Dynamic Response of Plates Under Moving Mass

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Abstract. The influence of loads moving on plates have been the prime objectives for structural experts in recent times. The immense application of this kind of loading in many industrial areas has enhanced the prominence of assessing the dynamic response of resonant structures under moving loads. Therefore, this paper focuses on the dynamic response of plates under moving mass using the finite element method (FEM). The deflections, velocities, and accelerations for each time step are computed using the Newmark integration method to arrive at the solution. An MATLAB code based on FEM is established to proffer a productive solution for this. The central deflection results of plates due to moving mass have been validated with previously published works. Deflections at any point of rectangular and curved plates due to a moving mass with constant velocity are explored.

Keywords: Rectangular plate · Curved plate · Dynamic response · Finite element method

1 Introduction

Dynamic analysis is one of the most required analyses among the modern engineering design such as Highway bridges, aerospace, ship structures, etc., since it is consistently subjected to dynamic loading. Among the dynamic analysis lots of work has been studied about the free vibration analysis and the dynamic response analysis under moving loads which are presented below. (Barik and Mukhopadhyay 1999; Barik and Mukhopadhyay 2002; Panda and Barik 2019; Mishra and Barik 2021) have analyzed the frequencies of various shaped stiffened plates using FEM. The free vibration of rectangular and curved stiffened plates have been explained (Sahoo and Barik 2020a, b; Sahoo and Barik 2021) under the influence of various parameters using FEM. The deflections of stiffened plate subjected to moving loads has been explained (Sahoo and Barik 2020a) using FEM. Also the dynamic deflection of bare plates has been presented (Taheri and Ting 1989; Taheri and Ting 1990) using structural impedance method and FEM respectively. Hence, the main focus of this paper is to investigate the dynamic response of rectangular and curved plates under moving mass.

Raske (1983) has predicted the deflections of a rectangular plate imposed with mass moving in a circular path using the Fourier series for simply-supported boundary condition. (Cifuentes and Lalapet 1992) have investigated the dynamic deflection of a plate

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under circularly moving mass employing FEM and the Lagrange multiplier formulation. This analysis has been found to accommodate any irregular shape and boundary conditions. (Shadnam et al. 2001) have performed the dynamic deflection of plates imposed with travelling masses along an arbitrary trajectory, applying a numerical-analytical method. (Gbadeyan and Oni 1995) have presented the dynamic deflections of a plate under the impact of moving mass using the finite difference method.

Dynamic deflection of a plate under with moving mass has been proposed by (Wu 2007) making use of FEM. The deflections of a circular plate under the impact of mass travelling in a circular direction have been suggested by (Ouyang 2011) offering an analytical solution. (Amiri et al. 2013) have analyzed a plate for dynamic deflection adopting the FSDT and the classical plate theory for various thickness values. The dynamic amplification factor (DAF) of a rectangular plate imposed with multiple masses travelling along rectilinear paths in opposite directions has been determined by (Nikkhoo et al. 2014).

(Esen 2013; Esen 2015) has explored the deflections of rectangular plates under the impact of moving mass, applying FEM. The dynamic deflections and the DAFs have been evaluated for cantilever and clamped boundary conditions. An analytical approach has been portrayed (Ghazvini et al. 2016) to analyse the deflections of a plate subjected to travelling mass for different plate thickness values. Demonstration of the deflections of a Kirchhoff plate imposed with massless and mass load travelling with acceleration in an arbitrary path with opposite directions have been made by (Dyniewicz et al. 2017) employing FEM.

Dynamic deflections of plate imposed with moving mass have been explained (Song et al. 2017) using a hybrid approach for different plate boundary conditions using an extended Rayleigh-Ritz technique. (Rad et al. 2020) have discussed the dynamic response of rectangular plate subjected to moving mass for arbitrary boundary conditions using Boundary Characteristic Orthogonal Polynomials (BCOP) technique.

The above literature has limited their study of dynamic response under moving masses for the rectangular plates only. Thus, this paper attempts to determine the dynamic response under moving mass of a rectangular and curved plate using the finite element method. An MATLAB code based on FEM is established to proffer a productive solution for the analyses of rectangular and annular sector plates by introducing an isoparametric quadratic plate bending element that can fit in with curved boundaries. The formulation of the plate and stiffener elements are dealt with separately. The shear deformation is accounted for in the formulation. The Newmark integration method is employed for each time step deflection. To show the method's effectiveness, the results are validated with the previously published results wherever possible. The deflection has been investigated for the plate subjected to a moving mass with constant velocity. Also, the deflections have been explored at various points of rectangular and curved plates.

2 Formulation

2.1 Plate Element Formulation

The transverse displacement and rotations along x- and y-directions at a node 'r' of a quadratic isoparametric plate bending element (Fig. 1) are w_r, θ_{xr} and θ_{yr} respectively. At any point within the element, we have

$$\begin{Bmatrix} w \\ \theta_x \\ \theta_y \end{Bmatrix} = \sum_{r=1}^{8} \begin{bmatrix} N_r & 0 & 0 \\ 0 & N_r & 0 \\ 0 & 0 & N_r \end{bmatrix} \begin{Bmatrix} w_r \\ \theta_{xr} \\ \theta_{yr} \end{Bmatrix} = \sum_{r=1}^{8} \begin{Bmatrix} \frac{1}{4}(1-\xi)(1-\eta)(-\xi-\eta-1) \\ \frac{1}{2}\left(1-\xi^2\right)(1-\eta) \\ \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1) \\ \frac{1}{2}(1-\xi)(1-\eta^2) \\ \frac{1}{2}(1+\xi)(1-\eta^2) \\ \frac{1}{4}(1-\xi)(1+\eta)(-\xi+\eta-1) \\ \frac{1}{2}\left(1-\xi^2\right)(1+\eta) \\ \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1) \end{Bmatrix} [I_3] \begin{Bmatrix} w_r \\ \theta_{xr} \\ \theta_{yr} \end{Bmatrix} \tag{1}$$

where N_r is the shape function for the node r expressed in non-dimensional coordinates.

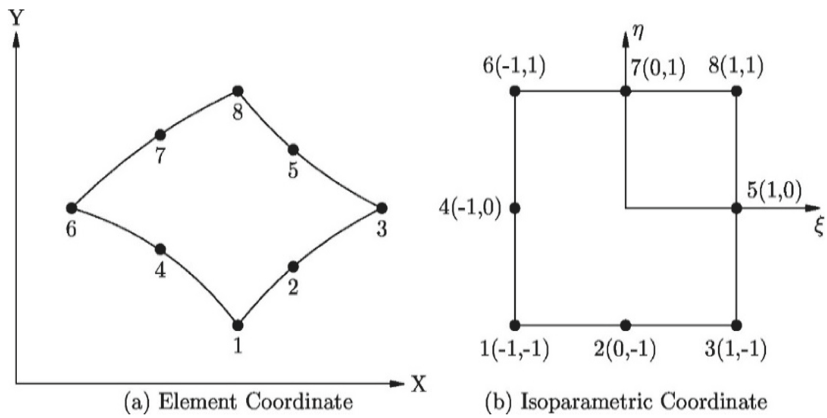

Fig. 1. Plate Element.

The plate's displacement field is expressed as

$$\{f\} = \begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = \begin{Bmatrix} -z\theta_x \\ -z\theta_y \\ w \end{Bmatrix} \tag{2}$$

Due to shear deformation, some warping occurs, which is illustrated in Fig. 2 and the rotations are now expressed as

$$\begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w}{\partial x} + \varphi_x \\ \frac{\partial w}{\partial y} + \varphi_y \end{Bmatrix} \tag{3}$$

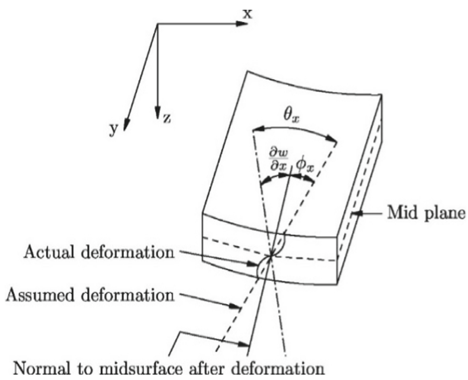

Fig. 2. Deformation of plate cross-section.

where θ_x and θ_y are the average rotations and φ_x and φ_y are the average shear deformations.

The strain components of the plate can be expressed with the help of Eqs. (2) and (3)

$$\{\varepsilon\} = \begin{Bmatrix} -z\frac{\partial\theta_x}{\partial x} \\ -z\frac{\partial\theta_y}{\partial y} \\ -z\left(\frac{\partial\theta_x}{\partial y} + \frac{\partial\theta_y}{\partial x}\right) \\ \left(\frac{\partial w}{\partial x} - \theta_x\right) \\ \left(\frac{\partial w}{\partial y} - \theta_y\right) \end{Bmatrix} = \begin{bmatrix} z & 0 & 0 & 0 & 0 \\ 0 & z & 0 & 0 & 0 \\ 0 & 0 & z & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} -\frac{\partial\theta_x}{\partial x} \\ -\frac{\partial\theta_y}{\partial y} \\ -\left(\frac{\partial\theta_x}{\partial y} + \frac{\partial\theta_y}{\partial x}\right) \\ \left(\frac{\partial w}{\partial x} - \theta_x\right) \\ \left(\frac{\partial w}{\partial y} - \theta_y\right) \end{Bmatrix} = [H]\{\overline{\varepsilon}\} \tag{4}$$

Combining Eqs. (1) and (4)

$$\{\overline{\varepsilon}\} = \sum_{r=1}^{8} \begin{bmatrix} 0 & -\frac{\partial N_r}{\partial x} & 0 \\ 0 & 0 & -\frac{\partial N_r}{\partial y} \\ 0 & -\frac{\partial N_r}{\partial y} & -\frac{\partial N_r}{\partial x} \\ \frac{\partial N_r}{\partial x} & -N_r & 0 \\ \frac{\partial N_r}{\partial y} & 0 & -N_r \end{bmatrix} \begin{Bmatrix} w_r \\ \theta_{xr} \\ \theta_{yr} \end{Bmatrix} = \sum_{r=1}^{8} [B]_r\{\delta\}_r = [B]\{\delta\} \tag{5}$$

$\frac{\partial N_r}{\partial \xi}$ and $\frac{\partial N_r}{\partial \eta}$ are related to $\frac{\partial N_r}{\partial x}$ and $\frac{\partial N_r}{\partial y}$ by

$$\begin{Bmatrix} \frac{\partial N_r}{\partial \xi} \\ \frac{\partial N_r}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \frac{\partial N_r}{\partial x} \\ \frac{\partial N_r}{\partial y} \end{Bmatrix} = [J] \begin{Bmatrix} \frac{\partial N_r}{\partial x} \\ \frac{\partial N_r}{\partial y} \end{Bmatrix} \tag{6}$$

The stress-strain relationship is defined by

$$\{\sigma\} = [D]\{\varepsilon\} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0 & 0 & 0 \\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 & 0 & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} & 0 & 0 \\ 0 & 0 & 0 & \frac{E}{2(1+\nu)} & 0 \\ 0 & 0 & 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \{\varepsilon\} \tag{7}$$

$$\{\sigma\}^T = \left\{ \sigma_x \ \sigma_y \ \ \tau_{xy} \ \tau_{zx} \ \tau_{yz} \right\} \tag{8}$$

The element stiffness matrix is

$$[k]_e = \iiint [B]^T [H]^T [D][H][B] dx\, dy\, dz \tag{9}$$

$$[k]_e = \iint [B]^T \left[\overline{D}\right][B] dx\, dy = \int_{-1}^{1}\int_{-1}^{1} [B]^T \left[\overline{D}\right][B]|J| d\xi\, d\eta \tag{10}$$

where

$$\left[\overline{D}\right] = \int_{\frac{-t}{2}}^{\frac{t}{2}} [H]^T [D][H]\, dz = \begin{bmatrix} D_{XF} & D_{IF} & 0 & 0 & 0 \\ D_{IF} & D_{YF} & 0 & 0 & 0 \\ 0 & 0 & D_{XYF} & 0 & 0 \\ 0 & 0 & 0 & S_X & 0 \\ 0 & 0 & 0 & 0 & S_Y \end{bmatrix} \tag{11}$$

$D_{XF} = D_{YF} = \frac{Eh^3}{12(1-\nu^2)}, \quad D_{IF} = \nu D_{XF}$
$S_X = S_Y = \frac{Eh}{2.4(1+\nu)}, \quad D_{XYF} = \frac{1-\nu}{2} D_{XF}$
h = Plate thickness, E = Young's Modulus of plate
ν = Poisson's ratio of plate, ρ = Mass density of plate
At any point within the element

$$\begin{Bmatrix} \ddot{w} \\ \ddot{\theta}_x \\ \ddot{\theta}_y \end{Bmatrix} = \sum_{r=1}^{8} N_r [I_3] \begin{Bmatrix} \ddot{w}_r \\ \ddot{\theta}_{xr} \\ \ddot{\theta}_{yr} \end{Bmatrix} \tag{12}$$

The plate's acceleration field is given by

$$\{f\} = \begin{Bmatrix} \ddot{U} \\ \ddot{V} \\ \ddot{W} \end{Bmatrix} = \begin{Bmatrix} -z\ddot{\theta}_x \\ -z\ddot{\theta}_y \\ \ddot{w} \end{Bmatrix} \tag{13}$$

The acceleration field may be exhibited by combining Eqs. (12) and (13),

$$\{f\} = \begin{bmatrix} 0 & -z & 0 \\ 0 & 0 & -z \\ 1 & 0 & 0 \end{bmatrix} \sum_{r=1}^{8} N_r [I_3] \begin{Bmatrix} \ddot{w}_r \\ \ddot{\theta}_{xr} \\ \ddot{\theta}_{yr} \end{Bmatrix} = [G][N] \sum_{r=1}^{8} \begin{Bmatrix} \ddot{w}_r \\ \ddot{\theta}_{xr} \\ \ddot{\theta}_{yr} \end{Bmatrix} \tag{14}$$

Applying D'Alembert's Principle, the inertia force associated with the acceleration can be expressed as

$$[F_e]_I = \begin{bmatrix} \rho & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \rho \end{bmatrix} \{f\} = [\rho_e]\{f\} \tag{15}$$

The element mass matrix is

$$[m]_e = \iiint [N]^T [G]^T [\rho_e][G][N] dx\, dy\, dz \tag{16}$$

$$[m]_e = \iint [N]^T \left[m_p\right][N] dx\, dy = \int_{-1}^{1}\int_{-1}^{1} [N]^T \left[m_p\right][N]|J| d\xi\, d\eta \tag{17}$$

where

$$\left[m_p\right] = \int_{\frac{-h}{2}}^{\frac{h}{2}} [G]^T [\rho_e][G] dz = \begin{bmatrix} \rho h & 0 & 0 \\ 0 & \frac{\rho h^3}{12} & 0 \\ 0 & 0 & \frac{\rho h^3}{12} \end{bmatrix} \tag{18}$$

2.2 Moving Mass

A load P_z having mass P_{zm} equal to P_z/g, is travelling with a velocity of v across the plate. The position of the mass, presented as a function of time t, is given by

$$x_1 = x_0 + v_x t_1 \tag{19}$$

$$y_1 = y_0 + v_y t_1 \tag{20}$$

where (x_0, y_0) is the initial coordinate at which the mass first enters the plate at $t = 0$, and v_x and v_y are the x and y components respectively of the constant velocity v of the mass along the plate. Due to the moving mass, the additional mass, damping, and stiffness element matrices which are position-dependent are given by

$$\{\overline{m}\}_e = P_{zm}\Big([N]^T [N]\Big)\Big|_{\substack{x = x_1 \\ y = y_1}} \tag{21}$$

$$\{\overline{c}\}_e = 2P_{zm}\big(v_x\left[N_{,x}\right] + v_y\left[N_{,y}\right]\big)\Big|_{\substack{x = x_1 \\ y = y_1}} \tag{22}$$

$$\left\{\overline{k}\right\}_e = P_{zm}[N]^T\Big(v_x^2\left[N_{,xx}\right] + 2v_x v_y\left[N_{,xy}\right] + v_y^2\left[N_{,yy}\right]\Big)\Big|_{\substack{x = x_1 \\ y = y_1}} \tag{23}$$

Hence, the new element mass, stiffness, and damping matrices become respectively

$$\{m\}_{new} = \{m\}_e + \{\overline{m}\}_e \tag{24}$$

$$\{k\}_{new} = \{k\}_e + \left\{\overline{k}\right\}_e \tag{25}$$

$$\{c\}_{new} = \{\overline{c}\}_e \tag{26}$$

2.3 Load Position

A plate of 6×6 mesh grid is shown in Fig. 3 with the path of the loads and their positions. The expression for the load position on the plate is $\frac{x}{a}$.

Where, $x =$ the position of the load

and $a =$ length of the plate (Fig. 3).

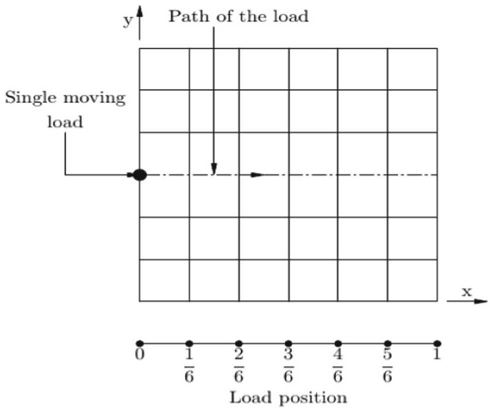

Fig. 3. Load position for moving load (mass).

3 Solution Procedure

The governing equation of motion is

$$[M]_{new}\{\ddot{u}\} + [C]_{new}\{\dot{u}\} + [K]_{new}\{u\} = \{F\} \tag{27}$$

3.1 Dynamic Response Due to Moving Force (Newmark Integration Method)

For the n-th time step at time $t_n = t_{n-1} + \Delta t$:

$$\overline{K} = K_{new} + a_0 M_{new} + a_1 C_{new} \tag{28}$$

$$\begin{aligned}\overline{F}_{t_n} = F_{t_n} + M_{new}\left[a_0 u_{t_{n-1}} + a_2 \dot{u}_{t_{n-1}} + a_3 \ddot{u}_{t_{n-1}}\right] \\ + C_{new}\left[a_1 u_{t_{n-1}} + a_6 \dot{u}_{t_{n-1}} + a_7 \ddot{u}_{t_{n-1}}\right]\end{aligned} \tag{29}$$

where $\ddot{u}_{t_{n-1}}$, $\dot{u}_{t_{n-1}}$ and $u_{t_{n-1}}$ are the initial acceleration, velocity and displacements of the structural system at time $t = t_n$. Displacements, velocities and accelerations at time t_n

$$u_{t_n} = \overline{K}^{-1}\overline{F}_{t_n} \tag{30}$$

$$\dot{u}_{t_n} = \dot{u}_{t_{n-1}} + a_4 \ddot{u}_{t_{n-1}} + a_5 \ddot{u}_{t_n} \tag{31}$$

$$\ddot{u}_{t_n} = a_0\left(u_{t_n} - u_{t_{n-1}}\right) - a_2\dot{u}_{t_{n-1}} - a_3\ddot{u}_{t_{n-1}} \tag{32}$$

where

$$\begin{array}{lll} a_0 = \frac{1}{\beta\Delta t^2} & a_1 = \frac{\gamma}{\beta\Delta t} & a_2 = \frac{1}{\beta\Delta t} \\ a_3 = \frac{1}{2\beta} - 1 & a_4 = \Delta t(1-\gamma) & a_5 = \gamma\Delta t \\ a_6 = \frac{\gamma}{\beta} - 1 & a_7 = \frac{\Delta t}{2}\left(\frac{\gamma}{\beta} - 2\right) & \end{array} \tag{33}$$

And the values of β and γ are 0.25 and 0.5.

4 Numerical Examples

4.1 Square Plate

Table 1. Material properties of a square plate

Properties	Value
E	206 GPa
ρ	7929 kg/m^3
ν	0.3
Mass	4.45 kN
Velocity	5.08 m/s
Δt	0.5×10^{-3} s

Table 2. Dimensions of a square plate

Dimensions	Value
a	1,524 m
b	1,524 m
h	0,00635 m

A square is imposed with a moving mass (Fig. 4) is analysed. The plate's material properties and dimensions are presented in Table 1 and 2 respectively. The convergence of the deflection result is explained for different mesh grids and illustrated in Fig. 5. Also, the dynamic deflections for the same time interval, are compared with the results of (Wilson and Tsirk 1967) in Fig. 6.

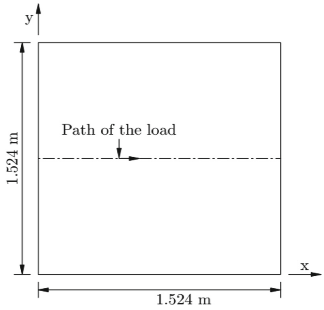

Fig. 4. Clamped square plate

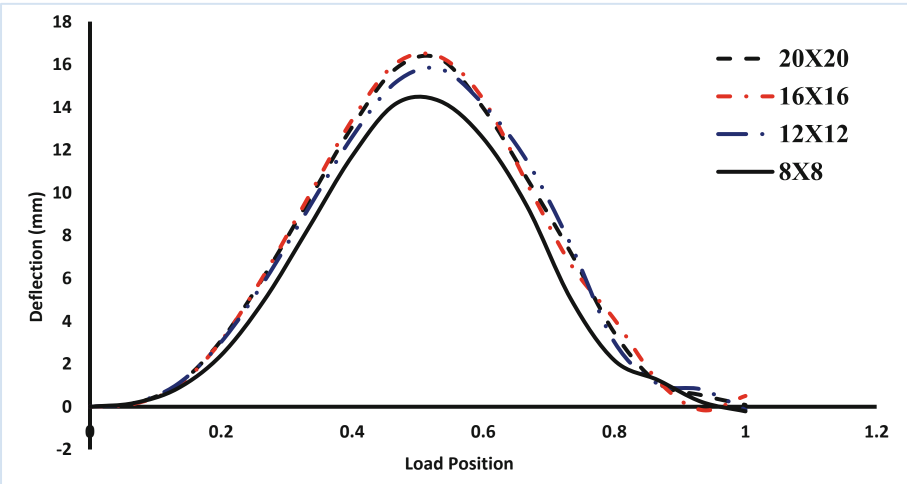

Fig. 5. Central deflection for various mesh grids

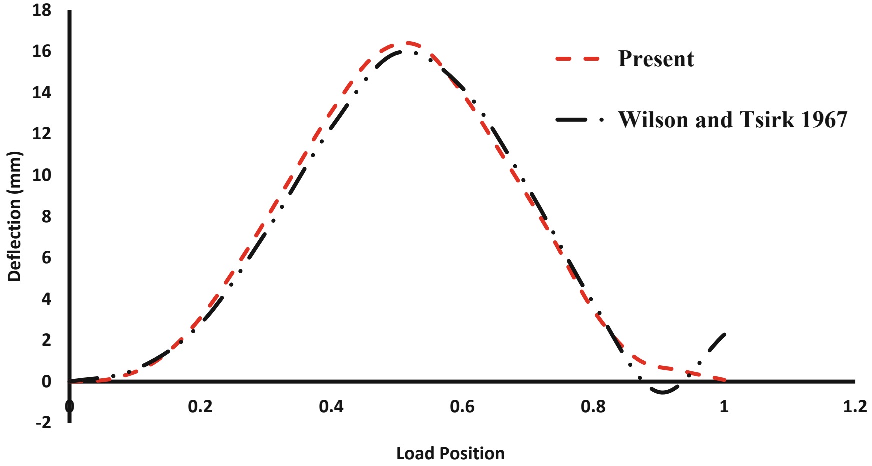

Fig. 6. Central deflection

4.2 Rectangular Plate

Table 3. Material properties of a rectangular plate

Properties	Value
E	200 GPa
ρ	$7850\,\mathrm{kg/m^3}$
ν	0.3
Mass	1 kN
Velocity	60 km/h
Δt	0.001 s

Table 4. Dimensions of a rectangular plate

Dimensions	Value
a	5 m
b	3 m
h	0.05 m

The dynamic response of a rectangular plate (Fig. 7) is explored. The plate's material properties and dimensions are presented in Table 3 and 4 respectively. The deflections are computed in various positions (1/4th, centre and 3/4th) of the plate (Fig. 7) and presented in Fig. 8.

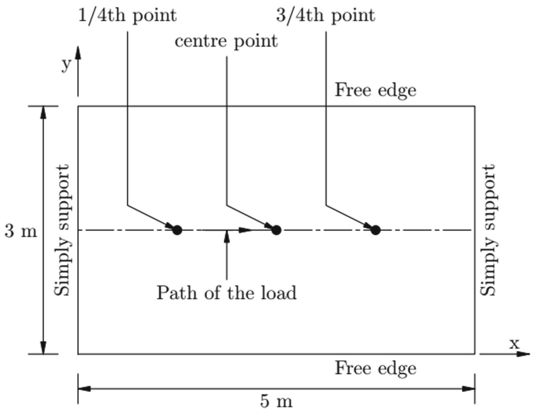

Fig. 7. Rectangular plate subjected to moving mass.

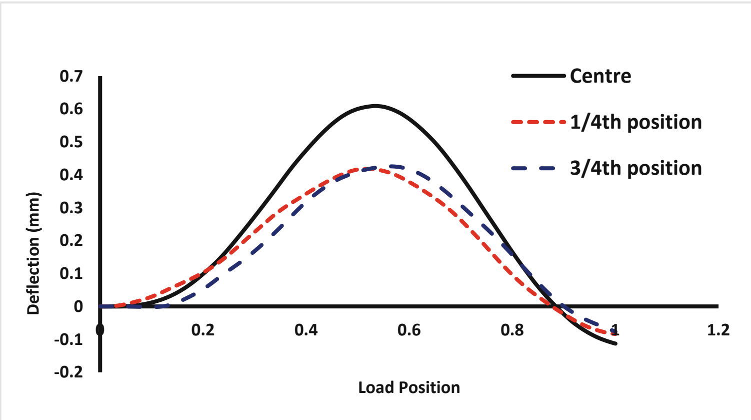

Fig. 8. Deflections at different positions.

Table 5. Dimensions of an annular sector plate

Dimensions	Value
Centre line length	5 m
b	3 m
h	0.05 m

4.3 Annular Sector Plate

The dynamic response of an annular sector plate with circumferential edges are free and radial edges are simply supported (Fig. 9) is explored. The plate's material properties and dimensions are presented in Table 3 and 5 respectively. The deflections with are

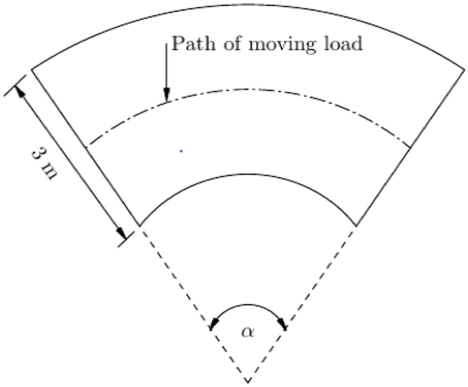

Fig. 9. Annular sector plate subjected to a moving mass.

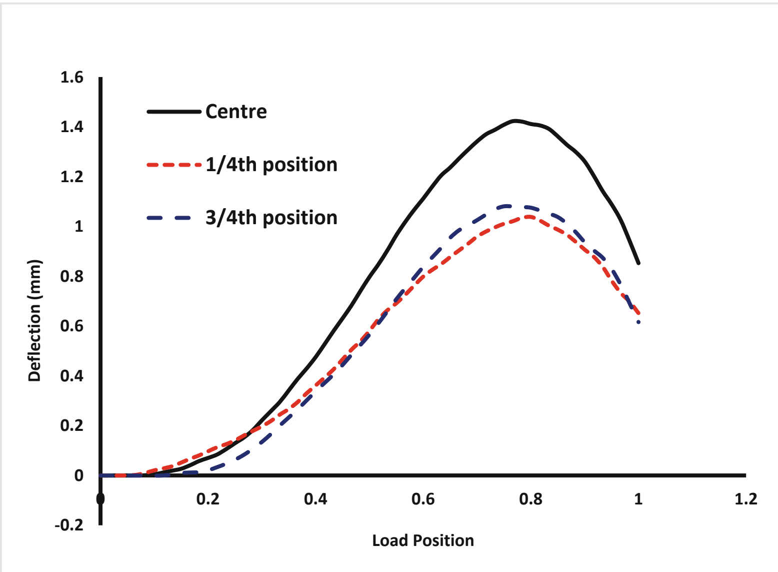

Fig. 10. Deflections at different positions of the annular sector plate.

computed in various positions (1/4th, centre and 3/4th) of the plate and presented in Fig. 10.

5 Conclusions

A quadratic plate element including shear deformation is reviewed for the dynamic response analysis using FEM. The formulation can be used for thin as well as thick plates. The deflections, velocities, and accelerations have been calculated for each time step using the Newmark integration method. Following conclusions can be drawn from the analysis:

- The deflections of a square plate are analyzed for the effect of moving mass which is validated through convergence study (Fig. 5) and also verified with the previously published works (Fig. 6). This study proves the method's efficacy.
- The dynamic deflections at different locations of rectangular (Fig. 8) and curved plate (Fig. 10) keeping the same center-line distance are explained under the effect of moving mass.
- The dynamic deflections are more for the curved plate than that of the rectangular one.
- The rectangular plate's maximum deflection is found at the central load position whereas in case of curved plate the maximum deflection load position shifted towards the end of the structure.
- The rectangular plate is showing a better result than that of the curved one for the dynamic response analysis under the influence of moving mass.

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