Girard Desargues and Geometry Applied to the Arts

Marta Salvatore(✉)

Sapienza University, Rome, Italy
marta.salvatore@uniroma1.it

Abstract. This study intends to examine the theoretical geometric contributions given by Girard Desargues to the applied arts, through the exploration of the related constructions that reveal their *manière universelle*. The applied arts, which played a fundamental role in 17th century society, involving craftsmen and artists, between the Renaissance and Baroque periods aroused the interest of mathematicians, who considered them the laboratories in real scale, where experimenting theories and validate results. Desargues' contribution is framed in this context, as the work of a geometer interested in practical questions to be resolved through the power and ability of generalizing theory. The research of universal methods capable of resolving constructions in the field of applied arts in a strictly operational way is the *fil rouge* that links perspective, stereotomy and gnomonics. These methods are the final result of the synergistic combination of the theory of form and methods of representation, which bring together the studies of conic sections and their properties with the idea of representation intended in its broadest sense, where "perspective" and "true form" are two sides of the same coin. The ultimate purpose of Desargues' method is to provide accessible constructions for the most ordinary workers, unfamiliar with geometry, by means of maximum simplification processes, reproducible with the tools they know and understand.

Keywords: Girard Desargues · *Manière universelle* · Perspective · Stereotomy · Gnomonics

1 Introduction

In the early years of the 17th century, the applied arts played a major role in society. Painting, sculpture, and architecture, close to the practice of crafts, assume a central position as privileged tools for representing reality. The operational nature of construction, which characterizes the practice of creating in all the applied arts, posed not easily resolvable problems, which found solutions in treatises and manuals containing numerous special cases. In those years, two parallel approaches were consolidated, one theoretical, the other practical, which nourished a printed production with a profoundly different approach. In fact, theoretical contributions oriented to the definition of synthesis solutions were flanked by a consistent production aimed at resolving especially particular cases. Thus, the applied arts aroused the interest of mathematicians and geometers, who sought to resolve operational problems through the generalization of processes, using the

M. A. Ródenas-López et al. (Eds.): EGA 2022, SSDI 22, pp. 121–130, 2022.
https://doi.org/10.1007/978-3-031-04703-9_13

power of theory. Between Renaissance and Baroque periods, perspective, stereotomic and gnomonics building sites were in fact considered as laboratories in real scale, in which to experiment theories and validate results. This is the context of Desargues' theoretical contribution, which considers applied geometry as the instrument for elaborating a universal theory capable of resolving construction problem from a specifically operational point of view. In fact, in the first half of the 17th century, Desargues elaborated a *manière universelle* for perspective, stereotomy and gnomonics, aimed at defining simple general procedures, practicable by ordinary workers with the instruments of common use, for resolving operational criticality in the field of applied arts. These procedures were the result of articulated geometrical reasoning that reveals its ability to work with form in space and to control it whit the drawing, using advanced overturning operations, that anticipate Monge's descriptive geometry.

Through the direct analysis of Desargues' texts and the critical studies that have followed his writings, this study aims to experiment the operational effectiveness of the *manière universelle* applied to the arts, explaining its theoretical foundations through digital reconstruction of the genesis processes at the basis of the proposed constructions. This contribution intends to evidence the *fil rouge* that links apparently distant application fields, which find a common ground in applied geometry, and which are the ultimate result of the synergic combination of the theory of form and "methods" of representation.

2 Applied Arts in Desagues' Work

The applied arts in Desargues' work are the expression of the synthesis between the knowledge of conic sections theory, addressed and developed in 1639 in the *Brouillon proiect d'une atteinte aux événemens des rencontres d'un cone avec un plan* [1] and a modern manner of understanding representation, which transversally permeates throughout his writings. Here drawing is understood in its broadest meaning, which considers the representation in plan and elevation and that in perspective as two species of the same nature [2]. In addition, drawing also acquires demonstrative as well as descriptive value in Desargues' work, assuming the role of a privileged tool for the affirmation of the theory over the practice [3]. This demonstrative capacity of drawing skillfully conveys ideal models, conceived in theoretical space, as in the case of perspective and stereotomy, or physical models, realized to bring life to the constructions, as in the case of gnomonics. Thus, the aspects that link perspective, stereotomy and gnomonics in the work of Desargues, are different from each other, but they are all referable to the theories of representation and to the theories of the form.

Desargues, in all three applications, relates the elements to be represented to a reference system in the space. About perspective, this is a plane system of Cartesian coordinates, that permits the association of the vertices of represented subject in a planimetric representation with the same vertices in a perspective representation. This system materialized the coordinates of an ideal perspective grid, realized through the movement of taut threads fixed on the drawing, capable of reproducing this biunivocal relationship, thus creating an effective perspective machine.

The same discretization of the subject in notable vertices involves stereotomy. The final objective of this construction, relating to the definition of the ashlars of a biased

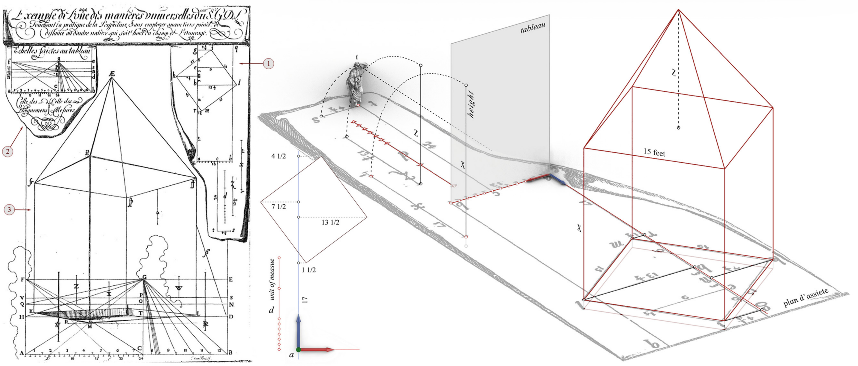

Fig. 1. Perspective of the cage and reconstruction of represented elements in the first drawing.

cylindrical vault placed on a scarp wall, was to determine the positional relationships between the vertices of each ashlar with respect to the positions of the planes passing through them, that could be perpendicular or oblique with respect to the axis of the cylinder. The reference system capable of defining these relations was composed of remarkable axes and planes, precisely derived from the geometric properties of the cylinder (axis, straight sections, generic sections). A similar approach to the problem allowed the representation of the hour lines of a sundial. The gnomonic question, as already observed by Poudra [2], follows the stereotomic one in its solution. In this case, rather than a cylinder, we are dealing with a cone that reproduces the apparent movement of the sun as seen from the earth's surface. Also in this case, the reference system is referred to the principal axis of the cone and to its significant sections.

3 About Perspective

Desargues' interest in practical perspective derives from the need to construct perspective images without the use of distance points, generally inaccessible with respect to the walls to be painted. This was a central issue at the time because the substantial production of architectural perspectives that frescoed the walls and ceilings of many churches and palaces. Perspective, appeared in 1636 in a twelve-page booklet entitled *Exemple d'une des manières universelles du S. G. D. L. touchant la pratique de la perspective sans emploier aucun tiers point, de distance ny d'autre nature, qui soit hors du champ de l'ouvrage* [4], is the first of Desargues' works devoted to the applied arts, and it is dedicated to the painters to teach them a universal operational method for constructing perspective.

The method is based on the bi-univocal correspondence between perspective and the representation in plan. Desargues imagines of discretizing the subject to be represented into a certain number of points, whose position can be referred to a system of Cartesian axes in the plane. This system lay on the *plan d'assiete* (geometric plane), with the axes respectively perpendicular and parallel to the *tableau* (picture plane). The notable points of the subject were represented in perspective using two perspective scales which allowed the perspective representation of their coordinates by means of tense threads.

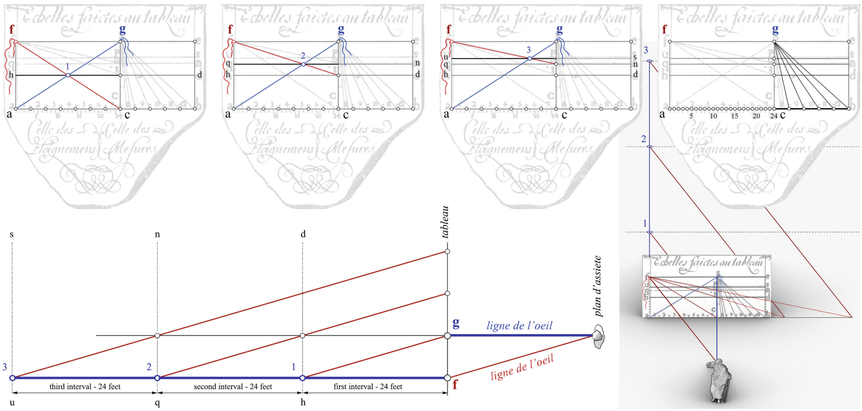

Fig. 2. Reconstruction of procedure for constructing perspective described in the second drawing.

Desargues proposes the construction of three preparatory drawings that compose the only figure of the booklet, representing the perspective of a cage. The first, executed on the *plan d'assiete*, concerns the planimetric representation of the perspective machine and of the subject, the other two, realized on the *tableau*, instead concern the construction of perspective (Fig. 1). The first drawing illustrates the true-form model of the perspective machine. It represents, in plan, the position of the observer with the relative overturned height, the position of the picture plane and that of the notable points of the subject through their coordinates. This same drawing showed the heights of the notable points of the cage overturned on it and the scale of representation, expressed in feet. The second and third drawings illustrate the perspective of the cage. The second, placed at the top left, describes, with demonstrative purposes, the construction of the perspective scales, while the third demonstrates their operational use, showing the perspective image of the cage. The discretization of the subject into points of known position resumes the universality of the method, reducing perspective construction to the representation of the subject's vertices through their coordinates, which are parallel and perpendicular to the picture plane. The only difficulty in the construction concerned the definition of the depths, resolved by introducing two perspective scales: the *échelle d'eloignement* and the *échelle de mesure* (to the left and right of the principal point in perspective figure). The first established the depths of lines parallel to the picture plane, while the second measured the intervals between lines perpendicular to the picture plane. The demonstration of the construction method of the *échelle d'éloignement* is not explained, but it can be reconstructed by considering the theoretical propositions that Desargues introduces in the last pages dedicated to the theorists (Fig. 2). In these pages, with surprising modernity, the *ligne de l'oeil* (line of the eye) is introduced; this line that issues from the observer eye is parallel to the straight lines to be represented in perspective. Desargues explains that the perspective images of generic lines parallel to each other, converge in the point where the *ligne de l'oeil*, parallel to them, meets the picture plane (today in the vanishing point of the lines in question).

In the *Exemple* Desargues proposes the construction of three straight lines parallel to the picture plane and placed at equal intervals to the observer's distance from the picture

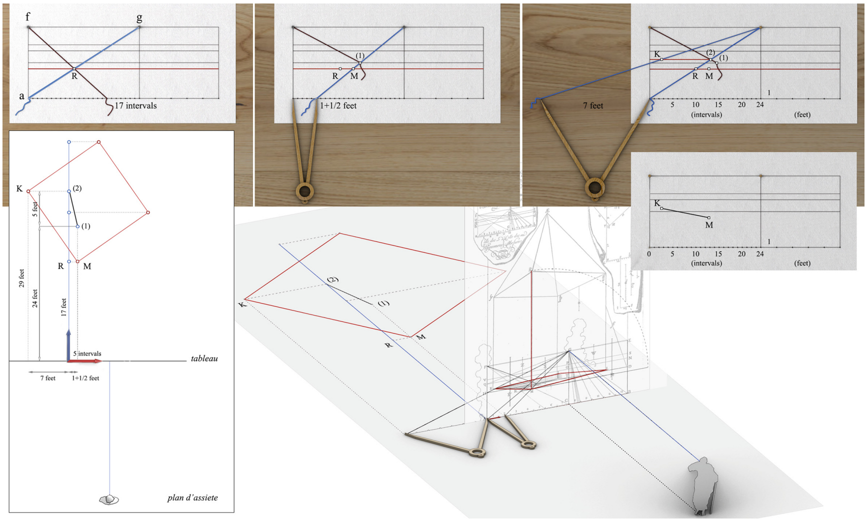

Fig. 3. The third drawing representing the perspective construction of the cage.

plane (24 feet). Therefore, he constructs the perspective of a straight line perpendicular to the picture plane passing to the left margin of its useful portion, assuming it as the axis on which to detach the searched intervals. Then, he constructs the perspective of a second straight line, inclined with respect to the first, capable of measuring segments equal to the sought distance. This second line, used in the manner of *ante litteram* measuring lines, can be arbitrarily chosen. However, to avoid using inaccessible points, Desargues places its point of concurrence on the horizon line in correspondence of the right margin of the picture plane available portion (**f**). Reconstructing the lines in the space and the corresponding *lignes de l'oeil*, we observe that to detach equal intervals, corresponding to the principal distance, the oblique line in question will have to pass through the intersecting point between the projection of principal distance on the geometric plane and the picture plane (point **c**). Reiterating the procedure on a second line perpendicular to the picture plane passing through **c** it is possible to obtain the next two intervals. Because the distance **ac** on the picture plane, between the line to measure and the line of measurement, determines a depth of 24 feet in space, by dividing this distance into 24 parts Desargues can represent progressive intervals spaced 1 foot of depth. Having thus established the *échelle d'éloignement*, Desargues determines the *échelle de mesure*, useful for establishing equal intervals between straight lines perpendicular to the picture plane. This scale is obtained by introducing lengths equal to the unit of measurement in true form according to the used scale of representation on the line of intersection between the *tableau* and the *plan d'assiete*. Therefore, the *échelle d'éloignement* and the *échelle de mesure* allowed the representation of an ideal grid of 1 foot, which permitted the location of the vertices of represented subject. In this way Desargues had built a sort of ideal perspective machine, in which the *échelle d'éloignement* operated through two mobile wires **ga** and **fc**, fixed at points **g** and **f**, while the *échelle de mesure* operated by a compass.

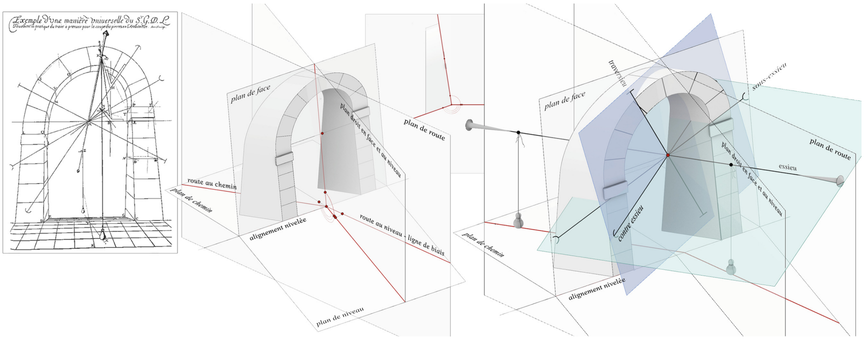

Fig. 4. Inclined planes that compose the vault and its straight section.

The definition of the method is followed by the construction of the cage, described in the third drawing, obtained through the representation of its vertices and the subsequent construction of their heights (Fig. 3). The measure of the heights is first degraded reversed on the *plan d'assiete* and brought back with the compass in the correct vertical position. The construction is enriched with the heights of the human figures and of hanging and oblique objects with the aim of conferring a character of maximum generalization to the illustrated procedures.

4 About Stereotomy

Desargues stereotomic theories were published in 1640 in the *Brouillon proiect d'exemple d'une maniere universelle du S. G. D. L. touchant la pratique du trait a preuves pour la coupe des pierres en l'architecture [...]* [5]. The contribution to stereotomy is related to a particular case, a biased rampant vault on a scarp wall, which encloses several conditions of obliquity in a single example [6]. In analogy with the case of the cage represented in perspective, the problem of the construction of the ashlars of a biased vault consisted in determination of the position of respective vertices in space. In general, the vertices of the ashlars of a barrel vault lie in the plane of the straight section of the cylinder that generates the vault. The vertices of the face arch are an exception, since they can be in a plane generically oriented in space, whose position is defined by the obliquity conditions of the vault. In order to determine the reciprocal position of these vertices, Desargues needs to represent the axis of the vault and the two planes where these vertices are located: the plane of the straight section of the cylinder and the oblique plane that sections the vault according to the arc of face. The representation of these planes is performed on a drawing sheet coincident with the plane of the face arc of the vault. The useful data for the solution of the problem are the three angles that the three planes component the arch form with each other, namely: the escarpment angle, formed by the *plan de face* (face plane) with the *plan de niveau* (horizontal plane), the bias angle, formed by the *plan de route* (vertical plane that contains the axis of the vault) with the *plan droit en face et niveau* (plane of the straight section of the dihedral formed by the *plan de face* and *de niveau*); the slope angle, formed by the *plan de niveau* and *the plan de chemin* (inclined plane that belongs to the axis of the vault and the horizontal

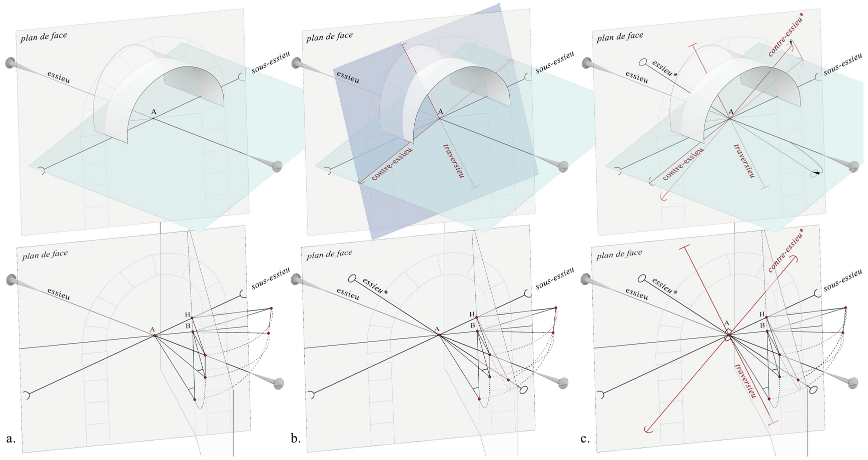

Fig. 5. Construction of four principal axes on the *plan de face.*

lines) (Fig. 4). These angles determine the direction in space of the *essieu*, i.e. the axis of the vault, with respect to its *plan de face*. Desargues represents on the drawing sheet the oblique *plan de face* and the straight section of the cylinder of the vault by means of two pairs of straight lines belonging to them. The first pair of lines is given by the *sous-essieu*, projection of the axis of the vault on the *plan de face* and by the overturning of the axis itself on the *plan de face*, obtained by rotating the projecting plane of the axis around the *sous-essieu* (Fig. 5a-b). The second pair of lines is given by the *traversieu*, intersection of the plane of the straight section of the vault with the *plan de face*, and by the *contre-essieu*, intersection of the plane of the straight section of the vault with the projecting plane, formed by the *essieu* and the *sous-essieu.* The *contre-essieu* is also overturned on the *plan de face* through the rotation of the plane that projects the *essieu* in the *sous-essieu.* Since the *contre-essieu* is perpendicular to the *essieu*, it will remain perpendicular even at the end of the overturning operation; the same condition of perpendicularity subsists between the *traversieu* and the *sous-essieu* (Fig. 5c). Given the position of the *sous-essieu* and the *essieu* overturned on the plane of the drawing, the construction of the other two axes is particularly simple because of this condition of perpendicularity.

Having represented the positions of the face arch and that of the corresponding right arch, Desargues proceeds with the construction of one of the vertices of a generic ashlar. This construction shows a repeatable procedure applicable for all vertices of the ashlars of the vault. Given an arbitrary vertex **P** on the face arc, Desargues constructs a line perpendicular to the *sous-essieu* in point **p** (Fig. 6a). From this point he constructs the perpendicular line to the *contre-essieu* in **p'**, that is a straight line parallel to the generatrices of the cylinder that have equal length to the distance between **P** on the face arc and **P'** on the right arc. Finally, in point **p'**, he constructs the perpendicular line to the *contre-essieu*, line **P'p'**, equal to the distance **Pp**. Once the vertexes of the ashlars have been determined, Desargues proceeds with the construction of the *panneaux*, plane developments of the relative faces. To perform this construction Desargues needs

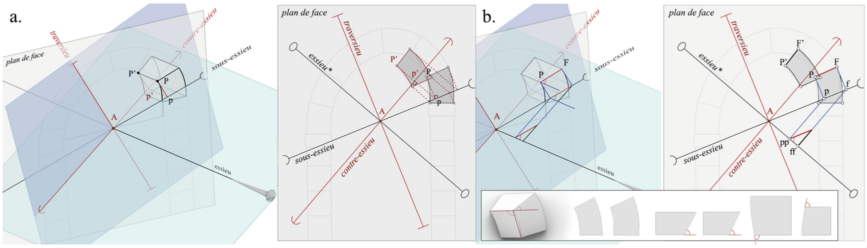

Fig. 6. Construction of vertexes of the ashlars and related *panneaux*.

to measure two angles: the first is the bias angle that the junction edges of the face arch form with the generatrices of the cylinder; the second is the angle that the chords subtended by the vertices of the intrados of the ashlar also form with the generatrices of the cylinder (Fig. 6b). To determine the angle formed by the joint edges with the generatrices of the cylinder Desargues projects the **PF** edge of given ashlar onto the projecting plane represented by the *essieu* and the *sous-essieu*. On this plane, overturned on the plan de face, it is possible to measure the angle in true form. The same procedure is used for the construction of the second searched angle. Once the angles in question are known, the *panneaux* are defined for all faces of the ashlar, which can be entirely constructed.

5 About Gnomonics

Desargues is author of two short writings on gnomonics dated 1640, dedicated to the construction of a sundial. One of the two closes the *brouillon* on the stone cutting, while the other came to us through a reconstruction performed by Poudra. The first of two, entitled *Maniere universelle de poser le style aux rayons du soleil en quelque endroit possible, avec la regle, l'esquerre et le plomb* [2], focuses on positioning and orienting the stylus with the exclusive use of ruler, square, compass, and plumb line. In analogy to stereotomy, gnomonics works with the planar sections of a cone. This is the round cone that reproduces the apparent movement of the sun, according to a geocentric model in which the earth is reduced to a point [3]. The position that the stylus must assume, coincides with that of the principal axis of this cone. Desargues operatively resolved the question by using some shafts oriented according to the generatrices of the ideal cone, whose position is given by the apparent movement of the sun. To execute this construction, he fixes a shaft to the ground and orients it in such a way that it projects shadows only at the exact point where it is planted. In two different times of the day, he observes the shadows brought to the ground by the shaft and, in correspondence of the relative extremes, he fixes two further shafts, having the free extreme coinciding with that of the first one (Fig. 7a). The three shafts in question materialize the generatrices of a quadric cone that has its vertex on the earth's surface and that belongs to the ideal circumference that describes the apparent motion of the sun. The axis of this cone indicates the position of the stylus, which will pass through the vertex of the cone and the center of its generic straight section. Because the cone is round, its straight section will be a circumference, which can be easily reproduced by detaching on the three shafts,

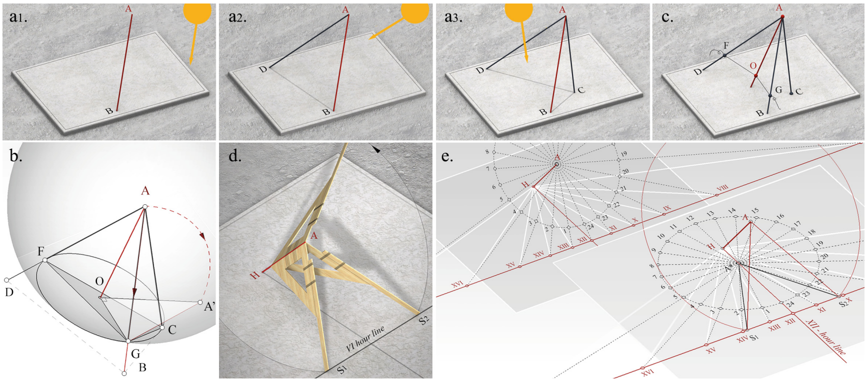

Fig. 7. Method for positioning the stylus and constructing the hour lines.

three segments of equal length starting from the shared vertex, whose extremes belong to the circumference sought. The center of the circumference is instead obtained by drawing, reproducing the construction in the plane starting from the representation of the triangle **CFG** and its circumscribed circumference. This center will be given by the intersection of the medians of the edges of this triangle. Given the center, it is possible to represent the position of the axis in the drawing, by overturning the plane that contains it and any of the three generatrices of the cone (Fig. 7b). Once the construction in the plane is defined, the problem could also be solved in space by materializing two radii of the circumference by means of a pair of metal wires fixed in **F** and **G** points of two of the shafts that represent the generatrices of the cone. The stylus was therefore fixed with one end at the vertex of the cone while the other tensioned the metal wires, assuming its correct position in space (Fig. 7c).

The second of the two writings *entitled Manière universelle de tracer au moyen du style placé, tous quadran plats d'heures au soleil, avec la regle, le compas, l'equierre et le plomb* [7], illustrates the "universal way" to draw the hour lines on the quadrant. The first lines drawn are the *ligne meridien* (XII-hour line) and the *ligne de l'équoteur* (VI-hour line). The line of the XII hours is the trace, on the plane of the quadrant, of the vertical plane passing through the stylus, obtained with the help of a plumb line. On the other hand, the VI-hour line, is given by the intersection of the quadrant plane with the equatorial plane. To construct this line Desargues uses a square with one edge fixed on the stylus. The other edge, free to rotate, reproduces the apparent movement of the sun and touches the plane of the quadrant at two points that belong to the VI-hour line (Fig. 7d). To construct the other hour lines, Desargues overturns the equatorial plane on the quadrant plane around the VI-hour line. Thus, he constructs a circle of any radius having its center at the extremity of the overturned stylus and divides it into 24 parts representing the hours of the day. These lines meet the VI-hour line at points which, joined with the base of the stylus, define the direction of the searched lines (Fig. 7e).

6 Conclusions

The writings of Desargues, as is known, had no success among his contemporaries. The specifically dedicated nomenclature of invention and the difficulty of orientation with respect to the reference systems adopted, made their reading particularly difficult. Equally difficult for the time was the theoretical reasoning behind the procedures used, referring to ideal models of geometric space, communicated through drawings difficult to interpret. The digital reconstruction of the construction processes of the ideal models allows to retrace Desargues' method, showing the algorithms of which graphic representations were an extreme synthesis. This kind of reading allows us to explore, through the analysis of the constructions, the relationships that link apparently distant fields of application that find a common ground in the interest in applied geometry, according to a distinctly modern approaches, typical of an *ante litteram* descriptive geometry.

References

1. Desargues, G.: Brouillon project d'une atteinte aux événement des rencontres d'un cone avec un plan (1639). In: Poudra, N.J., Oeuvre de Desargues reunites et analysées par M. Poudra. Leiber, Paris, pp. 103–238 (1861)
2. Poudra, N.J.: Oeuvre de Desargues reunites et analysées par M. Poudra. Leiber, Paris (1861)
3. Dhombres, J., Sakarovitch, J.: Desargues en son temps. Libraire scientifique A. Blanchard, Paris (1994)
4. Desargues, G.: Exemple d'une des manières universelles du S. G. D. L. touchant la pratique de la perspective sans emploier aucun tiers point, de distance ny d'autre nature, qui soit hors du champ de l'ouvrage (1636)
5. Desargues, G.: Brouillon proiect d'exemple d'une maniere universelle du S. G. D. L. touchant la pratique du trait a preuves pour la coupe des pierres en l'architecture; et de l'esclaircissement d'une manière de réduire au petit pied en perspective comme en géometral, et de tracer tous quadrans plats d'heures égales au soleil. (1640). In: Poudra, N.J., Oeuvre de Desargues reunites et analysées par M. Poudra. Leiber, Paris, pp. 305–351(1861)
6. Sakarovitch, J., Laurent, R.: Le fascicule de coupe des pierres de Girard Desargues. In : Encyclopédie des métiers: la maçonnerie et la taille de pierre, Tome 2. Presses du Compagnonnage, Paris, pp. 121–147 (2010)
7. Desargues, G.: Maniere universelle de tracer au moyen du style placé, tous quadrans plats d'heures égales au soleil, avec la regle, le compass, l'equierre et le plomb (1640). In : Poudra, N.J., Oeuvre de Desargues reunites et analysées par M. Poudra. Leiber, Paris, pp. 352–358 (1861)