



# Evolution History of Non-circular Gears

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**Abstract.** Recently variable-ratio gears are increasingly being used to drive the modern machines. For this purpose cams and non-circular gears are often used. In the past, the production of non-circular gears was a very complicated process, which explains their relatively limited use in practice. In the 20th century, Professor F. Litvin played a great role in the development of theory and manufacture of non-circular gears. Modern technologies of the 21st century make the manufacturing relatively cheap and easy, using high-precision machine-tools, 3D prototyping and spraying. The paper considers the history of non-circular gears evolution and application.

**Keywords:** Non-circular gear · History of machine and mechanism science · Variable gear ratio · Gear-cutting tool

## 1 Introduction

Non-circular gears (NCG) are designed for certain applications. While a regular (circle shaped) gear is designed to transmit torque to another engaged member of transmission with the least amount of noise and wear and the highest efficiency, a non-circular gear's primary goal could be to get a variable gear ratio, axle displacement oscillations, and other important properties and characteristics.

They were not used widely in the past because of the high prices for their production. Today, in the 21st century, due to the modern technologies and use of computer-controlled machine-tools as well as new mathematical models for calculations of the non-circular shape, the NCG have started actively to replace mechanisms with cams. Specially designed equipment, rapid prototyping and electron discharge machining allowed to establish non-circular gear manufacturing by duplicating or rolling. The enhanced potential in non-circular gear design and production explains the growing interest in this type of gears.

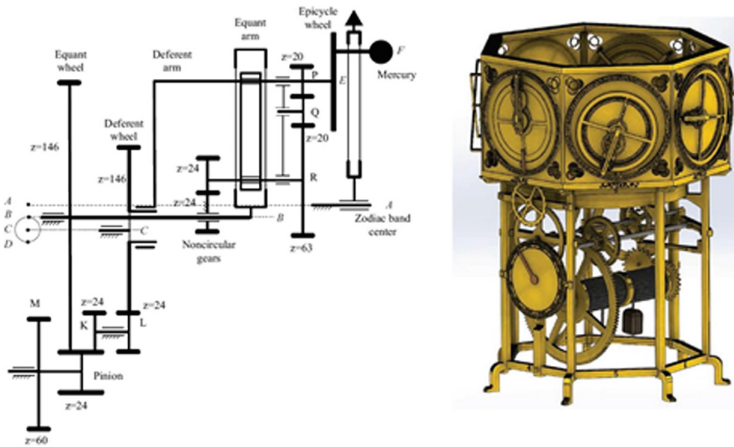
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## 2 The Earlier Application of Non-circular Gears

### 2.1 Dondi's Astrarium

Giovanni de Dondi (1330–1388) was one of the first, who used NCG while building his astronomical device, which simulated the solar system and is known as the Astrarium (Fig. 1). Also, he authored the manuscript «Tractatus Astrarii» documenting, with text and drawings, the design and manufacturing phases of a very sophisticated clock. In particular, the manuscript contains one of the earliest documented procedures for the design of non-circular gears” [1].



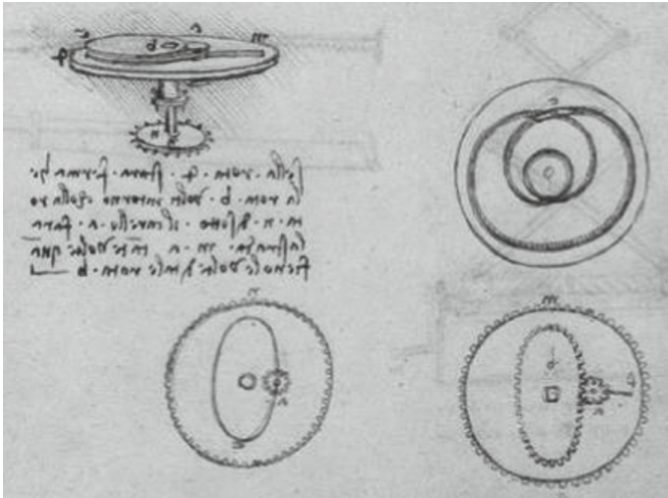
**Fig. 1.** Schematic diagram [23] and a general view of the Astrarium (drawing courtesy of Laura Guida [24])

### 2.2 Leonardo da Vinci and Non-circular Gears

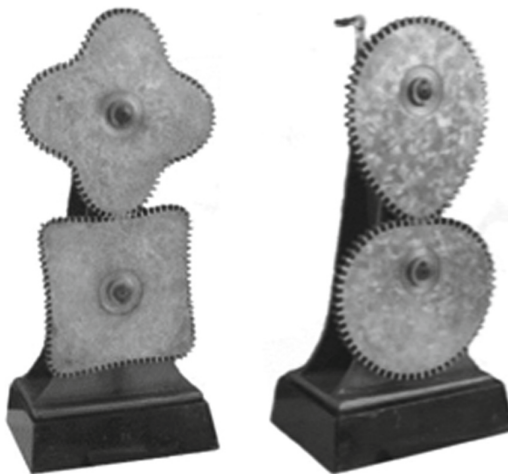
Reference to non-circular gears could be found in the works and sketches of Leonardo da Vinci (1452–1519) [2], who supposed to create a clockwork mechanism and elliptical shaped gears with a variable transmission ratio were ideal for this purpose (Fig. 2).

### 2.3 Redtenbacher's Non-circular Gear Mechanisms

Evolution of NCG in the 19<sup>th</sup> century was connected with Ferdinand Jakob Redtenbacher (1809–1863), one of the most prominent mechanics of the 19<sup>th</sup> century. He manufactured several models of non-circular gear mechanism to demonstrate them during the “Mechanics” courses he taught (Fig. 3) [7].



**Fig. 2.** Leonardo da Vinci's drawings [2]



**Fig. 3.** Models of NCG mechanisms by F. Redtenbacher [3]

### 2.4 Reuleaux’s Models

Later on, Franz Reuleaux (1829–1905) developed his teacher’s (Redtenbacher) ideas and found some new applications for non-circular gears (Fig. 4) in industry. Thus, he proposed to use them in textile machinery, mechanical presses and high-torque hydraulic motors as an alternative to cam mechanisms [4].

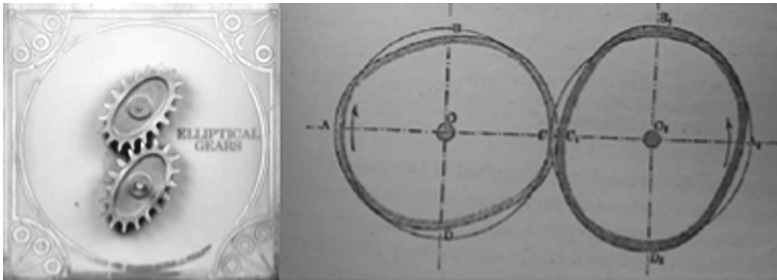


Fig. 4. Reuleaux’s models of NCG mechanism [4]

### 2.5 “Principles of Mechanism” by S.W. Robinson

The book “Principles of mechanism” by S.W. Robinson, published in 1903 (Fig. 5), shows in detail the main principles of NCG mechanism design and operation [5]. An entire chapter of the book is devoted to the profile and ratio calculations as well as application, with examples and photographs of various shaped non-circular gears used in Mechanical Engineering.

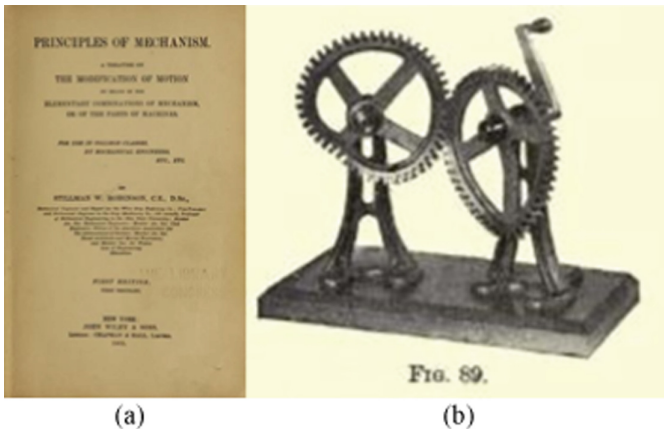


Fig. 5. A hardcover a) and a page b) of “Principles of mechanism” book by S.W. Robinson [5]

### 3 Non-circular Gears in the 20–21<sup>st</sup> Centuries

#### 3.1 Ivan Ivanovich Artobolevsky (1905–1977)

Descriptions of several mechanisms, composed of non-circular gears, can be found in the works of the Soviet (Russia) Professor Ivan I. Artobolevsky [9]. Basically, he showed the principle of operation of these mechanisms, giving a detailed description of their structure and properties, without touching on the problems associated with their use and manufacture. Artobolevsky considered NCG as an alternative to gears with a non-circular profile in order to show in the training course that the involute profile is not the only one, but is only the most successful for the current level of technology, but can be displaced in the future.

#### 3.2 Faydor Lvovich Litvin (1914–2017)

In the 20th century, Professor Faydor Litvin played a great role in the development of NCG theory and manufacture [6, 7]. In his book “Non-circular Gears: Design and Generation” [8] he represented the extension of the modern theory of gearing applied to the design and manufacture of the main types of non-circular gears: conventional and modified elliptical gears, eccentric gears, oval gears, gears with lobes, and twisted gears as well as updated theoretical descriptions of the methods of generation of non-circular gears by enveloping methods similar to those applied to the generation of circular gears. He also offers new developments intended to extend the application of non-circular gears for output speed variation and generation of functions.

However, Faydor Litvin understood that the main problem of developing and using NCG arose when cutting teeth on non-circular wheels. When using the run-in method, it is required to provide nonlinear dependencies between the movements of the tool and the gear, which led to the need to create a special gear-cutting machine, and a significant increase in the cost of manufacturing. Nevertheless, similar work was carried out. In the works of F.L. Litvin [6, 8] showed several machines that allow to obtain non-circular wheels, but the author noted that the existing methods of cutting gears need improvement for serial or mass production.

When using the run-in method, a gear rack was usually used. The task was to carry out such a movement of the tool, in which it was rolled with its centroid along the centroid of the gear being cut. For this purpose, an annular or rack-and-pinion milling cutter was used, which is a series of trapezoidal disk cutters, the axial distance between which should have been equal to the pitch between the teeth. However, the production of ring cutters was associated with great difficulties, as the exact value of the pitch of the teeth of the cutter was difficult to withstand. The cutting process turned out to be quite productive only in the manufacture of wheels of a small module. If the cutting of the teeth required several passes along the depth and length of the teeth, the cutting process was periodically interrupted to return the workpiece to its initial position. Also, this machine did not allow cutting oblique NCG.

Faydor Litvin recommended abandoning ring cutters and using conventional worm cutters, as well as developing machines that differ from existing mechanisms that carry out the required connection between the movement of the tool and the gear being cut.

In his works, he suggested using a device that allows you to engage an exemplary NCG with a rail. When cutting the teeth, the model and the cut gears, located on a common mandrel, took part in rotation around their axis, as well as in longitudinal and transverse movements. The tool rail moved only in the direction of the axis of rotation, performing a slotting motion. Thus, it was possible to realize continuous contact of the rail and the NCG.

Also, Faydor Litvin described a machine for cutting NCG with a chisel. Such a machine most effectively performed NCG with small centroid sizes (Fig. 6). The point  $O_1$  of the pantograph coincides with the center of rotation of the worm sector, which is the driver. The point  $O_2$  is the center of rotation of the wheel being cut. The point  $O_3$  is the center of rotation of the copier outlined by a curve, which is a transformed curve with respect to the centroid of the wheel being cut. The transformed curve is obtained by multiplying the radius of the centroid by the value  $m$ . When the driver rotates, the copier runs around a fixed circle, the diameter of which is  $m$  times larger than the diameter of the initial circle of the used chisel. In the process of cutting, the chisel makes a chiseling movement without having a rotational movement.

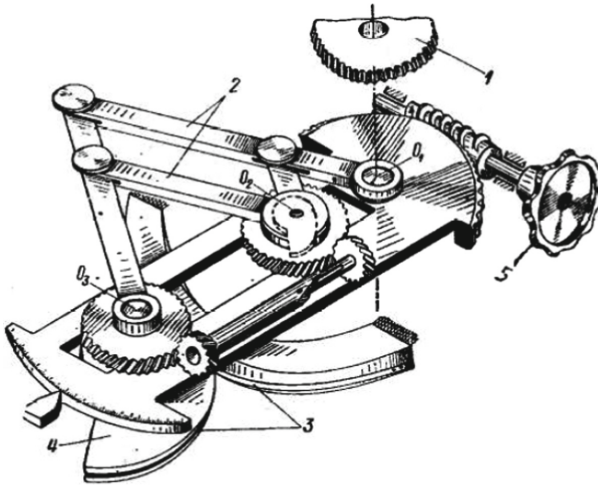


Fig. 6. NCG cutting device [8]

#### 4 Collection of NCG Mechanisms in Moscow

In Russia in Moscow at the Bauman Moscow State Technical University the unique collection [10] of five models of NCG mechanisms is stored (Fig. 7). Two of them were made in the workshop of Alexander Clair and purchased in the 19th century. One model was developed on the results of the dissertation work of the Soviet (Russia) Professor Karl Sergeevich Tarkhanov and mainly devoted to the methods of NCG manufacturing. The models are used during “Theory of Mechanisms and Machines” course to demonstrate

students the possibilities of non-circular gears. However, all five models were detailed described in the paper “Non-circular Gears: Innovative Manufacturing Technologies” [11].



Fig. 7. Mechanisms from the BMSTU collection [5]

## 5 Modern Non-circular Gear Manufacturing

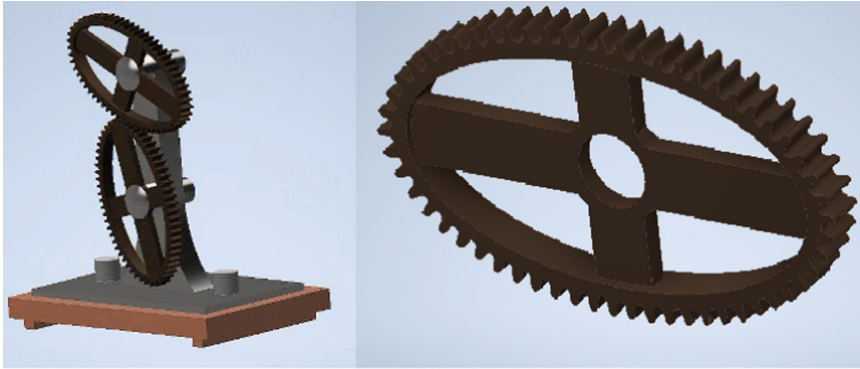
For non-circular gear profiles, ease of manufacturing is an important requirement. In this connection, a more efficient method of determining the coordinates of profile points has been proposed in [12, 13]. The method of profile normals has been used to determine the profiles of gear teeth.

The latest versions of software allow the coordinates of the tooth profile to be calculated automatically. For example, «Globas Group» offers a software product that allows the entire mechanism design to be calculated and simulated [14]. The results of the calculations can be fed directly into the machine tool software.

One more developing method is 3D modelling and subsequent printing of gears on a 3D printer. In this method, a 3D gear model is created in a computer-aided design (CAD) system. “Autodesk Inventor” program allows, for example, an approximate 3D model to be created (Fig. 8), by taking a profile from an existing gear [11]. “MathLab” program allows to describe the gear profile mathematically and create an accurate 3D model of a non-circular gear with any profile shape [15]. The advantages of both methods are their high accuracy and ease of manufacturing. In addition, modern software packages allow not only to create an accurate 3D model and to calculate the strength of even the most complex structures, taking into account their features by the FEM or other methods [16, 17].

## 6 Modern Designs with Non-circular Gears

Some designs with non-circular gears have been used in mechanical engineering, but they have not been widely used, primarily due to the complexity of profile calculating and manufacturing [18–21].



**Fig. 8.** 3D model with non-circular gears.

One application of NCG was the conveyor belt drive. The conveyor belt was driven by a Maltese cross and made an intermittent movement. The use of non-cylindrical gears can significantly reduce angular acceleration without reducing the angular velocity of the main shaft. This reduces the load on structural elements, reduces dynamic effects that lead to fatigue stresses and reduction of carrying capacity. This solution increases the reliability of the conveyor and increases the service life of its components.

Many crank-slide mechanisms need to move the slider at a constant speed, so the crank must be designed to rotate at a variable angular velocity. This can be achieved effectively by the use of NCG placed in the crank drive. These can be agricultural machine cutting units, piston pumps, oscillating conveyor drives, and etc. Application of NCG can improve both: the reliability of the crank-slide mechanism and its performance.

In some cases, non-circular gears are used in automatic lathes for slow rotation of the camshaft during working operations and fast rotation during idle strokes. Sometimes they are used in printing machines, in textile machines for obtaining fabrics with a certain pattern, in silk-winding machines for obtaining a barrel-shaped bobbin, and etc.

Also, a machine-tool for milling keys has been developed in which the rotation of the crank mechanism was communicated from NCG in order to feed at an approximately constant speed.

NCG are widely used in hydraulic machines. Rotary machines with improved characteristics based on hydraulic machines are gaining great popularity. A distinctive feature of such hydraulic machines is the absence of support reactions with certain combinations of geometric parameters, which makes it possible to significantly simplify the design of support units, mechanical seals and the system as a whole, and the geometric shapes of the parts and their mutual arrangement make it possible to create compact devices. In addition, the design of the new hydraulic machine such that it can easily be connected in series based on a single shaft. Switching of high and low pressure mains is carried out without the use of special devices [22]. Often, the use of NCG in the design of hydraulic systems and pumps makes it possible to reduce the dimensions of the system and increase its quality.

But the most important advantage of NCG is the ability to synthesize variators with any law of changing the gear ratio. When designing, as a rule, they rely on already



existing variators using gear wheels of an evolvent profile, cam mechanisms and other mechanisms, however, the use of NCG allows you to synthesize a car according to any given law of motion of input and output links, while NCG often make the car more compact, allow you to work at high speeds, in addition, in many works it has been shown that NCG, due to a larger contact area, have a greater fatigue life than other analogues. For example, in contrast to cam-type mechanisms, NCG allow high loads to be transmitted and are highly reliable. They also do not require special fixtures or springs to maintain contact continuity in the upper pair, resulting in a simplified design and increased manufacturability.

## 7 Conclusions

1. The main problem, namely the complexity of NCG manufacturing, which has existed for centuries, has been solved. Modern technologies make it relatively cheap and easy using high-precision machine-tools, 3D prototyping and spraying, while modern laser technology provides the necessary conditions for controlling the wear.
2. Non-circular gears have a longer service life and can handle heavier loads than comparable mechanisms. They will replace cams and could also reduce the risk of accidents, as jamming is much less common with NCG than with cams.
3. Non-circular gears make it possible to reproduce any, even the most complex law of motion. In addition, the NCG can operate at much higher speeds than their counterparts, thus increasing machine productivity.
4. NCG models give students an insight into a little-known area of Mechanics, allowing them to observe in person the movement of a non-regular gear, which greatly expands the horizons of future engineers and increases their interest in the study of engineering disciplines.
5. For some applications, planetary gearboxes with NCG are more efficient than planetary gearboxes with spur gears and could even provide a solution that cannot be achieved with spur gears.
6. NCG enable the development of more compact and efficient variable-speed drive, whose gear ratio can be derived naturally from the NCG geometry. Thus, the gear ratio change will be smoother and more shockless, which is a significant advantage compared to current variable-speed drives without NCG.
7. Studies have shown that NCG allow for the greatest range of gear ratio change, which also facilitates the development of new devices with large amplitudes of output link motion.

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