



Railway Bridge Model Calibration Using the Search Group Algorithm

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Abstract. In structural engineering, there are many cases where design data is insufficient or even nonexistent. Thus, there is a demand to assess the structural integrity of these constructions, usually depending on experimental field data. Given the limited design information, inverse approach can be an efficient tool in estimating the parameters of interest via structural optimization. In this work, it is proposed to estimate unknown design variables by means of the calibration of numerical models using experimental modal data. A real railway bridge structure is studied. Three numerical models of the railway bridge are studied in order to estimate four parameters of interest: stiffness of the supports, modulus of elasticity of concrete, modulus of elasticity of ballast and ballast density. For the efficiency of the calibration of numerical models via optimization, the robustness of the algorithm is essential. For this purpose, Search Group Algorithm (SGA) is employed to update the unknown model parameters. Finally, it is presented that in the creation of numerical models to assess real structures, one must be very sparing in order not to make a problem overly complex and, as result of, high computational cost. Further, simpler numerical models may be effective, both in relation to their computational, as well as obtaining accurate responses with error quantification in relation to the real problem.

Keywords: Finite element model updating · Structural optimization · Search Group Algorithm · Railway bridges

1 Introduction

Structural safety has gained attention in the field of engineering, peculiarly for structures such as bridges and viaducts, so that development of computers and mathematical tools was fundamental to overcome several imbroglios in structural analysis.

In the approach of structures with complex behavior, the use of numerical models based on Finite Element (FE) methods allows to simulate a real structure and evaluate its effects. The process of development a FE model can involve many simplifications related to geometry, discretization, boundary conditions and mechanical properties of the material that may cause errors (Ribeiro et al. 2012).

In these cases, the use of experimental data such as dynamic measurements are important to validate the numerical model as reported by Ren et al. (2004). Thus, the process of calibrating numerical models via optimization aims to match the numerical and experimental responses and to estimate the parameters of interest. The updated

numerical model can be used with confidence in structural analysis, to predict of dynamic responses under new operating speed or under new load scenarios, or for damage identification, proceeding from Structural Health Monitoring (SHM). The application of the SHM in railway bridges can be found in Kolakowski et al. (2011), Vagnoli et al. (2018) and Meixedo et al. (2021).

In the context of methods for model updating, the solution can be done by probabilistic inferences. In this case, the algorithm course the search space of the parameters of interest, that is, the bounds of design variables known a priori and, evaluates the discrepancy between the numerical and the experimental responses through the penalty function, called objective function or cost function. This process is iterative and it is repeated in order to reduce the difference between the answers to search the global optimal solution of the problem. The global optimum characterizes the solution of the problem and the identification of the parameters of interest. Several methods are presented in the literature to solve optimization problems. These includes gradient-based methods (Belegundu and Arora 1985); surface response methods (Roux et al. 1998); heuristic methods (Zanakis and Evans 1981), generally based on inspirations from nature; or in the combination of the methods mentioned (Lagaros et al. 2002).

Ribeiro et al. (2012) updated a numerical model of a railway bridge based on vibrational responses through the genetic algorithm (GA). The authors explain that GA has some advantages such as the ability to handle with a large number of parameters of interest and constrains, not dependent on a starting point for design variables, capability to deal with discrete variables and the ability to find the global optimum of the function. However, a low convergence rate compared to methods based on gradient is reported to be its main drawback. Furthermore, in the literature there are several hybrid methods that combine different optimization algorithms in order to improve the search for optimal solution. Papadrakakis et al. (2001) proposed the combination of gradient-based methods with evolutionary algorithms to solve optimization problems for structures under static and seismic loading. Kaveh and Talatahar (2008) implemented an algorithm based on the combination of the particle swarm with the ant colony algorithm for the optimization of trusses. In the context of hybrid methods for structural optimization of bridges, Jung and Kim (2013) applied a hybrid algorithm with the combination of the GA and the modified Nelder-Mead algorithm for updating a numerical model of a simply supported bridge. The objective function of the problem included displacement and modal responses. Deng and Cai (2009) proposed a combination genetic algorithm and the surface response technique to handle with FE model updating using static and modal responses. A hypothetical example of simply supported beam was performed and this method was also applied to a numerical model of a real bridge.

The Search Group Algorithm is a metaheuristic optimization algorithm, i.e. must have to able to find a reasonable solution related exploration and exploitation. Exploration, or global phase, is the capability of the algorithm to search promising regions in all design domain. Exploitation, also referred to as local phase, is the ability of the algorithm to refine the search only in these promising regions, as reported by Gonçalves et al. (2015). In this context, SGA have recognized advantages such as the high convergence rate compared to methods based on gradient and nature inspired algorithm, not dependent on a starting point for design variables, ability to manage a

huge number of design variables and constraints, which includes the possibility to handle with discrete and continuous variables, and capability to find the global minimum in functions that have several local minimum (Gonçalves et al. 2015). On the other hand, the input parameters of the SGA may vary depending on the characteristics of the problems to be solved which can make it exhausting and to be its main drawback.

This paper reports the numerical model calibration of a filler-beam railway bridge based on experimental vibrational responses. Three numerical models developed by Silva (2010) is detailed in order to estimate four parameters of interest: stiffness of the supports, modulus of elasticity of concrete, modulus of elasticity of ballast and ballast density. The calibration of the numerical models involves an optimization process using the SGA algorithm. This iterative process evaluates the objective function in order to reduce the difference between the numerical and experimental natural frequency responses of the bridge under study.

As far as the authors' knows, this is the first work aiming at numerical model calibration of railway bridge using the Search Group Algorithm. This may be related to the method being recent in the literature.

2 Structural Optimization

The optimization methods are, in general, numerical techniques for identifying parameters of interest. These techniques start from a starting point of the project and are carried out in small steps in order to improve the value of the objective function. The search is completed when a certain number of iterations is reached, when there is no progress in improving the objective function without the violation some of the constraints, or in some methods, when the convergence of the problem becomes very slow (Haftka and Gürdal 1992). Briefly, the general formulation of an optimization problem is given by:

$$\begin{aligned} & \text{minimize} && f(x) \\ & \text{subject to} && g_j(x) \geq 0, \quad j = 1, \dots, n_g \end{aligned} \quad (1)$$

where x is a vector of design variables with components $x_i, i = 1, \dots, n$; $f(x)$ denotes the objective function, $g_j(x)$ are the inequality constraints and $h_k(x)$ are the equality constrains.

2.1 Optimization Algorithms

The correct choice of the optimization algorithm is crucial for the identification of the variables of a given problem. In the literature, there is a diversity of algorithms to solve structural optimization problems. To name just a few, response surface techniques, methods based on gradient approximations (sequential programming, Quasi-Newton, augmented Lagrangian, etc.); heuristic and metaheuristic methods, generally inspired

by observations of nature (genetic algorithm, ant colony algorithm, particle swarm, etc.); in hybrid methods, based on the combination of optimization algorithms (Ribeiro et al. 2012).

Gradient-based algorithms depend on first and/or second order derivative calculations. These methods are usually built to converge to local minimum and guarantee the optimal global solution as long as it is proved that the objective function and constraints are convex in their design domains. If the convexity of an optimization problem is proven, it is guaranteed that there is only one minimum and that this is the global minimum (Carlson 2015). However, the gradient-based algorithms have some disadvantages such as the difficulty in identifying optimal global solutions, as they are very dependent on the starting point, and the difficulty of operating with non-differentiable functions.

Ren and Chen (2010) evaluated the application of the surface response method in the update of the bridge numerical model and describes that this is a mathematical tool to achieve the optimal solution of objective functions and constraints based on functional evaluations at the points in the design domain. Furthermore, the author reported that this technique may involve some strategies such as experimental procedures, statistical and probabilistic inferences that allow the analyst to make an efficient empirical exploration of the structure under study.

Regarding heuristic optimization methods, the algorithms have the characteristic of not depending on the gradient information of the objective function neither the gradient of the constraints. This occurs because heuristics use only the value of the objective function at certain points, which makes it possible to treat problems with discrete domains. However, for continuous domains, in general heuristics present a significantly slower convergence than gradient-based methods, as reported by Carlson (2015). A problem of worse convergence is translated into a higher computational cost in search for the optimal solution.

2.2 Search Group Algorithm

The heuristic method used in this work for the calibration of models is the Search Group Algorithm (SGA), and defines how the particles move in the domain due to a provoked disturbance. In this format, to generate the new position of each individual, it is added the current position to a defined constant disturbance and a random variable. Initially, for a high value it disturbs to be reduced until the local phase.

The first step of the method is to generate a random population \mathbf{P} , as shown in Fig. 1a, where each dot characterizes an individual of the initial population at the two-dimensional design domain. After to generate a population, each individual is evaluated by the objective function, as shown in Fig. 1b, which the biggest points represent the best objective functions. In sequence, there is a greater chance of selecting individuals with the best objective function. As illustrate in Fig. 2, the search group \mathbf{R} is created by selecting n_g individuals for \mathbf{P} , which, being disturbed, generates a number of new points through the disturbance α , which in turn is controlled during the iterations. In order to increase the ability of the algorithm to explore the design domain, a given

number of individuals of the search group \mathbf{R} is mutated at each iteration. The basic idea of the mutation is exploring new regions of the search design domain. Figure 2(b) shows the search group of Fig. 2(a) after mutation, which a mutated individual represented by the green circle replaced a member of the search group. The better the value of the objective function, the more points are created, and this process is repeated, characterizing the exploration or global search, which is also controlled by parameter α . Figure 3a and b illustrates of one of the first iterations of the algorithm. With each iteration, the value of α decreases, and in this way, the individuals generated by a given search group member tends to remain in its neighborhood so that in the local phase it becomes more refined, as it is shown in Fig. 4a and b. The Fig. 4b illustrates the families generated by each one of the five search group members of Fig. 4a.

As mentioned above, the algorithm is composed by the global and local phases. In the first it_{global}^{max} iterations, named the global phase or exploration, the algorithm aims to explore the most of the design domain. When the number of the iteration is higher than it_{global}^{max} , the algorithm goes to the local phase and tend to exploit the promising regions. This step is performed by the best n_g individuals among all the families that form a new search group. More details about the SGA may be found in Gonçalves et al. (2015).

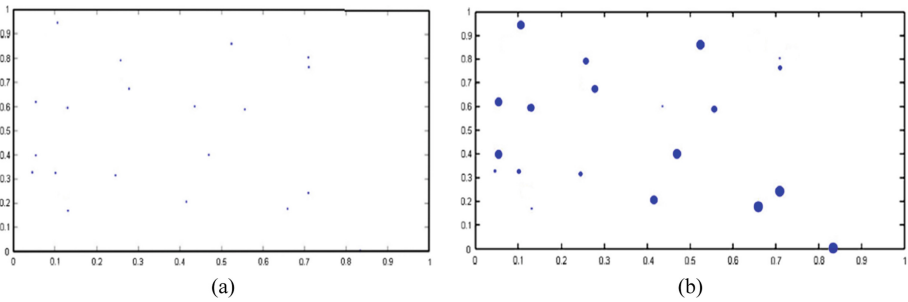


Fig. 1. Evaluation of the objective function of the initial population (Gonçalves et al. 2015).

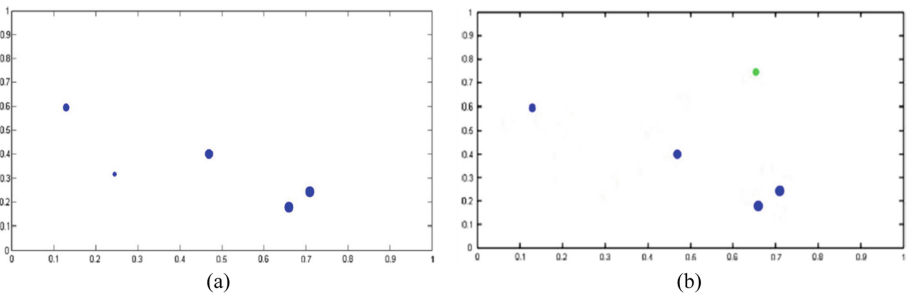


Fig. 2. Mutation of the initial search group of the initial population (Gonçalves et al. 2015).

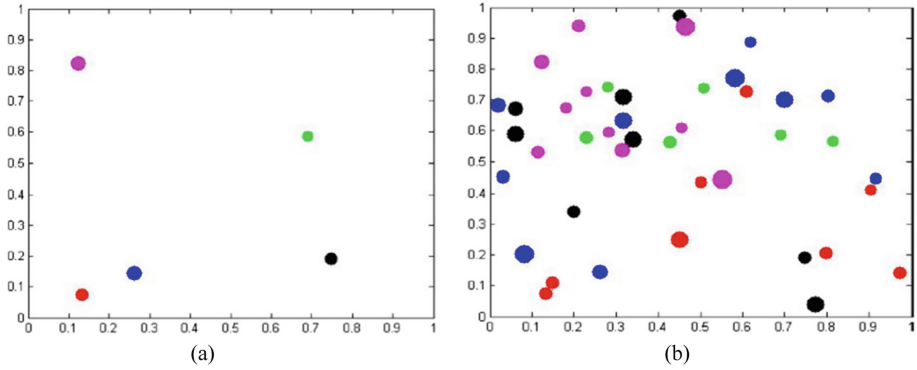


Fig. 3. Families generation of the search group member in the first iteration (Gonçalves et al. 2015).

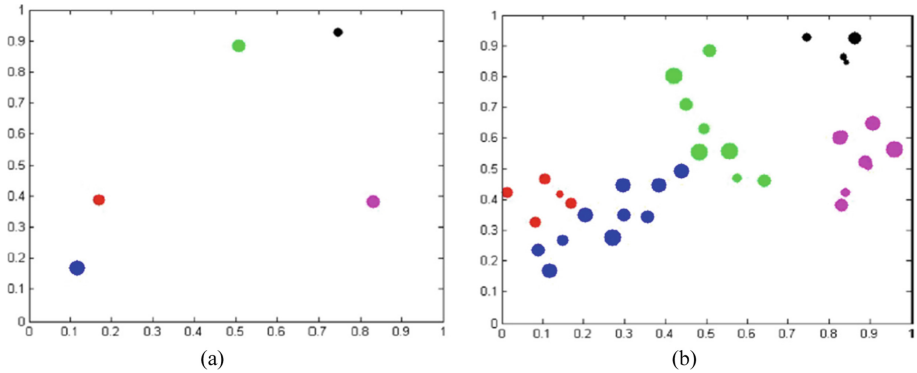


Fig. 4. Families generation of the search group member in a latter iteration (Gonçalves et al. 2015).

3 Canelas Railway Bridge

Canelas Bridge is selected for this study. The structure is part of the Portuguese railway line, near Aveiro, and has a total length of 72 m between the axes of supports. This structure consists of 6 simply supported spans, each 12 m long, Fig. 5. It is a composite steel-concrete of the type *filler beam*, composed of two half independent concrete slab decks, each supporting a railway track. Both slab decks are 6.2 m wide, with 4.5 m of 0.7 m thick slab with nine HE500B metal beams spaced 0.475 m apart. The ends of the slabs are composed of consoles 1.7 m wide and linearly variable in thickness between 0.3 and 0.7 m. The bridge deck is also equipped with a ballast guard consisting of a beam 0.60 m high and 0.3 m wide, located between the concrete slab and the side console. Ballast, sleepers and rails are placed on the board. The ballast guard causes the ballast structure to be confined to the board. All spans of the board are fixed on one side (side where the train is going) and mobile on the other side (side where the train is

going), made of neoprene involving stainless steel sheets and supported under each HE500B profile, totaling 9 support devices. In this work, the span called P5-E2 is studied, i.e., the gap closest to the landfill towards Porto. The longitudinal view and the respective cross section of this bridge are illustrated in Fig. 6. The main reason for choosing this type of bridge is due to the fact that the structural system of concrete is the most used in railway bridges in Brazil.



Fig. 5. General view of the Canelas railway bridge (Rocha et al. 2016).

3.1 Past Research

The Canelas bridge has been the target of experimental and numerical studies by some researchers. Presented in 2004, Rodrigues' doctoral thesis (Rodrigues 2005) was supported by the Portuguese laboratory LNEC and addressed the suitability of the Canelas railway bridge to increase the maximum operating speed of alpha-commuter trains from 140 km/h to 220 km/h. The study involved conducting in situ tests to assess the dynamic characteristics of the structure and the dynamic effects of rail traffic. The first vertical vibration mode and the first torsional mode of the bridge sections were identified.

Silva (2010) studied the dynamic behavior of the Canelas bridge using numerical models developed in ANSYS and coupled in Matlab for solving dynamic equilibrium equations. The dynamic analysis under moving loads corresponding to several trains that operate on the Canelas bridge, was applied and compared with results from major static analysis. Three finite element models of the bridge were developed, from the simplest to the most complete model. Based on the experimental responses of the first vibrational modes of structure, studied by Rodrigues (2005), numerical models were updated in relation to the parameters: elasticity module of the concrete, ballast elasticity module, ballast density and stiffness of supports. For this update, a sensitivity analysis of the parameters was used.

Rocha et al. (2016) published some studies on the dynamic behavior of the Canelas bridge were carried out by means of probabilistic approaches. The analysis of the

sensitivity of the bridge dynamic responses as a function of its structural parameters was considered, as well as the effect of rail irregularities. For such simulations, the Monte Carlo Method was applied. Thereby, Rocha demonstrated that the inertia of the cross section of the deck, the height of the ballast layer, the concrete elasticity module and the vertical stiffness of the supports were the parameters that most altered the bridge's response.

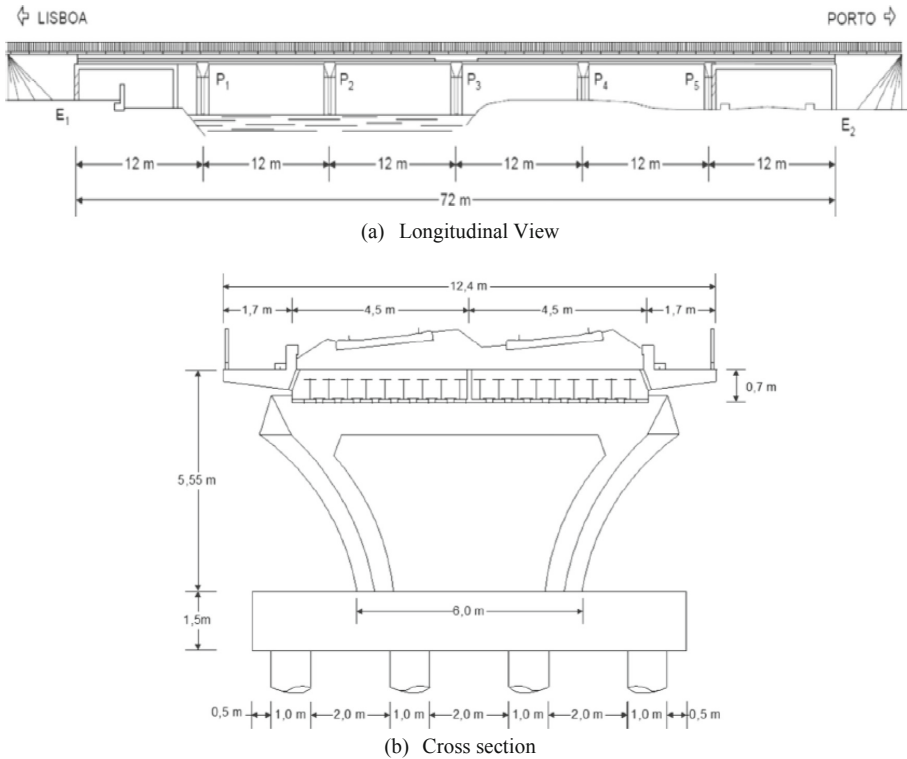


Fig. 6. Canelas railway bridge (Rocha et al. 2015).

3.2 Numerical Models

The numerical models of the bridge are developed by the Finite Element Method. Considering that this study basically focuses on the evaluation of the P5-E2 section, the representation of the entire length of the bridge and track becomes unviable due to the high computational cost of the numerical model. Thus, a simplification of the model with the inclusion of only the section under study and the continuity of the track in the adjacent section can guarantee an adequate model and with a significant gain in processing time. Thus, three numerical models developed by Silva (2010) are studied and presented in the following sections. A simpler numerical model, denominated C1, with only one half-span, where the track is represented by an equivalent distributed load on

the slab. A second, more complete than C1, the half-board model with two successive spans and an extension of the road to take into account the continuity effect is called C2. Finally, a more complete C3 numerical model, but with higher computational cost, is represented by two successive spans and an extension of the track, as in the model C2, plus the duplication of the half board, through the axis of symmetry of the structure represented by the longitudinal axis. In the three numerical models performed, the pillars and the bridge embankment were not considered. In view of the analyzes to be carried out, the ANSYS 16.0 software is used in the APDL language for modeling.

Subsequently, numerical models are calibrated by updating four properties of the bridge that are considered most relevant for possible changes in their dynamic behavior in modal terms: the stiffness of the supports, the modulus of elasticity of the concrete, the modulus of elasticity ballast and ballast density. Table 1 shows the variables of interest and their respective bounds, studied by Silva (2010).

Regarding computational modeling, shell elements (SHELL63) were used to simulate the concrete of the slab, console and ballast guard. The beam elements (BEAM44) were considered to represent the steel profiles of the slab and rails. Solid elements (SOLID45) were used to simulate ballast, sleepers and pads. In addition to these elements, rigid beam elements (MPC184) were introduced to connect the ballast to the slab, mass elements (MASS21) to simulate the guardrails and, finally, spring elements (COMBIN14) for elastic supports. The arrangement of the finite elements is given in relation to the center of gravity of the elements that make up the bridge structure, in the same way that the supports are located in the center of rotation.

Table 1. Variables considered in updating the models

Variables	Description	Unit	Bounds
K_v	Vertical Stiffness of the supports	MN/m	117–547
E_c	Modulus of the Young of the concrete	GPa	32–37
E_l	Modulus of the Young of the ballast	MPa	120–200
ρ_o	Ballast density	kg/m ³	1800–2700

3.2.1 Numerical Model C1

The numerical model C1, the simplest model of the span under study, represents the half board P5-E2 without the inclusion of track elements. In this case, the mass of the track; composed of ballast, sleepers and rails; it is simulated by means of a uniformly distributed load on the shell elements of the slab, that is, it is considered as the dead load in which the stiffness of the track does not influence the model's responses. The structure was discretized into 3335 elements and 2759 nodes. At Fig. 7 shows the numerical model C1 with its adaptive elements.

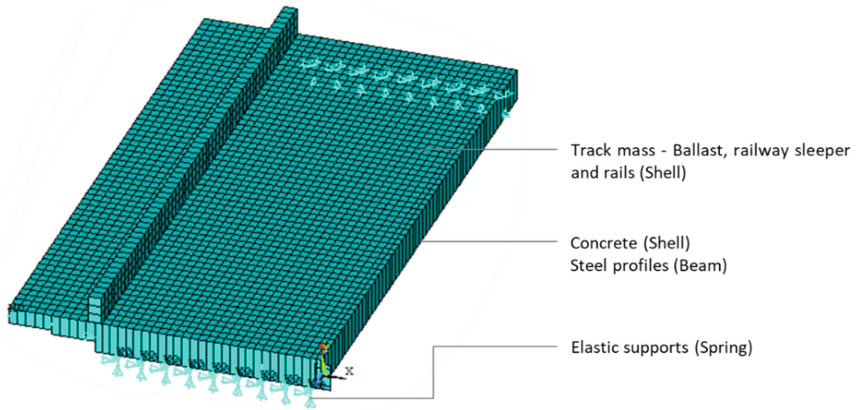


Fig. 7. Numerical model C1.

3.2.2 Numerical Model C2

In the numerical model C2, half slab is simulated with two successive spans, the span under study and the next span, and an extension of the road to simulate its continuity over the landfill. In this model, the elements of the track are considered, which implies the influence not only of the mass, but also of the stiffness of these elements in the simulated responses. The slab structure has no connection between the gaps, given by a 6 cm joint, this connection being made by the elements of the track. The model was discretized into 23709 elements and 29150 nodes. Figure 8 shows the numerical model C2 with its different elements.

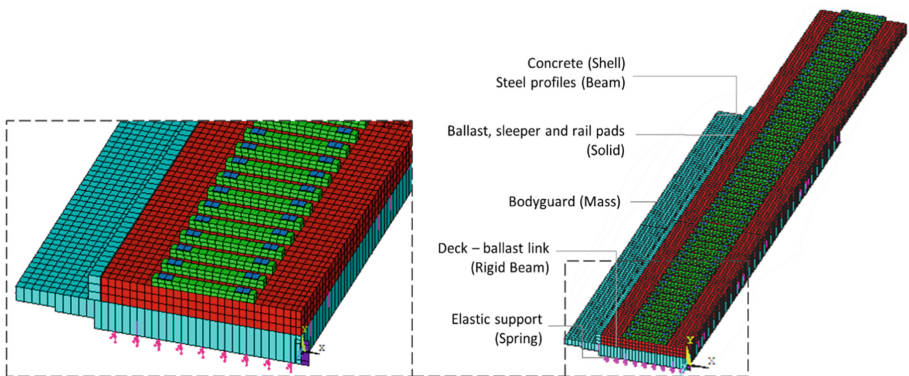


Fig. 8. Numerical model C2.

3.2.3 Numerical Model C3

The numerical model C3 is the most complete of the three, representing two half slabs and two successive gaps, in addition to the extension of the road over the embankment. As in the C2 model, the connection between the four-half slabs takes place via the

track, with no structural connection between the concrete trays. Likewise, the track elements also influence the stiffness of the structure. The model was discretized into 47860 elements and 58300 nodes. Figure 9 shows the numerical model C3 with its respective structural elements. A more detailed study of the numerical model of the Canelas bridge can be seen in Silva (2010).

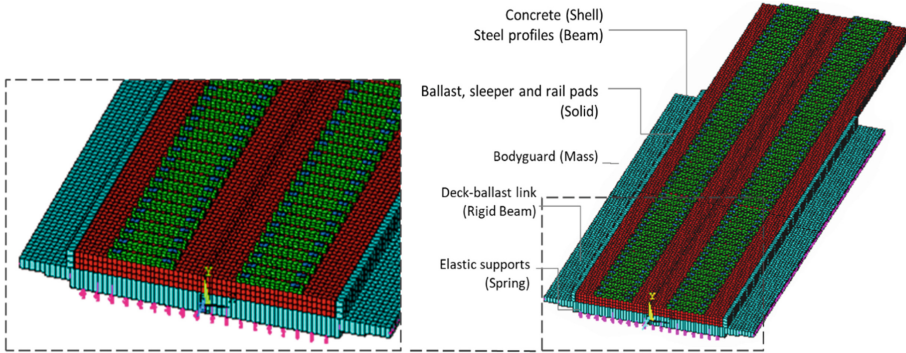


Fig. 9. Numerical model C3.

4 Numerical Model Calibration

In this section, the optimal values of the properties of the bridge are estimated using numerical models, in order to adjust the values of the natural frequencies obtained previously in experimental tests developed by Rodrigues (2005). The experimental values of the natural frequencies are shown in Table 2.

Table 2. Experimental natural frequencies (Rodrigues 2005)

1 st vertical bending mode (Hz)	1 st torsional mode (Hz)
8.60	15.60

The numerical frequencies are fitted by minimizing the objective function, aiming to approximate the values of the experimental natural frequencies and, thus, to identify the respective values of the parameters of interest. The objective function is given by Eq. (2):

$$f_{obj} = \sqrt{\left(\frac{f_1^{exp} - f_1^{num}}{f_1^{exp}}\right)^2 + \left(\frac{f_2^{exp} - f_2^{num}}{f_2^{exp}}\right)^2} \quad (2)$$

where f_1^{exp} and f_2^{exp} are the values of the experimental natural frequency of the first vertical bending mode and the first torsional mode, respectively, and f_1^{num} and f_2^{num} are the corresponding natural frequencies of the numerical model to be calibrated.

Figure 10 shows a flowchart that resumes the optimization loop of numerical model calibration through the SGA. This process involves combining two software packages:

ANSYS[®] and MATLAB[®]. Note that the closer to zero the objective function is, the smaller the error of the numerical response in relation to the experimental response.

In the numerical model C1, as described in the previous chapter, the elements of the track are not included, in such a way that the load of these elements on the board is simulated by means of a distributed load. In this way, three parameters of interest are estimated: the stiffness of the supports, the concrete elasticity module and the ballast density. Table 3 shows the value of the numerical frequencies of the model C1. The calibration of this model, due to its lower computational cost, was performed with a high number of population and iterations, represented by 100 individuals and 200 iterations. Table 4 presents the estimated values of the parameters of interest for the model C1. In minimizing the objective function for this model, despite the high number of population and iterations, the natural numerical frequencies not converges to a value very close to the experimental natural frequencies, having an error in the order of 0.0283.

Aiming to reduce the difference between the values of the numerical and experimental frequencies, the bounds of the parameters of interest, shown in Table 1, are expanded. In this case, the violation of the limits of the parameters of interest characterizes an error between the actual information of the bridge and the numerical model under study.

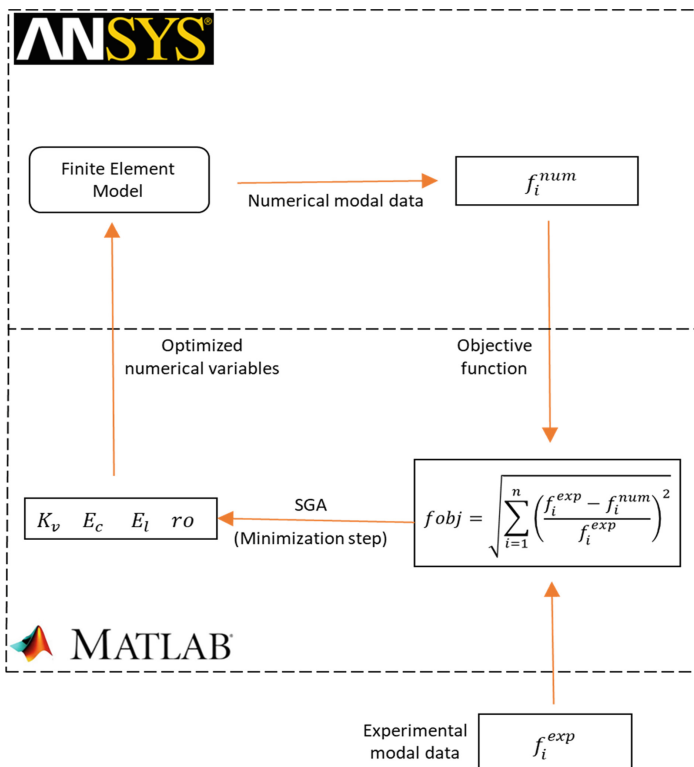


Fig. 10. Flowchart for numerical model calibration through the structural optimization.

Table 3. Numerical frequencies for model C1

Vibrational modes	Experimental value (Hz)	Numerical value (Hz)
1 st vertical bending	8.60	8.36
1 st torsional	15.60	15.67

Table 4. Updated variables for numerical model C1

Variables (output)	Unit	Estimated value
Vertical Stiffness of the supports	MN/m	344.1
Modulus of the Young of the concrete	GPa	37.0
Ballast density	kg/m ³	1800

In Table 5, the final values of the update of the numerical model C1 are presented and in Fig. 11, the first vertical bending vibration mode and the first torsional mode of the C1 model are illustrated. Due to the higher computational cost of the C2 model, the model updating was performed with a relatively low number of evaluations of the objective function, represented by 50 individuals and 20 iterations.

Table 5. Updated variables for numerical model C1 after the expanded bounds

Variables (output)	Unit	Estimated value
Vertical Stiffness of the supports	MN/m	309.6
Modulus of the Young of the concrete	GPa	38.8
Ballast density	kg/m ³	1556

In minimizing the objective function for the numerical model C2, the numerical natural frequencies converges to a value of the experimental natural frequencies. The convergence, in this case, although the model is more expensive compared to the C1 model, was given for a low number of population and iterations. This makes the model C2 more accurate in obtaining the parameters of interest. In Fig. 12, the first vertical bending vibration mode and the first torsional mode of the C2 model are illustrated, respectively.

In the numerical model C3, the most complete model of the three, the characteristics of the structure and its materials are kept as close as possible to reality. In this way, four parameters of interest are estimated: the stiffness of the supports, the concrete elasticity module, the ballast elasticity module and the ballast density. The optimization of the C3 numerical model was performed with a population of 100 individuals and 50 iterations.

In minimizing the objective function for the numerical model C3, as well as in the model C2, the natural numerical frequencies converges to a value of the experimental natural frequencies. This also makes the C3 model more accurate in obtaining the parameters of interest. On the other hand, this model has a very high computational cost. In Fig. 13 are illustrated the first vertical bending vibration mode and the first torsional mode of the C3, respectively. It is important to note that in the configuration of the vibration modes of the C3 model, the first torsional mode is visually similar to a

vertical flexion vibration mode. This is due to the geometrical complexity of the structure, which implies a difficulty in characterizing the modes it represents, already in its first modes of global vibration. However, in this work, it was decided to use the natural frequency value in a numeric of the first torsional mode, given its coherence in response compared to the experimental response. In addition, Silva (2010) more accurately evaluated the modal responses of the C3 model and characterized it this mode. Table 6 presents the values of the parameters of interest of the updated numerical models. Thus, it can be inferred that for numerical model C1 the disregard of the track stiffness implied a significant increase in the concrete's elasticity module and a sudden reduction in the ballast density, considered as dead load, since the natural frequency is directly proportional to the stiffness and inversely proportional to the mass. Analogously to the C3 model, the significant increase in natural frequencies in relation to the C2 model may be directly linked to the inclusion of the second half board, which implies an increase in the stiffness of the modeled structure.

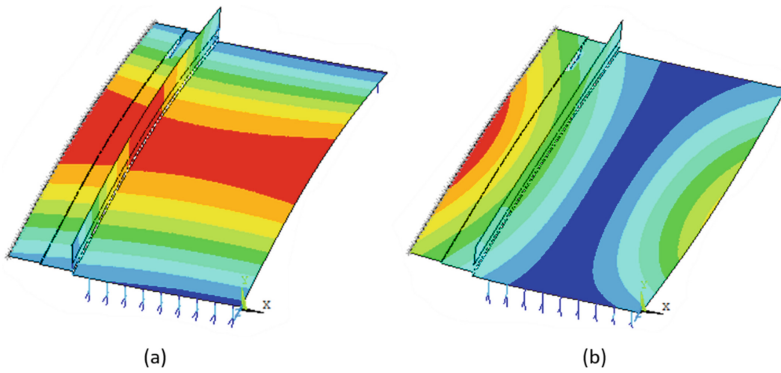


Fig. 11. Vibration modes of C1: (a) First vertical bending mode and (b) First torsional mode.

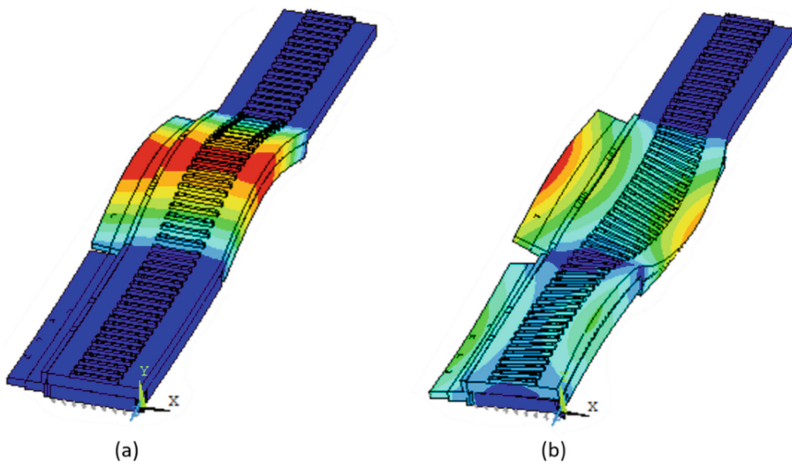


Fig. 12. Vibration modes of C2: (a) First vertical bending mode and (b) First torsional mode.

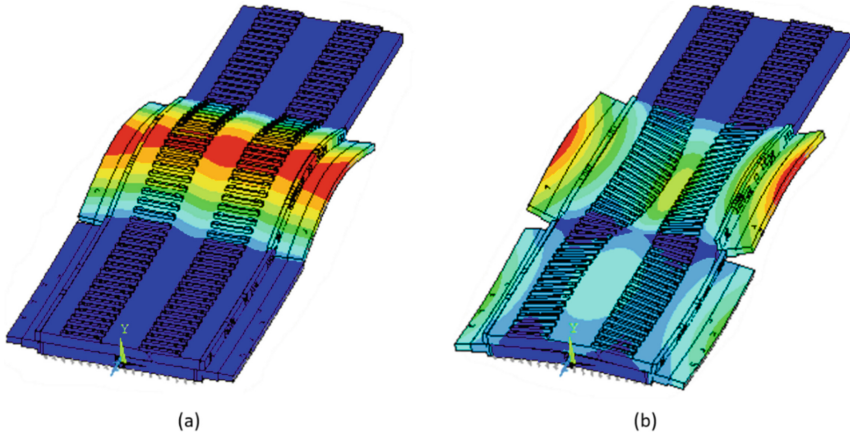


Fig. 13. Vibration modes of C3: (a) First vertical bending mode and (b) First torsional mode.

Table 6. Variable values of the optimized models

Variable	Unit	Bounds of the design	C1	C2	C3
K_v	MN/m	117–547	309.6	486.5	335
E_c	GPa	32–37	38.8	33.5	32.7
E_l	MPa	120–200	–	130	162.5
ρ_o	kg/m ³	1800–2700	1556	2571	2636

5 Conclusion

This work aims to estimate unknown design variables in structures from previously known responses. A real example of a short-span railway bridge is selected for this study and three numerical models are updated using the SGA optimization algorithm based on modal responses obtained in the field.

The simplest numerical model of the bridge, called C1, with the lowest computational cost, did not show a good convergence in minimizing the objective function within the lower and upper bounds of the parameters of interest, featuring a priori a numerical model with a grade of errors. Models C2 and C3, which are more complete and have a higher computational cost, presented numerical values of natural frequencies that converges to experimental values within the limits of the design variables.

Models C2 and C3 presented values of the parameters of interest that were closest to each other when compared to model C1. This is due to the influence of the track on the stiffness of the models. For the C1 model, due to the non-inclusion of the stiffness of the track elements, this showed a sharp drop in the value of the dead load that represents the ballast, in this case, the ballast density parameter.

Therefore, in the process to make numerical models to evaluate real structures, one must be very sparing in order not to make an exaggeratedly complex problem and, as a consequence, of high computational cost. Simpler numerical models can be efficient,

both in relation to its computational cost, and in obtaining accurate answers with the quantification of their errors in relation to the real problem.

Future research consists to estimate the total uncertainty regarding updated parameters and the errors between numerical models. Hence, reliable bounds for such uncertainties can be predicted. In addition, own error parameters can be included as quantities to be updated in the process.

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