



Nomograms: An Old Tool with New Applications

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Abstract. In this paper, we consider the history of nomograms as a computational tool in mechanical engineering, together with their potential applications for teaching purposes, and summarize the mathematical methods used to derive them. Nomograms are graphical descriptions of a mathematical problem, such that the desired solution may be derived through a simple geometric construction, which usually requires nothing more than a straightedge. This way, a reasonably accurate solution to a complex problem can be quickly obtained even in adverse environmental conditions by low-skilled users; moreover, a nomogram can provide immediate insight on the relationship between the variables. Nomograms date back to the 1800s and have been used by engineers for decades, due to their convenience over manual computation, before computers became widespread. While nomograms have now been largely superseded as engineering tools, our analysis shows that they can still have some applications in workshops and for teaching purposes.

Keywords: Nomograms · Graphical · Vibrations · History of engineering · Mechanisms

1 Introduction

In their work on a given problem, engineers usually develop an analysis which is ultimately condensed in one or more equations to be solved. These could be, for instance, the *ordinary differential equations* (ODEs) that describe the time-behavior of a system, or an algebraic equation that defines the optimal design parameters of a component to be realized. In the following, we will mostly focus on engineering equations having only one or finitely many solutions; conversely, we disregard equations that define a *constraint* (which correspond to a boundary in a vector space, with infinitely many points).

Once the right equation (whether algebraic or differential) for the model at hand has been found, finding the solution is usually a mechanical procedure. Indeed, after the mass advent of personal computers in the '60s and the corresponding development of prepackaged numerical algorithms that are not domain-specific (and can thus be used as “black boxes”), equation solving has become little more than a minor step in most cases.

This, however, was not the case up until a few decades ago; indeed, entire teams of human “computers” were sometimes hired by research centers to manually develop tedious arithmetical steps of numerical methods. Calculating by hand is however a long and risk-prone procedure, where a minor mistake can invalidate all the following work and which provides little to no insight that can be used to check the intermediate results.

Graphical methods have been used by engineers since before our profession was formally established; while their accuracy is physically limited by the tools used, such as pencil and straightedge, they offer interesting advantages. For example, they are quite immediate to understand and employ, such that they may be used by low-skilled operators; furthermore, they can provide reasonably precise results in a fraction of the time required for analytical procedures. Their strongest asset, however, is the intrinsic visualization of the data: this way, one has immediate insight over the results and especially over the relationships between variables, which is useful when exploring engineering alternatives.

Nomograms [5] are graphical methods developed for solving equations. Essentially, they are graphs drawn such that the mathematical relationship between the variables has a simple graphical equivalent (such as three points being collinear). Thus, they can be used to find a numerical result by graphical means: one “enters” the graph with given input parameters (corresponding, in general, to points on lines) and finds a solution by a geometrical construction, which usually requires no more than pencil and straightedge. A thin thread can also be used, to avoid drawing lines, so the nomogram can be reused.

Nomograms have been used extensively by engineers up until the ’60s due to their practical convenience in a number of applications, including those in the field of mechanical engineering. Here, we will offer a bibliographic retrospective on such methods, especially from the point of view of Machine Mechanics.

While no longer in widespread use, nomograms still offer interesting advantages and have some niche applications, for instance in shop-floor and open-field work, where digital tools would be less practical due to their lack of ruggedness. We also believe nomograms can have interesting advantages as teaching tools, again owing to their immediate visualization: thus, we present nomograms that we created specifically for such applications, as examples of how a nomogram can be developed and used. These nomograms have been designed through the open-source Python package “pyNomo” [4].

The goal of this work is to discuss the role of nomograms in the history of mechanical engineering, with a special attention to the field of Mechanisms and Machine Science. An application of nomograms for educational purposes is also presented, both for clarifying complex equations and for furthering interest in classical methods among students.

The rest of this paper is structured as follows. In Sect. 2, we present a history of the applications of nomograms in engineering, particularly in machine and mechanism design. In Sect. 3, we present the basic methods used to create nomograms and how they may be combined to solve more complex equations; we also present nomograms targeted for classical problems in Mechanisms and

Machine Science. Moreover, we discuss potential applications of nomograms, for example in industry and for educational purposes. Finally, in Sect. 4, we draw conclusions and offer directions for future work.

2 Nomograms in History of Machines and Mechanisms

Nomograms are a part of the larger history of *graphical methods* for computation: these were the most used techniques for numerical analysis in science and engineering, before computers became widespread. For more than a century, graphical methods were the object of research interest and were commonly a part of technical education programs; in this paper, we can only provide a very brief description of the landmarks in the development of nomograms, with a focus on their use for mechanical engineering. For a broader historical perspective, we refer the reader to [6, 13, 14, 18, 30]; we are not aware of any historical analyses of nomography specifically for our field.

To restrict the scope of our review, we only consider purely graphic approaches that can (at least in principle) be applied with nothing more than pen and paper. Thus, we disregard more general approaches using mobile elements, such as slide rules, another instrument (conceptually related to nomograms) that was in use until the '60s.

We also remark that the terminology is not unequivocal: several works on “nomograms” use in fact graphical methods that are quite different (and generally more cumbersome to use) than standard nomograms [17], for instance by using graphs with multiple lookup curves on squared paper; here, we only consider methods that require some form of geometrical construction. Moreover, especially in older sources, alternative terms such as “nomographs”, “alignment charts” and “abacs” are used interchangeably [6]. To avoid confusion, in this paper we only use the term “nomogram”, first introduced by French mathematician and engineer Maurice d’Ocagne (1862–1938) in the late 19th century [5] to distinguish his work from previous contributions: our bibliographic search on scholarly research engines shows that this definition remains the prevalent one.

2.1 Invention and Diffusion: 1800–1960

In the history of mathematics, nomogram (from Greek words νόμος, meaning “law”, and γραμμή, which means “line”) is a relatively recent term, which should strictly not be applied to calculating devices published before the seminal work by d’Ocagne [13]; at the same time, d’Ocagne was influenced in his research by several previous works, which are useful to understand the main concepts in nomography as he developed it.

Several mechanical devices have been developed since ancient times to ease complex computations; these mostly used movable components that allowed the user to keep track of numbers and of their respective relationships. An example is the abacus, using beads moving on wires. Devices that have been identified as precursors to nomograms include:

1. Astronomical calculating instruments [13,30], such as *astrolabes* and *equatoria*, used to compute the position of the Sun, the Moon and the planets; examples are the *Albion* by Richard of Wallingford (dating to the early 1300s) and the *jovilabe* 1600s) used by Galileo Galilei to study the orbits of the moons of Jupiter.
2. *Sundials* [13,16] and *moondials* [30], which, while not strictly computing instruments, are based on calculations and present graphically their results.
3. *Volvelles* and *slide charts* [13], related to the slide rules mentioned previously; composed of movable pieces of paper, they are designed for specialized computations.

The first necessary element was the development (in the 17th century) of coordinate geometry by Descartes, allowing the drawing of graphs of a function F . In the following, we consider mostly three-variable nomograms, which provide solutions for the equation

$$F(v_1, v_2, v_3) = 0 \tag{1}$$

We then define auxiliary functions F_i in the variables of interest v_i and in the coordinates x and y of a Cartesian plane: the F_i 's are such that the set of equations

$$\begin{cases} F_1(x, y, v_1) = 0 \\ F_2(x, y, v_2) = 0 \\ F_3(x, y, v_3) = 0 \end{cases} \tag{2}$$

holds if and only if Eq.(1) is satisfied. Then, three families of curves in the x - y plane can be graphed, one for each of the conditions in Eq. (2). Let us denote each curve by the corresponding value of the fixed parameter v_i : a point at the intersection of three curves is then a solution to Eq. (1). Most of the time, no exact intersection of three curves is available, as the values of v_i 's have been discretized (to avoid cluttering in the graph); the closest values are then interpolated by the user from simple graphical approximation. Some of the main features of the nomogram are already present in this simple concept:

- from two values v_i , the third can be derived graphically, even if the relationship in Eq. (1) cannot be inverted analytically; solving direct and inverse problems (depending on which variables are defined as input) is thus equivalently easy.
- the approximation of a given solution is immediately visible from the distance to the closest curve of the output variable, thus providing a visual feedback of the error;
- the precision is limited by the resolution of the graph and by the user's capabilities, especially in approximating a quasi-exact solution; however, large errors due to misreading are unlikely (and out-of-scale errors are impossible, since one only plots the curves corresponding to meaningful ranges for the v_i 's);
- graphical methods are targeted for a specific equation, while other calculating devices (such as slide rules) are more general; however, once a graph has been produced, its usage is immediate and less prone to user errors.

With reference to Eq. (2), suppose that $F_1 = v_1 - x$ and $F_2 = v_2 - y$; then, we can set $F_3 = F$. The nomogram is obtained by drawing the curves (called *isopleths*) on plane x - y corresponding to constant F_3 . See for example the Pouchet nomogram in Fig. 1a.

The first known example of such a graph (also called *intersection nomograms*) was presented by French manufacturer Louis-Ézéchiel Pouchet (1748–1809), who generalized the *discrete* multiplication tables (known since Pythagoras) to *continuous curves*, namely hyperbolas; this way, multiplying between non-integer numbers is also possible through interpolation (see Fig. 1a). Pouchet’s work is also historically significant as he was the first to introduce the expression “graphical calculus” to describe his approach [29].

An issue of the method above was that it required graphing complex curves on paper, which was a time-consuming process to do by hand. A further improvement was offered by French engineer Léon-Louis Lalanne (1811–1892), who suggested to use nonlinear scales for graphs. Consider again the multiplication graph introduced by Pouchet: the isopleths in this case are given by $xy = C$, with C being a constant value. If the x and y values are reported on logarithmic axes, however, the graph can be simplified: indeed, taking the logarithm of both sides, one has $\log(x) + \log(y) = \log(C)$, meaning that on a log-log plot the isopleths become straight lines. While drawing nonlinear scales is somewhat more complex, this is more than compensated by the advantage of drawing straight lines instead of curves, as the former can be defined by only two points and are thus much easier to plot. See Fig. 1b: here, the graph in Fig. 1a has been redrawn using a nonlinear scale for V . Lalanne described his concept as *anamorphosis*, in reference to a painting technique using exaggerated projective distortions. The use of nonlinear scales later became an essential tool in devising simple nomograms for complex equations.

The final element in this development was presented by d’Ocagne in several works, which in 1899 culminated in the first book on nomography [5]. There, the author put to use the tools of projective geometry, a branch of geometry that had recently been introduced by Poncelet and others, and in particular of the principle of *duality* between points and lines. Through this concept, each straight line in Lalanne’s anamorphic graph was represented by a point; the condition of having three curves through a point then corresponds to having three points aligned on the same line. This also simplifies the procedure to read the nomogram, which is much less cluttered: one only needs to draw a straight line through two points, corresponding to the two input variables, to find the third one, which corresponds to the output. The other idea advanced by d’Ocagne was to substitute the (orthogonal) coordinate axes with three curves C_i ($i = 1, 2, 3$) in the plane, thus introducing *alignment nomograms*. Each variable v_i then defines a point on the corresponding curve, which is graded along its length; the scale may be linear or nonlinear (Fig. 1c and 1d), depending on which approach makes the resulting nomogram simpler to draw and to use. If each curve is parametrically defined by $C_i = (f_{ix}(v_i), f_{iy}(v_i))$ in the Cartesian plane, the condition for alignment of three points is then given by

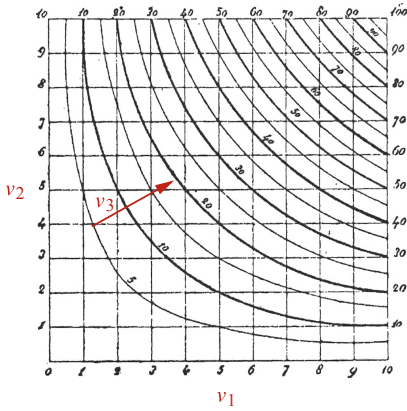
$$\begin{vmatrix} f_{1x}(v_1) & f_{1y}(v_1) & 1 \\ f_{2x}(v_2) & f_{2y}(v_2) & 1 \\ f_{3x}(v_3) & f_{3y}(v_3) & 1 \end{vmatrix} = 0 \tag{3}$$

If a function $F(v_1, v_2, v_3)$ can be rewritten in the form from Eq. (3), then a nomogram can be developed for solving Eq. 1. A further advantage of alignment nomograms is that they allow to easily solve equations in more than three variables; for instance, to solve $F(v_1, v_2, v_3, v_4, v_5)$ one may first derive v_3 from v_1 and v_2 and then obtain v_5 from v_3 thus obtained and v_4 ; the two nomograms (for the two steps of the solution) can then be combined, by having a curve C_3 that is common to both. While writing an equation in this form is a complex problem in general, the resulting nomogram is significantly easier to use and understand than an intersection nomogram. Another aspect to observe is that the nomogram for a given equation is not unique: with a different choice of scaling (for instance, logarithmic instead of linear) the scales are defined by different curves. Creating easy-to-use nomograms thus required skill and experience, a limitation that has now been largely superseded by software [4]. One can also modify a nomogram by multiplying the matrix in Eq. (3) by another (constant) matrix and then taking the determinant: geometrically, this is equivalent to applying a linear transformation in the plane (such as scaling, rototranslations, stretching and shearing), which results in a valid nomogram that still solves the original equation, but may be easier to read.

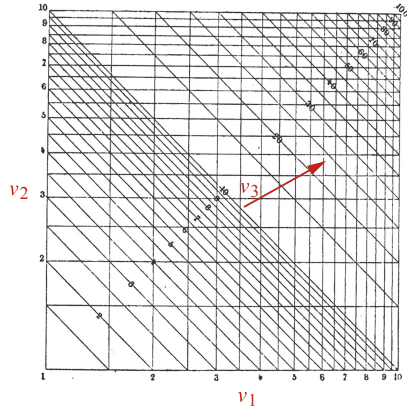
D’Ocagne greatly helped to popularize his invention through a number of publications, in which he presented example nomograms for several applications, such as physics, hydraulics, topography, navigation, aviation and accounting [30]. By the 1920s, nomograms had become the main research topic in graphic computation [29, p. 142, Fig. 3]; moreover, they were of interest also in terms of their mathematical analysis. For example, German mathematician David Hilbert, in his famed list of 23 unsolved problems presented in 1901 [20], posed the 13th one in terms of nomographic analysis, by asking whether a 7th-degree polynomial equation (which cannot be solved in closed form through algebraic functions) can be solved instead through nomograms. A solution was found by d’Ocagne himself, which however relied on a movable element in addition to a nomogram and was thus considered as unsatisfactory.

A practical problem in nomography was to determine whether a given equation of three variables can in fact be solved through nomograms, and if so, to write it in determinant form as in Eq. (3): this problem was finally solved in a practical form in 1959 by Polish mathematician Mieczyslaw Warmus [32], who also presented a useful classification of the functions that can be written in nomographic form.

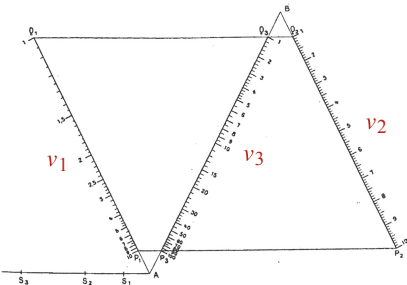
By the ’50s, research in nomography, at first mostly published in French [29, p. 141, Fig. 2] after the pioneering works by Pouchet, Lalanne and d’Ocagne, had become of interest at the global level; we observe, for example, a significant amount of works by authors from the former Soviet Union [13], possibly due to delayed access to computers.



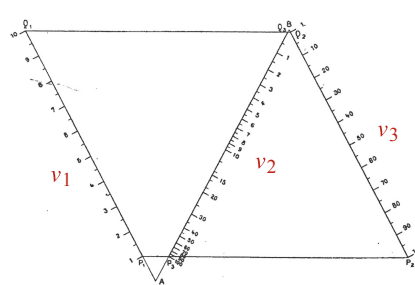
(a) Intersection nomogram.



(b) Intersection nomogram with nonlinear scales.



(c) Alignment nomogram, first version.



(d) Alignment nomogram, second version.

Fig. 1. For the same task (calculating the product of two variables, that is, $F = v_3 - v_2v_1 = 0$) different charts can be provided, with the alignment nomogram offering an elegant solution. Notice that this can be drawn in different ways; the one in Fig. 1d has two linear scales (which are easier to draw). Charts reproduced from [5], with some changes for clarity.

2.2 Decline: 1960–1990

The research on nomography began to decline in the 1960s [29, p. 140, Fig. 1], as computers became commonly used in academia and industry. Nevertheless, nomograms were still used by engineers; published papers frequently developed calculations in nomographic form for practical applications. Topics that saw a significant use of nomograms until very recently were the analysis of internal combustion engines, manufacturing technology, design of hydraulic systems and civil engineering (especially for the study of soil mechanical properties): in all these cases, one needs to work with complex relationships that are often approximated from numerical data, thus the reduction in accuracy is not an issue. On the other hand, the possibility of obtaining a quick result that can be used in the field makes nomograms appealing for these applications.

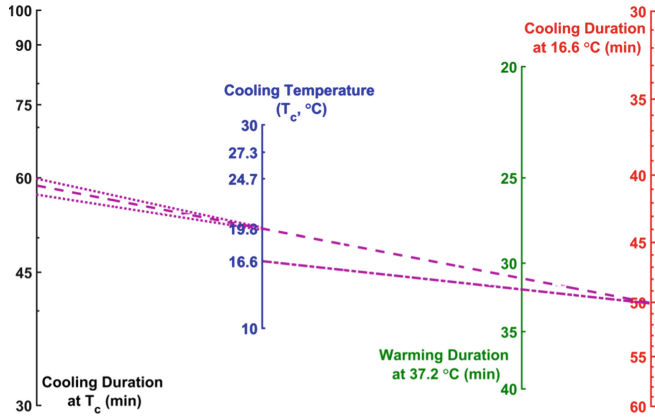


Fig. 2. Nomogram for thermal therapy of tissue injuries. Reprinted with permission from [24].

Regarding applications in Machine Mechanics, we observe that until the '70s nomographic solutions for mechanism design problems were appreciated for their practicality [1]: indeed, mechanism design generally leads to complex equations in which it is convenient to explore different design solutions, by designating different parameters as outputs. In particular, [1] showed a nomogram to optimize a four-bar linkage for function generation and observed how, after some practice, one can arrive in a relatively short time at a suitable solution with acceptably small errors. As it was common, the nomogram was also offered in large format on graph paper [1] for practical use.

Other nomograms were developed during this period specifically for mechanism design; for instance, planar, four-bar mechanisms were studied in [3,34], while [26] considered their spherical equivalents. A planar mechanism with higher (contact) pairs, namely a cam-follower linkage, was studied in [8] with the goal of preventing undercut. We also cite [7], on the vibration analysis of a spatial (Hooke's) joint.

Nomograms proved useful also in the design of gears [28,33], to present large amounts of experimental data in a form that is more convenient for designers (with respect to numerical tables). Some applications were also proposed for vibration analysis [21,27].

2.3 Resurgence? 1990–Today

After the '90s, nomograms have largely disappeared from engineering research, having been replaced by code listings that can be easily distributed. Nevertheless, nomograms are still present in published literature: for instance, they are still used in design standards and manufacturers' catalogs, to present results in a compact way. A common application is the Smith chart [16], used in electrical engineering for the design of transmission lines.

Outside of engineering, it should be noted that nomograms have had a significant impact on medicine [17, 23, 24]; see also Fig. 2. For instance, they are often used to quickly compute the BMI of a patient [16]. Note that the application of nomograms in medicine is somewhat controversial [17]; however, even in a 2008 review that was highly critical of their usage [17], the author noted that they were undergoing a “resurgence”, with more than 150 citations in the year 2007 alone. Nomograms, indeed, are observed to have a didactic value, which can be helpful in explaining a complex situation to a patient; this is especially true in comparison to PCs, which may hinder transparency. Nomograms are also low-cost, easily protected by environmental damage (by using waterproof paper) and allow users to compute results in an emergency situation. These advantages can also be useful for engineers in some industrial applications.

Finally, nomograms have a place where speed is essential: for example, they are used to predict the behavior of forest fires or the lift of hot-air balloons [16]. Artillery was another relevant application, in which nomograms were in use up to World War I [18].

Considering in particular the field of Machine Mechanics over the last 30 years, we found several works on nomograms for the design of gear trains [9–12]; they are also still used in some cases for mechanism design, for planar [19] or spherical [22, 25] linkages, for the optimization of motion laws in cams [15], for the analysis of efficiency in kinematic chain [2] or for studying tire dynamics in a vehicle [35].

3 Nomograms: Methods and Applications

In this Section, we present a very brief description of how nomograms can be produced, using modern tools, for a few example problems in Machine Mechanics. While a complete guide on drawing nomograms [6] is beyond the scope of this work, we believe some examples of nomograms can effectively show their advantages and limitations.

3.1 Modern Nomographic Tools: PyNomo

Traditionally, a notable limit of nomograms is the fact that they require skill in their design, not to mention the time needed to manually draw them in high resolution. Recently, however, some software tools and packages have been presented that allow us to automate nomogram design, which can help reintroduce nomography in practical usage. Indeed, as observed in [23], it is desirable to have a tool that “*combines computerization with classical nomograms*”. At the time of writing, the most complete tool for this task (that we are aware of) is pyNomo [4], a library for the Python programming language written to automate nomogram design. We therefore have created a few nomograms with pyNomo to illustrate some basic concepts in Machine Mechanics. The output of a pyNomo script is a PDF file containing the nomogram in high-resolution vector graphics; this is achieved by automatic compilation of a L^AT_EX program generated in Python.

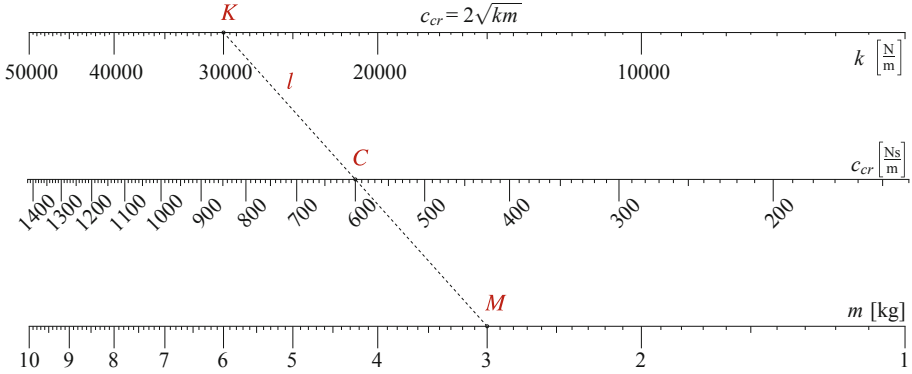


Fig. 3. Nomogram for calculating the critical damping coefficient c_{cr} of a 1-DoF damped harmonic oscillator; points K , C and M are defined by the values k , c and m of the stiffness and damping coefficients and of the system mass, respectively (each on the corresponding scale). For a critically-damped system, points K , C and M are aligned on a line l .

In the following, we refer to the classification in the pyNomo documentation, which identifies 10 types of nomograms. Traditionally, the most typical nomogram is the one defined by three parallel¹ straight lines (Type 1 in pyNomo), as shown in Fig. 3. The most general alignment nomogram determinant of Eq. (3) becomes simpler: if the j -th scale (along the line $x = x_j$) starts at $y = y_{jS}$ and ends at $y = y_{jE}$ ($j = 1, 2, 3$), one has

$$\begin{aligned}
 f_{jx}(v_j) &= x_j \quad (\text{constant}) \\
 f_{jy}(v_1) &= y_{jS} + \Delta y_j g_j(v_j)
 \end{aligned}
 \quad \text{with} \quad
 \begin{cases}
 g_j(v_{jS}) = 0 \\
 g_j(v_{jE}) = 1
 \end{cases}
 \quad \text{and} \quad
 \Delta y_j = y_{jE} - y_{jS} \quad (4)$$

where $v_j \in [v_{jS}, v_{jE}]$ is a variable corresponding to a point on the scale and g_j is a general monotonic continuous function; for linear scales, for example, it holds $g_j(v_j) = \frac{v_j - v_{jS}}{v_{jE} - v_{jS}}$. Substituting in the determinant from Eq. (3) and expanding, one has

$$(x_3 - x_2) \Delta y_1 g_1(v_1) + (x_1 - x_3) \Delta y_2 g_2(v_2) + (x_2 - x_1) \Delta y_3 g_3(v_3) + C_0 = 0 \quad (5)$$

where C_0 is a constant (once the nomogram has been defined): thus, any weighted sum of functions g_j (with weights $(x_i - x_k) \Delta y_j$ that can be set by a proper choice of the start and end points) can be achieved with a nomogram such as the one shown in Fig. 3.

A product of functions can also be computed by this type of nomogram: indeed, the equation $v_3 = v_1 v_2$ (for which Eq. (1) can be written as $F(v_1, v_2, v_3) =$

¹ In most nomograms of this type, the lines are vertical, for clarity. Here the lines are horizontal, due to lack of space; the y axis in the drawing plane is parallel to the scales.

$v_3 - v_1v_2 = 0$) becomes $\log(v_3) = \log(v_1) + \log(v_2)$ by taking the logarithm of both sides. We are thus again in the case defined by Eq. (5), with all three g_j 's being logarithmic functions.

In Fig. 3 we show a nomogram to compute the *critical damping coefficient* c_{cr} of a one-Degree-of-Freedom (1-DoF) system composed of a mass m (translating along a line) connected to a fixed frame through a linear spring of stiffness k and a viscous damper of coefficient c , acting in parallel; this is a classical topic in Vibration of Machinery which is commonly presented in undergraduate courses. From the definition of c_{cr} , one finds

$$c_{cr} = 2\sqrt{km} \Rightarrow \log(c_{cr}) = \log(2) + 1/2 \log(k) + 1/2 \log(m) \tag{6}$$

Multiplying by 1/2 the logarithms of the input values and adding $\log(2)$ to the result is “embedded” in the way the scales have been drawn (which is possible since 1/2 and $\log(2)$ are constant values) and does not complicate the nomogram. An isopleth has been added to Fig. 3 to show how a computation is performed: for $k = 30000 \frac{N}{m}$ and $m = 3$ kg, one has $c_{cr} = 600 \frac{Ns}{m}$, which is found by drawing a straight line l over the points K and M in Fig. 3 and finding the intersection C of l with the scale for c_{cr} .

A given equation can be represented in nomographic form in different ways. For example, another possible representation of an equation containing a product of functions $v_3 = v_1v_2$ can be achieved with a “N” (also called “Z”) type nomogram (Type 2 in pyNomo), corresponding to Figs. 1c and 1d; this nomogram is a graphical embedding of a multiplication between functions, but (unlike Fig. 3) requires using only one logarithmic scale. Consider three scales for the variables v_i , each starting at point $P_{jS} = (x_{jS}, y_{jS})$, corresponding to $g_j(v_j) = 0$. The direction of each scale is defined by vector $(\Delta x_j, \Delta y_j)$, which we assume to have unit magnitude, for simplicity. Equation (3) then becomes

$$\begin{vmatrix} x_{1S} + \Delta x_1 g_1(v_1) & y_{1S} + \Delta y_1 g_1(v_1) & 1 \\ x_{2S} + \Delta x_2 g_2(v_2) & y_{2S} + \Delta y_2 g_2(v_2) & 1 \\ x_{3S} + \Delta x_3 g_3(v_3) & y_{3S} + \Delta y_3 g_3(v_3) & 1 \end{vmatrix} = 0 \tag{7}$$

If the scale for variable v_2 passes through P_{1S} and P_{3S} , and if the scales for v_1 and v_3 are parallel, it can be shown that linear scales can be set for v_1 and v_3 , such that $g_1(v_1)$ and $g_3(v_3)$ are linear functions and that it holds $v_2 = v_3/v_1$ (and thus again $v_3 = v_1v_2$). Since the only nonlinear scale is the one for v_2 , the resulting nomogram is simpler to draw and, in some cases, more compact than the equivalent Type 1 nomogram for the same equation (obtained by using logarithmic scales, as explained above).

As another example application, we present a “concurrent scale” (or “angle”) nomogram in Fig. 4. This kind of nomogram (Type 7 in pyNomo) is defined by having three scales passing through a common point, which for simplicity is taken to be also the origin $P_{1S} = P_{2S} = P_{3S}$ of all three scales; the origin O of the coordinate plane is also set in this point. Defining the angles α_j formed by the j -th scale with the x -axis, one has

$$\frac{\sin(\alpha_3 - \alpha_2)}{g_1(v_1)} + \frac{\sin(\alpha_1 - \alpha_3)}{g_2(v_2)} + \frac{\sin(\alpha_2 - \alpha_1)}{g_3(v_3)} = 0 \tag{8}$$

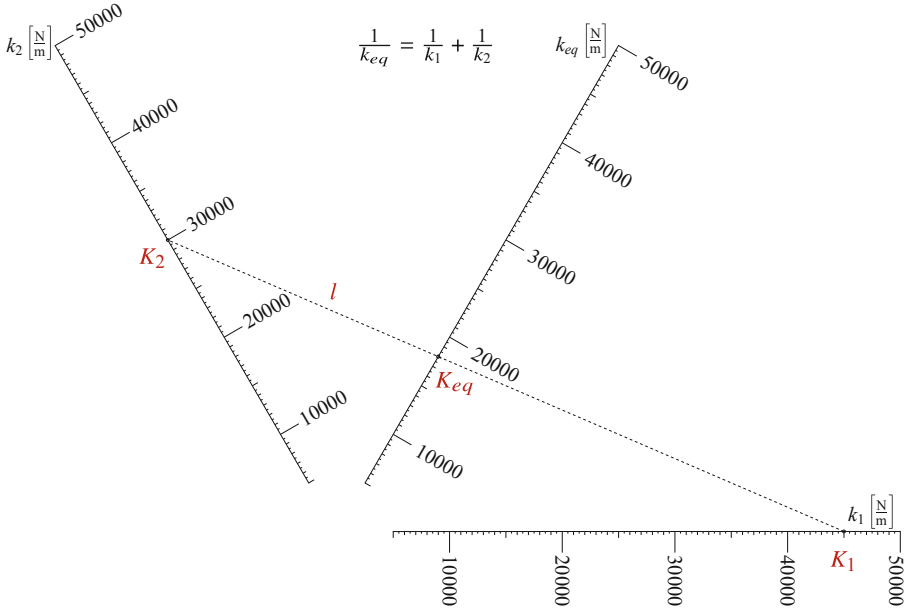


Fig. 4. Nomogram for the stiffness k_{eq} of a spring, equivalent to two springs in series having stiffnesses k_1 and k_2 , respectively. In this case, all three scales have the same units of measurement.

A possible application for this nomogram that is of interest for Machine Mechanics is computing the combined stiffness of a system of two linear springs (having stiffnesses k_1 and k_2 , respectively) attached in series. It can be shown that the resulting system is statically equivalent to a single spring of stiffness k_{eq} , having

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} \tag{9}$$

which can be directly implemented in a Type 7 nomogram. See Fig. 4, where the isopleth l has been drawn for an example problem with $k_1 = 45000 \frac{\text{N}}{\text{m}}$ and $k_2 = 30000 \frac{\text{N}}{\text{m}}$ (corresponding to points K_1 and K_2 , respectively); the stiffness $k_{eq} = 18000 \frac{\text{N}}{\text{m}}$ is found at K_{eq} . Notice that unrealistic cases having $k_j = 0$ are automatically excluded by the graphic procedure, making it impossible to obtain incorrect results from a mistake in the input. The angles α_j can be chosen according to convenience; having $\alpha_3 - \alpha_2 = \alpha_2 - \alpha_1 = 60^\circ$ leads to the same unit length being used for all three scales, as in Fig. 4.

3.2 Applications

To illustrate concrete applications of nomography in machines' and mechanisms' analysis, we present an example design of a more complex nomogram, devised to solve a practical problem in the kinematic analysis of a spatial mechanism. This

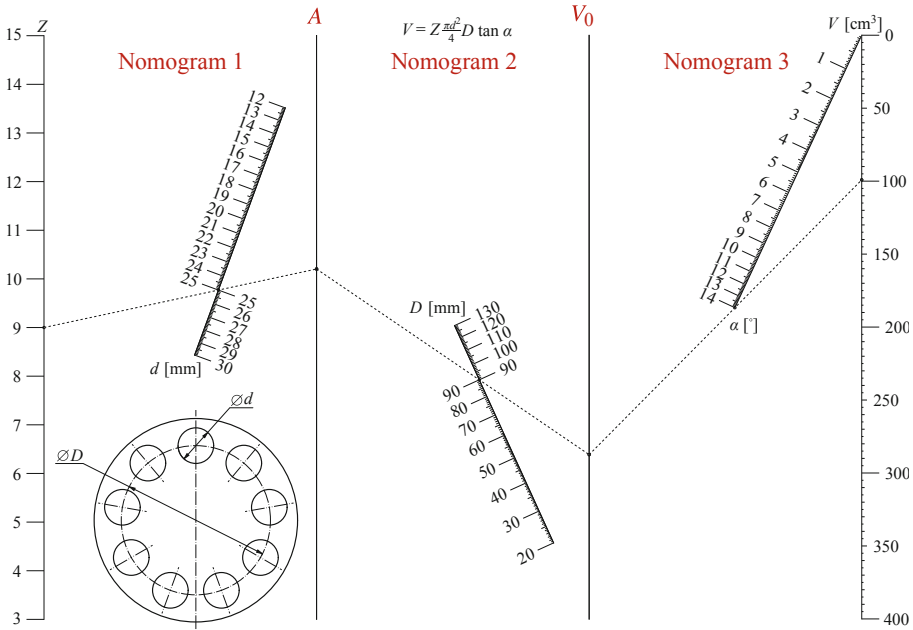


Fig. 5. Combined nomogram (composed of three “intermediate” Type 2 nomograms, denoted as 1, 2 and 3) for calculating the variable displacement of a volumetric axial piston pump.

design has been inspired by a collaboration with a company working on variable-displacement axial piston pumps (with a swashplate to control the displacement). One minor need reported by the company was to compute the pump displacement from measurements on its main parameters: this is useful when working on a shop floor, as the pump data (such as the serial number and product specification) is not directly available. The displacement V can then be computed from easily measured quantities, as

$$V = \frac{\pi d^2}{4} Z D \tan(\alpha) \tag{10}$$

where d is the diameter of the pistons, Z their total number, D the diameter of the cylinder through which the piston axes pass and α the swashplate inclination.

Clearly, Eq. 10 can be implemented in any computation software, for instance as a spreadsheet. However, operating a digital device when disassembling a pump (which is generally filled with lubricant) for maintenance operations is clearly inconvenient. Moreover, there is a significant risk of operator errors due to incorrect data entry.

Nomograms provide an interesting possibility for this task. However, Eq. (10) contains five variables; thus, it cannot be directly expressed as a nomogram of one of the types presented previously. We then divide the corresponding nomogram in parts, as follows.

1. We create a nomogram to compute $A = \frac{\pi d^2}{4} Z$; this is the total cross-sectional area of the pistons. This is achieved through a simple Type 2 nomogram with three variables, as described in Subsect. 3.1, where $v_1 = d^2$, $v_2 = Z$ and $v_3 = A \frac{4}{\pi}$.
2. The result from the previous nomogram is used for a second step, by joining two nomograms at the scale for A ; the corresponding expression is $V_0 = DA$. Again, being a product of two variables, this result can be computed with a Type 2 nomogram.
3. Finally, a third nomogram is introduced, joined at the previous one at the scale for V_0 . This computes the final result $V = V_0 (\tan(\alpha))$.

The final result is shown in Fig. 5; here, the two vertical scales in the middle correspond to variables A and V_0 , respectively. The values of these intermediate quantities are not required in the final result; thus, no ticks are displayed, to avoid cluttering the graph. The isopleth on the graph shows an example calculation for a realistic pump design; on one corner of the nomogram, a sectional view of the central cylinder block of the pump is shown for reference, to clarify the definition of diameters D and d .

This nomogram has been printed on laminated paper, to protect it from oil spills, and introduced in shop-floor practice for quick computations. The isopleths for each step can be found with any rigid element having a straight-line segment of sufficient length.

We also suggest an application in education. As noted in [6, 23, 30], nomograms allow us to easily understand the behavior of equations with complex algebraic expressions; also, nomography is a graphical approach that makes it unlikely to misinterpret inputs or results. Besides nomograms, other graphical methods, such as those used for planar mechanism kinematics and statics, are currently taught in mechanical engineering courses. While these methods could be replaced by purely analytical approaches, they are still used due to their pedagogical value. We thus expect nomograms to easily fit in existing courses; in particular, nomography extends graphical approaches to any equation that can be written in determinant form as Eq. (3). An experience on nomograms for math education in high schools was reported in [31]; a test on their usefulness for university-level education would then be worthwhile to understand their strengths and limitations. We also believe that mentioning examples of nomograms, together with their uses and applications, within a course in Machine Mechanics, could also enhance students' interest in historical methods for engineering analysis and in the broader historical development of our field.

4 Conclusion and Outlook

We have presented a brief history of nomograms, a graphical method once commonly used to solve equations. We focused on their applications in mechanical engineering, with the goal of performing a didactic experiment for a course

in Machine Mechanics, to appreciate their pedagogical potential. We also presented some original nomograms developed with pyNomo, a nomographic software package. We expect that computers, which once made nomograms obsolete, can indeed help in reviving interest in these tools.

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