System Reliability Models with Random Shocks and Uncertainty: A State-of-the-Art Review

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Abstract Reliability evaluation is an important task in safety–critical applications. The failure of a system is generally caused by random shocks resulting from adverse events or internal degradations. This chapter thus mainly focuses on the review of system reliability models with random shocks and the uncertainty of the degradation process. In the category of system reliability models with random shocks, we review system reliability models based on five random shock models that are commonly used in Reliability Engineering, cumulative shock model, extreme shock model, run shock model, δ-shock model, and mixed shock model. In addition, three sources of variabilities, commonly discussed in the literature, can result in the uncertainty of the degradation process, which are temporal variability in the degradation process, unit-to-unit variability, and measurement error caused by imperfect instruments or imperfect inspection. In the category of system reliability model with uncertainty, we review system reliability models using stochastic degradation models in terms of three stochastic processes, Wiener process, gamma process, and inverse Gaussian process.

Keywords Reliability model · Random shocks · Uncertainty · Stochastic process

1 Introduction

Reliability is defined as the probability that a product can function properly without failure during its designed life under the designed operating conditions [1]. The failure of a system has a wide-ranging societal impact. For example, a plane from Sudan lost control on the runway while landing due to the bad weather on June

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10, 2008, which caused the death of 1 crew member and 29 passengers. In addition, random incidents in our daily life, such as collisions with vehicles, are also very common, which can influence the lifespan of the product, and even influence human life. Thus, considering the random incidents into the reliability modeling can effectively improve the accuracy of the reliability evaluation in safety–critical applications. These random incidents can exert random stresses in the system, which can be modeled by random shocks received by a system in the field of Reliability Engineering.

Generally, random shock models are classified into five groups, cumulative shock model, extreme shock model, run shock model, δ-shock model, and mixed shock model. The reliability model with random shocks is first proposed by Esary and Marshall [2], in which the shock loadings are assumed to be independently distributed. Based on this, the development of the random shock-based reliability model is further investigated in many studies [1–16]. Gut [3] proposed a cumulative shock model, where the system fails when the cumulative shock damage is larger than a preset threshold. Later, Che et al. [4] developed a reliability model with a mutually dependent degradation process and shock process. Dong et al. [5] developed a multi-component system reliability model with generalized cumulative shocks and a stochastic degradation process. The extreme shock model is first developed by Shanthikumar and Sumita [6], in which the system failure is determined by the magnitude of the shock and further studied by [4, 8]. Based on the studies [3, 6], Gut [9] investigated a mixed shock model that considers the cumulative shock model and extreme shock model, which assumed that the system fails when the magnitude of the shock is larger than a threshold or the accumulative shock damage is larger than another critical threshold. Later, other types of shock models are introduced, which are run shock model [10] and δ-shock model [11]. Their applications in the reliability estimation can be referred to studies [12–16].

With the increasing complexity of the system engineering problems, the uncertainty caused by the internal product properties or external factors has also become significant. Generally, there are three sources of variabilities [17, 18]. The first source is the temporal variability representing the inherent uncertainty in the degradation path [17]. The second source is the unit-to-unit variability. Take the battery management system in electric vehicles as an example. One battery pack consists of numerous battery cells. The degradation of one battery cell may be different from the other cells even they are manufactured from the same production line. When considering the degradation of a battery cell, it is necessary to consider the degradation differences between cells. The third source is the measurement error caused by an imperfect instrument or imperfect inspection which is the distinction between the true value and the measured value. In many cases, regardless of the precision of the instrument, the experimental data is always contaminated in the experiment, which will influence the reliability prediction accuracy of the target system. In literature, many studies have considered the three sources of uncertainty in the system reliability modeling [19–34]. Three classic stochastic processes that are commonly used to address the uncertainty are Wiener process [19, 20], gamma process [21–23], and inverse Gaussian process [24].

The rest of this chapter is organized as follows. In Sect. 2, we first introduce the definitions and applications of random shock models and then review the related work on system reliability models considering different types of random shocks. Section 3 reviews the literature on system reliability models with uncertainty in terms of different stochastic processes. Section 4 concludes this chapter.

2 System Reliability Models with Random Shocks

Typically, five random shock models are widely used in the field of Reliability Engineering, cumulative shock model, extreme shock model, run shock model, δ-shock model, and mixed shock model. These models are generally defined by the inter-arrival time between consecutive shocks and/or the damage from shocks. Section 2 is divided into two parts. Section 2.1 reviews the definitions and applications of random shock models. Section 2.2 reviews the literature on system reliability models incorporating different types of random shocks.

The following notations are defined for Sect. 2. $R(t)$ is the system reliability function by time t. $M(t)$ is the system degradation function by time t. $X(t)$ is the internal degradation function by time t. $S(t)$ is the cumulative shock damage function by time t. $N(t)$ is the total number of random shocks by time t.

2.1 Shock Model Categorization

Shock models, cumulative and extreme shock models, are initially proposed in the 1970s, to apply for predicting the system reliability in a random environment [2]. Based on these two classic models, other shocks models, such as run shock model and δ-shock model, have been developed in the early 2000s. Meanwhile, the mixed shock model is proposed, which combines two types of random shock models, such as the combination of the extreme shock model and δ-shock model. Nowadays, the mixed shock model is not limited to the combination of two types of random shocks but extends to integrating three types of random shocks in order to predicate the complex engineering system reliability.

The cumulative shock model commonly use the following equation:

$$S(t) = \sum_{k=1}^{N(t)} Y_k \tag{1}$$

where Y_k is the damage caused by k^{th} shock. The system fails when the accumulative damage $S(t)$ exceeds a pre-specified threshold. This model is applied in the situation where the system is subject to a series of random shocks. Take an example in practice. A car accident can be regarded as a random shock for the engine. For a vehicle, it is likely to have more than one accident. To assess the damage from all accidents on the engine, it is indeed to summarize these damages. When the total damage on the engine is larger than a threshold, the engine will fail. Other applications in literature, for example, Che et al. [4] regarded the contamination lock in the jet pipe servo value as one random shock that can cause wear debris on the value. The cumulative wear debris will increase with time and finally results in failure when the total wear exceeds a threshold. Dong et al. [5] predicted the reliability of micro electro-mechanical systems that withstand three different kinds of shocks, mechanical vibration, piezoelectric stimuli, and magnetic stimuli. The shock damage from different kinds of shocks can be summarized. The micro electro-mechanical systems will fail when the damage exceeds a threshold.

The extreme shock model is commonly defined as the system fails when the magnitude of any shock exceeds the given level. In other words, the system lifetime is determined by the magnitude of individual random shock. The applications of the extreme shock model are presented as follows. In Che et al. [4], because the contamination lock can fail suddenly when there is sufficient friction generated to withstand the normal actuating force, the extreme shock model is utilized to model this application. Wang et al. [7] applied the extreme shock model to model the impact load in the microelectromechanical system because the load can cause the system failure directly. Hao and Yang [8] regarded the vessel collision on the bridge as one random shock and classified the shocks as fatal and nonfatal according to their magnitude based on the extreme shock model.

The run shock model is first proposed by Mallor and Omey [10], which defined the system breaks down when there are consecutive shocks whose magnitudes are above a threshold. This model is usually applied in mechanical and electronic systems, which generally suffered from fatigue damage. Specifically, fatigue damage refers to the situation that the system is under repeated shocks above a critical threshold. It is noted that the run shock model measures the magnitude of consecutive shocks instead of the magnitude of an individual shock.

The δ-shock model is first proposed by Li and Kong [11], which defined the system failure when the inter-arrival time between two consecutive shocks less than a pre-specified threshold δ. Compared with the traditional shock models, cumulative shock, and extreme shock models, there are some phenomena that are more suitable to use the inter-arrival time to define system failure. For example, when the damage caused by random shocks is hard to be determined, it is more suitable to use the δ-shock model since it pays more attention to the shock occurrence rate instead of the individual or cumulative damage of shocks.

The mixed shock model defines the system failure caused by two or more random shock models. For example, if cumulative shock and extreme shock are considered, the system will fail when the cumulative shock loadings are larger than one threshold or the magnitude of an individual shock is larger than another threshold, whichever occurs first. Parvardeh and Balakrishnan [35] proposed two mixed shock models based on the δ-shock model. One is the combination of the extreme shock model and δ-shock model, and the other is the combination of the cumulative shock model and δ-shock model. Lorvand et al. [16] combined the extreme shock model and run shock model as a mixed shock model. The mixed shock model is not limited to the combination of two shock models. Rafiee et al. [36] developed a mixed shock model that employed extreme shock model, δ-shock model, and run shock model.

2.2 System Reliability Models with Shock Models

2.2.1 System Reliability Models with Cumulative Shock Model

Systems are generally subject to two competing risks, degradation, and random shocks. Given many research efforts have been focused on modeling the dependent relationship between degradation and random shocks. We review the literature on system reliability models with cumulative shock model, considering the dependent relationship between degradation and random shocks. In general, random shocks are commonly assumed to have two types of impacts on the system, sudden incremental jump on the system degradation, and degradation rate acceleration [4, 37–40, 43]. The cumulative shock model is usually used to describe these impacts and further employed to calculate the accumulated system degradation. To represent the sudden incremental jumps, many studies [4, 38–40, 43] described the individual shock damage as an independent and identically distributed random variable. The cumulative shock damage is the summation of individual shock damages, which is shown in Eq. (1). Other studies [37, 41, 42] applied the shock magnitude instead of the individual shock damage to $S(t)$[37]. Assumed each shock damage is linear dependent with its shock magnitude, namely, $S(t) = \sum_{k=1}^{N(t)}(\alpha W_k)$, $N(t) > 0$, where W_k is the magnitude of k^{th} shock and α is the coefficient. In general, system degradation, $M(t)$, consists of the internal degradation $X(t)$ and cumulative shock damage $S(t)$:

$$M(t) = X(t) + S(t) \tag{2}$$

The failure happens when the system degradation exceeds a critical threshold. Generally, system reliability, $R(t)$, can be modeled as:

$$R(t) = P(X(t) + S(t) < H) \tag{3}$$

where H is the failure threshold.

The system may become more susceptible because of undertaking shocks, which makes degradation increase faster. Therefore, the degradation rate will not be ideally constant and will be accelerated by random shocks. Wang and Pham [38] developed a system reliability model with multiple degradation processes and random shocks. The arrival of random shocks follows a homogeneous Poisson process (HPP). The impacts of random shocks are classified into sudden incremental jump and degradation rate acceleration. The sudden incremental jump is adopted from Eq. (1). The degradation rate acceleration is illustrated by incorporating a time-scaled factor $G\left(t,\gamma^{(i)}\right)$ into the ith degradation process, in which $G\left(t,\gamma^{(i)}\right)=\gamma_1^{(i)}N_2(t)+\gamma_2^{(i)}S(t)$, where $\gamma_1^{(i)}$ and $\gamma_2^{(i)}$ denote the impact magnitude on the degradation rate of the ith degradation process and $N_2(t)$ is the total number of nonfatal shocks by time t. The internal degradation for the ith degradation process is modeled by a basic multiplicative path function, which is $X_i\cdot\eta_i(t;\theta_i)$, where X_i is the random variable representing the unit-to-unit variability, $\eta_i(t;\theta_i)$ is the ith mean degradation path function with a parameter vectorθ_i. The ith degradation function is thus modeled as $X_i\cdot\eta_i\left(te^{G\left(t,\gamma^{(i)}\right)};\theta_i\right)+\sum_{k=1}^{N(t)}w_{ij}$, where w_{ij} is the loading from shock j in the ith degradation process. The marginal reliability function $R_i(t)$ of the ith degradation process with no fatal shock is: $R_i(t)=P\left(X_i\cdot\eta_i\left(te^{G\left(t,\gamma^{(i)}\right)};\theta_i\right)+\sum_{k=1}^{N_2(t)}w_{ij}<H_i\right)$, in which H_i is the corresponding failure threshold of the ith degradation process. The system reliability model is [38]:

$$R(t)=C(R_1(t),R_2(t),\ldots,R_m(t)\Pr(N_1(t)=0)\tag{4}$$

where C is the joint copula of the marginal reliability function and $N_1(t)$ is the total number of fatal shocks by time t.

The degradation impact on random shocks can be reflected by shock arrival frequency. Fan et al. [41] assumed the number of random shocks follow the nonhomogeneous Poisson process (NHPP) with rate $\lambda(t)$. $\lambda(t)$ is assumed to be linearly related with the current internal degradation $X(t)$:

$$\lambda(t)=\lambda_0+\beta\cdot X(t)\tag{5}$$

where λ_0 is the initial intensity of NHPP and β is the dependence factor.

Because the shocks with different magnitude have different impacts on the system, they are classified into three zones, safety zone, damage zone, and fatal zone [41]. Noting that only the shocks in the damage zone can generate damage on the system, while the fatal shock will fail the system directly, and the shocks in the safety zone have no effect on the system. The system can function if the internal degradation does not reach one threshold, the cumulative shock damage in the damage zone does not exceed another threshold, and there is no shock in the fatal zone. Thus, the reliability function is [41]:

$$R(t)=\sum_{k=0}^{\infty}P(X(t)<H_1,S(t)<H_2,N_2(t)=0|N_1(t)=k)P(N_1(t)=k)\tag{6}$$

where k denotes random shock, H_1 is the threshold for the internal degradation, H_2 is the threshold for cumulative shock damage, $N_1(t)$ is the number of shocks in the damage zone, and $N_2(t)$ is the number of shocks in the fatal zone.

Based on the study [41], Che et al. [4] carried out that the shock intensity $\lambda(t)$ is not only affected by the current internal degradation but also by the shock occurrences by time t. The intensity after k random shocks is defined as $\lambda_k(t) = (1 + \eta k)\lambda_0(t)$, where $\lambda_0(t)$ is the initial intensity influenced by the current degradation level and η is the facilitation factor. The formulation of system reliability in reference [4] is based on Eq. (4).

Moreover, a few studies did not follow the common assumption of cumulative shock models, which is the shocks follow a distribution. In reality, such an assumption may not be practical because the frequency of the shock occurrence can be determined by many factors. For example, Gong et al. [12] developed a system reliability model incorporating the influence of shocks from different sources under the cumulative shock model. The system is subject to random shocks, which come from m sources, and the probability of each source is π_i. The magnitude of shocks from each source follows a phase-type (PH) distribution. The continuous PH distribution is a probability distribution constructed by the convolution of exponential distributions. According to the property of PH distribution, the summation of independent PH random variables still follows PH distribution, which is further utilized in modeling system reliability.

Ranjkesh et al. [44] proposed a new cumulative shock model considering the dependency between shock damage and inter-arrival time, and utilized this model to predict the system reliability of civil structures, such as bridges. A parameter δ, which represents the system recover time, is set to determine the shock damage level. When the inter-arrival time between two consecutive shocks, X_k, is larger than δ, the damage level, Y_k, is defined as mild since the system may recover itself from the previous shock. When the shock time-lapse is less than δ, the damage level is defined as severe because the system does not have enough time to recover from the shock:

$$Y_k = \begin{cases} Y_{k1}, X_k \leq \delta \\ Y_{k2}, X_k > \delta \end{cases} \tag{7}$$

where Y_{k1} is the severe damage of the kth shock and Y_{k2} is the mild damage of the kth shock. The system fails when the cumulative load and severe shock damage is larger than a certain threshold. Hence, the system reliability function [44] is as follows:

$$R(t) = \sum\nolimits_{m=0}^{\infty}\sum\nolimits_{m_1=0}^{m}\binom{m}{m_1}P\left(\sum\nolimits_{k=0}^{m_1}Y_{k1} + \sum\nolimits_{k=m_1+1}^{m}Y_{k2} < H\right)$$
$$P\left(X_1, \ldots, X_{m_1} \leq \delta, X_{m_1+1}, \ldots, X_m > \delta, N(t) = m\right) \tag{8}$$

Recently, Wang and Zhu [43] proposed a shock-loading based degradation model based on the magnitude of impacts caused by random shocks on degradation processes. Random shock are grouped into fatal shocks and nonfatal shocks. They incorporated a threshold H' to measure the temporal loading level. H' is a time-dependent critical ratio, calculated by $H' = S(t)/S$, where S is the cumulative shock loading that can cause the system failure. If $H^{'} < H_0$, where H_0 is the preset critical threshold, nonfatal shocks can only cause the degradation rate acceleration. If $H^{'} > H_0$, nonfatal shocks can cause both accelerate degradation rate and sudden incremental jump. The method to model system reliability with multiple dependent degradation process incorporating the proposed shock-loading based degradation model is based on Eq. (4).

2.2.2 System Reliability Models with Extreme Shock Model

The extreme shock model defined the system failure when the magnitude of any shock exceeds the given level [4, 37–41]. From this definition, many studies classified shocks based on the magnitude of shocks and further impacts on the system. For example, Wang and Pham [38] considered two types of shocks in the model, fatal and nonfatal shocks. Fatal shocks can fail the system directly, while nonfatal shocks can accelerate the degradation processes. Fan et al. [41] modeled the random shocks into three zones according to their magnitude, fatal, damage, and safety zones. Song et al. [40] classified random shocks into different sets according to their function, size, and affected components. Each component in the system has its own shock set, which indicates that only when the shock belongs to the shock set of that component, the damage will exist. Consider the magnitude of the kth shock that belongs to the jth shock set impacting component l, $W_{l,j,k}$, follows a normal distribution $W_{l,j,k} \sim N\left(\mu_{W_{l,j}}, \sigma^2_{W_{l,j}}\right)$, the reliability function of component l, $R_l(t)$, considering an extreme shock that belongs to the jth shock set is:

$$R_l(t) = P\left(W_{l,j,k} < H_l\right) = \phi\left(\frac{H_l - \mu_{W_{l,j}}}{\sigma_{W_{l,j}}}\right) \, for\, l = 1, 2, \ldots, n,\ j \epsilon \phi_l \tag{9}$$

where H_l is the failure threshold of the component l, $\phi(\cdot)$ is the cumulative density function (CDF) of a standard normally distributed function, and ϕ_l is the shock set for component l.

Some studies assumed that random shocks can be classified into fatal shocks and nonfatal shocks [38, 43, 45]. In this case, fatal shocks are extreme shocks. These studies [38, 43, 45] assumed that random shocks follow HPP with rate λ. The probability that a shock that could be fatal to the system at time t is $p(t)$. Thus, fatal shocks follow NHPP with rate $\lambda p(t)$. Rafiee et al. [39] proposed a system reliability model considering degradation and random shocks. The degradation rate is assumed to be changed by shocks because the system may become vulnerable. The first shock that leads to the degradation rate change is defined as a trigger shock, denoted as the kth shock. The overall degradation is represented as a linear path function: $X(t) = \beta t + \varphi + \varepsilon$, where φ is the initial degradation, β is the degradation rate, and ε is the measurement error. Considering the impact of the trigger shock, $X(t)$ is modeled as:

$$X(t) = \begin{cases} \beta_1 t + \varphi + \varepsilon, k > N(t) \\ \beta_1 T_k + \beta_2(t - T_k) + \varphi + \varepsilon, k \leq N(t) \end{cases} \tag{10}$$

where k is a random variable, T_k is the arrival time of the kth shock, β_1 is the initial degradation rate, and β_2 is the changed degradation rate. The reliability function with extreme shock model in reference [39] is developed based on two conditions, no shock occurs by time t and at least one shock occurs by time t.

Eryilmaz and Kan [46, 47] considered there are changes of distributions of the shock magnitude to propose a system reliability model. These models are preferable to use in the conditions that there is an urgent or a dramatic change in environments, which can cause a larger shock in the system. The change point is assumed to follow a certain distribution, for example, a geometric distribution with a given probability mass function. The reliability function can be derived based on the proposed extreme shock model.

2.2.3 System Reliability Models with Run Shock Model

System reliability models with run shock model is discussed in a few studies. For example, Gong et al. [48] assessed the reliability of the system under a run shock model with two thresholds H_1 and H_2, where $H_1 < H_2$. There are two cases that cause system failure: (1) more than k_1 successive shocks with the magnitude above H_1; (2) more than k_2 successive shocks with the magnitude above H_2. The inter-arrival time and the magnitude of shocks are modeled by PH distribution. Compared with the classic run shock model, adding one more threshold helps determine the severity of a shock. Ozkut and Eryilmaz [13] proposed a Marshall-Olkin run shock model to predict system reliability. The system is assumed to have two components subject to three sources of shocks. In this run shock model, the system failure occurs when k critical shocks arrive in succession and these shocks should come from the same source. Later, Wu et al. [14] proposed an N-critical shock model based on Markov renewal process. The run shock model is a special case of the developed model when N shocks occur consequently.

2.2.4 System Reliability Models with δ-shock Model

Wang and Peng [15] studied a generalized δ-shock model with two types of shocks, type 1 and type 2, with the recovery times are δ_1 and δ_2, respectively. Assume the arrival of shocks follow a HPP with rate λ, and the probability of being type 1 is p and type 2 is $q = 1 - p$. They also assume: (1) if either type of shock arrives during the recovery time, the system will fail; (2) if no shock occurs during the recovery time, the system will be recovered from the damage and shown as good as new. The reliability function of the δ-shock model is shown as [15]:

$$R(t) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P(T > t, N_1(t) = n_1, N_2(t) = n_2) \tag{11}$$

where $N_i(t)$ is the number of type i shock by time t[15]. Discussed several cases to compute Eq. (11). First, there is no shocks occurred by time t. Second, there is one type of shock occurred by time t. Third, there are two types of shocks occurred by time t, in which the authors proposed a reliability function with the generalized δ-shock model.

Poursaeed [49] developed a new δ-shock model with two thresholds δ_1 and δ_2, where $1 \leq \delta_1 < \delta_2$. When the time interval is smaller than δ_1, the system fails. When the time interval falls between δ_1 and δ_2, the probability of system failure is θ. When the time interval is larger than δ_2, the shocks do not cause damage to the system. Thus, the reliability function based on the proposed δ-shock model is:

$$R(t) = P\left(T_{\delta_1,\delta_2,\theta} > t\right) = P\left(\sum_{i=0}^{L_1} Y_i + \sum_{i=1}^{L_2} Z_i + W > t\right) \tag{12}$$

where $T_{\delta_1,\delta_2,\theta}$ is the system failure time, L_1 is the number of intervals in $[\delta_2, \infty)$, and L_2 is the number of intervals in (δ_1, δ_2). X_i is the time intervals between the ith and $(i+1)$th shock. $Y_i \sim X|X > \delta_2$,$Z_i \sim X|\delta_1 < X < \delta_2$ for $i = 1, 2, \ldots,$ and $W \sim X|\delta_1 < X \leq \delta_2$ or $W \sim X|X \leq \delta_1$, where $X \sim Y$ indicates that X and Y follow the same distribution.

Typically, the arrival of random shocks is modeled by HPP, in other words, the inter-arrival time between two consecutive shocks follows an exponential distribution, which is commonly adopted in many studies. HPP has the advantages of the simplicity of mathematical expressions; however, the limitation also exists. For example, Liu [50] pointed out that HPP can only fit the data which is equal-dispersion, that is, the mean should be equal to the variance. However, the mean and the variance of the shock inter-arrival time are not equal in most cases. Also, HPP can only represent the situation when the hazard rate is constant, while the rate can be either increasing or decreasing in the real life. Thus, a reliability model subject to degradation and random shocks is developed under the assumption that the inter-arrival time of shocks follows Weibull distribution. There are two advantages over HPP. First, Weibull distribution can model the under-dispersion data and over-dispersion data besides the equal-dispersion data. Second, Weibull distribution can specifically simulate the impact caused by related system failures. Under this assumption, the probability that n shocks occur is: $P(N(t) = n) = \sum_{j=n}^{\infty}\left[(-1)^{j+n}\left(\frac{t}{\lambda}\right)^{cj}\alpha_j^n\right]/\Gamma(cj+1), n = 0, 1, 2, \ldots,$ where $\Gamma(\cdot)$ is the Gamma function λ is the scale parameter of Weibull distribution, c is the shape parameter of Weibull distribution, $\alpha_j^0 = \Gamma(cj+1)/\Gamma(j+1)$, and $\alpha_j^{n+1} = \sum_{m=n}^{j-1}\alpha_m^n\Gamma(cj - cm + 1)/\Gamma(j - m + 1)$. Hence, the reliability function under the δ-shock model is expressed as [50]:

$$R(t) = \sum\nolimits_{n=0}^{\infty} P(\min(B_1, B_2, \ldots, B_n) > \delta, X(t) < H | N(t) = n) P(N(t) = n) \tag{13}$$

where B_k is the inter-arrival time between the $(k-1)$th shock and kth shock, $X(t)$ is the total degradation value, and H is the threshold. All B_k are independent and follow Weibull distribution.

Eryilmaz and Bayramoglu [51] investigated the system reliability under the δ-shock model by assuming the interarrival time follows a uniform distribution. This assumption is useful when the first-order effects of random changes are important to the result; in other words, when the difference between deterministic models and stochastic models is critical. Eryilmaz [52] studied the reliability properties of a discrete-time shock model. The inter-arrival time is assumed to follow a geometric distribution with a mean $1/p$.

In addition, the inter-arrival time in the δ-shock model in most studies is assumed to be independently and identically distributed. Some studies considered the inter-arrival time are dependent. For example, Eryilmaz [53] proposed a reliability model under the δ-shock model when the occurrence of shocks follows Polya process. In this case, $P\{N(t) = n\} = \binom{\alpha+n-1}{n}\left(\frac{t}{t+\beta}\right)^n\left(\frac{\beta}{\beta+t}\right)^\alpha$, for $n = 0, 1, \ldots,$ where α and β are parameters. In other words, $N(t)$ follows a negative Binomial distribution with parameters α and $\beta/(\beta + t)$. The reliability function is derived as [53]:

$$R(t) = P(T > t) = \left(\frac{\beta}{\beta + t}\right)^\alpha \sum\nolimits_{n=0}^{[\frac{t}{\delta}]} \binom{\alpha + n - 1}{n} \left(\frac{t - n\delta}{t + \beta}\right)^n \tag{14}$$

where $[x]$ is the integer part of x for $t \geq 0$.

Some studies consider the inter-arrival times are nonidentical. For example, Tuncel and Eryilmaz [54] described the inter-arrival times as a proportional hazard rate process which can apply to the situation that the inter-arrival time is stochastically increasing or decreasing. The reliability function of the interarrival time X_i is thus expressed as:

$$R_i(t) = P(X_i > t) = \left(\overline{G}(t)\right)^{\alpha_i}, \alpha_i > 0 \tag{15}$$

where $\overline{G}$ is the reliability function of a baseline random variable.

2.2.5 System Reliability Models with Mixed Shock Model

Parvardeh and Balakrishnan [35] proposed a mixed shock model which is the combination of extreme shock model and δ-shock model (extreme-δ mixed shock model). The system fails when the magnitude of any shock is larger than a threshold γ or the inter-arrival time between two consecutive shocks is smaller than another threshold

δ. In the extreme shock model, the time lapse between the $(k-1)$th shock and the kth shock X_k has the marginal distribution F and the magnitude of the kth shock, Z_k, has the marginal distribution G. X_k and Z_k are assumed to be dependent and has a joint distribution $\mathcal{H}$. The reliability function is derived as [35]:

$$
\begin{aligned}
R(t) = (F(\delta) - F(t)) I_{[0,\delta)}(t) \\
+ \sum\nolimits_{n=2}^{\infty}\left[\overline{F}(\delta) - \overline{\mathcal{H}}(\delta, \gamma)\right]^{n-1} \int_0^{\infty} P\left(S_{n-1}^* > t - x\right) dF(x) \\
+ \overline{\mathcal{H}}(\max\{\delta, t\}, \gamma) - \sum\nolimits_{n=2}^{\infty}\left[\overline{F}(\delta) - \overline{\mathcal{H}}(\delta, \gamma)\right]^{n} P(S_n^* > t)
\end{aligned}
\tag{16}
$$

where $\{S_n^*, n \geq 1\}$ is a renewal process with the time between successive renewals whose CDF is $F_{\delta,\gamma}(x) = (\mathcal{H}(x, \gamma) - \mathcal{H}(\delta, \gamma))/(G(\gamma) - \mathcal{H}(\delta, \gamma)), x > \delta$.

Lorvand et al. [55] proposed another extreme-δ mixed shock model by setting a new threshold δ_2, which can switch the system to a lower partially working state. Thus, there are three situations that can cause system failure: (1) the classic δ-shock model; (2) the classic extreme shock model; (3) when k out of interarrival times between two successive shocks are in (δ_1, δ_2). The extreme-δ mixed shock model has also been investigated by studies [56, 57].

Some studies considered the mixed shock models in the combination of more than two shock models. For example, Rafiee et al. [36] discussed the system failures can be caused by the internal degradation, or fatal shocks, in which the shock falls into any of three shock models, run shock model, extreme shock model, and δ-shock model. The system reliability function without degradation-based failure is expressed as [36]:

$$
R(t) = \sum\nolimits_{m=0}^{\infty} P(S > N(t), X(t) < H | N(t) = m) P(N(t) = m) \tag{17}
$$

where S is the number of fatal shocks, and H is the threshold of degradation failure.

Moreover, some studies considered the degradation rate and failure threshold can be changed multiple times as the changes of three mixed shocks patterns [59]. Jiang et al. [58] assumed the failure threshold will decrease as the increase of shocks. Specifically, when the inter-arrival time is smaller than δ or there are m shocks whose magnitude is larger than γ, the threshold will decrease. Zhao et al. [60] incorporated the system self-healing mechanism into random shock modeling to predict the system reliability. The system is assumed to have two stages by incorporating a change point, which is defined as the time when the cumulative number of valid shocks exceeds a threshold. Before the change point, the system is capable of self-healing from shocks. However, the system cannot recover from the damage after the change point.

3 System Reliability Models with Uncertainty

Generally, three sources result in the uncertainty of complex engineering systems, temporal variability, item-to-item variability, and measurement error [19–28]. Temporal variability represents the inherent uncertainty changed with the degradation progression over time [17]. Item-to-item variability refers to the diversity of degradation paths induced by manufacturing processes and service conditions. Measurement error represents the difference between the observed degradation data and the true degradation data [19]. This error is mainly due to the imperfect instrument, random environment, or imperfect inspection which is inevitable in the measurement process. In this section, we review system reliability models with uncertainty based on stochastic process, Wiener process [19, 20], gamma process [21–23], and inverse Gaussian process [24]. Typically, Wiener process is utilized when the degradation is non-monotonic, while gamma process and inverse Gaussian process are used to analyze the monotonic degradation processes [21, 61, 62].

The following notations are defined in this section. $Y(t)$ is the measured degradation by time t. $X(t)$ is the true degradation by time t. $\varepsilon(t)$ is the measurement error by time t.

3.1 System Reliability Models Based on Wiener Process

In Sect. 3.1, we define the following notations. $X(0)$ is the initial degradation value. θ is the drift parameter of Wiener process. δ_B is the volatility parameter of Wiener process. $B(t)$ is the standard Brownian motion.

In general, a Wiener-based degradation model is expressed as [18]:

$$X(t) = X(0) + \theta t + \sigma_B B(t) \tag{18}$$

Si et al. [18] considered three sources of uncertainty in Wiener process to reliability estimation. Stochastic dynamics of the degradation process is represented by $\delta_B B(t) \sim N\big(0, \delta_B^2 t\big)$, $t > 0$. Item-to-item variation is illustrated by assuming parameter θ as a random variable that follows a specific distribution. In most cases, θ is assumed to follow a normal distribution, denoted as $\theta \sim N(\mu_\theta, \sigma_\theta^2)$, which is s-independent of $\{B(t), t \geq 0\}$. System reliability models developed in studies [19, 20, 26] are also based on Eq. (18).

According to the property of Wiener process, the first passage time exceeding the critical threshold follows an inverse Gaussian distribution [63]. Thus, the probability density function (PDF) of the lifetime T is:

$$f_T(t) = \frac{H}{\sqrt{2\pi t^3 \sigma_B^2}} exp\left(-\frac{(H-\theta t)^2}{2\sigma_B^2 t}\right), t > 0, \theta > 0 \tag{19}$$

where H is a threshold. Then, system reliability models can be obtained.

Equation (18) is the general function of the linear Wiener process. For complex systems, it is necessary to take the degradation nonlinearity into account; thus, Liu and Fan [64] used a nonlinear Wiener-based degradation model to model the degradation process, $X(t) = X(0) + \theta\Lambda(t;\gamma) + \sigma_B B(t)$, where $\Lambda(t;\gamma)$ is a nonlinear function with unknown parameter γ. In their study [64], $\Lambda(t;\gamma)$ is represented by a power function, t^{γ}. Zheng et al. [65] developed a generalized form of Wiener process: $X(t) = X(0) + f(t;\theta_1)^T\theta_2 + \sigma_B B(t)$, where $f(t;\theta_1)^T$ is a n-dimensional vector with a group of fundamental functions, θ_1 and θ_2 are parameter vectors, and $\theta_2 \in \mathcal{R}^n$. The temporal variability and the item-to-item variation are represented by $B(t)$ and θ_2, respectively. Moreover, Wiener-based degradation model can be used to model measurement error $\varepsilon(t)$ following a normal distribution, Eq. (18) will be [64]:

$$Y(t) = X(t) + \varepsilon(t) \tag{20}$$

The widely used assumptions [19, 20, 26–28] to model measurement error are: (1) $\varepsilon(t)$ follows a normal distribution; (2) all measurement error terms are mutually independent and independent with the true degradation. Similarly, by using the concept of the first passage time, the lifetime of a system is modeled as [65]:

$$T = \inf\{t : X(t) \geq w | X(0) < w\} \tag{21}$$

where w is the predetermined degradation-based failure threshold. Then, system reliability models and remaining useful life models can be obtained [64, 65].

3.2 System Reliability Models Based on Gamma Process

A continuous-time stochastic process $\{X(t), t \geq 0\}$ is defined as a gamma process with the shape function $\eta(t)$ and scale parameter θ if the following properties can be satisfied [66]:

(1) $P(X(0) = 0) = 1$;
(2) The increment $\Delta X(t_1, t_2) = X(t_2) - X(t_1)$, for all $t_2 > t_1 \geq 0$, follows a gamma distribution with shape parameter $\Delta\eta(t_1, t_2) = \eta(t_2) - \eta(t_1)$ and scale parameter θ;
(3) The increments are independent.

Gamma process can be used to represent the degradation path function with the uncertainty, for example, temporal variability [21–23]. Moreover, a group of studies further develop the gamma process-based degradation model to capture the item-to-item variation. For example, Lawless and Crower [21] incorporated a random variable z into $X(t)$; thus, $X(t)$ follows a gamma distribution with shaped parameter $\eta(t)$ and scale parameter $z\theta$. Liu et al. [67] introduced the parameter vector θ_{Ga} following a gamma distribution with hyper-parameters $\theta_{Ga}^{H} = \left(\delta_{\mu_{Ga}}, \gamma_{\mu_{Ga}}, \delta_{\lambda_{Ga}}, \gamma_{\lambda_{Ga}}\right)$ to model random effects.

Gamma process can also be used to model degradation path function with measurement error [68]:

$$Y(t) = X(t) + \varepsilon(t) \tag{22}$$

where $\varepsilon(t)$ is assumed to follow a Gaussian distribution with the mean value as 0. The true increment is expressed as $\Delta X(t) = X(t + \Delta t) - X(t)$. The measured value of increment $\Delta Y(t)$ expressed as [68]

$$\Delta Y(t) = \Delta X(t) + \varepsilon(t + \Delta t) - \varepsilon(t) \tag{23}$$

where $\Delta X(t)$ follows a gamma distribution, denoted as, $X(t) \sim Ga(\alpha, 1/\lambda)$, and $\varepsilon(t + \Delta t) - \varepsilon(t)$ follows a normal distribution, denoted as $\varepsilon(t + \Delta t) - \varepsilon(t) \sim N(0, 2\sigma^2)$. Similar models are also studied in references [23, 69–73]. System reliability models and remaining useful life predictions can be obtained based on the assumption of gamma process [21–23, 69–73].

Measurement error is commonly assumed to be independent with degradation; however, it may not be realistic in practice [74]. For example, Pulcini [66] proposed a perturbed gamma process in which the measurement error is statistically dependent on the degradation state. The error term in Eq. (22) is assumed to follow a normal distribution with the zero mean and the variance equal to $\sigma^2(x_t)$, where x_t is the current degradation level. Under the condition that the true degradation is $x_t = X(t)$, the conditional PDF of the measurement error $\varepsilon(t)$ given the measured degradation level y_t is obtained [66]:

$$\begin{aligned} f_{\varepsilon(t)}(\varepsilon_t|y_t) &= \int_0^{\infty} f_{\varepsilon(t)}(\varepsilon_t|x_t) f_{X(t)}(x_t|y_t) dx_t \\ &= \frac{1}{\sqrt{2\pi}} \frac{\int_0^{\infty} \frac{x_t^{\eta(t)-1}}{\sigma_\varepsilon^2(x_t)} exp\left[-\frac{1}{2(\varepsilon_t/\sigma(x_t))^2} - \frac{1}{2}\left(\frac{y_t - x_t}{\sigma(x_t)}\right)^2 - \frac{x_t}{\theta}\right] dx_t}{\int_0^{\infty} \frac{x_t^{\eta(t)-1}}{\sigma(x_t)} exp\left[-\frac{1}{2}\left(\frac{y_t - x_t}{\sigma(x_t)}\right)^2 - \frac{x_t}{\theta}\right] dx_t} \end{aligned} \tag{24}$$

The system reliability model is then proposed in consideration of the false alarm caused by the degradation measurement error [66].

3.3 System Reliability Models Based on Inverse Gaussian Process

An inverse Gaussian process $\{X(t); t \geq 0\}$ with function $\Lambda(t)$ and parameters β and λ has the following properties [24, 75]:

(1) $P(X(0) = 0) = 1$;
(2) Each increment follows an inverse Gaussian distribution, expressed as $X(t + \Delta t) - X(t) \sim IG(\beta\Delta\Lambda(t), \lambda\Delta\Lambda(t)^2)$;
(3) Increments are independent.

Inverse Gaussian process can be used to model monotonic degradation process with uncertainty. For example, Pan et al. [75] assumed β as a random parameter to denote the variability among products. They assume the prior distribution of $1/\beta$ follows the normal distribution, expressed as $1/\beta \sim N(\mu_\beta, 1/\sigma_\beta^2)$, which is statistically independent of λ. By using the concept of the first passage time, the lifetime T of a system can be obtained. The CDF of the lifetime T with the random effect of β is formulated by using the monotonicity property of inverse Gaussian process [75]:

$$F_T(t) = P(X(t) > \mathrm{H}) = \Phi\left(\sqrt{\frac{\lambda}{H}} \cdot \frac{(\sigma_\beta t - \mu_\beta\sigma_\beta H)}{\sqrt{\sigma_\beta^2 + \lambda H}}\right) - exp\left(2\mu_\beta\lambda t + \frac{2\lambda^2 t^2}{\sigma_\beta^2}\right) \times \Phi\left(-\sqrt{\frac{\lambda}{H}} \cdot \frac{(\sigma_\beta^2 + 2\lambda H)t + \mu_\beta\sigma_\beta^2 H}{\sqrt{\sigma_\beta^4 + \lambda H\sigma_\beta^2}}\right) \quad (25)$$

where H is a threshold.

Peng [76] established a normal-gamma mixture of inverse Gaussian degradation model to incorporate the heterogeneity among products. To be specific, λ is assumed to have a gamma density function, expressed as $f(\lambda) = [\lambda^{\alpha-1}/\Gamma(\alpha)\tau^\alpha]\exp(-\lambda/\tau), \lambda, \alpha, \tau > 0$. Let δ, $\delta = \beta^{-1}$, have a conditional normal PDF with mean ξ and variance σ_β^2/λ. The PDF of δ is $f(\delta|\lambda) = \sqrt{\lambda/2\pi\sigma_\beta^2}\exp\left(-\lambda(\delta - \xi)^2/2\sigma_\beta^2\right), \delta, \xi \in R, \sigma_\beta^2 > 0$. Later, Hao et al. [77] relaxed the normal assumption of δ and assumed δ follows a skew-normal distribution, in which $\delta \sim SN(\mu, \sigma^2, \alpha)$. The PDF of δ is presented as [77]:

$$f_\delta(x) = \frac{2}{\sigma}\Phi\left(\frac{x - \mu}{\sigma}\right)\Phi\left(\alpha\frac{x - \mu}{\sigma}\right) \quad (26)$$

where μ is the location parameter, σ is the scale parameter, and α is the shape parameter. They [77] further model the CDF of the lifetime based on the skew-normal distribution assumption of δ. Recently, Sun et al. [78] predicted the system remaining useful life with the inverse Gaussian degradation model with measurement errors and further applied to the hydraulic piston pump. The errors are assumed to follow a normal distribution conditioning on the degradation level.

Meanwhile, the degradation path can be modeled by other distributions. For example, Zhai and Ye [29] discussed that Gaussian distribution has low probabilities in large values, which may result in a misleading result when some fatal errors are introduced during the observation process. Thus, the measurement error is assumed to follow a student's t-distribution. Shen et al. [30] assumed the measurement error follows a logistic distribution. This distribution has relatively heavier tails compared with the normal distribution, which is more suitable to use when there are large errors in the degradation data. Li et al. [32] considered that measurement errors are time-series data, which has the auto-correlation due to modeling errors or environmental changes especially when the time interval is short. Thus, a Wiener process degradation model with one-order autoregressive (AR(1)) measurement errors is established. The AR(1) measurement error is also considered in studies [31, 33]. Giorgio et al. [34] modeled $\varepsilon(t)$ as a three-parameter inverse gamma distributed random variable that is conditionally distributed on the degradation level.

4 Conclusion

Reliability evaluation of complex engineering systems is a critical task in many safety–critical applications. System failure is generally caused by random shocks and internal degradation. Typically, five random shock models are commonly used in the field of Reliability Engineering, cumulative shock model, extreme shock model, run shock model, δ-shock model, and mixed shock model. In addition, the uncertainty in the degradation process can influence the accuracy of the reliability estimation. In general, there are three sources of variability that can result in uncertainty, temporal variability in the degradation process, unit-to-unit variability, and measurement error caused by imperfect instruments or imperfect inspection. Considering the importance and popularity of considering random shocks and uncertainty in modeling system reliability, in this chapter, we first review system reliability models with random shock models and then system reliability models with uncertainty in terms of three classic stochastic processes, Wiener process, gamma process, and inverse Gaussian process.

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