






# A Strength Amplification Factor to Counterbalance P- $\Delta$ Effects in Steel Moment Resisting Frames

Francesca Barbagallo , Melina Bosco , Aurelio Gheresi,  
and Pier Paolo Rossi 

Department of Civil Engineering and Architecture, University of Catania,  
Catania, Italy

{fbarbaga, mbosco, aghersi, prossì}@dica.unict.it

**Abstract.** The importance of P- $\Delta$  effects is generally modest in the elastic range of behaviour. However, it becomes significant when the structure experiences large plastic deformations. Seismic codes indicate that P- $\Delta$  effects may be counterbalanced through an increase in the lateral strength required by a first order analysis. The expressions suggested in codes for this amplification factor are simplistic and often criticized by researchers. Hence, the paper proposes a formulation of the strength amplification factor, which is alternative to those of seismic codes. This formulation is a generalization of the one derived in the past by the authors based on the numerical response of SDOF systems.

To validate the proposed formulation, steel moment resisting frames characterized by different sensitivity to P- $\Delta$  effects are considered. Each structure is designed taking into account P- $\Delta$  effects in keeping with provisions of seismic codes or according to the proposed strength amplification factor. To comment on the effectiveness of the proposal, the nonlinear seismic response of these structures is determined by nonlinear static analysis. The increase in size of the cross-sections required to counterbalance P- $\Delta$  effects is also determined to compare the costs of structures designed according to the proposed and existing formulations.

**Keywords:** Steel frames · P- $\Delta$  effects · Seismic codes · Nonlinear analysis · Structural costs

## 1 Introduction

In the past, different design strategies were proposed for the control of P- $\Delta$  effects in seismic areas [1] aiming at compensating for P- $\Delta$  effects by means of the increase in either the lateral strength or stiffness of the system or by the increase in both properties of the structure. The approach generally preferred in studies and seismic codes is the first one [2–4].

### 1.1 Strength Amplification Factor in Seismic Codes

The strength amplification factor currently considered in EC8 and ASCE/SEI 2010 is calculated (at each storey of the building) by means of the following parameter

$$\alpha = 1/(1 - \theta) \quad (1)$$

where  $\theta$  is the interstorey sensitivity coefficient.

In EC8, the interstorey sensitivity coefficient is calculated at each storey as

$$\theta_{EC8} = P\Delta u/(V_d h) \quad (2)$$

where  $P$  is the total gravity load at and above the storey considered in the seismic design situation,  $V_d$  is the total seismic design storey shear,  $h$  is the interstorey height and  $\Delta u$  is the inelastic interstorey drift produced by the seismic design forces, i.e. the elastic design interstorey drift  $\Delta u_d$  produced by shear forces  $V_d$  times the displacement behaviour factor  $q_d$  (which is equal to the behaviour factor, with the exception of low period systems). No amplification of the seismic action effects is required if  $\theta_{EC8} \leq 0.1$ . If  $0.1 < \theta_{EC8} < 0.2$ , second order effects may be considered in a simplified way by the strength amplification factor given in Eq. (1). If  $\theta_{EC8} > 0.2$ , the simplified approach is not applicable, and a second order analysis has to be performed. Values of  $\theta_{EC8} > 0.3$  cannot be accepted.

In the American code, instead, the interstorey drift sensitivity coefficient is

$$\theta_{ASCE} = P\Delta u^*/(V_d h C_d) \quad (3)$$

where  $C_d$  is the deflection amplification factor and  $\Delta u^*$  is the elastic design interstorey drift times the deflection amplification factor. No amplification of the seismic action effects is required if  $\theta_{ASCE} \leq 0.1$ . Values of  $\theta_{ASCE} > 0.25$  cannot be accepted.

By comparing Eqs. (2) and (3), it is evident that the interstorey drift sensitivity coefficient adopted in EC8 is significantly larger than that used in ASCE because the coefficient used in ASCE is based on the elastic stiffness of the system.

## 1.2 Strength Amplification Factor in Research Studies on SDOF Systems

In most of the studies available in literature, the strength amplification is adjusted so as to counterbalance the effects of gravity loads on the displacement demand of SDOF systems [2, 4–6]. Studies carried out on SDOF systems pointed out that the strength amplification factor mainly depends on the interstorey drift sensitivity coefficient, on the ductility demand and on the equivalent viscous damping ratio of the system. The same studies have shown that the formulation suggested in EC8 is conservative (i) for systems on firm soil with ductility demand lower than 6; (ii) for systems founded on soft soil; and (iii) for systems characterized by high values of the equivalent viscous damping ratio.

Based on the response of SDOF systems, a new formulation of the strength amplification factor  $\alpha_m$  is proposed in reference [6]. According to this formulation,  $\alpha_m$  is the product of the static amplification factor  $\alpha_{el}$  times a coefficient  $C_m$  depending on the equivalent viscous damping ratio  $\xi$ , ductility demand  $\mu$  and inelastic interstorey sensitivity coefficient  $\theta$

$$\alpha_m = C_m(\xi, \mu, \theta) \alpha_{el} \quad (4)$$

where

$$\alpha_{el} = 1/(1 - \theta/q_d) \quad C_m = 1 + \theta(-0.025\mu^2 + 0.45\mu - 2.6\xi - 0.65) \alpha_{el} \quad (5)$$

## 2 Extension of the Proposed Strength Amplification Factor for the Design of Moment Resisting Frames

To apply the strength amplification factor proposed in [6] to MDOF systems, some issues should be pointed out. Indeed, the seismic response of Moment Resisting Frames (see the capacity curve in Fig. 1a) mainly differs from that of SDOF systems (see Fig. 1b) for the following reasons:

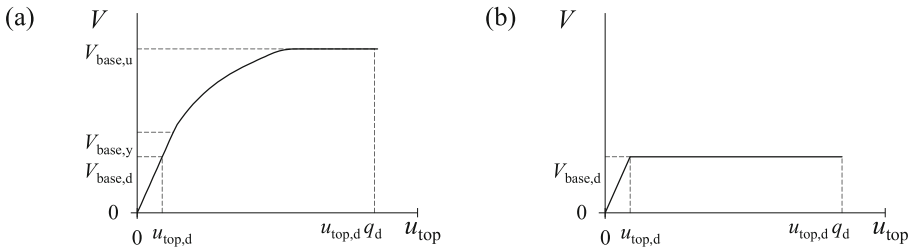
1. SDOF systems are designed only to sustain the design base shear force  $V_{base,d}$  while MDOF systems are designed based on different design situations. As a result, these latter systems are characterised by an overstrength, i.e. the lateral strength at the first yielding  $V_{base,y}$  is generally larger than  $V_{base,d}$ ; this overstrength is further increased because of the adoption of cross-sections available in the market;
2. The nonlinear response of SDOF systems is generally represented by an elastic-perfectly plastic relationship. Thus, once yielding has developed a sudden decrease of stiffness occurs.; Conversely, in MRF systems designed according to capacity design principles, yielding gradually spreads at the ends of beams and at the base of first order columns. This leads to a nonlinear hardening behaviour. Hence, the ultimate storey strength  $V_{base,u}$  corresponding to the top inelastic displacement  $u_{top,in}$  is significantly larger than  $V_{base,d}$ .

Based on the considerations above, the authors propose the following procedure to consider P- $\Delta$  effects.

1. MRF is first designed neglecting P- $\Delta$  effects
2. A pushover analysis of the designed structure is carried out. The peak ground acceleration (PGA) associated to each point of the pushover curve is determined according to the N1 method [7]
3. The base shear corresponding to the top displacement associated with the design value of the PGA is assumed as the ultimate lateral strength  $V_{base,u}$
4. The interstorey drift sensitivity coefficient to be adopted in Eq. (4) is calculated as

$$\theta = \theta_{EC8} V_{base,d} / V_{base,u} \quad (6)$$

5. The pushover curve is converted into a bilinear relationship in the relevant range of displacements and the ductility demand  $\mu$  is calculated as the ratio of the inelastic displacement demand to the top displacement at yielding
6. Internal forces in members and floor displacements provided by the adopted linear method of analysis are increased by the amplification factor given in Eq. (4).



**Fig. 1.** Capacity curve of a) Moment resisting frames; b) SDOF systems.

### 3 Methodology

To validate the proposed formulation, steel MRFs characterized by different values of the interstorey drift sensitivity coefficient are analysed following this procedure:

- Step 1: Each structure is designed by modal response spectrum analysis neglecting P-Δ effects
- Step 2: The seismic response of the obtained structures is determined by nonlinear static analysis in which P-Δ effects are neglected. The obtained response and the performance levels achieved by these structures are assumed as target.
- Step 3: Each structure is redesigned taking into account P-Δ effects in keeping with provisions of seismic codes or according to the proposed formulation.
- Step 4: The nonlinear seismic response of the structures designed in Step 3 is determined taking into account P-Δ effects.

If P-Δ effects are properly considered in the design phase, the performance levels obtained by structures designed in step 3 and analysed in step 4 should be similar to the target values (i.e. to the performance of structures designed in Step 1 and analysed in Step 2). Finally, the increase in size of the cross-sections required to counterbalance P-Δ effects is determined to compare the costs of structures designed according to the proposed and existing formulations.

### 4 Considered Structures

The considered buildings have a number of levels equal to either 4 or 6 and are located on soft soil (Soil C according to Eurocode 8), in high seismicity area characterised by a peak ground acceleration PGA equal to 0.35g for seismic events with a probability of exceedance of 10% in 50 years. The span length  $L$  and the interstorey height  $h$  are equal to 8.0m and 3.3m, respectively (Fig. 2a). To investigate systems characterised by different values of overstrength related to the design for the non-seismic design situation, the considered steel MRFs have to sustain moderate (MRF-M) or high (MRF-H) gravity loads. Independently of the amount of gravity loads, the seismic storey mass is equal to 97.86 kNs<sup>2</sup>/m. The first situation is typical of structures where MRFs are located only along the perimetric frames of the buildings, whereas pinned beam-to-

column connections are considered for interior frames; the second situation is typical of buildings where MRFs are mainly located in interior frames. Characteristic values of dead and live loads supported by the beams are provided in Fig. 2b.

Moment resisting frames are first designed to sustain gravity loads in the non-seismic design situation. Second, buildings are designed according to EC8, but neglecting P- $\Delta$  effects, to sustain seismic actions with reference to both the strength requirement for seismic events with a probability of exceedance of 10% in 50 years and the Damage Limitation requirement (DL) for seismic events with a higher probability of exceedance. Consequently, the design interstorey drift  $\Delta u$  is reduced by a coefficient  $v$  to consider the lower return period of the seismic action associated with the damage limitation requirement. Based on the analysis of seismic parameters on the Italian territory, the parameter  $v$  is assumed equal to 0.28.

In particular, the fulfilment of the DL requirement has been verified under the hypothesis of buildings having non-structural elements fixed so as not to interfere with structural deformations. Hence  $v \Delta u$  is verified to be lower than a limit value equal to 0.010 times the storey height  $h$  equal to 3.3 m. A value of the behaviour factor  $q$  equal to 5.5 has been used to reduce the elastic response spectrum. This value, which is lower than the maximum value suggested in EC8, has been selected because structures designed by higher values would not satisfy the DL requirement.

The main properties of the 4 designed structures are reported in Table 1. In particular, the period of the first mode of vibration is reported alongside with the maximum value of the interstorey drift sensitivity coefficient calculated according to EC8, ASCE and the proposed formulation. Further, the structural weight  $W_s$  of the frame is calculated as well. To discharge the effect of different steel grades used for members, the weight of the single members is multiplied by the ratio  $f_y/235$ , where  $f_y$  is the yield strength in MPa of steel.

According to EC8, no one of the designed buildings is acceptable, indeed  $\theta_{EC8}$  is always larger than 0.10. Further, only in one case the approximated procedure is applicable (i.e. MRF-B-4st). All the other MRFs should be made significantly stiffer to have  $\theta_{EC8}$  lower than 0.3. Conversely, all the buildings are characterised by  $\theta_{ASCE}$  lower than 0.1. Thus, no modification is required for these buildings according to ASCE/SEI 2010. An intermediate behaviour is achieved when the proposed value of  $\theta$  is considered.

In a following step, internal forces in members and displacements provided by the linear design method of analysis are amplified to take into account P- $\Delta$  effects according to prescriptions of either EC8, ASCE and the proposed approach. To evaluate these effects an amount of gravity loads equal to  $m \times g$  has been considered.

When required, the cross-sections of members are modified to sustain the increased internal forces. Further, when EC8 prescriptions are considered, the stiffness of the frame is increased so that the maximum value of  $\theta_{EC8}$  be lower than 0.30. The percentage increase in the structural weight of frames designed taking into account P $\Delta$  effects is reported in Table 2. The table shows that when EC8 is considered, the increase in structural weight is significant, with the sole exception of MRF-H-4st. Indeed, in this latter system, cross-sections defined in step 1 are able to sustain the amplified internal forces provided by P $\Delta$  effects because of the overstrength produced by the non-seismic design situation. When ASCE procedure is used, no increase in the cross-section is required. Finally, in a single case (MRF-M-6st) the internal forces derived by the proposed procedure require an increase in the cross-section size.

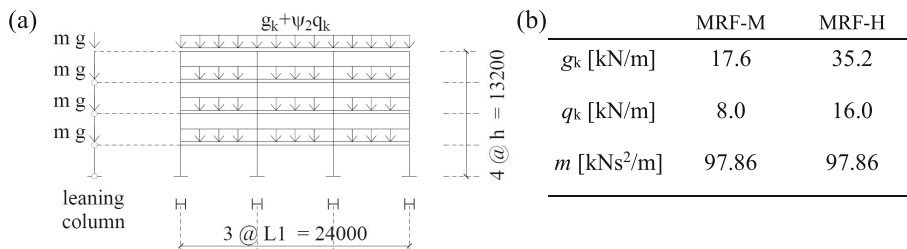


Fig. 2. (a) Layouts of the considered 4-storey MRF; b) loads and seismic storey mass.

Table 1. Properties of the structures designed neglecting P-Δ effects (Step-1).

Building	MRF-M-4st	MRF-M-6st	MRF-H-4st	MRF-H-6st
$T_1$ [s]	1.578	2.162	1.029	1.594 s
$W_s$ [kN]	100	139	158	236
$\theta_{EC8}$	0.40	0.54	0.17	0.31
$\theta_{ASCE}$	0.07	0.10	0.03	0.06
$\theta_{PRO}$	0.19	0.32	0.06	0.11

Table 2. Percentage increase in structural weight because of P-Δ effects.

Building	MRF-M-4st	MRF-M-6st	MRF-H-4st	MRF-H-6st
EC8	33.5%	79.0%	0.0%	14.1%
ASCE	0.0%	0.0%	0.0%	0.0%
PROPOSED	0.0%	29.6%	0.0%	0.0%

## 5 Numerical Analyses and Response Parameter

The seismic response of the considered frames has been determined by nonlinear static analysis. The single analysis is carried out by the OpenSEES computer program. Beams and columns are modelled as ‘Beam with Hinge’ elements. The length  $L_{pl}$  of the plastic hinge is equal to the cross-section height. Within the hinge region, the cross-section is discretised into fibres. The Steel02 uniaxial material is assigned to the fibres. A linear transformation is assigned to beams and columns. When P-Δ effects are taken into account (analyses of structures designed in Step 3), a leaning column is added in the numerical model. The gravity load applied to the leaning column at each storey is that corresponding to the seismic mass.

The frames are subjected to a distribution of forces proportional to the first mode of vibration. The top displacement is increased up to the achievement of a top displacement demand equal to 5% of the height of the building. The PGA corresponding to each point of the pushover curve is determined by the N1 method.

At each step of the pushover analysis, the plastic rotation  $\theta_{pl}$  at the end of members is

$$\theta_{pl} = (\chi - \chi_y)L_{pl} \tag{7}$$

where  $\chi$  is the curvature at the end of members and  $\chi_y$  is the curvature at yielding. This latter curvature is calculated as a function of the plastic bending resistance of members  $M_{pl}(N)$  (eventually reduced because of the axial force according to the relevant prescription of Eurocode 3), the moment of inertia  $I$  of the cross-section and the elastic modulus of steel  $E_s$

$$\chi_y = M_{pl}(N)/(E_s I) \tag{8}$$

The plastic rotation is compared with the rotation capacity provided in EC8- part 3 [8] for the Significant Damage (SD) and the Near Collapse (NC) limit states.

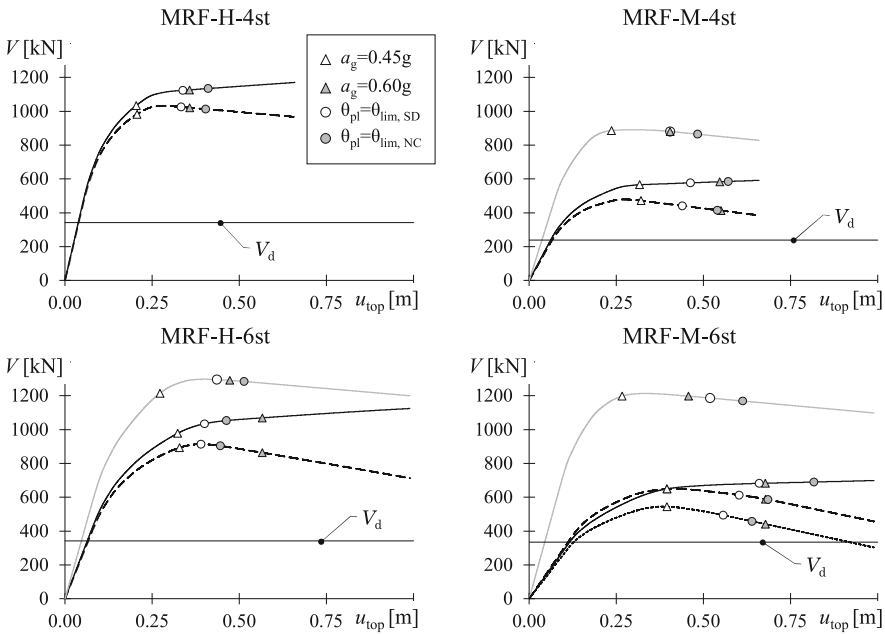


Fig. 3. Seismic response of the considered structures.

## 6 Response and Conclusions

The response of the considered structures in terms of pushover curve is represented in Fig. 3. For each structure, a maximum number of 4 curves is reported. The continuous solid line represents the response of structures designed and analysed neglecting P- $\Delta$  effects. Instead, the dashed line, the continuous grey line, and the dotted line represent the response of the structures re-designed considering P- $\Delta$  effects using the proposed

formulation, EC8 and the ASCE code, respectively. In cases where EC8 and ASCE codes lead to a structure equal to that obtained by the proposed procedure, the corresponding curves are not reported.

In each pushover curve four points are highlighted. Specifically, white and grey dots pinpoint the first achievement of a plastic rotation demand equal to the plastic rotation capacity at the SD and NC limit states, respectively. The triangles pinpoint the seismic demand related to PGA equal to 0.35 g (white triangle) and 0.60 g (grey triangles). The first value of the PGA is the one considered in the design phase and for this seismic intensity the structures are expected to fulfil the SD limit state. The PGA equal to 0.60 g corresponds to seismic excitations with a probability of exceedance of 2% in 50 years. For this intensity level, structures are expected to fulfil the NC limit state. Finally, the design value of the base shear is reported by a horizontal line.

Independently of the procedure adopted to counterbalance P- $\Delta$  effects, all the structures fulfil the SD limit state (with dots located at the right of white triangles). In regard to the NC limit state, in a single case (MRF-H-6st) the adoption of the proposed procedure leads to a structure that does not fulfil code requirements. However, it should be noted that even the structure designed and analysed without P- $\Delta$  effects is not able to fulfil this limit state. It is noteworthy the case MRF-M-6st, because it is characterised by the highest value of  $\theta$ . In this case, the proposed procedure leads to the fulfilment of NC limit state while the same limit state is not fulfilled if strength and stiffness are not increased as required by ASCE code.

## References

1. Humar J, Mahgoub M, Ghorbanie-Asl M (2006) Effect of second-order forces on seismic response. *Can J Civ Eng* 33:692–706
2. Tremblay R, Duval C, Léger P (1998) Effects of viscous damping models, hysteretic models and ground motion characteristics on seismic P-delta strength amplification factors. In: Usami T, Itoh Y (eds) *Stability and Ductility of Steel Structures*. Elsevier, Oxford, pp 103–118
3. Black EF (2011) Use of stability coefficients for evaluating the P- $\Delta$  effect in regular steel moment resisting frames. *Eng Struct* 33:1205–1216
4. Bernal D (1987) Amplification factors for inelastic dynamic P- $\Delta$  effects in earthquake analysis. *Earthq Eng Struct Dynam* 15:635–651
5. MacRae GA (1994) P- $\Delta$  effects on single-degree-of-freedom structures in earthquakes. *Earthq Spectra* 10(3):539–568
6. Amara F, Bosco M, Marino EM, Rossi PP (2014) An accurate strength amplification factor for the design of SDOF systems with P- $\Delta$  effects. *Earthquake Eng Struct Dynam* 43:589–611
7. Bosco M, Ghersi A, Marino EM (2009) On the evaluation of seismic response of structures by nonlinear static methods. *Earthq Eng Struct Dyn* 38:1465–1482
8. (2005) CEN. EN 1998–3. EuroCode 8: design of structures for earthquake resistance. Part 3: Assessment and retrofitting of buildings, Bruxelles: European Committee for Standardization