Chapter 12
In-plane Vibration Measurement of an Aluminum Plate Using a Three-Dimensional Continuously Scanning Laser Doppler Vibrometer System

Ke Yuan and Weidong Zhu

Abstract A three-dimensional (3D) continuously scanning laser Doppler vibrometer (CSLDV) system that contains three CSLDVs and an external controller is developed to conduct full-field scanning to measure 3D vibrations of an aluminum plate with free boundary conditions under sinusoidal excitation. A reference object parallel to the plane of the plate is used as the measurement coordinate system to obtain in-plane vibration components. Calibration among three CSLDVs in the 3D CSLDV system based on the geometrical model of its scan mirrors is conducted to adjust their rotational angles to ensure that three laser spots can continuously and synchronously move along the same 2D scan trajectory on the plate. The demodulation method is used to process the measured response to obtain operating deflection shapes (ODSs) of the plate. By using frequencies that are close to damped natural frequencies of the plate as sinusoidal excitation frequencies, four in-plane ODSs, including shear and longitudinal ones, are obtained in the frequency range between 0 and 5000 Hz. These ODSs are in good agreement with those obtained by traditional stepwise scanning and theoretical undamped mode shapes of the plate calculated from its finite element (FE) model. Modal assurance criterion (MAC) values between the first four in-plane ODSs from 3D CSLDV measurements and those from stepwise scanning measurements are larger than 95%. MAC values between ODSs from 3D CSLDV measurements and corresponding mode shapes from the FE model are larger than 90%. However, the 3D CSLDV system can scan more measurement points in much less time than 3D stepwise scanning.

Keywords Plate · In-plane vibration · Operating deflection shape · Three-dimensional continuously scanning laser Doppler vibrometer system · Demodulation method

12.1 Introduction

There is no doubt that transverse, or called out-of-plane, vibration plays an important role in modal analysis of plate structures, since their out-of-plane mode shapes are usually found at low natural frequencies and can be easily excited. There are many studies on out-of-plane full-field vibration measurements of structures by using accelerometers [1–2]. However, the significance of in-plane vibration in wave transmission of high-frequency vibration in built-up structures and its effects on energy flow were also studied by some researchers [3–5]. Many studies focused on developing methods to improve accuracy of analytical solutions of in-plane vibration of platelike structures with various shapes and boundary conditions [6–8]. There is still a scarcity of study on modal test of in-plane vibration of plate structures.

One possible reason for the lack of experimental investigation of in-plane vibration of plate structures is that usual vibration measurement devices, such as accelerometers and single-point or one-dimensional (1D) scanning laser Doppler vibrometers (LDVs), are designed for measuring vibration normal to the surface of a structure, and it is not easy to directly measure in-plane vibration of the structure. Larsson [9] investigated the feasibility of measuring in-plane vibration of a rectangular plate with an aspect ratio of 1:2 and free boundary conditions by using accelerometers. In the study, three accelerometers were attached along the direction parallel to the plate surface and moved around the plate to measure its in-plane vibration. Twelve in-plane modes were identified in the frequency range from 1600 to 7000 Hz; however, some errors would be induced by mass loading of the accelerometers and large signal noise at interior positions of the plate. Batista et al. [10] measured in-plane mode shapes of a plate with free boundary conditions by using a single-point LDV and a light hammer. The laser beam of the LDV was parallel to the plate surface in the study, and a cubic block was attached to the plate

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surface at its different positions to receive the laser spot during the experiment. However, it took much time to move and reattach the block, and the block would also induce mass loading. Xu and Zhu [11] developed an operational modal analysis method by using a single-point LDV and a free-field microphone as well as an experimental modal analysis method by using the LDV and an impact hammer to measure both in-plane and out-of-plane vibrations of a plate in a noncontact way. In the study, the laser beam was shinned on the plate surface with a proper incident angle, and in-plane vibration of the plate was obtained from velocities along the laser beam by calculating their components parallel to the plate surface.

Although a triaxial accelerometer was developed and applied to various fields to measure three-dimensional (3D) vibrations of structures, mass loading is still a problem, especially for lightweight structures. Miyashita and Fujino [12] developed a noncontact 3D vibration measurement system by calibrating positions of three LDVs, and reported good agreement between results from the 3D LDV system and a triaxial accelerometer in measuring ground motion, wave propagation of a plate, and vibration of a steel railway bridge. To automate a vibration test, a scanning laser Doppler vibrometer (SLDV) was developed by adding a pair of orthogonally mounted galvanometer mirrors to the LDV [13]. A 3D SLDV system, which contains three SLDVs and controls three laser spots to synchronously move in a stepwise manner along the surface of a structure, has been developed to measure three perpendicular velocity components of the structure and commercialized as Polytec PSV-400-3D and PSV-500-3D systems. The 3D SLDV system has been applied in many areas, such as vibration analysis of power tools with 3D complex surfaces [14], identification of fatigue cracks [15], detection of damage in a beam by using its longitudinal vibration shapes [16], modal analysis of a whole vehicle body [17] and a towerlike skeletal structure [18], full-field strain response measurement of an underwater structure [19–20], modal testing of a sandwich structure [21], and modal identification of a three-bladed wind turbine assembly [22]. Some researchers focused on developing 3D vibration measurement systems by attaching a single SLDV to a multi-axis positioning system [23] or moving the single SLDV to three different locations to reduce the cost [24–26]. However, it usually takes a long time to obtain high spatial resolution in SLDV measurement when the surface of a structure is large and measurement points are dense [18], and most previous studies focused on in-plane components of 3D vibrations at relatively low out-of-plane frequencies, which usually have larger magnitudes than those at high in-plane frequencies. An improved method is to continuously move the laser spot over the surface of a structure, which was developed as a continuously scanning laser Doppler vibrometer (CSLDV) system [27–29]. Dense operating deflection shapes (ODSs) and mode shapes of structures can be used to detect damages in them [30–33].

For 3D vibration measurements by using three CSLDVs, the main challenge is to calibrate positions of the three CSLDVs to continuously and synchronously direct their laser beams along the same scan trajectory. A novel 3D CSLDV system, which contains three CSLDVs, was developed to address the challenge through a calibrating procedure, and 3D vibrations of a beam under white-noise [34] and sinusoidal [35] excitations were studied and showed good agreements with results from a commercial 3D SLDV system. Also, 3D OSDs of a composite plate under sinusoidal excitation with a frequency of 148 Hz were studied in Ref. [36]. However, 3D vibrations of the plate in the study were obtained by resolving its out-of-plane vibration at a low excitation frequency through an arbitrary measurement coordinate system (MCS). One still needs to study whether the 3D CSLDV system can be used to measure in-plane vibration of a plate at excitation frequencies close to its in-plane natural frequencies, which are usually much higher than its lowest out-of-plane frequency, with enough accuracy. With different vibration information from that of out-of-plane ODSs of a structure, in-plane ODSs of the structure can be used to some advantages to detect its damage in some future study.

The objective of this work is to propose a noncontact and fast way to measure dense full-field in-plane vibration of an aluminum plate, which has high frequencies and low response magnitudes. A novel 3D CSLDV system is developed to measure 3D full-field vibrations of the aluminum plate under sinusoidal excitation at frequencies close to its in-plane natural frequencies, which are identified by its finite element (FE) model and modal test by using a Polytec PSV-500-3D SLDV system. A two-dimensional (2D) zigzag scan trajectory is designed on the plate surface to conduct full-field scanning, a shaker is placed at a side of the plate to excite its in-plane vibration, and a reference object parallel to the plate plane is used to calibrate the 3D CSLDV system to ensure that the in-plane vibration can be measured. The difference between the proposed 3D CSLDV system and existing commercial 3D SLDV systems lies in their calibration method. Existing commercial 3D SLDV systems, such as a Polytec PSV-500-3D, calibrate three laser spots by a video triangulation procedure when moving laser spots from one measurement point to the next one, which increases test time and limits its application to continuous scanning. The demodulation method is used to process in-plane response of the plate from 3D CSLDV measurements to obtain its in-plane ODSs. Four obtained in-plane ODSs at three excitation frequencies of the plate from 3D CSLDV measurements are compared to those from 3D SLDV measurements and corresponding in-plane mode shapes from its FE model. Modal assurance criterion (MAC) values between experimental results from 3D CSLDV and 3D SLDV measurements of the plate are larger than 95%, and MAC values between experimental results from its 3D CSLDV measurements and numerical results from its FE model are larger than 91%. The 3D CSLDV system can scan much more measurement points in much less time than the 3D SLDV system.

12.2 Instrument and Method for Measuring In-plane ODSs of a Platelike Structure

A novel 3D CSLDV system, which consists of three CSLDVs and an external controller, is developed to measure 3D full-field vibrations of a plate. As shown in Fig. 12.1, three CSLDVs can be referred to as Top, Left, and Right CSLDVs based on their positions during measurement. A zigzag scan trajectory on the surface of the plate is predesigned, and corresponding signals are inputted into scan mirrors in each CSLDV to direct three laser spots to continuously and synchronously move along the scan trajectory.

As shown in Fig. 12.2, a reference object, which has some known accurate coordinates, is used as the MCS to calibrate the 3D CSLDV system. It is noted that the reference object needs to be parallel to the plane of the plate, so that response measured by three CSLDVs can be projected onto the o-xy plane of the MCS, which includes in-plane ox and oy directions of the plate. Two orthogonal scan mirrors in a CSLDV, which can be referred to as X and Y mirrors, are used to build the vibrometer coordinate system (VCS) to describe the position of the CSLDV during measurement in this work. One can see from Fig. 12.2 that the rotating center of the X mirror $o^{'}$ is also the origin of the VCS, and the rotating center of the Y mirror o'' is on the $o^{'}z^{'}$ axis; so the X mirror can rotate about the $x^{'}$ axis, and the Y mirror can rotate about the y'' axis that is parallel to the $y^{'}$ axis. Variables α and β, which can be controlled by inputting voltages to X and Y mirrors through the external controller, are rotational angles of X and Y mirrors from their initial positions, respectively. The distance between centers of the two scan mirrors $|o^{'}o''| = d$ is a known parameter for the CSLDV.

Based on the geometrical model of the CSLDV in Fig. 12.2, coordinates of a point P in the VCS can be expressed as

$$\mathbf{P}_{\text{VCS}} = [-d\tan(\beta) - r\sin(\beta), -r\cos(\alpha)\cos(\beta), -r\sin(\alpha)\cos(\beta)]^T, \tag{12.1}$$

where r is the distance from the point P to the incident point of the laser path on the X mirror $P^{'}$. The unit vector $\mathbf{e}$ in the laser beam direction in the VCS can be written as

$$\mathbf{e} = [\sin(\beta), \cos(\alpha)\cos(\beta), \sin(\alpha)\cos(\beta)]^T. \tag{12.2}$$

It can be assumed that coordinates of a selected calibrating point P in the MCS are $\mathbf{P}_{\text{MCS}} = [x, y, z]^T$, whose values can be easily obtained from the reference object. The relation between $\mathbf{P}_{\text{MCS}}$ and $\mathbf{P}_{\text{VCS}}$ can be written as

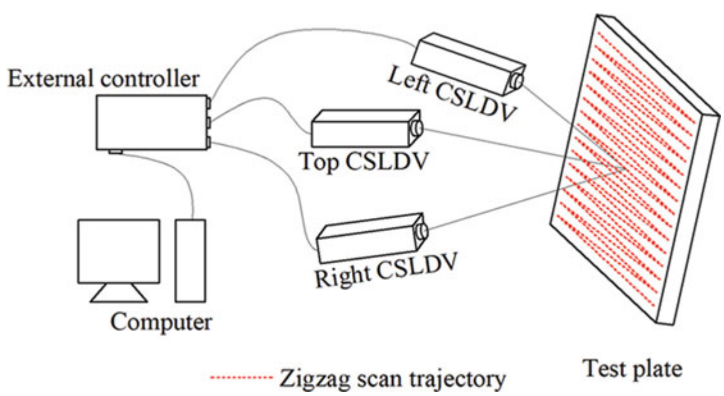
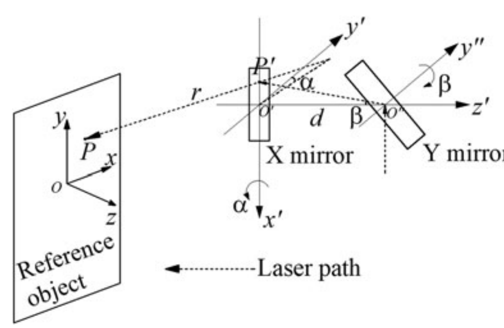


Fig. 12.1 3D CSLDV system and zigzag scan trajectory for measuring 3D full-field vibrations of an aluminum plate

Fig. 12.2 Geometrical model of X and Y mirrors of a CSLDV, in which the reference object is used as the MCS to provide points with known coordinates



$$\mathbf{P}_{\mathrm{MCS}} = \mathbf{T} + \mathbf{R}\mathbf{P}_{\mathrm{VCS}}, \tag{12.3}$$

where $\mathbf{T} = [x_{o'}, y_{o'}, z_{o'}]^T$ is the translation vector that denotes coordinates of the origin o' in the MCS and $\mathbf{R}$ is the direction cosine matrix from the MCS to the VCS. By using the method proposed in Refs. [34–36], which includes procedures of solving an overdetermined nonlinear problem and an optimization problem, exact values of r for all selected calibrating points, the error of the geometrical unit in the Top CSLDV, and $\mathbf{T}$ and $\mathbf{R}$ matrices for each CSLDV can be calculated. In this work, six points including four corner points and one reference point on either of two poles on the reference object were selected as reference points, which ensures that all the reference points are not on the same plane.

A zigzag scan trajectory is usually used in CSLDV measurements of platelike structures. For the 3D CSLDV system in this study, the approach of equal division that is used in 1D CSLDV measurements can generate signals to control three laser spots to move along the same scan line, but not synchronously, since it is difficult to keep all the laser beams from three CSLDVs to be absolutely perpendicular to the test plate and any slight angle changes would lead to asynchronous scanning. For the 3D CSLDV system used in this study, the first step to design the zigzag scan trajectory is to calculate values of r for all measurement points along the scan trajectory. Since the plate is flat, the bisection method proposed and used to design a 1D scan line on a beam in Ref. [34] can be extended to design a 2D scan trajectory on a plate by repeating the same process for multiple lines. Based on the fact that coordinates of a measurement point P^k are constant in the MCS for three CSLDVs, positional relations among three CSLDVs for the point can be established by

$$\mathbf{T}_{\mathrm{Top}} + \mathbf{R}_{\mathrm{Top}}\mathbf{P}^k_{\mathrm{VCS(Top)}} = \mathbf{T}_{\mathrm{Left}} + \mathbf{R}_{\mathrm{Left}}\mathbf{P}^k_{\mathrm{VCS(Left)}} = \mathbf{T}_{\mathrm{Right}} + \mathbf{R}_{\mathrm{Right}}\mathbf{P}^k_{\mathrm{VCS(Right)}}, \tag{12.4}$$

where subscripts Top, Left, and Right denote the Top, Left, and Right CSLDVs, respectively, and the superscript k denotes the k-th measurement point on the scan trajectory. Hence, coordinates of the point P^k in VCSs of Left and Right CSLDVs can be obtained by

$$\mathbf{P}^k_{\mathrm{VCS(Left)}} = \mathbf{R}^{-1}_{\mathrm{Left}}\left[\left(\mathbf{T}_{\mathrm{Top}} - \mathbf{T}_{\mathrm{Left}}\right) + \mathbf{R}_{\mathrm{Top}}\mathbf{P}^k_{\mathrm{VCS(Top)}}\right], \tag{12.5}$$

$$\mathbf{P}^k_{\mathrm{VCS(Right)}} = \mathbf{R}^{-1}_{\mathrm{Right}}\left[\left(\mathbf{T}_{\mathrm{Top}} - \mathbf{T}_{\mathrm{Right}}\right) + \mathbf{R}_{\mathrm{Top}}\mathbf{P}^k_{\mathrm{VCS(Top)}}\right], \tag{12.6}$$

respectively. Therefore, rotational angles of X and Y mirrors of Left and Right CSLDVs for the point P^k can be obtained by

$$\begin{aligned} \alpha^k_{\mathrm{Left}} &= \arctan\left(z^k_{\mathrm{VCS(Left)}}/y^k_{\mathrm{VCS(Left)}}\right) \\ \beta^k_{\mathrm{Left}} &= \arctan\left(x^k_{\mathrm{VCS(Left)}}/\left(y^k_{\mathrm{VCS(Left)}}/\cos\left(\alpha^k_{\mathrm{Left}}\right) - d\right)\right), \end{aligned} \tag{12.7}$$

and

$$\begin{aligned} \alpha^k_{\mathrm{Right}} &= \arctan\left(z^k_{\mathrm{VCS(Rightt)}}/y^k_{\mathrm{VCS(Right)}}\right) \\ \beta^k_{\mathrm{Right}} &= \arctan\left(x^k_{\mathrm{VCS(Right)}}/\left(y^k_{\mathrm{VCS(Right)}}/\cos\left(\alpha^k_{\mathrm{Rightt}}\right) - d\right)\right), \end{aligned} \tag{12.8}$$

respectively.

With the proposed methodology described above, three laser spots from three CSLDVs can continuously and synchronously move along the same scan trajectory by inputting corresponding rotational angles to their scan mirrors. Vibration components of the point P^k in x, y, and z directions of the MCS can then be obtained by

$$\left[V^k_x, V^k_y, V^k_z\right]^T = \left[\left[\mathbf{R}_{\mathrm{Top}}\mathbf{e}^k_{\mathrm{Top}}, \mathbf{R}_{\mathrm{Left}}\mathbf{e}^k_{\mathrm{Left}}, \mathbf{R}_{\mathrm{Right}}\mathbf{e}^k_{\mathrm{Right}}\right]^T\right]^{-1}\left[V^k_{\mathrm{Top}}, V^k_{\mathrm{Left}}, V^k_{\mathrm{Right}}\right]^T, \tag{12.9}$$

where V^k_{Top}, V^k_{Left}, and V^k_{Right} are measured velocities of the point P^k by using Top, Left, and Right CSLDVs, respectively. The process described by Eq. (12.9) can be repeated for each point on the prescribed scan trajectory. Finally, 3D vibrations of the plate in the MCS can be obtained by the 3D CSLDV system.

The demodulation method can be used to process the steady-state response of a structure under sinusoid excitation from CSLDV measurement to obtain its ODS at the excitation frequency [37]. The steady-state response along the scan trajectory u can be expressed as

$$u\left(\mathbf{x},t\right)=U\left(\mathbf{x}\right)\cos\left(\omega t-\varphi\right)=U_I\left(\mathbf{x}\right)\cos\left(\omega t\right)+U_Q\left(\mathbf{x}\right)\sin\left(\omega t\right), \quad (12.10)$$

where $\mathbf{x}$ denotes the position information of measurement points along the scan trajectory, $U(\mathbf{x})$ are responses at measurement points along the scan trajectory that have two components, which are the in-phase component $U_I(\mathbf{x})$ and quadrature component $U_Q(\mathbf{x})$, ω is the excitation frequency, and φ is a phase variable. To obtain in-phase and quadrature components of $U(\mathbf{x})$, multiplying $u(\mathbf{x}, t)$ by $\cos(\omega t)$ and $\sin(\omega t)$ yields:

$$\begin{aligned}u\left(\mathbf{x},t\right)\cos\left(\omega t\right)&=\Phi_I\left(\mathbf{x}\right)\cos^2\left(\omega t\right)+\Phi_Q\left(\mathbf{x}\right)\sin\left(\omega t\right)\cos\left(\omega t\right)\\&=\tfrac{1}{2}\Phi_I\left(\mathbf{x}\right)+\tfrac{1}{2}\Phi_I\left(\mathbf{x}\right)\cos\left(2\omega t\right)+\tfrac{1}{2}\Phi_Q\left(\mathbf{x}\right)\sin\left(2\omega t\right),\end{aligned} \quad (12.11)$$

and

$$\begin{aligned}u\left(\mathbf{x},t\right)\sin\left(\omega t\right)&=\Phi_I\left(\mathbf{x}\right)\sin\left(\omega t\right)\cos\left(\omega t\right)+\Phi_Q\left(\mathbf{x}\right)\sin^2\left(\omega t\right)\\&=\tfrac{1}{2}\Phi_Q\left(\mathbf{x}\right)+\tfrac{1}{2}\Phi_I\left(\mathbf{x}\right)\sin\left(2\omega t\right)-\tfrac{1}{2}\Phi_Q\left(\mathbf{x}\right)\cos\left(2\omega t\right),\end{aligned} \quad (12.12)$$

respectively. A low-pass filter can then be used to eliminate $\sin(2\omega t)$ and $\cos(2\omega t)$ terms in Eqs. (12.11) and (12.12), and corresponding results can be multiplied by a scale factor of two to obtain $U_I(\mathbf{x})$ and $U_Q(\mathbf{x})$.

12.3 Experimental Investigation

A rectangular aluminum plate, which has dimensions of 400 mm × 500 mm × 4.75 mm, was used as the specimen in this work. Three SLDVs from a Polytec PSV-500-3D SLDV system were used to build the 3D CSLDV system in this work. The Polytec PSV-500-3D system has no continuously scanning function, but has the interface connector in each SLDV, which can be connected to an external controller. A dSPACE MicroLabBox and the ControlDesk software were used as the external controller to generate a series of signals to control a total number of six scan mirrors to continuously rotate in the configured 3D CSLDV system. The external controller can be removed from the 3D CSLDV system to obtain the original Polytec PSV-500-3D system and conduct stepwise scanning measurements. As shown in Fig. 12.3a, the longitudinal axis of the scan head of the Top CSLDV needs to be perpendicular to the scan surface of the plate. Left and Right CSLDVs can be placed at arbitrary locations as long as they are not too close to the Top CSLDV and their laser spots can cover the complete area to be scanned in the input voltage range. The Polytec PSV-A-450 reference object parallel to the plate was used to calibrate the 3D CSLDV system. From Fig. 12.3b, one can see that two strings were used to hang the plate to simulate its free boundary conditions, and a shaker was placed at one side of the plate to excite it along its in-plane directions by using frequencies that are close to in-plane natural frequencies of the plate. The shaker in Fig. 12.3b excited the plate along the x direction, and the plate could rotate about the oz axis by 90° in the o-xy plane when measuring its ODSs along the y direction. In-plane ODSs at frequencies close to identified natural frequencies of the plate were obtained in the 3D stepwise scanning manner of the Polytec PSV-500-3D system in the same MCS and used as references for comparison purposes to validate in-plane ODSs obtained from 3D CSLDV measurements.

A FE model of the plate with free boundary conditions was created by using the commercial finite element software Abaqus to obtain its theoretical natural frequencies and mode shapes. The material of the FE model of the plate is aluminum that has a Young's modulus of 70 GPa, a density of 2750 kg/m^3, and a Poisson's ratio of 0.33. As shown in Fig. 12.4, three in-plane natural frequencies of the plate can be calculated in the frequency range from 0 to 5000 Hz. One can see that mode shapes at natural frequencies of 3924.3 Hz and 4388.3 Hz are in-plane shear modes along the x direction, and the mode shape at the natural frequency of 4788.1 Hz is an in-plane longitudinal mode that has components along both x and y directions.

A modal test was first conducted by using the Polytec PSV-500-3D system in its stepwise scanning mode to identify in-plane natural frequencies of the plate, where a sweeping signal with a frequency band from 0 to 5000 Hz was used as excitation. A total number of 289 measurement points were selected in the modal test, which formed a 17 × 17 grid on the plate. The summed in-plane FRFs of all the 289 measurement points is shown in Fig. 12.5 where in-plane natural frequencies of the plate are marked. The first three in-plane natural frequencies of the plate identified from the modal test are shown in Table 12.1 and compared to those from its FE model. One can see that the maximum error between the first three in-plane

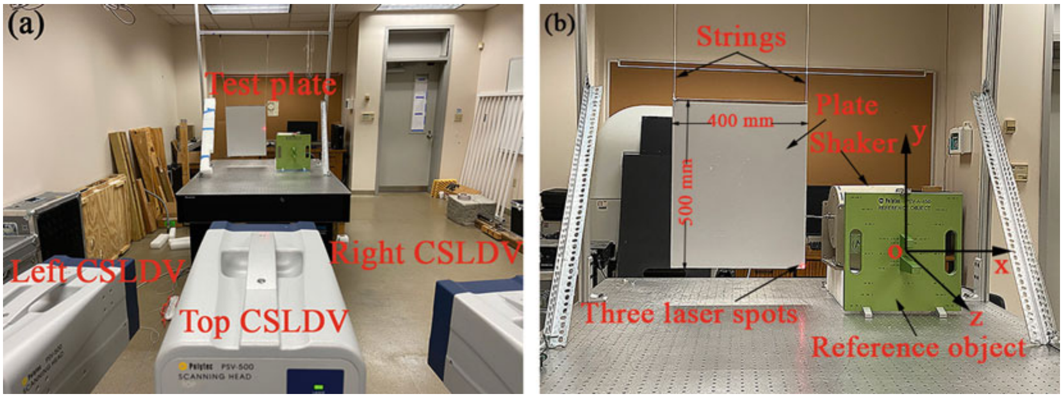


Fig. 12.3 (**a**) Arrangement of three CSLDVs with respect to the test plate and (**b**) the test plate with free boundary conditions and in-plane excitation from a shaker

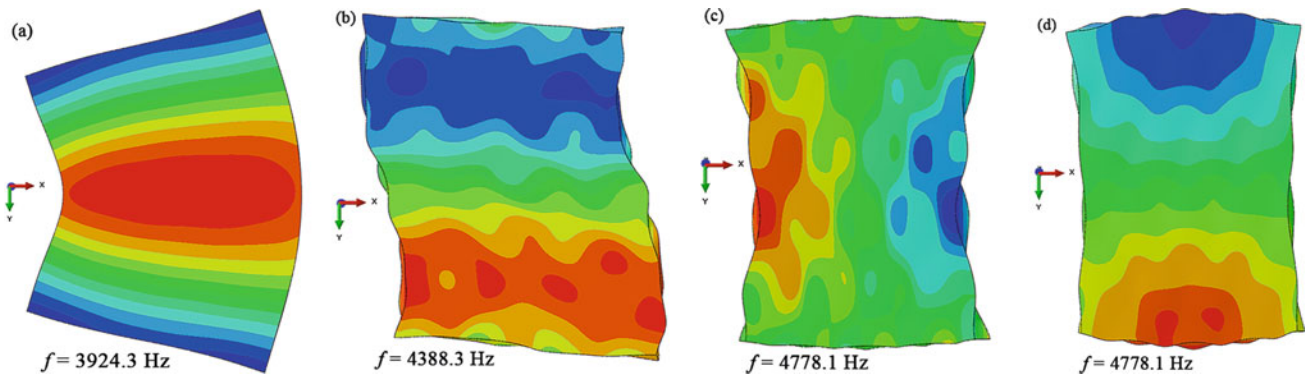


Fig. 12.4 Numerical results of in-plane mode shapes of the free aluminum plate along the x direction at frequencies of (**a**) 3924.3 Hz, (**b**) 4388.3 Hz, and (**c**) 4778.1 Hz, and (**d**) along the y direction at the frequency of 4778.1 Hz

Table 12.1 Natural frequencies of the first three in-plane mode shapes of the free aluminum plate from its FE model and the modal test by using the 3D SLDV system, and corresponding errors between them

In-plane mode number	Natural frequency (Hz)		Error (%)
	FE model	Modal test	
Mode 1	3924.3	3915.6	0.2
Mode 2	4388.3	4366.4	0.5
Mode 3	4778.1	4715.6	1.3

natural frequencies from the modal test of the plate and its FE model is 1.3%, indicating that these identified in-plane natural frequencies of the plate are accurate and can be used as excitation frequencies to obtain corresponding in-plane ODSs. The ratio of the largest measured in-plane rigid-body natural frequency of the plate to its first in-plane elastic natural frequency is 0.4%, which is less than 10%, indicating that boundary conditions simulated by strings in this experiment can be considered as free boundary conditions [38]. Note that some relatively low frequencies at peaks in Fig. 12.5 are natural frequencies of out-of-plane mode shapes of the plate.

A detailed description for calculating rotational angles of scan mirrors in Left and Right CSLDVs and calibrating the 3D CSLDV system is provided below:

1. Control scan mirrors in three CSLDVs to successively direct their laser spots to six reference points on the PSV-A-450 reference object, record their coordinates in the MCS and corresponding rotational angles, and obtain translation vectors and direction cosine matrices of the three CSLDVs.
2. Design a zigzag scan trajectory on the test plate, record rotational angles of scan mirrors in the Top CSLDV for start and end points on each scan line, and measure distances from the Top CSLDV to the start and end points.
3. Use Eqs. (12.4, 12.5, 12.6, 12.7 and 12.8) to calculate rotational angles of scan mirrors corresponding to Left and Right CSLDVs.

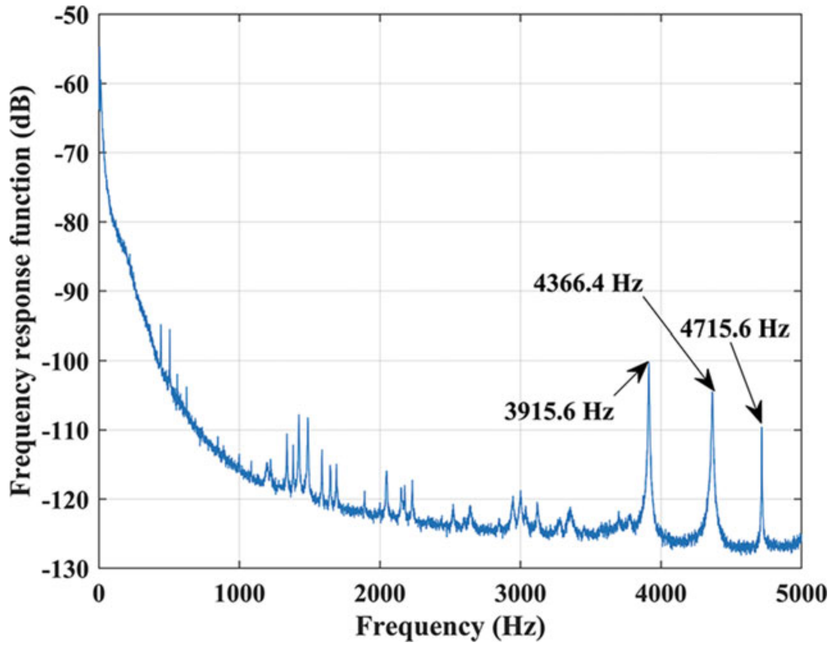


Fig. 12.5 Summed in-plane FRFs of all the 289 measurement points from the modal test by using the 3D SLDV system

Table 12.2 Comparison between test times and numbers of measurement points of 3D CSLDV and 3D SLDV measurements

Measurement method	Test time (s)	Number of measurement points
3D CSLDV measurement	82.5	25,000 × 33 = 825,000
3D SLDV measurement	1200	17 × 17 = 289

4. Feed signals corresponding to rotational angles calculated in the above steps to three CSLDVs, and visually examine if three laser spots can continuously and synchronously move along the same scan line.

To ensure laser spots continuously move along the whole zigzag scan trajectory, they need to be designed to move through one scan line n times, where n is odd. In this work, the scan frequency of the 3D CSLDV system is $f_{sc} = 1$ Hz, which means that it takes 0.5 s to move laser spots through one scan line, and the sampling frequency of the system is $f_{sa} = 50{,}000$ Hz, which means that 50,000 data are sampled in 1 s. The full-field zigzag scan trajectory on the plate in this experiment includes 34 scan lines and n is set to 5. Therefore, the total time to complete the whole scanning is $t = 0.5 \times 5 \times 33 = 82.5$ s, and the total number of measurement points is $k = 0.5 \times 50000 \times 33 = 825, 000$. Comparison of the total scanning time and number of measurement points between the 3D CSLDV system and 3D SLDV system is shown in Table 12.2. It can be seen that the 3D CSLDV system can measure much more points in much less time than the 3D SLDV system in full-field vibration measurement.

Results of four in-plane ODSs of the free aluminum plate at three excitation frequencies from both 3D CSLDV and 3D SLDV measurements are shown in Figs. 12.6, 12.7, 12.8, and 12.9. Note that the coordinate system in the above figures is the same as that in the FE model of the plate, where the x-axis is along the horizontal direction and the y-axis is along the vertical direction. It can be found that in-plane ODSs at frequencies of 3915.6 Hz and 4366.4 Hz are shear ones along the x direction and those at the frequency of 4715.6 Hz are longitudinal ones along both x and y directions, which have similar patterns to corresponding mode shapes in Fig. 12.4 from the FE model of the plate.

To further check correlations between in-plane ODSs of the plate from 3D CSLDV measurements and in-plane mode shapes from its FE model, and those between in-plane ODSs of the plate from 3D CSLDV and 3D SLDV measurements, their MAC values are calculated and shown in Table 12.3; correlation is high when a MAC value is close to 1 and low when it is close to 0 [38]. One can see that MAC values between in-plane ODSs from 3D CSLDV and 3D SLDV measurements are larger than 95% for all the four modes, indicating that 3D CSLDV measurements in this study have the same accuracy as that of 3D SLDV measurements. MAC values between in-plane ODSs from 3D SLDV measurements and in-plane mode shapes from the FE model of the plate are larger than 89% for all the four modes. MAC values between in-plane ODSs from 3D CSLDV measurements and in-plane mode shapes from the FE model of the plate are larger than 91% for all the four modes. The possible reason for the slightly lower MAC value of mode 3 along the x direction is that the mode shape from

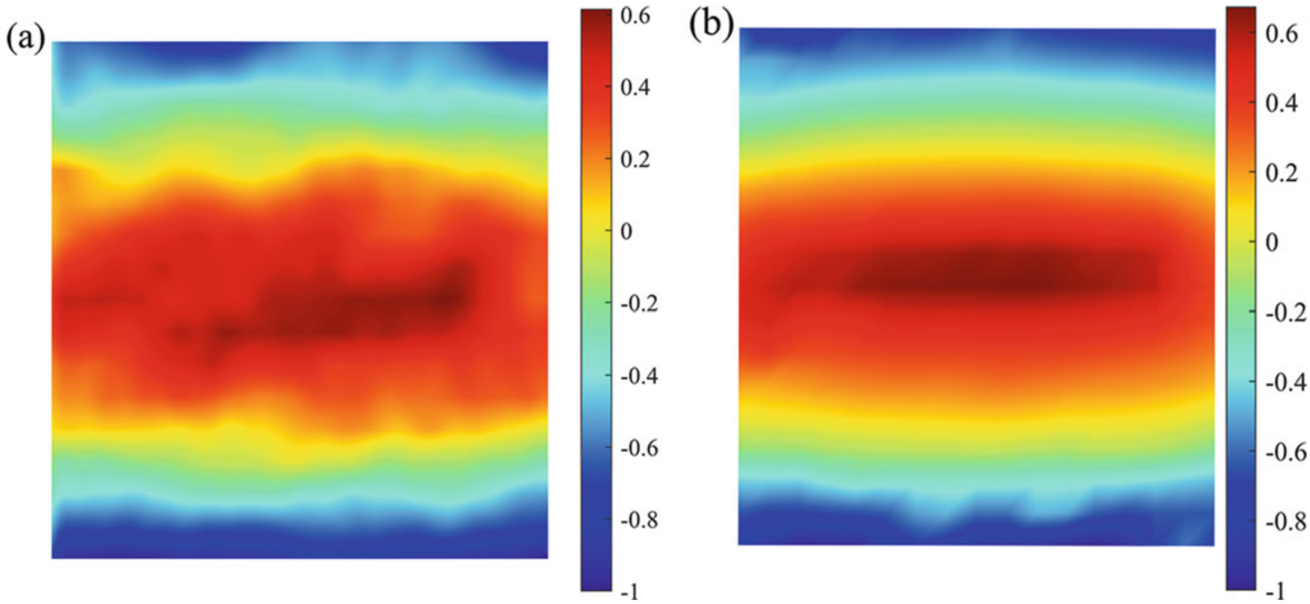


Fig. 12.6 In-plane ODSs along the x direction of the free aluminum plate at the excitation frequency of 3915.6 Hz from (**a**) 3D CSLDV and (**b**) 3D SLDV measurements

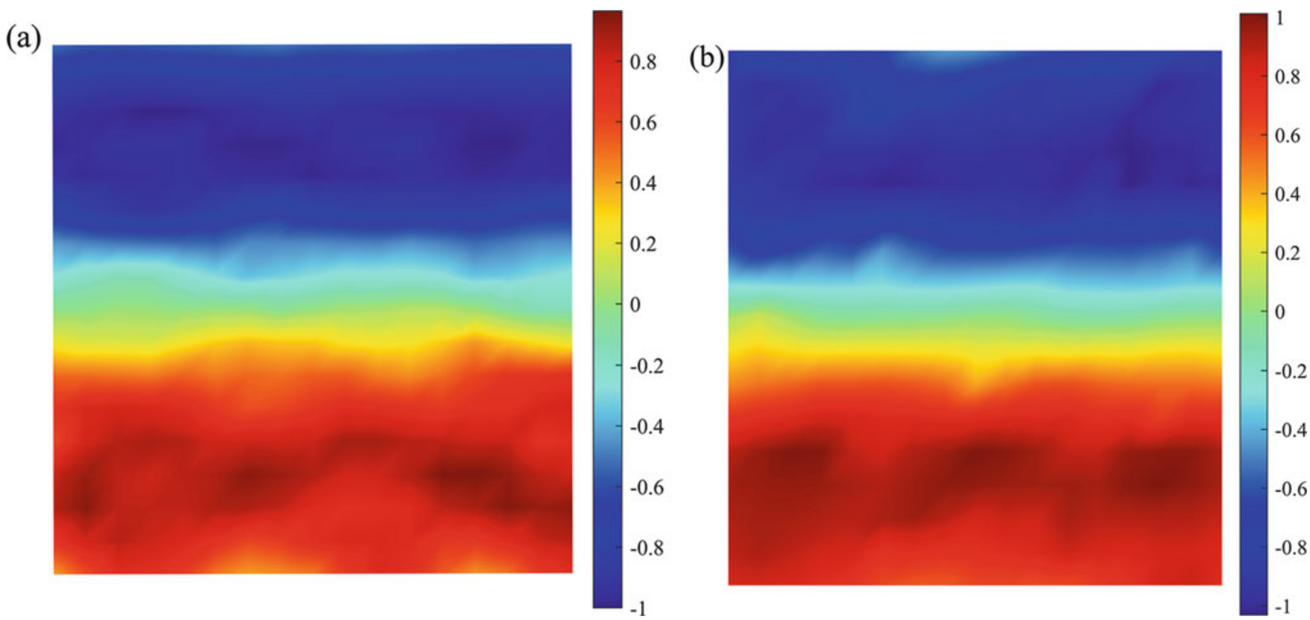


Fig. 12.7 In-plane ODSs along the x direction of the free aluminum plate at the excitation frequency of 4366.4 Hz from (**a**) 3D CSLDV and (**b**) 3D SLDV measurements

the FE model at the frequency of 4788.1 Hz is a coupled mode shape that has components along both x and y directions, but is dominated by the component along the y direction.

12.4 Conclusion

A novel measurement method using a 3D CSLDV system is developed in this study to obtain 3D full-field vibrations of a plate under sinusoidal excitations at frequencies close to its in-plane natural frequencies. A 2D zigzag scan trajectory is designed on the plate surface to conduct a full-field scanning, a shaker is placed at a side of the plate to excite its in-

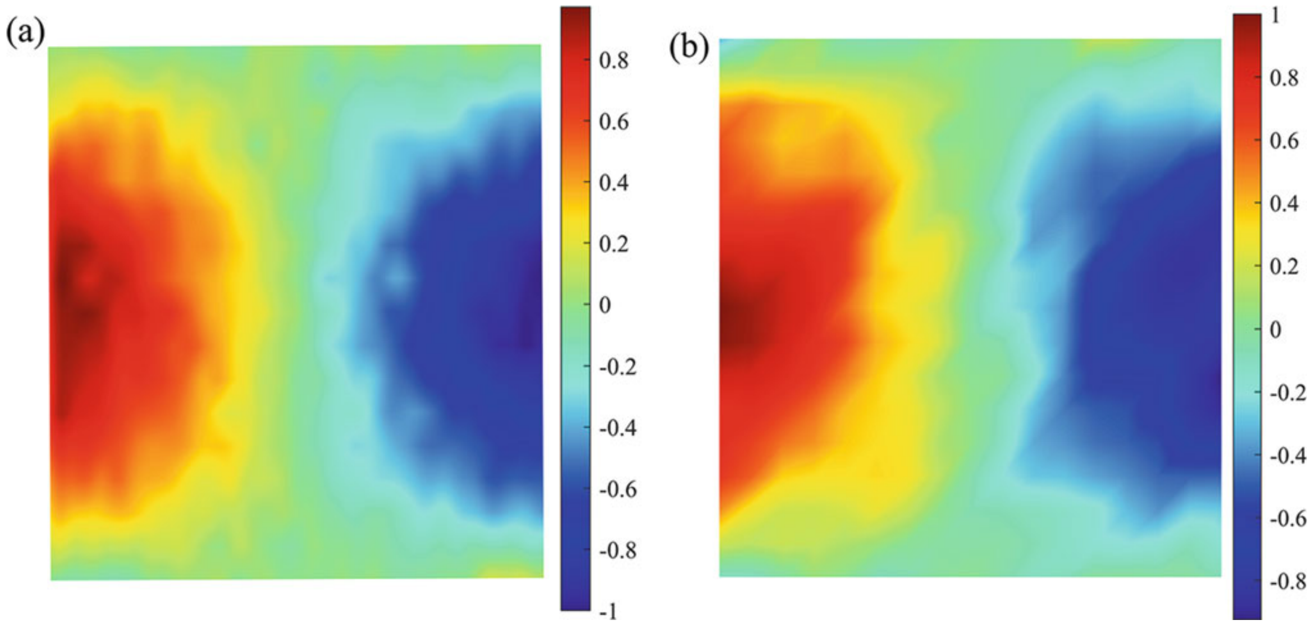


Fig. 12.8 In-plane ODS along the x direction of the free aluminum plate at the excitation frequency of 4715.6 Hz from (**a**) 3D CSLDV and (**b**) 3D SLDV measurements

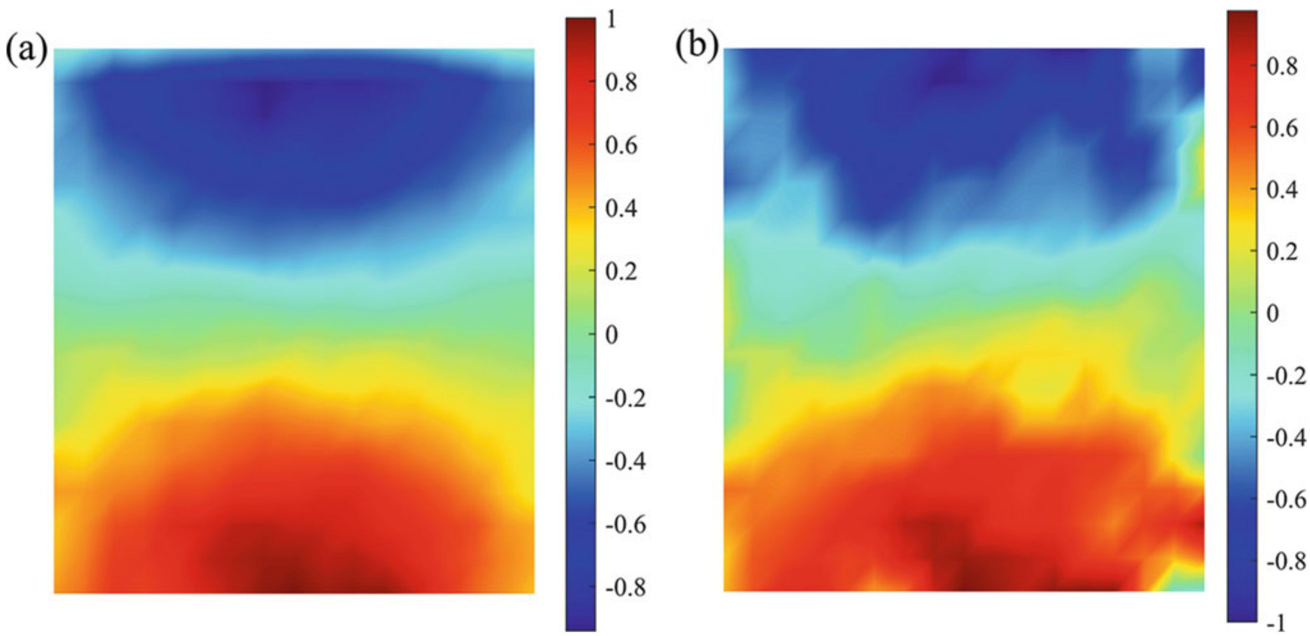


Fig. 12.9 In-plane ODS along the y direction of the free aluminum plate at the excitation frequency of 4715.6 Hz from (**a**) 3D CSLDV and (**b**) 3D SLDV measurements

plane vibrations, and a reference object parallel to the plate is used to calibrate the 3D CSLDV system to ensure that in-plane vibrations can be measured. The demodulation method is used to process response of the plate from 3D CSLDV measurements to obtain its in-plane ODSs. In the frequency range from 0 to 5000 Hz, four obtained in-plane ODSs of the plate at three excitation frequencies from 3D CSLDV measurements are compared to those from 3D SLDV measurements and corresponding in-plane mode shapes from its FE model. MAC values between experimental results from 3D CSLDV and 3D SLDV measurements are larger than 95%, and MAC values between experimental results from 3D CSLDV measurements and numerical results from the FE model of the plate are larger than 91%, indicating that 3D CSLDV measurements in this study have the same accuracy as that from 3D SLDV measurements. The 3D SLDV system can measure 289 measurement points in 1200 s, while the 3D CSLDV system can measure 825,000 measurement points in 82.5 s, which means that the

Table 12.3 MAC values between in-plane ODSs of the plate from 3D CSLDV measurements and in-plane mode shapes from its FE model, and those between in-plane ODSs of the plate from 3D CSLDV and 3D SLDV measurements

Mode number	MAC values between 3D SLDV measurement and the FE model (%)	MAC values between 3D CSLDV measurement and the FE model (%)	MAC values between 3D CSLDV and 3D SLDV measurements (%)
Mode 1	98.0	97.9	96.3
Mode 2	98.1	97.1	98.4
Mode 3 (x)	89.3	91.5	96.7
Mode 3 (y)	93.8	95.8	97.1

3D CSLDV system can measure much more points in much less time than the 3D SLDV system in full-field vibration measurement.

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