From Historical and Theoretical Analysis of Representation and Geometry to Topology for Structural Optimization

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Abstract. To study Geometry and Representation from a theoretical point of view, it is certainly necessary to refer critically to the historical-anthropological climate of their genesis. This training approach allows to see and understand their evolution/impact for new approaches to the genesis of architectural/engineering artefacts, both from a configurative and structural point of view. On the other hand, if we consider the typical primitive/intuitive approaches of Representation, up to its rigorous elaborations based on a consistent knowledge of Optics and Geometry, it is possible to recognize the strong links between artistic experience, mathematical contribution and scientific elaboration. It is therefore possible to offer a broad overview of the "state of the art" relating to critical sector studies, conducted both in Italy and abroad, to underline:

– how awareness in the multiple fields of Geometry is expressed in the methods and process of realization of architecture and engineering, from conception to its realization;
– how Representation stands as a means between theory and construction.

In this sense we focus on Topology and its genesis, as an area of Geometry that can be concretized in Structural Optimization, in order to verify the promising results that these procedures ensure in terms of reducing the use of material and design iterations, without neglecting the architectural/engineering quality, also in configurative terms. For this reason, one of the main factors in the growing popularity of this topic is the development, in computational terms, of modern computers, which allow to reliably solve complex analyzes based on FEM (Finite Element Analysis).

Keywords: Topology · History of geometry/representation · Structural optimization

1 Introduction

To study Geometry and Representation from a theoretical point of view, it is certainly necessary to refer critically to the historical-anthropological climate of their genesis. This

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training allows to see and understand their evolution/impact for new approaches to the genesis of architectural/engineering artefacts, both from a configurative and structural point of view. If we consider the typical primitive/intuitive approaches of Representation, up to its rigorous elaborations based on a consistent knowledge of Optics and Geometry, it is possible to recognize the strong links between artistic experience, mathematical contribution and scientific elaboration [1]. It is, therefore, possible to offer a broad overview of the state of art relating to significant research fields conducted both in Italy and abroad, to underline:

- how awareness in multiple areas of Geometry is expressed in implementing architecture and engineering assets, from the ideation to its realization;
- how Representation is placed as a medium between theory and built environment.

This validates the idea that an architectural/engineering structure - conceived in the mind of the architect/engineer - finds its configuration, even before its constructive reality, within its Representation [2]. It anticipates not only its spatial, metric, or functional values, but also it incorporates a whole series of cultural, theoretical and historical values that belong to it. It is in this sense that we intend to historically analyze the different forms of Representation by connecting them critically with the design process. Necessarily, in this multifaceted process, Geometry is therefore an indispensable "tool" for the architect/engineer. Firstly, it allows not only a correct and effective description of particularly complex structures and spatial configurations. Secondly, Geometry inspires every design operation in a creative sense, thus reviving the ancient combination of art with science [3]. For this reason, the knowledge of the historical evolution of Geometry influences an inventive and design awareness. In particular, the comprehension of Topology - even in historical terms - has been advances in "structural optimization", a general concept historically consolidated in Science and Art of building [4]. In this paper, it is intended as a sum of design condition that allows the optimal exploitation of the physical-mechanical characteristics of construction elements through reduced material use and highest performance level.

Precisely the deep connection between Science and Art is fundamental, renovating/highlighting in this research the link between Representation and Geometry to obtain a correct scientific congruence between configuration and structure in architecture and engineering.

The proposed practice implies that structural efficiency must be an integral part of design. The balance between engineering and architecture is one of the most imminent problems facing the construction industry [5]. In the design practice of the construction sector the established principle often places first the architecture (form) and then the structure (stability). Usually the structural engineer focuses on finding the best solution – from the point of view of stability and efficiency – based on the spatial and functional distribution defined by the architect. The main role of structural engineers is often limited to the selection of materials or the determination of the dimensions of the elements by structural analysis [6], however, they should be an integral part of the decision-making process of the formal design of a building-structure by integrating their own skills with those of the architect [7]. As a result, structural engineers should gain greater digital knowledge and make larger use of advanced computational analysis techniques in the design process. Engineering/architectural design is influenced by multiple factors, such as aesthetic requirements, client needs, geotechnical properties, climate, materials availability, duties etc. This multitude of factors presupposes an adequate evaluation that deviates from a standard practice from time to time and must adapt to each individual situation, including the loads acting on the structure. Current computerized simulation tools make it possible to take into account various load situations and to generate the optimal structure for each specific situation.

2 Historical/Theoretical Investigation of Geometry to "Scientificize" the Representation Systems

The aim of this paper is to highlight the common nature and properties of Geometry and Representation, and vice-versa show how computation will diversify them. In this way it is possible to reach a generalization which indicate their fundamental role for the education and training for the architect/engineer [8].

It is important to underline, therefore, that Gaspard Monge marks the beginning to "make scientific" the Representation methods encoding Descriptive Geometry [9, 10]. During the nineteenth century, a further step towards the maximum generalization of perspective, within the Projective Geometry, will lead to the method of Central Projections, by Henry Jean Victor Poncelet. The perspective is thus freed of all the operational limitations, such as that of the optical cone and visual likelihood. It becomes a mathematical tool capable of establishing a one-to-one correspondence between entities and figures of three-dimensional space and the related plane images, where the properties of the space are translated into equivalent properties in the plane [11–13].

We are therefore witnessing - thanks to Monge and Poncelet - the mathematization of graphic techniques, operated by two types of geometric transformations: the central projection and the cylindrical projection. Tracing the main lines of the mathematization of graphic techniques means, therefore, to look for the origins of perspective theory and Descriptive Geometry, precisely through the systematization of two categories of graphic procedures, on the one hand those of designers, painters, sculptors, engravers, on the other hand those of architects, builders, stone cutters and carpenters.

These methods are still adopted to represent architecture, whether built or in progress, partially loosing those features that distinguished them, that are, to give a scientific foundation to the architect/engineer ability to figure and therefore recreate reality, filtered by his personality and his culture.

If, on the one hand, it is possible to recognize that from the nineteenth century to today the birth of new non-Euclidean geometries - such as elliptic and hyperbolic ones, or Topology - has contributed to renewing the architectural and engineering projects, on the other hand, the representation itself has been transformed into a computed routine.

The advent of the computer, as a *deus ex machina,* led to some improvements, despite the fact that orthogonal projections, axonometry, perspective are used once again - and automatically - in design software. We are witnessing the transition, in the general context of a scientific changing process of Representation, from the mathematization of graphic techniques to their computerization. Algorithms thus systematically and globally organize all the necessary data, transforming those techniques into information immediately/quickly available.

3 Topology: Properties of Shapes/Relations of Things

Topology is an abstraction of specific geometric concepts such as ‘continuity’ and ‘proximity’. The word ‘topology’ derives from the Greek “τοποσ”, a place, and “λογοσ”, a discourse [14]. It was introduced by J.B. Listing (1848), a student of C.F. Gauss, in the title of the first book on the subject. Another adopted name was Analysis situs [15], because the term topology is not usually found in the literature before 1920 [16]. Further research by Poincaré provided the first systematic treatment of Topology and revolutionized the subject by using algebraic structures to recognize non-homeomorphic topological spaces, founding the field of Algebraic Topology [16, 17]. Another popular term used to convey the more intuitive aspects related to Topology is ‘rubber-sheet geometry’.

Topology, unlike Euclidean Geometry, studies invariants under continuous/invertible transformations. For example, we can shape and stretch a sphere into a filled cube by such transformations, but not into a toroidal shape. Topology is discussed especially in relation to traditional geometry and topological space both from an algebraic and spatial point of view [18–20]. To understand its meaning, consider modeling surfaces as if they were flexible spatial models in a three-dimensional (or higher-dimensional) space.

It is considered that two surfaces are equal and homeomorphic if one of the spatial patterns can be continuously distorted – by bending, stretching, crushing without splitting or gluing the stitches – to resemble the other [14]. According to these principles, the surfaces of a torus and a cup of tea are homeomorphic. Similarly, a circle and an ellipse, for example, or a square and a rectangle, can be considered topologically equivalent, since both the circle and the square could be respectively deformed into an ellipse or a rectangle.

The homeomorphism is particularly interesting, since the focus is on the relational structure of an object and not only on its geometry [21]. Topology is an abstraction of the operable features, based on geometry of objects [22]. When we talk about Topology, we are often interested in how spaces and shapes are joined and connected, not how they appear [23]. Topology is therefore also the study of the intrinsic/qualitative properties of geometric shapes, not normally affected by changes in size or shape [21]. In this sense, Rozvany defines Topology as “the model of connectivity or spatial sequence of members or elements in a structure” [24]. Eschenauer and Schumacher stated that “the topology of any constructions, i.e., the position and arrangement of structural elements in a given design space, has strong influence on its structural behaviour” [24].

The relational structure that describes a topology can be exploited to identify the links that are established within a continuous body subject to stresses and constraints. The search for an optimal topology is crucial, is we consider that the primary objective of engineering techniques (in general ideative) is realizing a design to obtain the best system to meet certain needs, within the available resources [25]. It is then interesting to exploit a method that allows to predict the ideal topology – spatial relationship of the form – of 1 any component, in a generic sense. Moreover, using a general formulation that allows to predict the layout of a structure based on the assigned boundary conditions. This process can be defined as the material distribution method used for engineering design. The method improves efficient estimation of the optimal topology, optimal shape and optimal use of established conditions [26].

4 Searching for Optimum Design

Thanks to digital paired with geometric awareness, it is possible to achieve greater manipulability and simulation of any architectural/engineering component, strengthening the creative potential. Simulation have to contribute to translate/create reality, being able to affirm that the purpose of the digital machine (as for drawing) therefore lies in its creative potential. Consequently, the purpose of this paper is to outline a process to achieve an optimization that, with the study of Topology, allows us to understand, interpret and finally create optimal spatial configurations, ensuring reduction of design iterations and use of material and decreasing the waste of resources.

Furthermore, the quality in architectural and civil design, defined by the development of formal possibilities that such awareness can offer. The application of structural optimization techniques can be implemented creatively to both increase the visual appeal of a project and improve the readability of structural actions. This introduces a new way of thinking about shape, not only as a result of a creative genesis but also as a result of a physical process obtained from the forces acting on it, which allows to design the structural configuration and its spatial layout. Among many experiences, it is important to highlight the work by Arata Isozaki and Mutsuro Sasaki, who proposed the use of topology optimization to create an architecture based on structural efficiency [27].

Topology optimization is widely required in multiple manufacturing sectors, such as automotive and aerospace, to enhance mechanical efficiency of components. In the AEC industry, the use of these methods are less numerous. During last years, the cases have increased, particularly focused on developments of single components, as the work done by steel structural joints by Arup [28].

Generally, considered a structure as a solid body subjected to tensional, compressional and shear stress and deformation and geometrical constraints, the aim of a structural optimization is to minimize, or maximize, an objective function [29]:

$$\begin{aligned} &\text{minimize}\, f(x) \\ &\text{such that}\;\; h_j(x) = 0, \quad j = 1, 2, \cdots, n_h \\ &\qquad\qquad\;\; g_k(x) \le 0, \quad k = 1, 2, \cdots, n_g \\ &\qquad\qquad\;\; x_i^l \le x_i \le x_i^u,\; i = 1, 2, \cdots, n \end{aligned} \tag{1}$$

The structural optimization – properly shape optimization or generally topology optimization [30] – began in 20th century thanks to A. Mitchell [31]. From the analytical principle expressed by J. C. Maxwell (1864), Mitchell explored the limits of material usage for trusses. Recently, Hemp uses those concepts to demonstrate the importance the advantages of structural optimization, to reduce costs of design and construction [32].

There are three types of structural optimization: size, shape e topology (Fig. 1). In the size (or sizing) optimization the design variable is some type of structural thickness, i.e., cross-sectional areas of truss members [33]. In the shape optimization the design variable represents the form or contour of some part of the boundary of the structural domain [34–36].

The Topology optimization is the most general form of structural optimization and has the complex features of both size and shape optimization problems [37]. Typical problems in topology optimization are represented by specific features, such as number of holes and their position, in the 2D and 3D domain [26, 38].

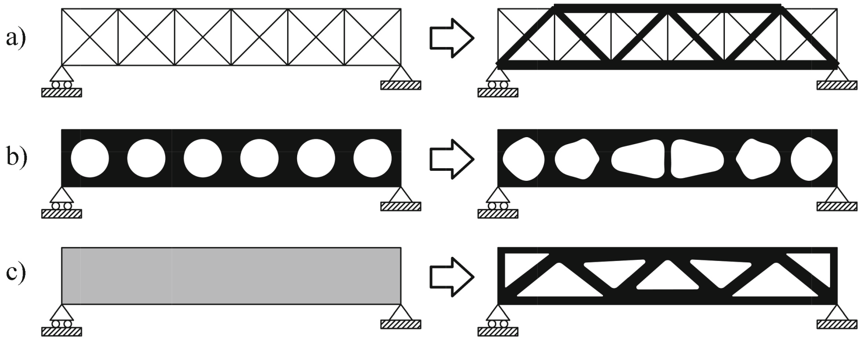

Fig. 1. Three categories of structural optimization. a) Sizing optimization of a truss structure, b) shape optimization and d) topology optimization.

Solutions to topological optimization problems are of two types: exact/analytical or discrete/finite element (FE) based [24, 39]. The emergence of practical FE-based topology optimization for high-volume fractions was determined by extensive research by Bendsøe [40, 41]. The topology design problem was expressed as to find the optimal distribution of material density in a domain, modeled with a FEM network (Fig. 2 and Fig. 3) [26].

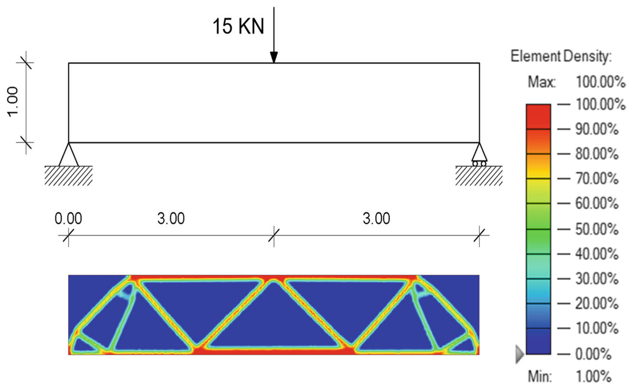

Fig. 2. After the topology optimization iterations the finite elements of the domain have different density values.

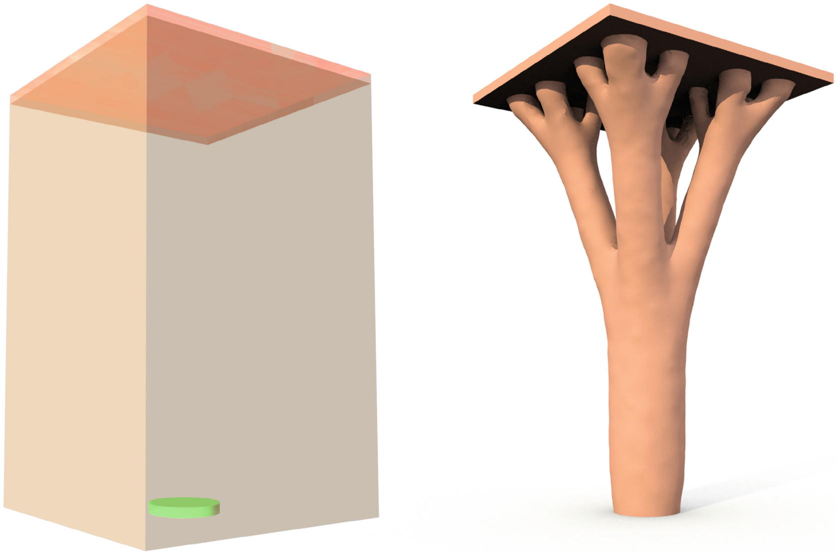

Fig. 3. Initial domain of a floor with central support (volume: 40,5 m^3) and topology optimization result (volume: 4,321 m^3), with 10% of volume target value (image by Gianluca Rolle).

Any topology optimization process is composed by the following steps: defining a geometric domain and a constituent material - Isotropic-Solid/ Empty (ISE), Anisotropic-Solid/Empty (ASE), and Isotropic-Solid/Empty/Porous (ISEP) -, setting loads and constraints, setting a mass target. Our study proposed a series of tests on typical architectural/structural components such as beam, wall, slab, dome (Fig. 4). Starting geometrical

Fig. 4. Catalogue of optimized shapes (image by Gianluca Rolle).

components are then simulated under different load and constraint systems (continuous or singular actions), a catalogue of generically optimized structures is created. Structures are then assembled to build an architectural space (Fig. 5).

Fig. 5. Assembly of generic topology optimized shapes (image by Gianluca Rolle).

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