and keratometry values (Holladay, Hoffer Q, SRK/T) or axial length and anterior chamber depth (Haigis).

The ELP is usually significantly deeper than the actual IOL position. Its implicitly includes factors that have nothing to do with the geometric position of the IOL, such as refractive indices (corneal model), pupil size, decentration and tilting of the IOL, pseudoacocommodation, asphericity of cornea and lens, atypical ratio of anterior to posterior eye segment, surgical technique, etc. This compensates for systematic errors in the ELP by taking different quantities into consideration, which is correct on average but can be occasionally incorrect when evaluated on an individual basis. If ray tracing is used instead of Gaussian optics, these adjustments become less important, but here too, the calculation accuracy is affected by the modelling of the individual optical surfaces and by measurement errors, centration, and tilt.

Formulae

First generation

The first-generation formulae mentioned above shared the problem that the effective position of the lens was assumed to be constant. This meant that the refractive power was calculated as much too high for short eyes and too low for long eyes. Therefore, these formulae are obsolete.

Second generation

The unsatisfactory results of the early formulae led to the development of a purely empirical formula that established a relationship between IOL power, axial length and corneal power ("k readings") via a bivariate linear regression [10]. To adjust the prediction error, an offset "A" was used, which can still be found today on almost every lens package and is supposed to describe specific properties of the lens. The formula is:

$$P_{IOL} = A - 2.5 \cdot AL - 0.9 \cdot K$$

This formula can only achieve good results if the eye is statistically similar to the "standard eye". Myopic and hyperopic eyes as well as deviating anterior segments in particular can result in extremely large errors.

Third generation

The first published formula of the so-called third generation was the Holladay formula [11]. For the first time, a formula was presented that provided useful results for almost all eyes except for very long axial lengths, depending on the correct input data. The systematic errors are small for axial lengths less than 25 mm and are almost negligible when no other anomalies are present, especially for steep and flat corneal radii. Adjustment is done by a "surgeon factor" (SF), which is supposed to describe the axial position of the IOL in relation to the cornea. The SRK authors followed suit and developed a similar formula: SRK/T [12]. At the same time, the authors declared their earlier statistical formulae obsolete. In 1993 the Hoffer Q-formula was introduced though they all have a common weakness in very high myopic eyes (see model error above). Another common feature is the adjustment via a single offset (sf, A, pACD), which requires at least 50 refractions per lens model in order to be optimized.

Haigis [13] pursued a somewhat different approach with a modification of the Gernet formula. He used axial length and anterior chamber depth as predictors for ELP estimation. Thereby certain systematic errors were avoided. The adjustment is done by three variables a_0, a_1, a_2 (offset, coefficients for axis length and anterior chamber depth), but requires at least 200 data sets per IOL type. The Haigis formula is somewhat more robust than its American counterparts and for axial lengths over 26 mm it is superior to the American formulas.

Newer approaches

Multivariate formulas

Based on our clinical experience, the Holladay 2 formula, with its numerous coefficients, has no advantages over the classical ones, even if the optically measured lens thickness and corneal diameter are additionally included.

Very good results for most eyes except high hyperopic ones (see Sect. 27.1) are obtained with the Barrett Universal II formula [14]. Although the lens thickness is processed, it has, as with Holladay 2, almost no effect on the result.

The Kane formula [15] is also said to be superior to the classical ones, but the data published so far is so sparse that, apart from statistical significance, there is no clinical relevance. However, preliminary results of our own research indicate that the Kane formula could consistently deliver very good results even with hyperopic eyes and is close to the ray tracing results. In particular, the optically measured lens thickness is processed appropriately. Unfortunately, this formula is not published either.

Thick Lens formulas

Gaussian optics with "thick lenses", i.e. those in which the cornea and lens are not approximated by a single infinitely thin surface, have advantages, especially in the case of abnormal corneas or those whose front-to-back surface ratio deviates from what is typical. In addition, well-known modelling errors are avoided. The best-known representative in this group is the Olsen formula which is either pre-installed in Gaussian optics on biometric devices or available "stand-alone" in the PhacoOptics software.

Of course, this approach requires an exact measurement of both curvatures of the cornea.

The general lens formula can be adapted easily "do it yourself" to a "thick cornea", for that see Sect. 27.1. The simplified variant of the Haigis formula provides results that are at least equivalent to the Kane or Olsen formula, but the re-parameterization of the constants is quite complex.

Ray tracing

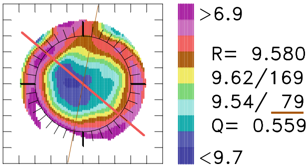
Ray tracing or beam tracing is nothing more than the application of Snell's law at all optical boundaries. In the optical industry such calculations have been standard for decades. The angles and refractive indices must be known for each interface, which also includes internal information from IOL manufacturers. Different devices can be used as data source, e.g. Okulix can import topography/tomography from devices of different manufacturers. This is important for abnormal corneas, for example after refractive surgery, and provides valuable additional information, e.g. for the selection of the individual best IOL asphericity. The quality of the imported data is absolutely crucial for the quality of the calculation. In comparison to the classical formulae, a greater amount of data and more complex data are included in the calculation (Fig. 1).

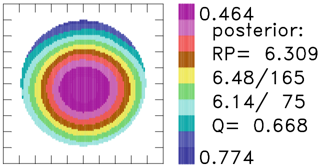
"Artificial intelligence"

A new approach is to calculate the refractive power without any formula at all based on pattern recognition by a software that has been trained with large amounts of data from past results. This can be realized with the "Hill Radial Basis Function". Unfortunately, only axial length, corneal radii and anterior chamber depth are included in the calculation. Other inputs such as lens thickness and corneal diameter

Patient: 860234 OS 2019/11/22

Topo–Radius [mm] Pachy [mm]





OS

22/11/19

R1= 9.62mm

R2= 9.54mm 0.06D/139.9°

RP= 6.31mm

AL=26.98mm –>26.96mm

ACDP=3.08mm, LT=3.58mm

Tomey OA–2000

HOYA: Vivinex iSert XY1/XC1
ACD=4.29mm

	[parax]	bf pup 2.5	
22.00 –>	[1.03]	0.98	(1.01/ –0.06/ 50)
22.50 –>	[0.65]	0.60	(0.63/ –0.06/ 50)
23.00 –>	[0.31]	0.26	(0.29/ –0.06/ 50)
23.50 –>	**[–0.07]**	**–0.12**	(–0.09/ –0.06/ 50)
24.00 –>	[–0.45]	–0.50	(–0.47/ –0.06/ 50)
24.50 –>	[–0.83]	–0.87	(–0.85/ –0.06/ 50)
25.00 –>	[–1.22]	–1.26	(–1.23/ –0.06/ 50)
25.50 –>	[–1.56]	–1.60	(–1.57/ –0.06/ 50)

Bausch&Lomb: enVista MX60
ACD=4.47mm

	[parax]	bf pup 2.5	
22.00 –>	[1.35]	1.17	(1.20/ –0.06/ 50)
22.50 –>	[0.86]	0.70	(0.72/ –0.06/ 50)
23.00 –>	[0.50]	0.34	(0.37/ –0.06/ 50)
23.50 –>	**[0.14]**	**–0.03**	(0.00/ –0.06/ 50)
24.00 –>	[–0.22]	–0.39	(–0.36/ –0.06/ 50)
24.50 –>	[–0.58]	–0.76	(–0.73/ –0.06/ 50)
25.00 –>	[–1.00]	–1.18	(–1.15/ –0.06/ 50)
25.50 –>	[–1.36]	–1.54	(–1.52/ –0.06/ 50)

J&J: Sensar AAB00
ACD=4.24mm

	[parax]	bf pup 2.5	
21.50 –>	[1.32]	1.11	(1.14/ –0.06/ 50)
22.00 –>	[0.94]	0.73	(0.76/ –0.06/ 50)
22.50 –>	[0.55]	0.34	(0.37/ –0.06/ 50)
23.00 –>	**[0.17]**	**–0.05**	(–0.02/ –0.06/ 50)
23.50 –>	[–0.20]	–0.42	(–0.39/ –0.06/ 50)
24.00 –>	[–0.59]	–0.81	(–0.78/ –0.06/ 50)
24.50 –>	[–0.99]	–1.22	(–1.19/ –0.06/ 50)
25.00 –>	[–1.37]	–1.60	(–1.58/ –0.06/ 50)

J&J: Tecnis ZCB00/ZMB00
ACD=4.51mm

	[parax]	bf pup 2.5	
22.00 –>	[1.26]	1.28	(1.30/ –0.06/ 50)
22.50 –>	[0.89]	0.91	(0.93/ –0.06/ 50)
23.00 –>	[0.51]	0.53	(0.56/ –0.06/ 50)
23.50 –>	**[0.14]**	**0.16**	(0.18/ –0.06/ 50)
24.00 –>	[–0.25]	–0.22	(–0.20/ –0.06/ 50)
24.50 –>	[–0.65]	–0.62	(–0.60/ –0.06/ 50)
25.00 –>	[–1.02]	–1.00	(–0.97/ –0.06/ 50)
25.50 –>	[–1.41]	–1.39	(–1.36/ –0.06/ 50)

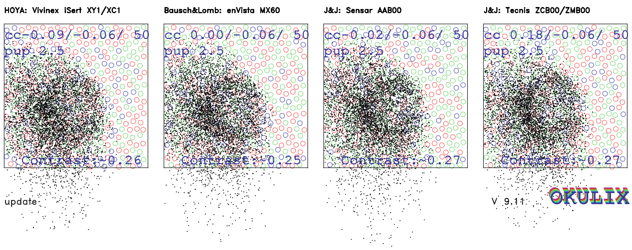


Fig. 1 Printout of the raytracing software Okulix 9.11 with a data set of the Heidelberg Anterion ssOCT. The eye has undergone myopic LASIK of approx. -8 dpt. The central flattening is clearly visible, the ratio of the anterior to posterior radii is only 0.66 and the anterior surface is an oblate asphere. In addition to the calculation of refractive power, the retinal image is also simulated with a measure. This allows an estimation which IOL would probably achieve the highest local contrast on the retina

seem to serve primarily to collect data for the operator, but do not influence the result.

Practically achievable accuracy

Firstly, on the basis of postoperative refractions, the "constants" must be adjusted such that either the mean or median error become zero. The appropriate measure of the accuracy of the method is the standard deviation or variance of the prediction error. With the classical formulae, standard deviations of the prediction error of 0.4–0.5 dpt. can be achieved (corresponds to MAE ≈ 0.3–38 dpt., ≈ 80% within 0.5 dpt.), provided that the basis conditions are well designed. With aspherical lenses and high visual acuity, the accuracy will be better, because spherical aberration is reduced and refraction precision is improved (Fig. 2).

In statistically abnormal eyes (long axis, steep or flat radii, anterior segment not corresponding to the axial length), the error of the classical formulae is considerably greater, but not in raytracing, Olsen or Kane.

If the optically measured lens thickness is included and the problems of Gaussian optics are avoided, improvements of 6–20% can be achieved depending on the algorithm, visual acuity level and lens type [9]. In the best case (high visual acuity, aspherical lens) the standard deviation is 0.29 dpt and the MAE 0.21 dpt, more than 90% of eyes are within 0.5 dpt [9].

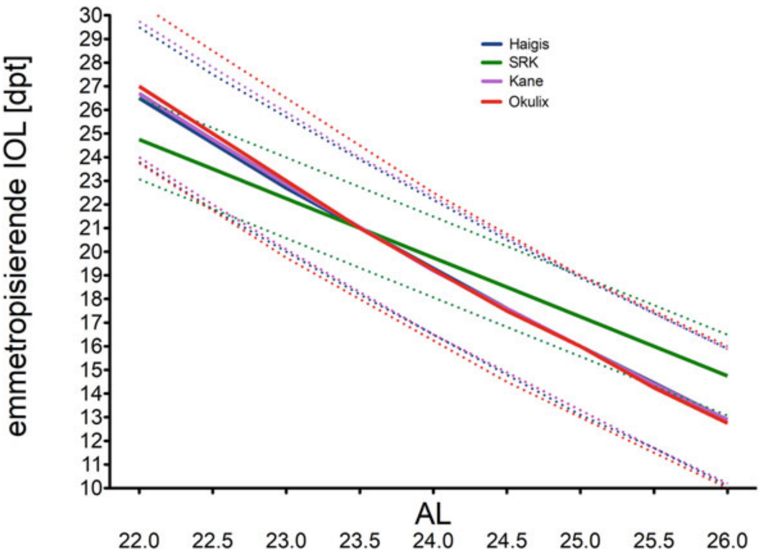


Fig. 2 Calculation of emmetropizing IOL power for axial lengths 22 - 26 mm ("normal" eyes). Solid line: Corneal radii and anterior chamber depth on the 50th percentile, dashed lines: 10th and 90th percentile. It is clear to see that the old SRK formula differs greatly, while the classical formula (Haigis), the latest formula (Kane) and raytracing (Okulix) differ only very slightly and are not clinically relevant. The biggest differences are shown in hyperopic eyes with a large anterior segment. For reasons of clarity, only one method per category has been listed

Second eye

Since the refractive prediction error of both eyes is positively correlated with r ≈ 0.5, the result of the first eye can be used to improve the calculation of the second one [16–18]. In practice, 50% of the error of the first eye is removed. If, for example, a prediction error of + 0.6 dpt occurred, the target refraction in the 2nd eye would be -0.3 dpt instead of 0.0 dpt.

In ray tracing programs, the anatomical IOL position of the 1st eye can also be used directly for the 2nd eye [8]. The optically measured anterior chamber depth is entered directly into the pACD field for the 2nd eye. The potential for improvement of both methods is ≈10% on average and significantly more for short eyes.

Author's recommendation

Optimal biometry is essential for good refractive results. Without precise input data, no IOL calculation can yield good results. ssOCT is currently the most precise and reliable method for measuring axial length and other sections of the optical path. The measurement of corneal radii or calculation of corneal refractive power remains the main source of error in diagnostics of "normal" eyes. The choice of formula is of secondary importance in the vast majority of eyes, provided that the adjustment is correct for the individual types of IOL. Larger deviations can occur with statistically abnormal combinations of values. If raytracing (our preference) is not available, the Kane formula currently seems to process the data generated by modern biometers in the most meaningful way and may have the smallest deviations at the edges of the distribution.

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