Identifying Zones of Targeted Feedback with a Hyperbolic Cosine Model

Ye Yuan and George Engelhard

Abstract Formative assessments are used to identify student strengths and weaknesses, but they frequently do not identify targeted feedback strategies for improving student achievement. The study introduces the concept of Zones of Targeted Feedback for identifying the sets of optimal feedback strategies that can be used for improving student writing. The study suggests a non-cumulative responses process for linking feedback strategies and achievement levels. We offer an unfolding model as an alternative measurement paradigm to identify and improve the effectiveness of teacher feedback strategies linked to levels of student achievement. The study presents two examples (an illustrative one and an empirical one) of using an unfolding model called the Hyperbolic Cosine model to illustrate our conceptual framework.

Keywords Writing · Feedback · Unfolding model · Formative assessment

1 Introduction

Feedback is one of the most powerful influences on student achievement, and feedback can be viewed as a "consequence of performance" (Hattie & Timperley, 2007, p. 81). A significant body of research stresses the importance of feedback effectiveness in assessment and instruction (Bangert-Drowns et al., 1991; Kluger & DeNisi, 1996; Kulhavy, 1977; Kulhavy & Stock, 1989; Narciss & Huth, 2004; Shute, 2008; Willingham, 1990). Important tasks for teachers include the identification of a student's level of achievement, and then selection of feedback strategies that can move the student forward in their learning. Feedback strategies on different areas may not be appropriate for a given student based on their current level of achievement. Therefore, educators must target their feedback for each student.

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The purposes of the study are to explore the use of an unfolding model to identify the Zones of Targeted Feedback, and to discuss how teachers might use Zones of Targeted Feedback to provide scaffolding to assist in the development of appropriate feedback strategies. The study first introduces the concept of Zones of Targeted Feedback, and then describes an unfolding model that can be used to link levels of student achievement with recommended feedback strategies. Next, the approach is illustrated with two examples. The first example is an illustration of the idea, and the second example is an application of our idea with an empirical study in the context of writing assessment. The results of this empirical study are briefly described.

2 Zones of Targeted Feedback

Feedback holds promise for improving student achievement, but the promise is not guaranteed and depends in no small part on the care taken in choosing an appropriate mode of feedback (Hattie & Timperley, 2007). Appropriate feedback can be related to the concept of Zone of Proximal Development (ZPD; Vygotsky, 1978). Vygotsky defined ZPD as "the distance between the actual developmental level as determined by independent problem-solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (1978, p. 86). In this study, we propose adapting this idea to identify zones of targeted feedback (ZTF) that can assist teachers in identifying effective feedback strategies based on student levels of achievement. Formative assessments provide the identification of achievement levels, while the method proposed in this study can be used to identify the targeted feedback strategies to match an individual student's ZPD with the goal of improving each student's level of achievement.

Figure 1 shows the conceptual map of the study. The achievement levels of three students (Students A, B, and C) are shown on the first line from low to high achievement. The second line defines the locations of feedback strategies (S1, S2, and S3). The ZTF can be defined based on the recommended feedback strategies identified by teachers for students located at different levels on the achievement continuum. Take student B as an example, the bell curve in the middle can be recognized as the ZTF for student B. Comparing other feedback on the second scale, the feedback strategies in this range are judged to relatively effective and appropriate for student B. The optimal feedback for student B is the "peak" of the ZTF, which is feedback strategy S2. Teachers are less likely to provide the other two feedback strategies (S1, S3) as they are not in student B's ZTF. In the context of writing, a feedback strategy *below* a student's level might be related to feedback on the neatness of their handwriting, while a feedback strategy *above* a student's proficiency level might focus on more complex matters of organization, such as transitions between sentences. The key idea is that feedback strategies below or above a student's achievement level may not provide optimal feedback for improving student writing, so it is important to connect student achievement levels to a set of feedback strategies that define the ZTF for each student.

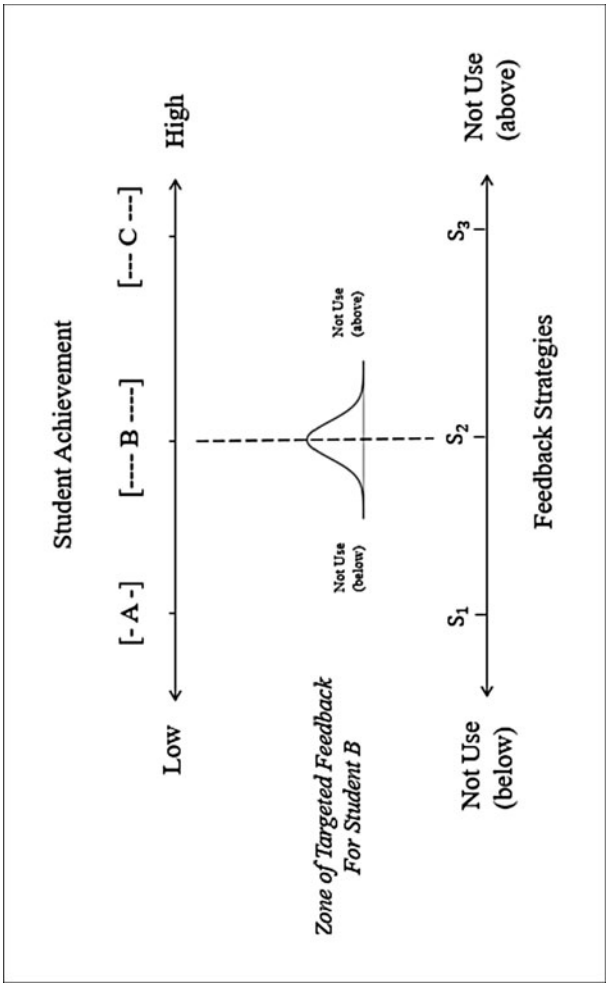
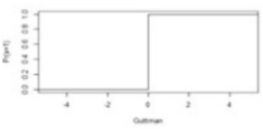
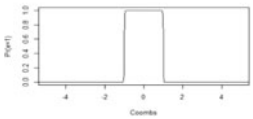


Fig. 1 Conceptual map of connection between levels of student achievement and zones of targeted feedback

Table 1 Ideal response pattern for cumulative and unfolding scales

	Panel A Ideal Response Patterns													
	Cumulative Scale: Scalogram (Guttman, 1950)							Unfolding Scale: Parallelogram (Coombs, 1964)						
Person	A	B	C	D	E	F	G	A	B	C	D	E	F	G
1	1	0	0	0	0	0	0	1	1	0	0	0	0	0
2	1	1	0	0	0	0	0	1	1	1	0	0	0	0
3	1	1	1	0	0	0	0	0	1	1	1	0	0	0
4	1	1	1	1	0	0	0	0	0	1	1	1	0	0
5	1	1	1	1	1	0	0	0	0	0	1	1	1	0
6	1	1	1	1	1	1	0	0	0	0	0	1	1	1
7	1	1	1	1	1	1	1	0	0	0	0	0	1	1
	Panel B Operating Characteristic Functions													
	Deterministic							Deterministic						
														

The study suggests viewing this bell curve of ZTF as an unfolding response process because the probability of a positive response is a single-peaked function (Andrich, 1997), which is different from cumulative response processes. Most measurement models are based on cumulative response processes. In the cumulative response processes, the probability of a positive response is a monotonic function of the relevant parameters. A comparison of a cumulative scale and an unfolding scale is shown in Table 1 (Andrich, 1988). Panel A in Table 1 illustrates a cumulative scale for seven persons and seven items (A–G). The persons and items are ordered by row and column scores. This ordering yields the distinctive triangular pattern that defines a Guttman scale (positive responses highlighted). The unfolding scale is also ordered, and the responses exhibit a parallelogram structure that is iconic for an unfolding scale (positive responses highlighted). Panel B in Table 1 shows the underlying operating functions for the two scales. A Guttman scale has a distinctive stairstep pattern, while the unfolding scale can be represented by distinctive top-hat pattern. The comparison helps to illustrate the distinction between the cumulative and unfolding principles.

3 Hyperbolic Cosine Model

This study proposes to use a Hyperbolic Cosine model as an innovative method to identify the appropriateness of feedback. The Hyperbolic Cosine Unfolding Model (HCM; Andrich & Luo, 1993) can be viewed as a probabilistic model for

non-cumulative scales that can be used to identify an ideal point on a continuum that represents a person's location. It implies a single-peaked response function where a person has a higher probability of endorsing a subset of items, and these items identify the location of a person on the unfolding scale. The probability of endorsement increases when the person's location gets closer to the item's location. This feature reflects the basic characteristic of an ideal point process (Coombs, 1964) with the probability of a person endorsing a statement dependent on the distance between the person's location and the item's position. The HCM takes the following form (Andrich, 1996; Luo, 2001):

$$P\left(X_{ij}=k\right)=\frac{\left[\cosh\left(\theta_i-\lambda_j\right)\right]^{m-k}\prod_{l=1}^{k}\cosh\left(\rho_i\right)}{\sum_{k=0}^{m}\left[\cosh\left(\theta_i-\lambda_j\right)\right]^{m-k}\prod_{l=1}^{k}\cosh\left(\rho_i\right)},$$

when, $k=0, \prod_{l=1}^{k}\cosh\left(\rho_i\right)\equiv 1$;
where, in the context of writing,

$k=0, \ldots, m$, and $m+1$ is the number of rating categories,
$X_{ij}=$ observed rating given to student essay i on feedback strategy j,
$\theta_i=$ location of the student essay i,
$\lambda_j=$ location of the feedback strategy j,
$\rho_j=$ threshold/unit parameter for feedback strategy j (these threshold parameters reflect the ZTF for each strategy), and

the underlying unfolding scale for the feedback strategies used by teachers for essays is called the *joint* (J) scale (Coombs, 1964). Each feedback strategy (items) and essay have a unique location on the J scale. The relative distances between feedback strategies and an essay are important and meaningful in the unfolding scaling; therefore, we also construct *individual* (I) scales for each essay by folding the J scale at the ideal point (i.e., HCM location) of each essay on the J scale. The I scale reflects an ordering of strategies based on their relative location for each essay on the unfolding continuum.

4 A General Illustrative Example

An illustrative example is discussed in this section. Let us assume student achievement is represented by different achievement levels as shown in the first scale in Fig. 1. Meanwhile, some feedback strategies focus on different aspects or levels of student achievement. We simulate three possible situations of teachers providing feedback to students: they will not provide a type of feedback strategy, they may provide this type of feedback strategy, and they will provide it to the students. We use a rating scale (0–2) to indicate how likely they would be to provide this type of

feedback at this achievement level: 0 = No, 1 = Possibly, and 2 = Yes. We used the *RateFOLD* program (Luo & Andrich, 2003) to run the HCM analysis.

Teacher responses are shown in Table 2 (Panel A). For student response A, the teacher chooses to use Feedback 1 and 2, possibly to use Feedback 3 and 4, and not to use Feedback 5 and 6. The HCM scale ranges from −3.53 for Feedback 6 to 2.92 for Feedback 1. Table 2 (Panel B) provides information in terms of the distance between feedback strategies and the locations of the answers on the HCM scale. The entries in this table are the absolute values of these distances. Smaller values are highlighted because they are more likely to be endorsed than the others. They indicate the most recommended feedback strategies by the teachers for each essay. For example, the smallest distance for student response B is .16 for Feedback 2 with this being the most recommended feedback strategy. For response C, Feedback 3 is the most recommended one with the smallest distance of .19. Figure 2a shows a HCM map for the feedback strategies. In the HCM map, the ZTF for student response C is highlighted. A useful check on the appropriateness of the HCM model is to fit a polynomial model for the relationship between the location of feedback

Table 2 Illustration based on six feedback strategies for five essays

Panel A: Illustrative ratings						
	Essays					
Feedback strategies	**A**	**B**	**C**	**D**	**E**	**Feedback location**
1	2	1	1	1	0	2.92
2	2	2	2	1	1	1.10
3	1	2	2	2	1	-0.02
4	1	1	2	2	1	-0.56
5	0	1	2	2	1	-1.15
6	0	0	1	1	2	-3.53
Essay location:	2.73	1.26	-0.21	-0.68	-3.10	
Panel B: Absolute values of differences between essays and feedback locations						
	Essays					
Feedback strategies	**A**	**B**	**C**	**D**	**E**	**Feedback location**
1	1.63	1.66	3.13	3.60	6.02	2.92
2	1.63	0.16	1.31	1.78	4.20	1.10
3	2.75	1.28	0.19	0.66	3.08	-0.02
4	3.29	1.82	0.35	0.12	2.54	-0.56
5	3.88	2.41	0.94	0.47	1.95	-1.15
6	6.26	4.79	3.32	2.85	0.43	-3.53
Essay location:	2.73	1.26	-0.21	-0.68	-3.10	

Note: Cell entries are the absolute values of the differences between essay and feedback locations on the unfolding scale. The feedback strategies with smaller distances are highly recommended by the teachers

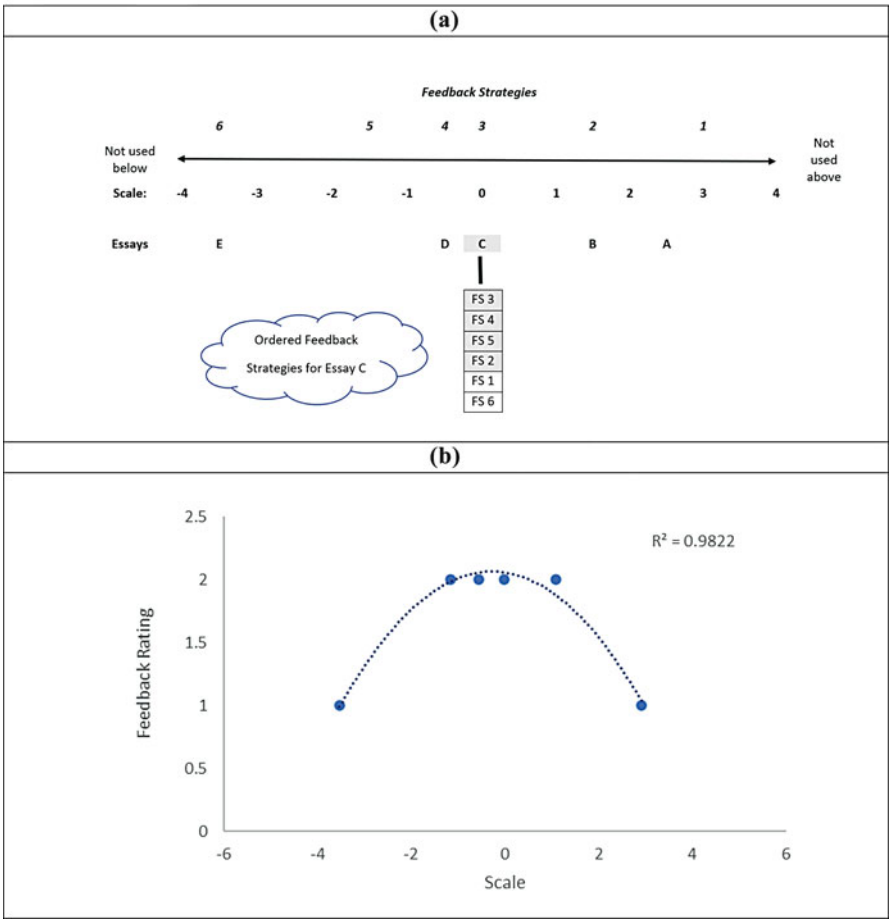


Fig. 2 Illustrative example. (**a**) Recommend feedback strategies for essay C. (**b**) Relationship between location for essay C (second degree polynomial)

strategies and the proportion of teacher judgments for each feedback strategy. This is shown in Fig. 2b with a R-square value of .9822 that supports the use of an unfolding process for these feedback data.

5 Application to Writing Assessment

To develop a deeper understanding of the conceptual map and the approach discussed in the previous section, an empirical study that examined feedback in the context of writing assessment is briefly introduced in this section. In this

Table 3 Questionnaire

Short labels	Items (feedback strategies)
A. *Organization*	
Introduction	1. Feedback should focus on how to create a more effective introduction to the response.
Organization	2. Feedback should focus on how to organize the response more clearly.
Conclusion	3. Feedback should focus on how to create a more effective conclusion to the response.
B. *Development*	
Relevant evidence	4. Feedback should focus on how to incorporate relevant evidence from the source texts into the response.
Elaborate evidence	5. Feedback should focus on how to elaborate more effectively on the evidence incorporated from the source texts.
Tone	6. Feedback should focus on how to create a tone that is appropriate for the task.
C. *Language usage and conventions*	
Create sentences	7. Feedback should focus on how to create clear, complete sentences.
Vary sentences	8. Feedback should focus on how to vary sentence structure.
Usage	9. Feedback should focus on usage (e.g., subject-verb agreement, pronoun-antecedent agreement, and using correct forms of homonyms).
Mechanics	10. Feedback should focus on mechanics (e.g., use of internal punctuation, spelling, capitalization, paragraph indentations, etc.)

Notes: Teachers responded to these questions using a 4-point scale indicating how likely they were to provide feedback in each area: 1 = Definitely not, 2 = Probably not, 3 = Probably and 4 = Definitely

empirical study, we conducted both a qualitative design to collecting the data and a quantitative analysis to demonstrate the application of HCM analysis. Essays written by middle school students with different writing proficiencies are used in this study ($N = 20$). A questionnaire was constructed based on a focus group of English teachers' recommendations of the possible feedback strategies for the essays (see Table 3). The ten feedback strategies in the questionnaires focus on three aspects: (1) organization (e.g., feedback should focus on how to create a more effective introduction to the response), (2) development, and (3) language usage (e.g., feedback should focus on how to create clear, complete sentences). Next, middle school ELA teachers ($N = 20$) responded to this questionnaire for identifying feedback strategies for the set of 20 essays. The HCM analysis was done in the *RateFOLD* program after we collect the responses to the questionnaire from the teachers.

It is beyond the scope of this chapter to describe the empirical study results in detail; however, Fig. 3 shows the calibration of essays and feedback on the unfolding continuum. In Table 4, the absolute values of the distances between HCM scale and person ability are showed. Smaller values indicate more highly recommended feedback strategies by the teachers for each essay. Distances less than 2 were highlighted as the ZTF for each essay. These strategies were more likely to

HCM Scale	Feedback Items	Feedback Categories: Organization	Development	Language Usage
6	6		Tone	
5				
4				
3	8, 5		Elaborate Evidence	Vary Sentences
2				
1	3	Conclusion		
0	1	Introduction		
-1	4,2	Organization	Relevant Evidence	
-2				
-3	7,10			Mechanics, Create Sentences
-4				
-5				
-6	9			Usage

Fig. 3 Map of feedback strategies

be recommended than the others for each essay. The preliminary results from this study suggest that this is a promising approach for identifying ZTF. This program of research uses the HCM to model the recommended feedback strategies, and the next step is to extend the approach to other content areas including mathematics, science, and social studies.

6 Discussion

The study introduces the concept of Zones of Targeted Feedback, and illustrates how to use a Hyperbolic Cosine unfolding model to identify and improve the effectiveness of feedback. The study also briefly presents empirical work in the field of middle school writing assessment. Our illustration indicates that unfolding models can be used as measurement tools to identify the optimal strategies for

Table 4 Results of an empirical study

Feedback items:	9	10	7	2	4	1	3	5	8	6	Essay locations		
Scale / Essays	-5.30	-2.89	-2.54	-0.86	-0.78	-0.19	1.11	2.76	2.89	5.80	Mean	Variance	Chi-squared
Essay 1	7.05	4.64	4.29	2.61	2.53	1.94	0.64	1.01	1.14	4.05	1.75	3.80	1.81
Essay 2	6.75	4.34	3.99	2.31	2.23	1.64	0.34	1.31	1.44	4.35	1.45	9.99	5.20
Essay 3	6.72	4.31	3.96	2.28	2.20	1.61	0.31	1.34	1.47	4.38	1.42	3.06	2.10
Essay 4	6.51	4.10	3.75	2.07	1.99	1.40	0.10	1.55	1.68	4.59	1.21	9.99	4.54
Essay 5	6.24	3.83	3.48	1.80	1.72	1.13	0.17	1.82	1.95	4.86	0.94	1.80	1.44
Essay 6	6.17	3.76	3.41	1.73	1.65	1.06	0.24	1.89	2.02	4.93	0.87	1.51	.63
Essay 7	6.13	3.72	3.37	1.69	1.61	1.02	0.28	1.93	2.06	4.97	0.83	2.40	2.28
Essay 8	5.54	3.13	2.78	1.10	1.02	0.43	0.87	2.52	2.65	5.56	0.24	0.86	2.24
Essay 9	5.53	3.12	2.77	1.09	1.01	0.42	0.88	2.53	2.66	5.57	0.23	3.92	3.27
Essay 10	5.53	3.12	2.77	1.09	1.01	0.42	0.88	2.53	2.66	5.57	0.23	3.88	1.53
Essay 11	5.45	3.04	2.69	1.01	0.93	0.34	0.96	2.61	2.74	5.65	0.15	5.71	12.36
Essay 12	5.21	2.80	2.45	0.77	0.69	0.10	1.20	2.85	2.98	5.89	-0.09	1.32	1.87
Essay 13	5.11	2.70	2.35	0.67	0.59	0.00	1.30	2.95	3.08	5.99	-0.19	1.66	1.02
Essay 14	4.62	2.21	1.86	0.18	0.10	0.49	1.79	3.44	3.57	6.48	-0.68	2.13	1.69
Essay 15	4.54	2.13	1.78	0.10	0.02	0.57	1.87	3.52	3.65	6.56	-0.76	9.00	18.07[a]
Essay 16	4.53	2.12	1.77	0.09	0.01	0.58	1.88	3.53	3.66	6.57	-0.77	2.07	2.07
Essay 17	4.32	1.91	1.56	0.12	0.20	0.79	2.09	3.74	3.87	6.78	-0.98	6.76	1.22
Essay 18	4.10	1.69	1.34	0.34	0.42	1.01	2.31	3.96	4.09	7.00	-1.20	1.69	8.50
Essay 19	3.97	1.56	1.21	0.47	0.55	1.14	2.44	4.09	4.22	7.13	-1.33	2.89	9.38
Essay 20	3.82	1.41	1.06	0.62	0.70	1.29	2.59	4.24	4.37	7.28	-1.48	2.25	20.18[a]

Note: Cell entries are the absolute values of the differences between essay and feedback locations on the HCM scale. The labels for the feedback strategies: 9 = Usage, 10 = Mechanics, 7 = Create Sentences, 2 = Organization, 4 = Relevant Evidence, 1 = Introduction, 3 = Conclusion, 5 = Elaborate Evidence, 8 = Vary Sentences, and 6 = Tone

[a]Approximate chi-squared statistics are evaluated with df = 9 and $p < .01$

students at different writing proficiency levels, and to define ZTF that may be effective for improving student writing.

Further analyses of ZTF can identify strategies in different content areas. Moreover, examining students' reception of the feedback and exploring empirical evidence of effective learning is essential in the coming future. As researchers generalize from this study to broader contexts, examining appropriate feedback strategies may help fill the gap between formative assessments and what teachers can do in practice to improve student achievement. Research is also needed to explore other unfolding models, such as the nonparametric unfolding IRT model (Post & Snijders, 1993), and the generalized graded unfolding model (Roberts et al., 2000). In summary, the identification and use of zones of targeted feedback (ZTF) offer a promising strategy for moving students forward to a higher level of achievement.

References

Andrich, D. (1996). A hyperbolic cosine latent trait model for unfolding polytomous responses: Reconciling Thurstone and Likert methodologies. *British Journal of Mathematical and Statistical Psychology, 49*, 347–365.

Andrich, D. (1997). A hyperbolic cosine IRT model for unfolding direct response of persons to items. In W. J. van der Linden & R. K. Hambleton (Eds.), *Handbook of modern item response theory* (pp. 399–414). Springer.

Andrich, D. (1988). The application of an unfolding model of the PIRT type to the measurement of attitude. *Applied Psychological Measurement, 12*(1), 33–51.

Andrich, D., & Luo, G. (1993). A hyperbolic cosine latent trait model for unfolding dichotomous single-stimulus responses. *Applied Psychological Measurement, 17*, 253–276.

Bangert-Drowns, R. L., Kulik, C. L. C., Kulik, J. A., & Morgan, M. (1991). The instructional effect of feedback in test-like events. *Review of Educational Research, 61*(2), 213–238.

Coombs, C. H. (1964). *A theory of data*. Wiley.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 7*(1), 81–112.

Kluger, A. N., & DeNisi, A. (1996). The effects of feedback interventions on performance: A historical review, a meta-analysis, and a preliminary feedback intervention theory. *Psychological Bulletin, 119*(2), 254.

Kulhavy, R. W. (1977). Feedback in written instruction. *Review of Educational Research, 47*(2), 211–232.

Kulhavy, R. W., & Stock, W. A. (1989). Feedback in written instruction: The place of response certitude. *Educational Psychology Review, 1*(4), 279–308.

Luo, G. (2001). A class of probabilistic unfolding models for polytomous responses. *Journal of Mathematical Psychology, 45*(2), 224–248.

Luo, G., & Andrich, D. (2003). *RateFOLD computer program*. Social Measurement Laboratory: School of Education, Murdoch University.

Narciss, S., & Huth, K. (2004). How to design informative tutoring feedback for multimedia learning. In *Instructional design for multimedia learning* (pp. 181–195). Waxmann.

Post, W. J., & Snijders, T. A. (1993). Nonparametric unfolding models for dichotomous data. *Methodika, 7*(1), 130–156.

Roberts, J. S., Donoghue, J. R., & Laughlin, J. E. (2000). A general item response theory model for unfolding unidimensional polytomous responses. *Applied Psychological Measurement, 24*(1), 3–32.

Shute, V. J. (2008). Focus on formative feedback. *Review of Educational Research, 78*(1), 153–189.

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scriber & E. Souberman, Eds. & Trans.). Harvard University Press

Willingham, D. B. (1990). Effective feedback on written assignments. *Teaching of Psychology, 17*(1), 10–13.