Chapter 2
A Comparative Assessment of Online and Offline Bayesian Estimation of Deterioration Model Parameters

Antonios Kamariotis, Luca Sardi, Iason Papaioannou, Eleni N. Chatzi, and Daniel Straub

Abstract Many preventive maintenance schemes for managing structural deterioration rely on stochastic deterioration models. In this context, continuous structural health information can be employed within a Bayesian framework to update the distributions of the time-invariant deterioration model parameters. Bayesian parameter estimation can be performed either in an online or an offline fashion. In this contribution, we investigate different online and offline algorithms implemented for learning the model parameters, and their uncertainty, considering a probabilistic model of fatigue crack growth that is updated with continuous crack monitoring measurements. The numerical investigations provide insights on the performance of the different algorithms in terms of accuracy of the posterior estimates and computational cost.

Keywords Bayesian inference · Particle filter · Markov chain Monte Carlo · Uncertainty quantification · Structural deterioration

2.1 Introduction

The tracking and tackling of deterioration is a major challenge throughout the structural life cycle. To address this challenge, stochastic models describing the various deterioration processes can be employed, which typically contain time-invariant parameters with prior uncertainty. The deployment of sensors on structures allows for a continuous monitoring of such deterioration processes. Efficient use of continuous monitoring data within a Bayesian framework can lead to posterior estimates of the time-invariant deterioration model parameters, which is indispensable for the task of performing informed predictions on the deterioration process evolution. An important distinction can be made between online and offline Bayesian parameter estimation [1], which is the main focus of this contribution. Although the typical use of online methods, such as the particle filter [2], targets the tracking of the system's response (dynamical state) by means of a state-space formulation, these can also be used in pure recursive estimation of time-invariant parameters, such as the system properties. The task of estimating time-invariant parameters is most commonly performed with the use of offline Markov chain Monte Carlo (MCMC) methods. However, use of offline methods in setups where the measurements are obtained sequentially at different points in time can become computationally unaffordable.

2.2 Offline and Online Bayesian Parameter Estimation

This work is based on the premise that a stochastic deterioration model is available, with $\boldsymbol{\theta} \in \mathbb{R}^d$ a vector containing the d unknown time-invariant parameters. In a Bayesian framework, $\boldsymbol{\theta}$ is modeled as a vector of random variables (RVs), with their prior uncertainty described by a prior distribution $\pi_{pr}(\boldsymbol{\theta})$. Monitoring of the deterioration process leads to a set of noisy measurements $\{y_1, y_2, \dots\}$ obtained in a sequential manner at different points in time $\{t_1, t_2, \dots\}$. The measurements can be

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used to learn the posterior distribution of $\boldsymbol{\theta}$ through application of Bayes' rule. Different posterior distributions might be of interest; for instance, one might be interested in updating the distribution of $\boldsymbol{\theta}$ in a sequential manner, i.e., at every time step t_n when a new measurement y_n is obtained (given all the set of measurements $\{y_1, y_2, \ldots, y_n\}$ available up to t_n, denoted as $y_{1:n}$). In such settings, the use of offline MCMC algorithms is not practical, as a different chain needs to be generated for each posterior $\pi_{pos}(\boldsymbol{\theta}|\, y_{1:n})$, and the previously obtained posterior $\pi_{pos}(\boldsymbol{\theta}|\, y_{1:n-1})$ is not accounted for. In a static scenario, one might seek estimation of a single posterior density $\pi_{pos}(\boldsymbol{\theta}|\, y_{1:N})$. Using Bayes' rule, this posterior distribution of interest can be estimated

$$\pi_{pos}\,(\boldsymbol{\theta}|y_{1:N}) \propto \pi\,(y_{1:N}|\boldsymbol{\theta})\;\pi_{pr}\,(\boldsymbol{\theta}) \tag{2.1}$$

where $\pi(y_{1:N}|\,\boldsymbol{\theta})$ is the likelihood function. For the latter task, typically offline MCMC methods can be employed to obtain the posterior distribution. In structural deterioration setups, where each measurement is obtained at a different point in time, each evaluation of the likelihood function within the MCMC process requires the whole set of measurements to be processed, which induces a significant computational cost.

Online particle filter methods operate in a sequential fashion, i.e., they use $\pi_{pos}(\boldsymbol{\theta}|\, y_{1:n-1})$ to obtain $\pi_{pos}(\boldsymbol{\theta}|\, y_{1:n})$ via importance resampling [2], having to account only for the new measurement y_n. They can be used in exactly the same way both for updating the distribution of $\boldsymbol{\theta}$ in an online sequential manner and for static scenarios, where only a single posterior density $\pi_{pos}(\boldsymbol{\theta}|\, y_{1:N})$ is of interest. In the latter case, they use the sequence of measurements to sequentially arrive to the final posterior density of interest via estimating all the intermediate distributions. Online particle filtering methods suffer from two distinct issues. In most cases, after a certain number of update steps, almost all the particles comprise zero (or close to zero) weights, the so-called degeneracy problem [2]. This problem is alleviated by the use of adaptive resampling procedures based on the effective sample size. When using online particle filters to estimate a posterior distribution of interest for time-invariant parameters, for which the process noise is formally zero, one runs into the issue of sample impoverishment [2]. This means that after the resampling step, most (or in extreme cases all) of the particles in the sample set end up having the exact same value, i.e., the particle set consists of only few (or one) distinct particles. The degeneracy and sample impoverishment issues render estimation of static parameters with online particle filters a challenging task.

Herein, we implement an offline MCMC-based particle filter (SMC); an online particle filter (PF), which performs Gaussian mixture (GM)-based resampling to counteract the degeneracy and sample impoverishment; and the online iterated batch importance sampling algorithm (IBIS) [3], which employs offline MCMC steps after resampling for counteracting the abovementioned issues. We apply these three algorithms on the numerical example that is described below.

2.3 Numerical Investigation

A fracture mechanics-based model serves as a use case. This describes the crack growth evolution under increasing stress cycles [4]. The crack growth follows an ordinary differential equation, known as the Paris-Erdogan law, with solution

$$a(n) = \left[\left(1 - \frac{m}{2}\right)\exp\left(C_{ln}\right)\Delta S^m \pi^{m/2} n + {a_0}^{(1-m/2)}\right]^{(1-m/2)^{-1}} \tag{2.2}$$

a [mm] is the crack length, n [−] is the number of stress cycles, ΔS [Nmm^{-2}] is the stress range per cycle when assuming constant stress amplitudes, and C_{ln} and m represent empirically determined model parameters. To express the crack size as a function of the number of stress cycles n, the boundary condition $a(n = 0) = a_0$ is imposed. We assume that noisy measurements of the crack y_n are obtained sequentially at different values of n. A multiplicative error is assumed for the measurement equation, i.e., $y_n = a_n \exp\left(\varepsilon_n\right)$. Under this assumption, the likelihood function for a measurement at a given n is shown in Eq. (2.3). Table 2.1 shows the prior probability distribution model for each of the random variables $\boldsymbol{\theta}$ of the deterioration model of Eq. (2.2), as well as the assumed probabilistic model of the measurement error.

$$L\,(y_n; a_n) = \frac{1}{\sigma_{\varepsilon_n}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln\left(y_n\right) - \mu_{\varepsilon_n} - \ln\left(a_n\right)}{\sigma_{\varepsilon_n}}\right)^2\right] \tag{2.3}$$

Table 2.1 Prior model for the deterioration model parameters and the measurement error

Parameter	Distribution	Mean	Standard deviation	Correlation
a_0	Exponential	1	1	–
ΔS	Normal	60	10	–
C_{ln}, m	Bi-normal	$(-33; 3.5)$	$(-0.47; 0.3)$	$\rho_{C_{ln},m} = -0.9$
$\exp(\varepsilon_n)$	Lognormal	1.0	0.1508	–

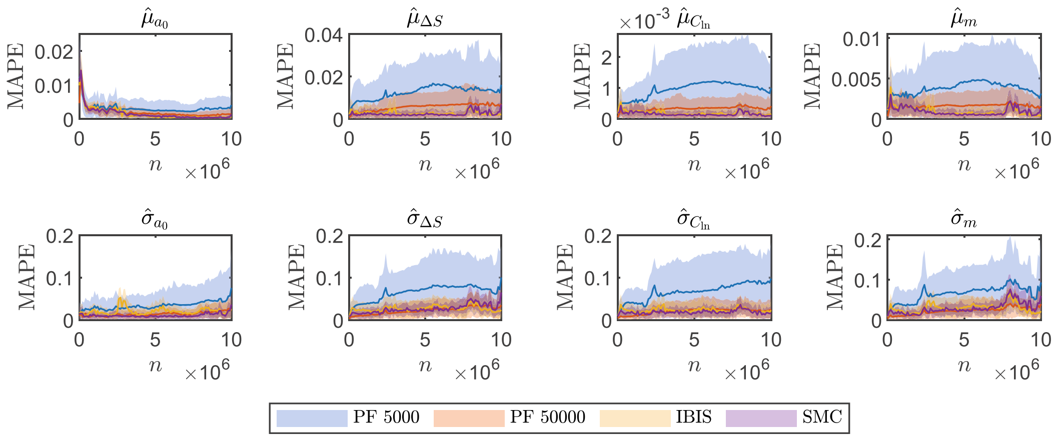

Fig. 2.1 Comparison of the MAPE evaluated for the three different applied filters. The full lines show the mean and the shaded areas the 90% credible intervals from 50 different runs of the different algorithms

An underlying "true" deterioration process $a^*(n)$ for $n = k\Delta n$, with $k = 1, \ldots, 100$ and $\Delta n = 10^5$ is generated for $a_0^* = 2.0$, $\Delta S^* = 50.0$, $C_{ln}{}^* = -33.5$, and $m^* = 3.7$, and a synthetic noisy crack measurement y_n is generated at each $\Delta n = 10^5$. We are interested in estimating the complete set of 100 posterior densities $\pi_{pos}(\boldsymbol{\theta}|\, y_{1:n})$. A reference posterior solution for each of the $\pi_{pos}(\boldsymbol{\theta}|\, y_{1:n})$ is generated using acceptance-rejection sampling [5].

We apply the offline MCMC-based SMC filter with 5000 particles, the online GM-based PF filter with 5000 and 50,000 particles, and the online IBIS filter, which performs resampling using offline MCMC steps with 5000 particles to estimate all the 100 $\pi_{pos}(\boldsymbol{\theta}|\, y_{1:n})$ posterior densities. We evaluate the performance of each filter by taking the mean absolute percentage error (MAPE) with respect to the reference posterior solution. Figure 2.1 reveals that SMC and IBIS yield superior performance, with similar results, as expected, since they both use MCMC steps in their solution. However, the IBIS is an intrinsically online algorithm, which performs offline MCMC steps only when the effective sample size drops below a threshold; hence it has a much lower computational cost. The online PF with 5000 particles performs worst, however at the lowest computational cost. Using 50,000 particles, the online PF performance increases significantly, and it provides results of a quality comparable with the SMC and IBIS results. The IBIS with 5000 particles and the PF with 50,000 particles have comparable computational costs.

2.4 Conclusion

For the specific low-dimensional numerical investigation presented herein, it is demonstrated that online particle filters lead to time-invariant deterioration model parameters posterior results of comparable quality to the results obtained with an offline MCMC-based filter, but at a significantly lower computational cost. In problems with higher dimensionality, it should be expected that the use of purely online filters will lead to posterior estimates of reduced quality.

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