Chapter 3
Finite Element Model Updating Using a Shuffled Complex Evolution Markov Chain Algorithm

Marwan Sherri, Ilyes Boulkaibet, Tshilidzi Marwala, and Michael I. Friswell

Abstract In this paper, a probabilistic-based evolution Markov chain algorithm is used for updating finite element models. The Bayesian approaches are well-known algorithms used for quantifying uncertainties associated with structural systems and several other engineering domains. In this approach, the unknown parameters and their associated uncertainties are obtained by solving the posterior distribution function, which is difficult to attain analytically due to the complexity of the structural system as well as the size of the updating parameters. Alternatively, Markov chain Monte Carlo (MCMC) algorithms are very popular numerical algorithms used to solve the Bayesian updating problem. These algorithms can approximate the posterior distribution function and obtain the unknown parameters vector and its associated uncertainty. The Metropolis-Hastings (M-H) algorithm, which is the most common MCMC algorithms, is used to obtain a sequence of random samples from a posterior probability distribution. Different approaches are proposed to enhance the performance of the Metropolis-Hastings where M-H depends on a single-chain and random-walk step to propose new samples. The evolutionary-based algorithms are extensively used for complex optimization problems where these algorithms can evolve a population of solutions and keep the fittest solution to the last. In this paper, a population-based Markov chain algorithm is used to approximate the posterior distribution function by drawing new samples using a multi-chain procedure for the Bayesian finite element model updating (FEMU) problem. In this algorithm, the M-H method is combined with the Scuffled Complex Evolution (SCE) strategy to propose new samples where a proposed sample is established through a stochastic move, survival for the fittest procedure, and the complex shuffling process. The proposed SCE-MC algorithm is used for FEMU problems where a real structural system is investigated and the obtained results are compared with other MCMC samplers.

Keywords Bayesian model updating · Markov chain Monte Carlo · Scuffled complex evolution · Finite element model · Evolutionary algorithm

3.1 Introduction

Finite element methods (FEMs) are the common tools for engineering analysis [1–3]. These approaches are adopted for different physical disciplines, including structural dynamics, heat transfer, fluid flow, and electromagnetic potential. In structural engineering, FEMs are sufficient to predict the response of simple structural system. In contrast, the reliability of the FEM results might be decreased when dealing with complex structural systems. Therefore, the finite element model updating (FEMU) techniques are employed to minimize the dissimilarities between the FEM solutions and the real structural

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data [4–7]. The FEMU methods are classified in two main classes: (i) direct updating approaches and (ii) indirect updating approaches. Direct updating methods are computationally efficient since they only use one updating step to achieve the structural updating goal. In these methods, the system properties (i.e., mass and stiffness matrices) are directly equated to the structural measured mode shapes. However, the dimensionality of the acquired solutions by the FE analysis is normally larger than the measured data. Thus, direct updating methods may produce unrealistic physical components. On the other hand, the indirect updating methods, also refer to iterative techniques, are guaranteed to produce physically realistic updating results and to increase the correlations between FE solutions and the actual experimental data.

The Bayesian framework is well-known for quantifying the uncertainties that are associated with the system parameters. In Bayesian theory, the uncertainties of the modeled parameters are defined through the posterior probability density function (PDF). The function defines the updating parameters as random variables. Unfortunately, the analytical solutions for this function cannot be attained. That is due to the complexity and the high-dimensionally of this function. Alternatively, sampling methods are used to approximate solutions for complex distribution. Among these methods, the Markov chain Monte Carlo (MCMC) [8, 9], represented by the Metropolis-Hastings (M-H) algorithm [10, 11], is the most known approach to draw samples from complex probability distributions. The generated samples (proposals or solutions) are then used to approximate solutions for the posterior PDF, thereby updating the required FEM. However, MCMC methods are limited when several updating parameters are involved, as well as when the structure under consideration is sufficiently complex. For that reason, further updating methods are advised to merge with the MCMC procedure to improve the ability of drawing new proposals.

The evolutionary algorithms (EAs) are well recognized natural-inspired methods used for optimization problems [12–14]. Generally, the evolution-based search algorithm utilizes the availability of the current generation, known as solution or population, to produce a potentially more accurate solution. The interacting population at each step (iteration) is modified through specific operators to produce the next generation. Each individual solution is evaluated by a fitness function. This function is defined as the objective function of the optimization problem. The estimation of the function reflects the solution which the individual provides. EAs are promising techniques for the optimization scope. The evolutionary paradigm is implemented through different classes and for wide range of application.

In this paper, a shuffled complex evolution Markov chain algorithm is used to update a 2-D frame structure with measured data. The algorithm uses a population of solution and evolves through shuffling and partitioning into complexes. Next, the algorithm applies the metropolis acceptance rule to accept or reject the proposed solutions. As a result, this combination of the two procedures generates more accurate updating parameters. The following section explains the Bayesian inference for FEMU problem. Section 3.3 details the implementation of shuffled complex evolution Markov chain. In addition, Sect. 3.3 shows the implementation of the algorithm to sample from the posterior PDF of a Bayesian approach. Section 3.4 shows the application of the algorithm to update a 2-D frame structure with real data. Finally, the paper is concluded in Sect. 3.5.

3.2 Bayesian Inference

The Bayesian approach defines the uncertain parameters as random vector. Each parameter is determined as a stochastic variable as given by Bays' rule [3, 15–17]:

$$P\left(\boldsymbol{\theta}|\,\mathcal{D},\mathcal{M}\right) \propto P\left(\mathcal{D}|\,\boldsymbol{\theta},\mathcal{M}\right)\ P\left(\boldsymbol{\theta}|\,\mathcal{M}\right) \tag{3.1}$$

$\boldsymbol{\theta}$ is the vector of updating parameters $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathcal{R}^d$. $\mathcal{D}$ is the experimental data of the structural system, represented by the natural frequencies f_i^m and mode shapes $\boldsymbol{\phi}_i^m$. $\mathcal{M}$ represents the model class of the system, where each model class is defined by a different set of updating parameters. $P\left(\boldsymbol{\theta}|\,\mathcal{M}\right)$ refers to the prior PDF that offers the previous knowledge of the uncertain parameters $\boldsymbol{\theta}$ given $\mathcal{M}$ and with the absence of $\mathcal{D}$. $P\left(\mathcal{D}|\,\boldsymbol{\theta},\mathcal{M}\right)$ is the likelihood function that describes the difference between the measured data and the analytical solutions. $P\left(\boldsymbol{\theta}|\,\mathcal{D},\mathcal{M}\right)$ is the posterior PDF of the updating parameters given a model class $\mathcal{M}$ and the measured data $\mathcal{D}$. Since only one model class is considered by this paper, $\mathcal{M}$ is removed for simplicity.

The likelihood function is given by:

$$P\left(\mathcal{D}|\,\boldsymbol{\theta}\right) = \frac{1}{\left(\frac{2\pi}{\beta_c}\right)^{N_m/2}\prod_{i=1}^{N_m} f_i^m} \exp\left(-\frac{\beta_c}{2}\sum_i^{N_m}\left(\frac{f_i^m - f_i}{f_i^m}\right)^2\right) \tag{3.2}$$

N_m is the number of measured modes, and β_c is an arbitrary constant. f_i^m and f_i are the ith measured and analytical natural frequencies.

The prior $P(\boldsymbol{\theta})$, that provides the initial knowledge of the updating parameter $\boldsymbol{\theta}$, is defined by a Gaussian probability distribution:

$$P\left(\boldsymbol{\theta}\right) = \frac{1}{(2\pi)^{Q/2}\prod_{i=1}^{Q}\frac{1}{\sqrt{\alpha_i}}}\exp\left(-\sum\nolimits_{i}^{Q}\frac{\alpha_i}{2}\left\|\theta^i-\theta_0^i\right\|^2\right) = \frac{1}{(2\pi)^{Q/2}\prod_{i=1}^{Q}\frac{1}{\sqrt{\alpha_i}}}\exp\left(-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\theta}_0)^T\,\Sigma^{-1}\,(\boldsymbol{\theta}-\boldsymbol{\theta}_0)\right) \tag{3.3}$$

where Q is the number of the uncertain parameters, $\boldsymbol{\theta}_0$ represents the mean value of the uncertain parameters, α_i is the coefficient of uncertain parameters, $i = 1, \ldots, Q$, and the Euclidean norm of $_*$ is noted by $\|_*\|$.

The posterior PDF $P\left(\boldsymbol{\theta}|\,\mathcal{D}\right)$ of the updating parameters $\boldsymbol{\theta}$ given the measured data $\mathcal{D}$ can be now described by substituting Eqs. (3.2) and (3.3) into Eq. (3.1):

$$P\left(\boldsymbol{\theta}|\,\mathcal{D}\right) \propto \frac{1}{Z_s\left(\alpha,\beta_c\right)}\exp\left(-\frac{\beta_c}{2}\sum\nolimits_{i}^{N_m}\left(\frac{f_i^m-f_i}{f_i^m}\right)^2-\sum\nolimits_{i}^{Q}\frac{\alpha_i}{2}\left\|\theta^i-\theta_0^i\right\|^2\right) \tag{3.4}$$

where,

$$Z_s\left(\alpha,\beta_c\right) = \left(\frac{2\pi}{\beta_c}\right)^{N_m/2}\prod\nolimits_{i=1}^{N_m}f_i^m\,(2\pi)^{Q/2}\prod\nolimits_{i=1}^{Q}\frac{1}{\sqrt{\alpha_i}} \tag{3.5}$$

Equation (3.4) represents the final form of the posterior PDF. The same function is employed as the objective function for evolutionary procedure. The dimensionality of the unknown parameters along with the complexity of the uncertain search bounds may affect the accuracy of the solutions. However, in the next section, the shuffled complex evolution Markov chain method is explained and detailed to solve the Bayesian FEMU problem.

3.3 Shuffled Complex Evolution Markov Chain Algorithm

The shuffled complex evolution Markov chain (SCE-MC) [18, 19] is a population-based algorithm for global optimization problems. It aims to infer the target function by combining the advantages of the Metropolis-Hastings sampler with the shuffled complex evolution (SCE) approach [20, 21]. The algorithm applies the controlled random search with competitive evolution and complex shuffling to continuously update the proposal distribution. In addition, the SCE-MC's evolution procedure evolves the proposed samples to the posterior PDF through sequence of iterations. As a result, the SCE-MC method has the ability to improve the best parameter set found in the search space every iteration. Thus, the population (Markov chains) is continuously evolved toward the optimal region in the search. The main advantage of this algorithm, in comparison to the standard SCE method, is to avoid trapping into a region with lower posterior density, which prevents most of the population to converge toward a single mode or to collapse into relatively small region of a single best parameter set (local minimum) (Fig. 3.1).

In addition, the SCE-MC algorithm uses a large number of population, N, which offers the ability for wide exploration of the parameters space, thereby raising the chance to find the global optimum of the parameter set in few iterations. Usually, a population size $N = 10d$ is sufficient to approximate a complex target distribution. The parallel sequence provides a

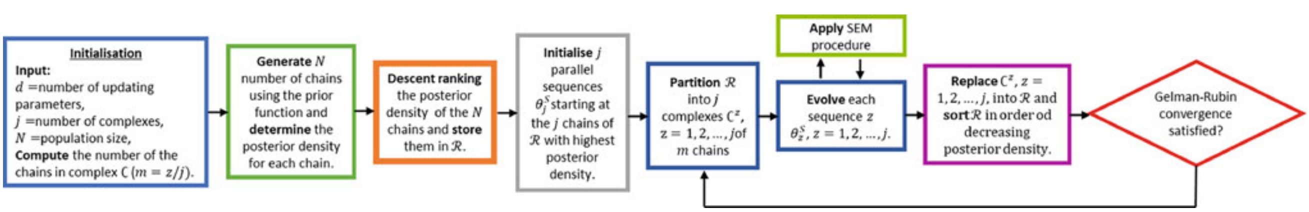

Fig. 3.1 Flow chart of the SCE-MC algorithm

mechanism for a simultaneous different region exploration. Thus, the number of j (number of parallel sequencing) enables the algorithm to seek several regions of attraction and allows to apply the heuristic test to check the convergences. A default set $j = d$ is enough for wide exploration. The user can define a greater number of sequences as the complexity of the posterior function increased. Partition into complexes brings the possibility for each individual (Markov chain) sequence to collect more knowledge about the search space during the evolution step. The shuffling process for these complexes assists the surviving for the sequence through a global sharing of the information which is already collected by each parallel sequence. The implementation of this process produces a robust sampling procedure that uses the MCMC principles for global optimum solutions.

The SCE-MC algorithm uses the Sequence Evolution Metropolis (SEM) procedure to produce new candidates (samples) in each parallel sequence θ_z^S. This can be done by creating draws from an adaptive proposal distribution through the use of the information induced in m chains of $\mathbb{C}^z$. This paper applies the default SEM procedure as explained in [18]. Another important feature of the SCE-MC algorithm is the convergence to the stationery posterior distribution. The algorithm applies the Gelman-Rubin rule [18] to estimate the convergence status. This rule is to determine the scale reduction $\sqrt{SR}$ between the chains (sequences) and the variances as given by:

$$\sqrt{SR} = \sqrt{\frac{t-1}{t} + \frac{j+1}{j.t}\frac{V}{U}} \tag{3.6}$$

where t is the number within each sequence, V is the variance between the j sequence means, and U is the average of the j within sequence variance for the parameters under consideration, respectively. Note that the product of j and t is identical to the total number of derived samples (number of algorithmic iterations T). The convergence of each parameter occurs when $\sqrt{SR}$ is close to 1. However, since this score is difficult to achieve, Gelman-Rubin suggested using value less than 1.2 to declare convergence to a stationary distribution. On the other hand, the algorithm can work through pre-defined number of iterations, which can be used to compare two updating procedures at the same number of iterations.

The SCE-MC algorithm for Bayesian FEMU is summarized as follows:

1. **Create population**: generate N number of chains $\{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_N\}$ randomly from the prior distribution $P(\boldsymbol{\theta}_i)$.
2. **Compute posterior**: calculate the posterior density for each generated chain $\{P(\boldsymbol{\theta}_1|D), P(\boldsymbol{\theta}_2|D), \ldots, P(\boldsymbol{\theta}_N|D)\}$.
3. **Descent ranking**: sort N chains in order of decreasing the posterior density and store them in matrix $\mathcal{R}\ [1:N, 1:d+1]$, where d is the number of the updating parameters and the first row of $\mathcal{R}$ represents the chain with highest posterior density.
4. **Initialize parallel sequence**: start the parallel sequencing of the chains, $\boldsymbol{\theta}_1^S, \boldsymbol{\theta}_2^S, \ldots, \boldsymbol{\theta}_j^S$, such that $\boldsymbol{\theta}_z^S$ is $\mathcal{R}\ [z, 1:d+1]$, where $z = 1, 2, \ldots, j$.
5. **Complex partitioning**: partition the N chains of the matrix $\mathcal{R}$ into $z_{\mathbb{C}}$ complexes $\mathbb{C}^1, \mathbb{C}^2, \ldots, \mathbb{C}^z$, where each complex $\mathbb{C}$ contains k chains, such that the first complex includes $j(i-1)+1$ ranked chains, the second complex includes every $j(i-1)+2$ ranked chains of the matrix $\mathcal{R}$, and so on, where $i = 1, 2, \ldots, k$.
6. **Sequence evolving**: evolve each of the parallel sequences according to the Sequence Evolution Metropolis procedure.
7. **Shuffle complexes**: unpack all complexes $\mathbb{C}$ back into $\mathcal{R}$, then rank the chains in order of decreasing posterior density and reshuffle the N chains into j complexes according to the procedure specified in step 5.
8. **Termination**: check the Gelman-Rubin convergence criteria or use maximum number of iterations. If any of these conditions is satisfied, stop. Otherwise return to step 6.

3.4 Application: Updating the Young's Modulus

A 2-D steel frame structure, as shown by Fig. 3.2, is used to demonstrate the performance of the SCE-MC algorithm for Bayesian FE model updating. The structure is fixed to the ground from its bottom side [22]. The overall height of the frame is 1.5 m and the span of the beam is 0.5 m. Each story of the structure has the same height of 0.5 m. The structure is assembled with five beams; two vertical beams, making up the right and the left side of the structure, are noted as Beam-1and Beam-2, respectively. The other three are horizontal beams noted as Beam-3 at the bottom side, Beam-4 in the middle, and Beam-5 at the top. All the beams have the same cross-sectional dimensions: their width is 75 mm, and their thickness is 5 mm. The material density of the frame is 7850 kg/m^3, and the elastic modulus is 2.0×10^{11}N/m^2.

An instrumental hammer with a rubber tip was used to excite the frame, where the frequency range of interest is about 0–100 Hz. Aaccelerometers with magnet bases are mounted on the vertical and horizontal beams to measure the response. The

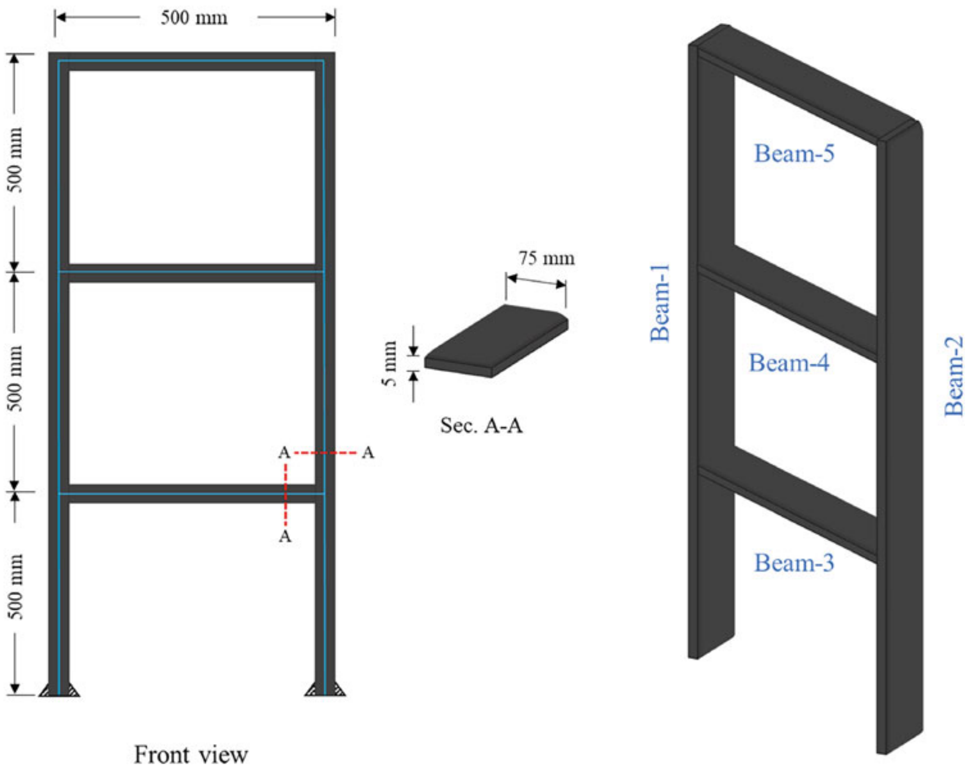

Fig. 3.2 the 2-D frame structure

first five natural frequencies and mode shapes in the range of 0 – 100 Hz were extracted. The measured natural frequencies are $\omega_m = \{4.23, 14.03, 25.45, 44.81, 58.12\}$ Hz. the FEMU application was implemented through the MATLAB environment [23], and the FE model was developed using the structural dynamic toolbox [24]. A 2-D geometrical beam element is used to simulate the frame model. The obtained analytical frequencies with the mode shapes are illustrated in Fig. 3.3.

In this application, the modulus of elasticity of the five structural beams (Beam-1 to Beam-5) is selected as the updating parameters. The updating vector is $\boldsymbol{\theta} = \{E_{m1}, E_{m2}, E_{m3}, E_{m4}, E_{m5}\}$, where E_{m1} represents the elastic modulus of Beam-1 and so on. Then, $\boldsymbol{\theta} = \{2 \times 10^{11}, 2 \times 10^{11}, 2 \times 10^{11}, 2 \times 10^{11}, 2 \times 10^{11}\}$. The updating parameters are bounded by $\boldsymbol{\theta}_{\max}$ and $\boldsymbol{\theta}_{\min}$ to keep the updated values realistic, where $\boldsymbol{\theta}_{\max} = \{2.4 \times 10^{11}, 2.4 \times 10^{11}, 2.4 \times 10^{11}, 2.4 \times 10^{11}, 2.4 \times 10^{11}\}$ and $\boldsymbol{\theta}_{\min} = \{1.6 \times 10^{11}, 1.6 \times 10^{11}, 1.6 \times 10^{11}, 1.6 \times 10^{11}, 1.6 \times 10^{11}\}$. The variance vector is $\sigma^2 = \{5 \times 10^{11}, 5 \times 10^{11}, 5 \times 10^{11}, 5 \times 10^{11}, 5 \times 10^{11}\}$, and the total number of iterations is $t_s = 500$. Meanwhile, the number of complexes $z_{\complement}$ in the SCE-MC algorithm is set to 5 and the population size $N = 50$. On the other hand, the M-H method is set as its default settings with increasing the size of its move.

In this application, the SCE-MC algorithm and the canonical M-H method are applied to update the same structure, this aims to compare the two algorithms and to highlight the improvements gained when combining the M-H criterion with the evolutionary mechanism. Both algorithms were tested to update the frame structure for several independent runs. Each updating trial was set to the same algorithmic settings and parameters. Table 3.1 presents the updated parameters by the SCE-MC algorithm and the M-H method. The initial values of the elastic modulus are given in the table for the five updating elements. The final updated values as shown in the table are different to the initial states. This shows that both algorithms produced realistic updated parameters that keep physical meaning for the updating feature. The table also includes the coefficient of variation (cov) of the updated parameters from each algorithm. This coefficient measures the level of precision within the samples produced by the updating methods. The cov is defined as the ratio of the standard deviation A to the mean value θi (the value of the final updated parameter), i.e., cov= $\frac{\sigma_i}{\theta_i}$.

Generally, the cov values obtained by the SCE-Mc algorithms are better than the M-H method. The sampling precision that happens in the SCE-MC procedure is due to the use of large number of population. Thus, the selected solutions reflect the best results among the other solutions. Contrary, the M-H applies a single move only to propose the new solutions state. However, the cov of the still reflect a quiet acceptable range.

Figure 3.4 illustrates the boxplots of the generated samples by SCE-MC algorithm. On each box, the central dark blue line represents the median, and the light blue box spans indicate the 25th and 75th percentile (the range that bounds 25%

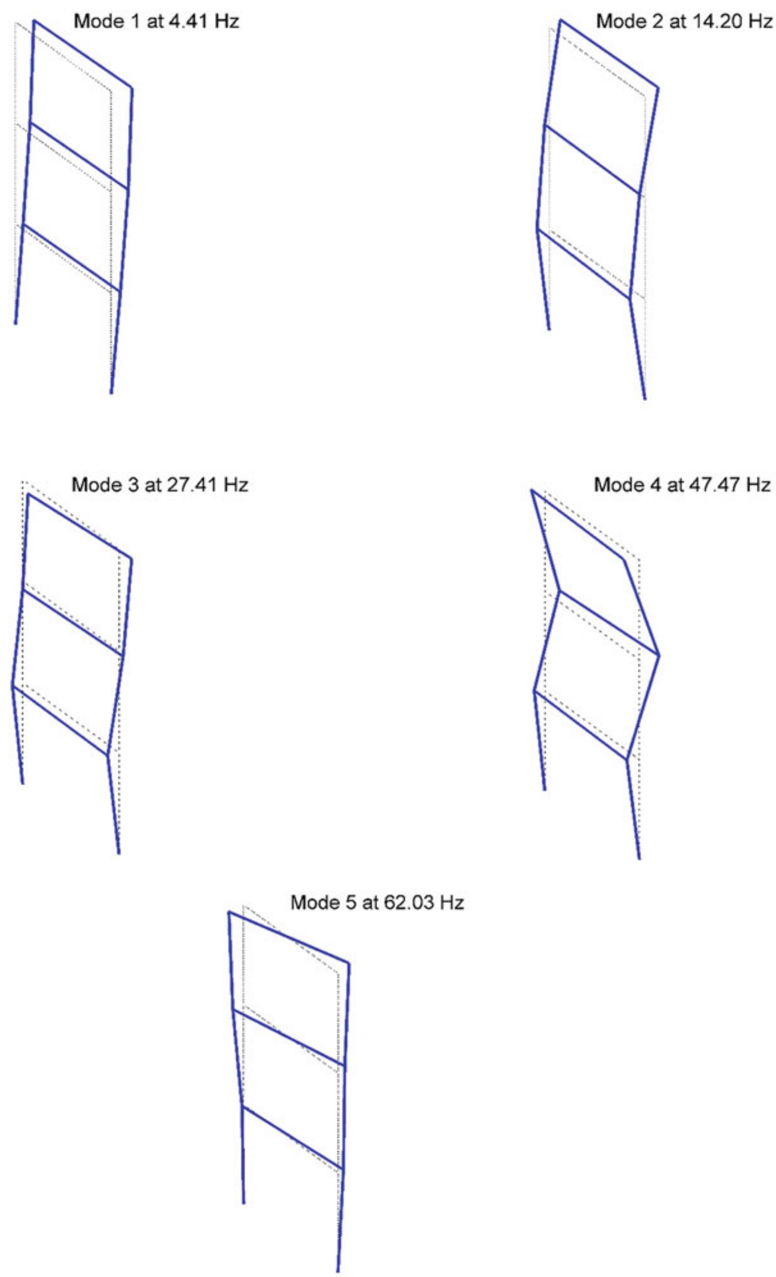

Fig. 3.3 The analytical frequencies and the mode shapes of the 2-D frame structure

Table 3.1 The updated elastic modulus of the 2-D frame structure using the M-H and the SCE-MC

Updating the elastic modules E_m (N/m^2)					
	Initial FEM	M-H method	cov (%) $\frac{\sigma_i}{\theta_i}$	SCE-MC method	cov (%) $\frac{\sigma_i}{\theta_i}$
E_{m1}	2.0×10^{11}	1.929×10^{11}	3.15	1.987×10^{11}	2.02
E_{m2}	2.0×10^{11}	2.050×10^{11}	3.77	1.945×10^{11}	2.63
E_{m3}	2.0×10^{11}	2.196×10^{11}	4.31	2.287×10^{11}	3.54
E_{m4}	2.0×10^{11}	2.087×10^{11}	2.31	2.107×10^{11}	2.18
E_{m5}	2.0×10^{11}	2.127×10^{11}	3.44	2.163×10^{11}	3.07

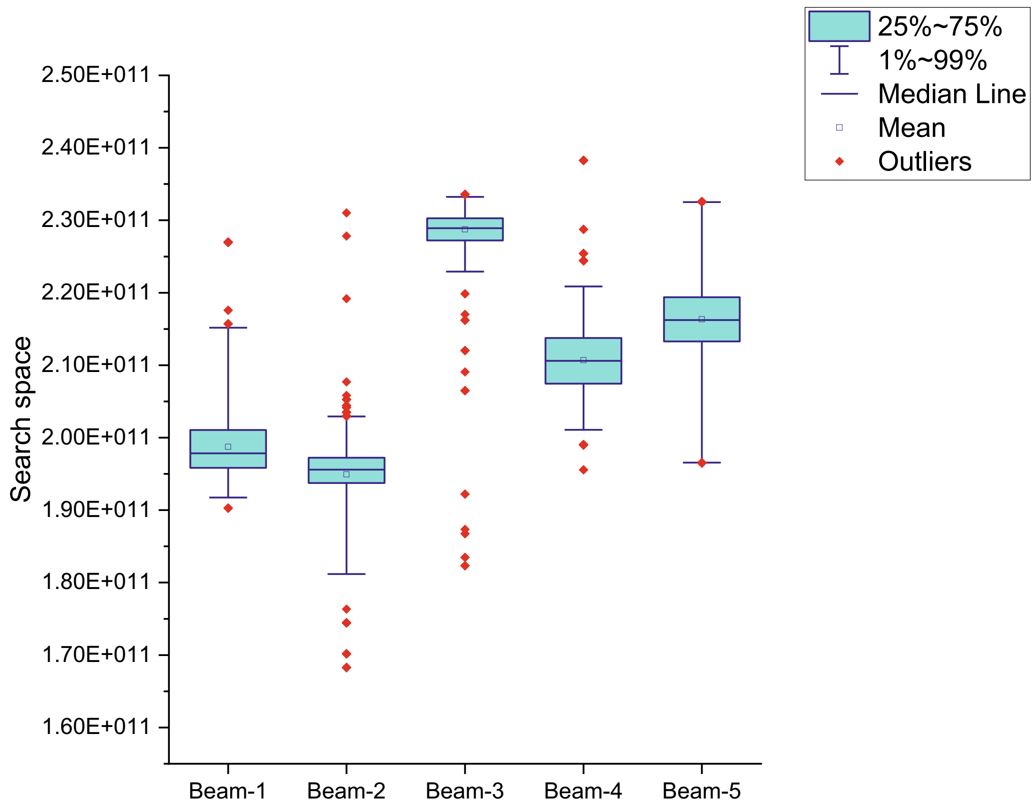

Fig. 3.4 Boxplot of the random samples by the SCE-MC algorithm

to 75% of samples). The whiskers (dark blue horizontal lines) extend to the most extreme samples position (from 1% to 99% of samples are placed within this range), and *red dot* symbol denotes the outliers. This figure indicates the sampling performance of the SCE-MC algorithm. The convergence toward the high region of solutions can be seen from the figure as 25~75% of the samples are concentrated around the mean (a small dark blue square at the central). The outliers are expected to appear, since the algorithm uses a population size of 50 Markov chains, which creates a challenge to converge all set of chains simultaneously.

The correlations between the updated parameters are shown in Fig. 3.5. A small rate correlation demonstrates that both updating parameters are weakly correlated (<0.3), while a larger rate means a highly correlation (>0.7). The positive or negative correlation indicates that the variables relatively affect each other. The SCE-MC algorithm has successfully sampled from the posterior PDF since no highly absolute correlation is observed.

The updated natural frequencies are given in Table 3.2. The results indicate that the SCE-MC updating method has updated the natural frequencies for the five modes with outcomes that are better than the initial FE model. The percentage absolute mode error $\frac{|f_i^m - f_i|}{f_i^m}$ is included in the table to compare the differences between the measured and the updated frequencies. The table also presents the total average error (*TAE*), which gives a general overview of the updated modes, and is determined due $TAE = \frac{1}{N_\omega}\sum_{i=1}^{N_\omega}\frac{|f_i^m - f_i|}{f_i^m}$, where N_ω is the number of the updated frequencies. The SCE-MC has successfully minimized the *TAE* from its initial state to 2.67%. On the other hand, the M-H algorithms has completed the updating process with 3.8% for the *TAE*. It can be shown here the SCE-MC has the advanced ability to update the frame structure and to produce efficient updating solutions.

Figure 3.6 shows the convergence of the *TAE* for the SCE-MC and the M-H algorithms when updating the natural frequencies of the 2-D frame structure. The SCE-MC algorithm utilizes 50 Markov chains and 5 evolutionary complexes to implement the updating process. Thereby, the algorithm tests and shuffles a number of 50 solutions at each iteration. This mechanism provides the algorithm with the ability to reach a quick convergence. It shows in the plot that the algorithm started with *TAE* which is higher than the *TAE* of initial FE model. This indicates the effectiveness of the evolving process which is implemented by the algorithm to explore a wider region of the search space and iteratively select the best set of solutions. The convergence of the SCE-MC method, as seen in the figure, is achieved before the first 100 iterations.

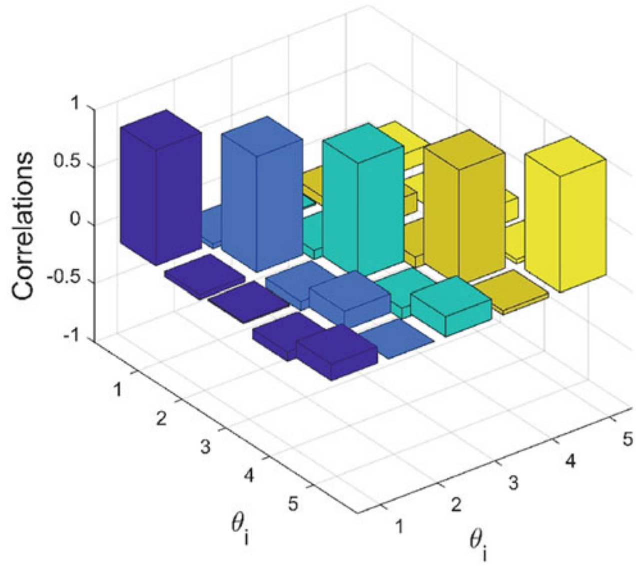

Fig. 3.5 The correlation between the updating parameters obtained by SCE-MC algorithm

Table 3.2 The updated natural frequencies using M-H and the SCE-MC algorithms

Modes	Measured frequency (Hz)	Initial frequency (Hz)	Error (%)	M-H Method frequency (Hz)	Error (%)	SCE-MC Method frequency (Hz)	Error (%)
1	4.23	4.41	4.36	4.29	1.42	4.326	2.26
2	14.03	14.2	1.2	14.46	3.09	14.209	1.27
3	25.45	27.40	7.69	27.31	7.33	26.58	4.46
4	44.81	47.47	5.94	45.81	2.23	43.98	1.83
5	58.12	62.03	6.73	61.0	4.95	60.17	3.53
TAE	–		**5.18**	**–**	**3.80**	**–**	**2.67**

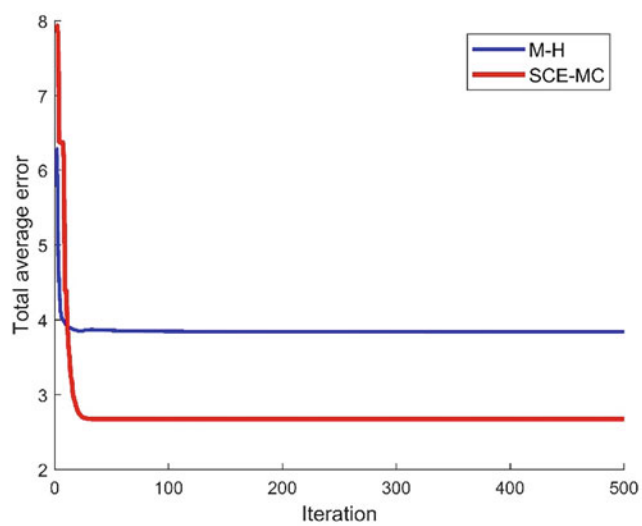

Fig. 3.6 Convergence of the total average error of the 2-D frame structure using the SCE-MC algorithms

For the purpose of comparing the results of both updating methods, the M-H and the SEC-MC methods, the M-H algorithm is set to have a relatively large jump at iteration, which facilitates the M-H procedure to have its maximum performance and quickest convergence within 500 iterations only for the FEMU application. This is used to highlight the limitations for each updating method when used to solve the same updating conditions.

3.5 Conclusion

This paper has introduced the shuffled complex evolution Markov chain algorithm for Bayesian updating problem. The presented algorithm merges the advantages of the Metropolis-Hastings method, controlled stochastic search, competitive evolution, and complex shuffling to evolve population of samples to approximate the posterior PDF of the Bayesian model. The algorithm has been tested to update a FE model of a 2-D frame structure. The Young's modulus is chosen to be updated. Five updating elements are considered for the updating problem. In addition, the posterior function includes five natural frequencies which are collected from modal experiment. It is found that the SCE-MC algorithm requires large number of population to propose desired samples and that is correlated with the complexity of the posterior function. The algorithm is also affected by the number of the complexes used to partition and sequencing the population. Increasing the number of the complexes offers more opportunities to rank and sequence the solutions, therefore giving the chance to converge to a higher region of solutions. However, this may raise the computational demand of the problem.

The updated parameters have reflected realistic updating values. The obtained set of solutions has minimized the total average error of the updated natural frequencies. The obtained results were compared with the outcomes of the M-H algorithm and found that the performance of the SCE-MC method is more efficient. It's advised for future work to investigate the SCE-MC algorithm with another evolutionary Markov chain method.

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