11 Valuing Flexibilities in Power Systems as Optionalities

The concepts of flexibilities and optionalities in electricity systems have become increasingly popular over the last two decades. There are two major but distinct drivers for this development: the first one is related to the financial trading of electricity products on future and other derivative markets. In that context, it has apparent merits to consider flexibilities in physical assets, like power plants analogously to financial contracts with embedded flexibilities. The latter are named options, and hence, it has become popular to consider power plants, storages and other assets as "real options".

The other driver is increasing shares of fluctuating renewables that are expected to dominate in the future sustainable energy systems. Here, a lack of flexibilities is perceived as a potential challenge: increasing shares of renewables imply, other things being equal, higher uncertainties due to growing forecast errors. And at the same time, they go along with decreasing shares of controllable conventional power plants.

The two perspectives on flexibilities have somewhat different starting points, and dealing with them in a common framework is not an easy exercise. The most striking difference is that the real options perspective takes prices as exogenous to the decision-maker. In contrast, the second perspective takes a system view, where prices are necessarily a result of interactions between system elements – as in the fundamental equilibrium models of Sect. 7.1. A complete synthesis of these two perspectives is beyond the scope of this textbook. Yet, some elements are put forward after a concise introduction to the financial perspective on flexibilities, i.e. real options. We start thereby by modelling prices as stochastic processes (cf. Sect. 11.1). Then, we introduce the concept of the hourly price forward curve to link future and spot prices in electricity markets in Sect. 11.2. Section 11.3 uses these concepts to value simple options, followed by a digression to financial options and the seminal Black–Scholes model in Sect. 11.4. Section 11.5 discusses the

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/978-3-030-97770-2_11.

C. Weber et al., *Economics of Power Systems*, Springer Texts in Business and Economics, https://doi.org/10.1007/978-3-030-97770-2_11

merits and limits of the Black–Scholes model for electricity market modelling, whereas Sect. 11.6 describes an approach to model thermal and hydropower plants as options in view of valuation. Section 11.7 then applies this approach, and Sect. 11.8 finally comes back to how to bridge the gap between the asset valuation and the system perspective.

Key Learning Objectives

After having gone through this chapter, you will be able to

- Describe and apply key stochastic processes that are used to model price changes in energy and other markets.
- Explain the concept of the hourly price forward curve and how it is used to price electricity supply contracts.
- Discuss key concepts underlying the valuation of options using methods from mathematical finance.
- Discuss the concept of real option and apply a simple valuation model for a thermal power plant.

11.1 Prices as Stochastic Processes

For financial assets like stocks, the price reflects the value attributed to that asset in the market. This price may change over time. E.g. if a company announces unexpected losses, the price of its shares on the stock exchange will go down. Mathematically, the price of an asset may then be described as a stochastic process, i.e. a sequence of realisations of a stochastic variable. One may wonder: why is the price considered a **stochastic process**? This is closely related to the efficient market hypothesis (see Sect. 7.2.5). If a market is efficient, it uses all available information at time t (the information set Ω_t) to determine the asset price. Any new information arriving after time t may change the price. But it would not be **new** information if it did not come as a surprise, i.e. randomly, from the perspective of time t. Put differently: with hindsight (ex-post), we may pretend that we knew before, but ex-ante, we as rational decision-makers will include all available information (even vague expectations, etc.) in our decisions and valuations.

To describe stochastic processes in general, it is helpful to start with a straightforward process that may serve as the basis for multiple generalisations, namely the **Wiener process**. The Wiener process may be best understood as a kind of a **random walk** in continuous time. A random walk consists of a sequence of steps Δz_k that are taken during subsequent time intervals k of length Δt and are randomly and independently chosen. Additionally, we impose for the mathematical description that the steps correspond to stochastic variables ε that are normally distributed with zero mean and standard deviation proportional to $\sqrt{\Delta t}$. This leads to the following mathematical description:

$$\Delta z_k = \varepsilon \quad \varepsilon \sim N(0, \sqrt{\Delta t}). \tag{11.1}$$

Applying standard rules of calculus for normally distributed random variables, it can be shown that for any time interval $T = K \cdot \Delta t$ (i.e. composed of K time steps Δt), the following relationship holds

$$z_{t+T} - z_t = \sum_{k=1}^{K} \Delta z_k \sim N(0, \sqrt{K \cdot \Delta t}). \tag{11.2}$$

That means that for a time interval of arbitrary length T, the change in the **stochastic process** variable z_k is still normally distributed with mean zero and standard deviation $\sqrt{T}$. The process is hence "self-similar", independently of the time granularity considered.

This property may then be generalised to infinitesimal time steps $\mathrm{d}z$, leading to the formulation:

$$\mathrm{d}z = \lim_{\Delta t \to 0} \Delta z_k. \tag{11.3}$$

The so defined $\mathrm{d}z$ is then the (infinitesimally small) increment of a Wiener process $z(t)$ and using a somewhat loose mathematical notation, we may write $\mathrm{d}z \sim N\left(0, \sqrt{\mathrm{d}t}\right)$. Besides being normally distributed, the increments $\mathrm{d}z$ are independent of each other, again irrespective of the time scale considered. The stochastic process variable $z(t)$ itself is then described as a stochastic integral of the increments

$$z(t) - z(0) = \int_0^t \mathrm{d}z. \tag{11.4}$$

One application of this Wiener process in physics is the description of the random movement of particles in a (non-flowing) gas or liquid. This movement was first observed by Scottish nineteenth century scientist Robert Brown and is also known as Brownian motion.

The self-similarity of the Brownian motion becomes apparent in Fig. 11.1, where one single realisation of the Brownian motion is depicted at different discretisation levels. The highest discretisation in the top panel includes 2000 time steps of equal length, whereas the middle panel highlights 100 discrete steps. And the bottom panel is further zoomed out with just 5 discrete steps over the same overall time period. Yet, at each discretisation level, the process includes random steps upwards and downwards of different size. Another property that is also visible is the absence of any mean-reverting effect. The observed realisation of the random process moves

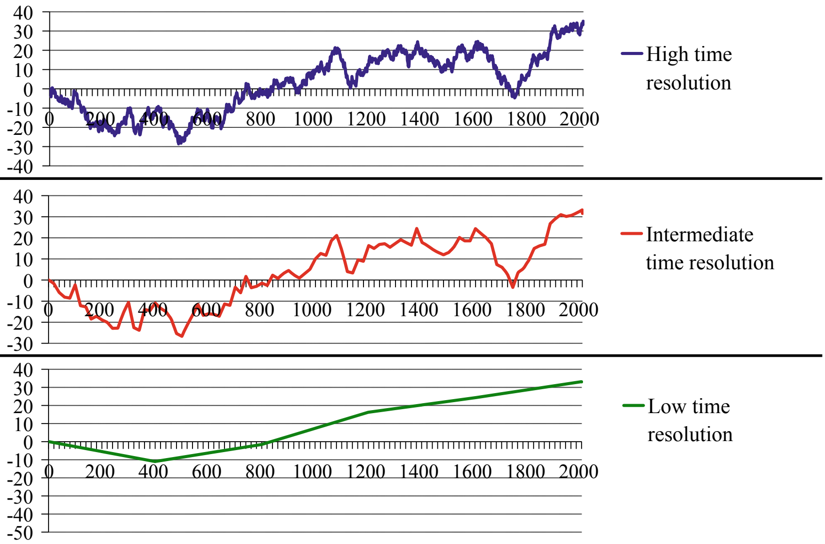

Fig. 11.1 One realisation of a Wiener process observed at different scales of time discretisation

away from the starting value of 0. Independently of the level attained, the probability of going up or down the next step remains unchanged. This property is related closely to the fact that the resulting time series is "non-stationary". We will come back to this point after introducing some generalisations.

A straightforward generalisation of the Wiener process is to introduce a drift – in physics, the equivalent would be an (average) flow direction – and a scaling of the stochastic component so that it may be of arbitrary variance. This leads to the following definition of a **generalised Wiener process** dx:

$$dx = a \cdot dt + b \cdot dz. \tag{11.5}$$

The (positive or negative) parameter a is called the drift rate – it is an average rate of change of x over time. The positive parameter b is named variance rate – although it rather scales the standard deviation (i.e. the square root of the variance) of the stochastic process x.

The impact of the drift rate becomes obvious in Fig. 11.2. With a positive drift rate, the stochastic process moves on average upwards – although this does not preclude that certain increments are negative. As indicated by Eq. (11.5), the overall change is the sum of the deterministic drift part (first term) and the stochastic process part (second term) and the sign depends on the sign and magnitude of the stochastic realisation.

Suppose a price process is expected to oscillate around some average value. In that case, an alternative specification is required for a stochastic process since neither the Wiener process nor a fortiori its generalisation tend to return to some

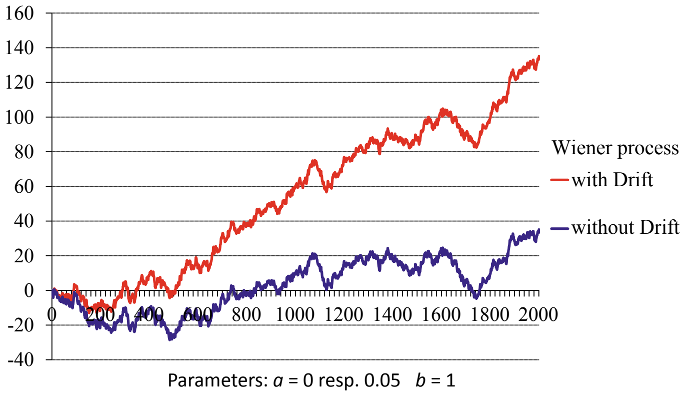

Fig. 11.2 Realisations of a Wiener process with and without drift with identical stochastic components

prespecified mean value. Equation (11.6) specifies a so-called mean-reversion process, also called **Ornstein–Uhlenbeck process**:

$$dx = \kappa \cdot (\mu - x) \cdot dt + \sigma \cdot dz. \tag{11.6}$$

The stochastic second term consists again of a Wiener process multiplied by a standard deviation parameter σ. So the difference lies in the deterministic first term, which includes the factor $(\mu - x)$, which is positive when x is smaller than μ and negative in the opposite case. With a positive factor κ (called mean-reversion rate), this induces a tendency for x to return to the mean value μ. The higher the mean-reversion rate κ, the faster the return to the equilibrium value μ – similar to the pull-back force of a mechanical spring. Yet again, we have a stochastic component superposed on this mean-reversion component, and thus, the resulting incremental changes may go in both directions as illustrated in Fig. 11.3.

As a last relatively simple stochastic process, we introduce the so-called **geometric Brownian motion** (or GBM for short). It is notably used in standard finance models to describe the movement of stock prices $S(t)$. The increments dS of this stochastic process are described by the following stochastic differential equation:

$$dS = \mu \cdot S \cdot dt + \sigma \cdot S \cdot dz. \tag{11.7}$$

Besides the use of different symbols both for the stochastic process variable and the parameters, there are two salient differences of this equation compared to the one describing the generalised Wiener process (Eq. 11.5), namely the multipliers "$\cdot S$" in both the deterministic first term and the stochastic second term. Rewriting the previous equation slightly, we get the following formulation:

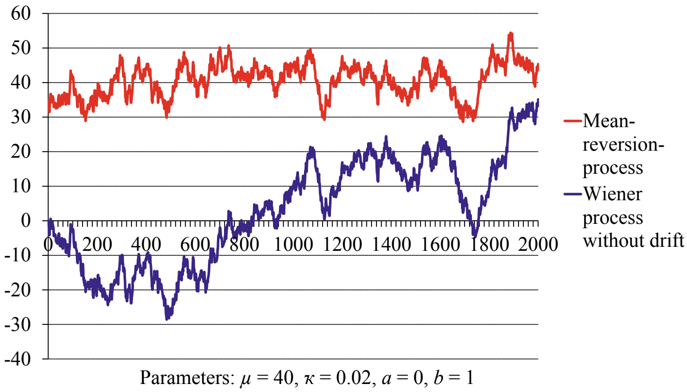

Fig. 11.3 Realisations of a mean-reversion process and a Wiener process with identical stochastic components

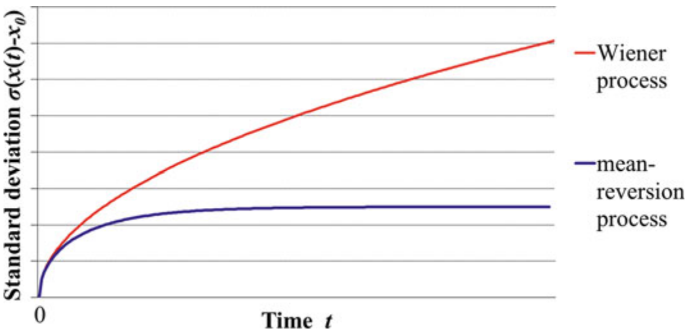

Fig. 11.4 Standard deviation of stochastic process increments as a function of the time step length for Wiener and mean-reversion processes

$$\frac{\mathrm{d}S}{S} = \mu \cdot \mathrm{d}t + \sigma \cdot \mathrm{d}z. \tag{11.8}$$

This highlights that the relative changes in prices $S(t)$ are composed of a mean rate of change μ and stochastic deviations around that mean with standard deviation σ. This is considered appropriate for stock prices because it implies that the expected return on the currently invested capital is independent of the current stock price. E.g. with t measured in years and $\mu = 0.07$, the expected annual return will be somewhat above 7% (due to compound interest effects), independently of whether the current share price is 50 or 500 €. So this may be easily connected to standard asset pricing models like the seminal **capital asset pricing model (CAPM)**. Closer mathematical scrutiny also reveals that prices under the geometric

Brownian motion (GBM) model will always remain positive if the starting price is positive. This property seems very obvious for stock prices. Yet, it is less for electricity prices, where technical constraints and market regulations have induced repeatedly negative prices, particularly in market areas with high proportions of renewables (cf. Sect. 10.1).

This indicates that a pure transposition of approaches developed in the mathematical finance literature to electricity markets may not be adequate. On the other hand, one must acknowledge that electricity is the clear exception to the rule – negative prices are almost unthinkable for storable commodities like oil and gas.[1] And even the GBM model may be considered a reasoned choice for these commodities, as notably the **Hotelling model** of price formation for exhaustible resources suggests a constant return on assets (cf. Sect. 2.3), at least at constant interest rates.

Before proceeding further, four observations are essential:

1. Mathematical finance mainly defines stochastic price processes in continuous time, as sketched above. This enables an elegant analytical treatment using stochastic calculus. Alternatively, stochastic processes may be defined in discrete time, as is current practice in econometrics. The mathematical treatment, especially for valuation purposes, is then in general less elegant. Yet when it comes to numerical estimation and simulation procedures, a discretisation of continuous time is required, and computational techniques for discrete problems have rapidly evolved over the last few decades. Hence, both approaches have their merits, and it is worth considering in applications which approach is more convenient.
2. There exist many more general and more complicated stochastic process specifications than those discussed above. Directions that have been explored by research notably include:

   - Time-varying mean: Especially, when it comes to modelling electricity spot prices, the time-varying scarcity of electricity should be captured by time-varying parameters, e.g. a time-varying mean in a mean-reversion process.
   - Time-changing volatility: In discrete time, so-called **GARCH processes** (cf. Bollerslev 1990) have become very popular to describe the volatility-clustering observable in stock and other asset prices. Several specifications like the **Heston model** (Heston 1993) exist in continuous time, which capture shifts between periods with weak and strong price changes.
   - Increments that are not normally distributed, e.g. jumps. An interesting, general model class in that field are so-called Lévy processes (Bertoin 1996), which build on independent and identical increments yet drop the normality

[1] There was an exemption during the beginning of the Corona crisis in April 2020, as oil demand suddenly sharply decreased resulting in negative prices for the US standard oil variety WTI (West Texas Intermediate). In fact, the strong demand shock coincided with a lack of spare physical storage at the delivery point – and this combination drove prices below zero given that WTI futures are settled physically, contrarily to the common practice mentioned in Sect. 8.6.

assumptions. Any **Lévy process** may be decomposed in a Brownian motion, a drift term and a pure jump process.

- **Multi-factor processes**: Prices may be driven by more than one influencing factor, e.g. electricity prices by fuel prices and scarcity of generation capacities. Correspondingly, different stochastic processes may also be needed to describe actual price characteristics, e.g. different time constants for mean reversion or a combination of mean-reverting and non-stationary components (cf. below). If some of these price components are not directly observable, we are in the presence of so-called "latent variables" which pose additional challenges in identification and estimation.

3. A fundamental property of stochastic processes is stationarity respectively its absence. This is closely linked to the **stationarity** of time series in econometrics. A stochastic process $x(t)$ is (strictly) stationary, when the distribution of $[x(t_1+\tau), x(t_2+\tau)\ldots x(t_k+\tau)]$ is independent of τ, i.e. notably, the mean and the variance of $x(t)$ are independent of t. This is the case for the mean-reverting process described above but neither for the (generalised) Wiener process nor the geometric Brownian motion. An important implication of stationarity is that the price uncertainty remains bounded when the time step length is extended (cf. Fig. 11.4). That means that even for several years ahead, prices under a mean-reverting process only have a limited range of expected values. Whether this is an appropriate property has to be checked in each application.
4. There are multiple links between stochastic price models and neighbouring disciplines like econometrics and control theory worth exploring in more advanced modelling. As with finance models, one should be thoughtful and precise when adapting approaches, e.g. from control theory to pricing issues. Societal and economic systems are made up by persons who make purposeful, individual decisions. And these may hence be described by relationships similar to those governing technical systems only under specific assumptions.

11.2 Hourly Price Forward Curves to Link Future and Spot Prices

As discussed in Sect. 8.5, future contracts are usually written at time t for delivery at time T. Yet for electricity futures, delivery is generally not specified for one single point in time but rather over a time interval $\tilde{T} = [T_1 T_2]$, e.g. a month or a year. The question then arises how the price $\tilde{F}(t, \tilde{T})$ for the future contract over the interval $\tilde{T}$ links to the future prices $F(t, T)$ at different points in time T with $T \in \tilde{T}$. Theoretically, we may argue that such a future market is not complete, meaning that not every **idiosyncratic risk** in each hour of the delivery period may be insured (or hedged) through a specific trading product. Practically, this goes along with the fact that there is not one unique rule to derive the single hour prices $F(t, T)$ from the observed prices $\tilde{F}(t, \tilde{T})$. Practitioners have, therefore, designed various approaches to overcome the gap and to construct what is known as the **hourly price forward curve** (**HPFC**). The two most important methods are those based on

(a) econometric procedures or
(b) the **typical day** approach.

Both methods take observed past spot prices as the basis for constructing a time profile of electricity prices. This profile is then adjusted to the current level of the future prices. In such a way, **arbitrage-free** hourly expected prices are obtained which may then be used to value both delivery contracts to final customers and generation profiles. Note that the obtained prices are future prices for short (hourly) periods and need to be adjusted by the adequate risk premium to obtain expected spot prices (cf. Sect. 8.5.4).

We subsequently focus on the typical day method, which may be summarised in the following five steps:

1. **Define the typical time segments** $\boldsymbol{s} \in \boldsymbol{S}$ to be used for the analysis.
   Example: each hour of the day, differentiated by day of the week, constitutes a separate time segment. Hence, there are a total of 168 (24 × 7) different time segments.
2. **Select the historical observation period** $\tilde{\boldsymbol{T}}_H$ to be used for the establishment of the HPFC.
   Example: the three preceding calendar years.
3. **Define the mapping function** $\boldsymbol{m}(t)$ **linking historical observations** $\tilde{\boldsymbol{T}}_H$ **and future time steps** $\tilde{\boldsymbol{T}}$ **to the typical time segments**.

$$m : \begin{array}{ccc} \tilde{T}_H \cup \tilde{T} & \mapsto & S \\ \tau & \rightarrow & s \end{array} \tag{11.9}$$

   Example: assign to each time step the time segment with the corresponding weekday and the corresponding time of day.
4. **Compute the average historical prices** $\boldsymbol{p}_s$ **for each time segment** using the formula:

$$p_s = \frac{1}{\sum_{\tau \in \tilde{T}_H} \mathbf{1}_{m(\tau)=s}} \sum_{\tau \in \tilde{T}_H} \mathbf{1}_{m(\tau)=s} \cdot p_\tau. \tag{11.10}$$

   The indicator function $\mathbf{1}_{m(\tau)=s}$ is thereby equal to one if and only if the mapping function $m(\tau)$ maps the time step τ to the time segment s, otherwise it is zero.
   Example: compute the average price in hour 8 on Mondays over the last three years

5. **Compute the average price $p_H(\tilde{T})$ for the considered future period $\tilde{T}$** based on historical prices, taking into account the occurrence frequency of the different time segment in the period $\tilde{T}$:

$$p_H(\tilde{T}) = \frac{1}{\operatorname{card}(\tilde{T})} \sum_s \left(\sum_{\tau \in \tilde{T}} \mathbf{1}_{m(\tau)=s} \right) \cdot p_s. \tag{11.11}$$

   Example: determine the average price for next year based on the frequency of the days of the week and hours of the day during next year and the previously computed prices for the time segments.

6. **Based on the price $p_H(\tilde{T})$ and the current future price $\tilde{F}(t, \tilde{T})$, the calibration factor for future hourly prices $g(t, \tilde{T})$ is determined** as follows:

$$g(t, \tilde{T}) = \frac{\tilde{F}(t, \tilde{T})}{p_H(\tilde{T})}. \tag{11.12}$$

   Example: if the current future price is 30 €/MWh and the average price based on historical values is 25 €/MWh, the calibration factor is 1.2.

7. **The calibration factor $g(t, \tilde{T})$ is used together with the mapping function to determine the hourly price $F(t, T)$** for each hour in the future from the historical average price for the corresponding time segment:

$$F(t, T) = g(t, \tilde{T}) \cdot \sum_s \mathbf{1}_{m(t)=s} \cdot p_s \tag{11.13}$$

   Example: with the factor computed previously, the future price for hour 8 on Mondays would be 1.2 times higher than the observed historical prices for this hour.

Note that a more detailed application example for this method is provided in Sect. 11.7. The adequacy of this method mainly hinges on two prerequisites:

- the appropriate selection of typical time segments and
- the absence of structural breaks between historical price structures and the expected future price structures.[2]

The first prerequisite implies a good balance between a sufficient distinction of different time segments and a sufficient number of observations per time segments to avoid substantial impacts from single outliers. Typically, one might choose every

[2] Yet all statistical and econometric methods rely in one way or another on the assumption of the absence of structural breaks.

weekday in each month as a separate typical day. But then, the question arises how public holidays should be treated: Are the Christmas holidays or Easter or regional holidays like All Saints to be treated as one single day type, or should there be a different day type for each of these holidays?

The second prerequisite leads to a preference for short historical periods, but again this has to be traded off against the limited number of observations in short periods.

A more fundamental inconvenience of this approach is that it only provides estimates of the expected hourly future prices but not the possible variability around that mean value. If this is searched for, the HPFC has to be complemented by a stochastic process describing the variations around that mean. This issue will be addressed in the following subsection.

11.3 Valuing Simple Options on a Stochastic Spot Price

Given the preceding discussion, we may now wonder what the value of a flexible generation (or demand side) option is considering future prices. To answer this question, we have to combine the elements outlined in the previous two subsections. Yet, a first terminological disambiguation is necessary: there are (at least) two meanings of the term "future prices" that we have to distinguish. The first meaning is "prices in the future", the second "prices of future contracts". To be more precise: when assessing the value of physical flexibility options in the electricity market, the key question is about "possible spot prices in the future" rather than on "current prices of future contracts". The focus is on spot prices since the physical options are to be used in the actual operation of the system – and spot prices (should) reflect the value of actual operations (cf. Sect. 7.2.3.2). The loose qualification of "possible" spot prices emphasises that the value of these physical flexibility options is related to the uncertainty surrounding operations and prices in the future.

Having this in mind, a standard recipe for valuing simple flexibility options may consist of five steps:

1. **Define the flexibility option under study**.
   In the simplest case, the flexibility option is fully characterised by its **variable cost** c^{var} in €/MWh at which it supplies additional electricity (or reduces demand) and its **capacity** K describing the achievable output rate in MW. Taking into account operational constraints or energy volume constraints (storage-type flexibilities) makes the valuation exercise more demanding (cf. below).
2. **Determine the expected spot price(s) for the valuation period**.
   Here, the method for constructing an HPFC described in Sect. 11.2 may be used.

3. **Describe the distribution of the spot price(s) around its expected value**.
   Here, stochastic price processes as discussed in Sect. 11.1 may be used. It is then essential to incorporate the time-varying mean as specified in step 2 into the formulation of the stochastic processes
4. **Determine the expected payoffs of the flexibility option at exercise time under the spot price distribution**.
   This requires a set of valuation formulas that are discussed subsequently for the case of a simple flexibility option.
5. **Obtain the current value of the flexibility option through discounting and aggregation**.
   The present value of the flexibility option is obtained by discounting the value at the time of delivery (so-called exercise time in finance slang). Moreover, the value may be aggregated over the relevant valuation period if it consists of more than one time step (hour).

Note that in step 2, the future prices obtained through the HPFC need in principle to be adjusted by the corresponding market risk premium to obtain expected spot prices (cf. Sect. 7.2.5.3). Conversely, the discount rate used in step 5 should in principle include not only the risk-free rate but also the risk premium. Yet practitioners tend to neglect the risk premium given the difficulty to obtain reliable estimates for it. From a theoretical perspective, one may argue that the effects in steps 2 and 5 at least partly cancel out each other, so the assumption of a zero risk premium is generally defendable.

Having clarified the preliminaries and prerequisites, we now turn towards the valuation of a simple flexibility option characterised by its variable cost c^{var} and capacity K (step 4). At given spot price S_T, the option will be used at full capacity if $S_T \geq c^{\text{var}}$, and it will not be used (by a profit-maximising operator) if $S_T < c_{\text{Var}}$. Under uncertain spot prices, the expected payoff of the option at exercise time is then given by the relationship:

$$
\begin{aligned}
V_{T|t}(T) &= K \cdot \int\limits_{-\infty}^{+\infty} \max(x - c^{\text{var}}; 0) f_{S_T|t}(x)\mathrm{d}x \\
&= K \cdot \int\limits_{c_{\text{Var}}}^{+\infty} (x - c_{\text{Var}}) f_{S_T|t}(x)\mathrm{d}x.
\end{aligned}
\tag{11.14}
$$

The notation $V_{T|t}(T)$ emphasises that the option is exercised at time T (function argument T), and the value of the payoffs is also considered at time T (subscript T), yet based on the information available at time t (subscript $|t$). Note that this value is not dependent on the actual price process used for S_T, but only on the probability distribution for the prices at exercise time, here characterised by the probability density function $f_{S_{T|t}}$ and the corresponding cumulative distribution function $F_{S_{T|t}}$.

Explicit results for the option value may inter alia be obtained, if prices are normally distributed, i.e. $S_{T|t} \sim N\left(\mu_{T|t}, \sigma_{T|t}\right)$. This will notably be the case if prices result from a generalised Wiener process as given in Eq. (11.5) or of a mean-reversion price process as described in Eq. (11.6). Then, we obtain the following formula for the value:

$$\begin{aligned} V_{T|t}(T) &= K\left(\left(\mu_{T|t} - c^{\text{var}}\right)\left(1 - F_{S_T|t}(c^{\text{var}})\right) + \sigma_{T|t}{}^2 f_{S_T|t}(c^{\text{var}})\right) \\ &= K \cdot \sigma_{T|t} \cdot (\mathrm{d}\Phi(d) + \phi(d)) \end{aligned} \tag{11.15}$$

With $d = \frac{\mu_{T|t} - c^{\text{var}}}{\sigma_{T|t}}$.

Thereby Φ is the cumulative distribution function and ϕ the probability density function associated with the standard normal distribution. One may note that this result corresponds to the one obtained in finance for option values under the so-called Bachelier model (e.g. Schachermayer and Teichmann 2008). Furthermore, this total option value exceeds always the so-called **intrinsic value**, which is defined as

$$V_{T|t}^{\text{Intr}}(T) = K \cdot \max\left(\mu_{T|t} - c^{\text{var}}; 0\right). \tag{11.16}$$

This would be the option value if it were executed at the current expected price $\mu_{T|t}$. The difference between the total option value according to Eq. (11.15) and the intrinsic value is then labelled **extrinsic value** or **time value** − time value because it disappears as the exercise of the option gets closer, i.e. the uncertainty about future prices is reduced. Similar considerations have been established in finance for the Black–Scholes model that we discuss in the following section.

A small example may illustrate the point right here: Consider a flexibility option with variable costs c^{var}= 50 €/MWh, e.g. a combined cycle plant. With an expected price in the future $\mu_{T|t}$ = 60 €/MWh, the intrinsic value of the option is 10 €/MWh (cf. Eq. 11.16). If we consider a period T in the distant future, the uncertainty regarding the future price is large, e.g. the standard deviation reaches $\sigma_{T|t}$ = 20 €/MWh. Using Eq. (11.15), we then obtain the total value of the option as $V_{T|t}(T)$ = 13.96 €/MWh. This is almost 40% higher than the intrinsic value, and the extrinsic (or time) value equals 3.96 €/MWh. This value vanishes gradually if the price level remains constant while the price uncertainty decreases as the exercise time T approaches.

11.4 Analytical Approaches for Option Valuation: The Black–Scholes Model

The previously described valuation approach has the advantage that it combines rather standard methods and analytical tools of medium complexity. However, both practitioners and scientists in the field have in the past been more turned towards another option valuation approach, the famous Black–Scholes model (cf. Black and Scholes 1973, Merton 1973), respectively, its variant considering options on futures published by Black (1976).

The Black–Scholes model was originally developed for options on stocks and correspondingly, it does not consider normally distributed prices but a geometric Brownian motion as underlying stochastic price process (cf. Sect. 11.1, Eq. 11.7). Furthermore, its derivation is placed in the context of efficient, arbitrage-free markets and dynamic hedging and replication strategies (Schachermayer and Teichmann 2008). The objective of the model is to determine a "fair price" for so-called **European option**s on stocks or similar financial papers.[3] There are two types of European options (cf. Sect. 8.6):

Call options provide the holder the right (but not the obligation) to buy the underlying (the stock) at some point of time T in the future (called exercise or strike time) at a predefined price X, the so-called exercise or strike price.

Put options conversely provide the holder the right (but not the obligation) to sell the underlying at exercise time T in the future at the predefined price X.

It may be noted that the simple flexibility option discussed in Sect. 11.3 (e.g. a controllable power plant) with specified variable costs c^{var} is a real option analogy to a call option if all technical operation restrictions are disregarded. The (much less common) equivalent to a put option would be a pure flexible consumer willing to consume additional electricity below a specific price threshold – one may think of an electrolyser producing pure hydrogen and selling it at a given market price. But one has to be aware that electricity spot prices are usually not adequately modelled based on a geometric Brownian motion (cf. Sect. 11.1). Therefore, the Black–Scholes analysis is not directly transposable to flexibility options in the electricity system. Nevertheless, it is worthwhile to discuss the principles of financial option valuation based on the seminal Black–Scholes analysis.

This analysis focusses on the above-mentioned fair price, which is a price upon which sellers and buyers may agree. To be acceptable for both sides, such a price should be derived solely from objective market information and not depend on individual subjective preferences. By providing such a fair price, the Black–Scholes model has paved the way for a tremendous increase in financial derivatives trading

[3] A broad variety of options is traded on financial markets. The most standard options are labelled European and **American options**. European options may only be exercised at the exercise date, whereas American options may be exercised any time up to the exercise date. So for American options "early exercise", i.e. a use before the agreed exercise date is possible whereas it is not for European options. Real options involve a physical activity and hence obviously may not exercised in advance—they correspond to European options, or often rather to a sequence of European options (cf. Sect. 11.6).

in the four decades after its publication – until the global financial crisis in 2008 led to a deep questioning of many valuation practices. A major consequence for corporate and regulatory risk management concerning this and other similar models has been to take "model risk" seriously – and model risk arises notably from deviations between model assumptions and the real world.

This being said, the assumptions underlying the Black–Scholes model have to be scrutinised critically. On the other hand, the mathematical elegance and application simplicity of the Black–Scholes formula strongly hinge on these assumptions, which may be summarised as follows (cf. Hull 2018):

1. The price of the underlying asset follows a geometric Brownian motion.
2. Short selling of assets is possible, and there are no limitations to the use of corresponding revenues.
3. Transaction costs and taxes are negligible, and shares are infinitely divisible.
4. No dividend payment on the stock occurs [extension with dividends in Black (1976)].
5. There are no risk-free arbitrage opportunities.
6. Trading is done continuously.
7. The risk-free interest rate is constant and identical for all expiry dates.

Extensions of the Black–Scholes model aim to deal with less simplifying assumptions, yet we focus subsequently on the original model since it captures key features of option pricing. A complete mathematical treatment of the Black–Scholes model is out of scope for this book. We limit ourselves to sketching the key elements of the reasoning [for a more detailed but still accessible treatment cf. Hull (2018)]. The derivation of the valuation formula relies mainly on the three following elements:

1. Construction of a **risk-free portfolio** consisting of the option and the according underlying[4] in an appropriate ratio.
2. **No-arbitrage argument**: the risk-free portfolio will offer the same return rate as a risk-free bond.
3. **Risk-neutral evaluation**: the value of options on stocks is independent of the risk appetite of investors. Options can, therefore, be evaluated under the simplifying assumption of risk neutrality.

Considering the value $V(S,t)$ of the option as a function of the price of the underlying stock S and time t, the two first elements allow to derive the following stochastic partial differential equation, also known as the **Black–Scholes–Merton differential equation**:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV. \tag{11.17}$$

[4] The term underlying is used in finance to designate the asset, which a derivative is based on, e.g. the shares of a particular company, cf. also Sect. 8.2.

A first important point to note on this equation is that it describes the changes in value V for all financial products[5] with the underlying S (e.g. also for forwards or complex options).[6] The differential equation has multiple solutions. These are obtained by adding specific boundary conditions to the equation. We will come back to that point later.

To provide some intuition, we take a closer look at the terms of the differential equation: the right-hand side describes the value change corresponding to interest payments based on the risk-free interest rate r. For a risk-free derivative, i.e. when both the first derivative $\frac{\partial V}{\partial S}$ and the second derivative $\frac{\partial^2 V}{\partial S^2}$ for S are zero, the interest payment corresponds to the value change over time $\frac{\partial V}{\partial t}$, as is to be expected in an arbitrage-free world. Another particular case arises for $\frac{\partial V}{\partial S} = 1$ and $\frac{\partial^2 V}{\partial S^2} = 0$. An obvious solution satisfying these boundary conditions is $V \equiv S$, i.e. the considered product is equal to the underlying (or at least always has the same value). Then, obviously $\frac{\partial V}{\partial t} = 0$, i.e. the (partial) derivative with respect to time at given asset price S is zero. While $\frac{\partial V}{\partial S}$ describes the direct dependency of the product value on the value of the underlying, the third term on the left side is less intuitive: its magnitude is determined by the variance σ^2 of the stochastic process, i.e. it is related to the stochasticity of prices. This term is labelled diffusion term. An intuitive understanding may be derived from considering the expected value change for a product with a positive second derivative $\frac{\partial^2 V}{\partial S^2} > 0$ in the presence of a discrete uncertainty for the underlying S (cf. Fig. 11.5).[7] If an up-movement $+\Delta S$ and a down-movement $-\Delta S$ of similar magnitude may occur with similar probability, the expected change in S is zero. Given the positive curvature of the value function, the expected change in V will be strictly positive, other things being equal.

With positive S and positive $\frac{\partial V}{\partial S}$ (as in Fig. 11.5) and typical magnitudes for these terms, a solution to the differential equation will then require $\frac{\partial V}{\partial t} < 0$, i.e. a product with positive second derivative with respect to S will lose value over time. This holds, other things being equal, notably for a given S. This value decrease corresponds to the loss in time value for an option. Explained differently: in the setting of Fig. 11.5, the likely up and down movements until expiry $\pm\Delta S$ decrease in size as the expiry date approaches. Then also the difference between the ex-ante expected value $\frac{(V(S_0 - \Delta S, t) + V(S_0 + \Delta S, t))}{2}$ and the realised value $V(S, t)$ shrinks – this is (a discretised version of) the loss in time value.

At the boundaries of the definition domain for the value function, boundary conditions have to be added, and these boundaries determine the specific solutions.

[5] These products are frequently subsumed under the term "**derivatives**" (cf. Chap. 8). Yet we avoid this nomenclature in the following to avoid confusion with the mathematical concept of derivatives of a function.

[6] Note that there are no indices $T|t$ or likewise to the value function V as in the previous subsection. In fact, we consider here always the value at time t evaluated with information at the same time t. Therefore, we drop these unnecessary, identical indices.

[7] Mathematically, it is a consequence of Ito's lemma, which is a fundamental theorem in stochastic calculus.

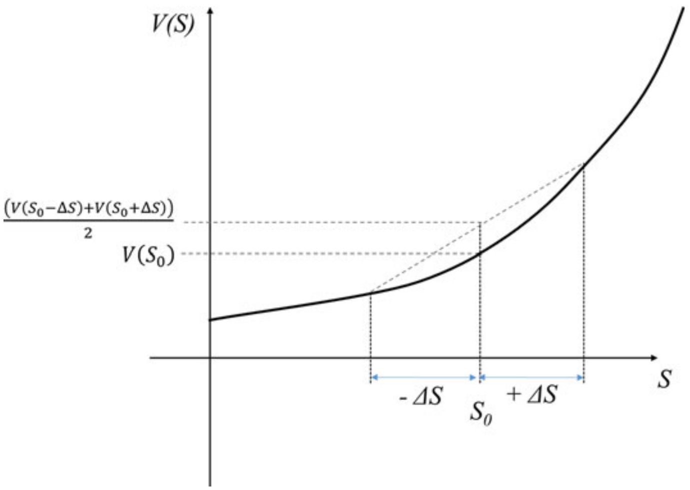

Fig. 11.5 Illustration of the diffusion term in the Black–Scholes–Merton differential equation

For the most common European options, the key boundary conditions are given by the **payoffs** at exercise time.

- for a **call option** (purchase option), this payoff may be written as follows:

$$V^{\text{Call}}(S,T) = \max(S - X, 0). \tag{11.18}$$

  This condition summarises the definition of a European call option: the call option will be exercised at maturity T, if the price of the underlying S exceeds the strike price X. Then, the payoff will be equal to the positive difference $S - X$. At prices below the strike price, the option is not exercised and no payoff occurs. Additionally, the following boundary conditions are specified: $V^{\text{Call}}(0,t) = 0$ and $\lim_{S \to +\infty} \left(V^{\text{Call}}(S,t) - S\right) = 0$, i.e. the call option value is bounded by zero at low prices and by S at high prices.
- for a **put option** (sell option), the payoff at exercise time is

$$V^{\text{Put}}(S,T) = \max(X - S, 0). \tag{11.19}$$

  Again, this condition describes mathematically the payoff of a European put option at maturity: it will provide a positive payoff if and only if the strike price exceeds the spot price at maturity, i.e. when it is more profitable to sell the underlying at the strike price to the option writer (seller of the option) than to the market at the current spot price. The payoff is in that case equal to the difference $X - S$.

Table 11.1 Limiting cases for option values according to the Black–Scholes formula

Limiting case		Implication	Value limit
Just before delivery of the option			
$\boldsymbol{t \rightarrow T}$	$S > X$	$d_1 \rightarrow +\propto$, $d_2 \rightarrow +\propto$	$\boldsymbol{V^{Call} \rightarrow S(t) - X}$, $\boldsymbol{V^{Put} \rightarrow 0}$
	$S < X$	$d_1 \rightarrow -\propto$, $d_2 \rightarrow -\propto$	$\boldsymbol{V^{Call} \rightarrow 0}$, $\boldsymbol{V^{Put} \rightarrow X - S(t)}$
Current price far above exercise price			
$\boldsymbol{S \gg X}$		$d_1 \rightarrow +\propto$, $d_2 \rightarrow +\propto$	$\boldsymbol{V^{Call} \rightarrow S(t) - Xe^{-r(T-t)}}$, $\boldsymbol{V^{Put} \rightarrow 0}$
Current price far below exercise price			
$\boldsymbol{S \ll X}$		$d_1 \rightarrow -\propto$, $d_2 \rightarrow -\propto$	$\boldsymbol{V^{Call} \rightarrow 0}$, $\boldsymbol{V^{Put} \rightarrow Xe^{-r(T-t)} - S(t)}$
Almost risk-free option			
$\boldsymbol{\sigma} \rightarrow 0$	$S > Xe^{-r(T-t)}$	$d_1 \rightarrow +\propto$, $d_2 \rightarrow +\propto$	$\boldsymbol{V^{Call} \rightarrow S(t) - Xe^{-r(T-t)}}$, $\boldsymbol{V^{Put} \rightarrow 0}$
	$S < Xe^{-r(T-t)}$	$d_1 \rightarrow -\propto$, $d_2 \rightarrow -\propto$	$\boldsymbol{V^{Call} \rightarrow 0}$, $\boldsymbol{V^{Put} \rightarrow Xe^{-r(T-t)} - S(t)}$

Further boundary conditions are again imposed—derived from limit case considerations: $V^{\mathrm{Put}}(0,t) = X \cdot e^{-r(T-t)}$ and $\lim\limits_{S \to +\infty} V^{\mathrm{Put}}(S,t) = 0$. Note that the limiting value for an underlying price of zero considers the discount of the terminal payoff to the valuation time.

With these boundary conditions and under the assumptions above, Black and Scholes derive the following value formulas for European put and call options:

$$V^{\mathrm{Call}}(S,t) = S \cdot \Phi(d_1) - X \cdot e^{-r(T-t)}\Phi(d_2) \tag{11.20}$$

and

$$V^{\mathrm{Put}}(S,t) = X \cdot e^{-r(T-t)}\Phi(-d_2) - S \cdot \Phi(-d_1). \tag{11.21}$$

Thereby, the cumulative distribution function Φ of the standard normal distribution and the parameters given in the following formula are used.

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \\ d_2 &= \frac{\ln\left(\frac{S}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \end{aligned} \tag{11.22}$$

These formulas are best understood by considering various limiting cases, as summarised in Table 11.1. The first example given there is an option approaching expiry. As price uncertainty gets smaller and the boundary of the definition set is reached, the value approaches the final payoff for the option. Similarly, the reader is invited to consider the other cases listed there and to make use of Eqs. (11.20–11.22) to validate the results, cf. also Exercise 11.3.

11.5 Merits and Limits of the Black–Scholes Model for Electricity Market Analyses

The Black–Scholes model is generally considered as the reference model for valuing options in financial markets. Yet, there are multiple off-springs and alternatives to that standard model, too numerous to name. However, two are worth mentioning. Black (1976) discusses options on futures and includes a discussion of dividend-paying stocks whereas Margrabe (1978) generalises the valuation formula to options with two underlyings. The former is interesting for electricity markets (and more generally energy markets) since options therein are usually not written on the physical underlying but on futures. The latter provides a conceptual frame that allows dealing with thermal power plants as real options. We will come back to that in the next section.

In general, option valuation approaches derived from finance have found the following applications in the electricity industry and more generally the energy sector:

1. Valuation of financial options and similar products traded on the energy markets.
2. Valuation of optionalities embedded in contracts or complex products.
3. Support for hedging decisions for real options such as power plants.
4. Valuation of real options in medium to long-term perspective.

The first application field is rather straightforward yet it suffers in the case of electricity from a lack of liquidly traded options in most market places. For oil markets, this is, however, a typical usage of option price models. The second field encompasses a broad range of concrete applications – including, e.g. the evaluation of flexibility clauses in gas supply contracts. The third and fourth applications are most directly linked to the physical and system perspective on electricity markets: the applied model's assumptions must fit the actual market conditions to obtain reliable results. For the use of the Black–Scholes or similar formulas, two aspects are thereby critical:

- Given the non-storability of electricity, each spot delivery period corresponds to a separate product. For this product, price distribution parameters have to be assessed, and the corresponding real option is to be evaluated.
- Furthermore, it is questionable whether a geometric Brownian motion may adequately describe the price process for electricity spot prices. Notably, negative prices and prices of zero are not compatible with the assumption of a geometric Brownian motion process. Therefore, any application of Black–Scholes, Black (1976) or Margrabe formulas in the context of hedging or valuation of real options should be aware of the necessarily approximate nature of the results. In the following, we, therefore, follow a somewhat different route.

11.6 Thermal and Hydropower Plants as Real Options

From what we have discussed in the previous sections, five key elements may be distilled when it comes to conceptualising power plants as real options:

1. **Power plants** do not correspond to a single option on one underlying. Rather they correspond to a series of options – also called a "**strip of options**": a power plant provides production options for every delivery period of the spot market. A similar reasoning holds for demand-side flexibilities.
2. **Technical constraints** such as minimum operation times or start-up costs **limit the usage of these options**. They also prevent using simple analytical option formulas such as the ones discussed in Sects. 11.3 and 11.4.
3. If a **power plant** burns commercially traded fuels such as **hard coal or natural gas**, then it should be considered as an **option dependent on two underlyings**. Both the output electricity price and the input fuel price are time-varying and may be described by stochastic processes. If additionally emission certificates are to be used, then the option depends on three underlyings.[8]
4. **Storages** are a type of real option that does not have a common equivalent in financial options. They are usually assimilated to so-called **swing options**. Swing options describe the right to take more or less of a specified commodity over a time period.[9]
5. To value all these real options, an **adequate modelling of the price process** is vital. Assessing the value of flexibility options in the future electricity systems is particularly challenging since this requires an anticipation of the future prices, including their stochasticity.

These are key takeaways for anyone trying to link the challenging issue of valuing generation flexibilities in electricity systems to the broad literature stream of financial option valuation. By and large they are also applicable when it comes to valuing demand-side flexibilities. A few additional remarks may, however, be useful:

First, one should be aware that our treatment so far has focussed on analytical approaches to financial option valuation. Yet research in finance has also developed a broad range of numerical methods, cf. Hull (2018) for an overview. The most important classes are **Monte Carlo simulations**, (binomial) tree approaches, finite difference methods and the so-called **least-squares Monte Carlo** approach, cf. Longstaff and Schwartz (2001). Notably, the latter has emerged as a very flexible and computationally feasible method for evaluating path-dependent options such as storages or thermal power plants with operation restrictions.

[8] Pushing even further, a CHP plant with heat as second output besides electricity is dependent on four underlyings.

[9] Swing options have been introduced in the finance literature mostly to describe the characteristics of common gas contracts, which include minimum and maximum delivery quantities, cf. e.g. Jaillet et al. (2004).

Especially for storage valuation, numerical methods are crucial since there are no analytical valuation formulas readily available neither for swing options nor in general for storage plants. For thermal power plants, it may be quite useful to disregard operation restrictions and use analytical formula to obtain an upper bound to the flexibility value.

When the dependency of thermal power plant valuation on input factor prices is to be taken into account, then considering the spread between input factor costs and output prices is advantageous. For the Black–Scholes model, a corresponding generalisation has been developed by Margrabe (1978). He develops an analytical formula for an option dependent on the spread between two underlyings. Thereby, the option value is driven by the volatility of the price ratio of the two underlyings. There is then also not a specific strike price. Rather the exercise of the option depends on the ratio of the two commodity prices. The corresponding spread is called "spark spread" for gas-fired power plants, which corresponds to the gross margin at given commodity prices. For coal-fired power plants, the term "dark spread" is used. For an application to European power plants, an extension is required to include besides fuel also CO_2 certificates as input factor with separate price risks. This is then a "clean spark spread", respectively, a "clean dark spread". Yet such models are still based on several questionable assumptions, and therefore, we subsequently rather pursue a different approach – namely the application of the previously developed simple models to an actual flexibility valuation for a power plant.

11.7 Application: HPFC and Parsimonious Real Option Valuation for Thermal Power Plants

To assess the future value for a power plant, we have to first link the available market quotes for derivative products (in occurrence for quarter 3 of 2016) to hourly expected spot prices. This is done by establishing first an hourly price forward curve (cf. Sect. 11.2). Then, the flexibility value of an (idealised) CCGT plant for the considered period, here from July to September 2016, is determined based on historical data, in occurrence those available by the end of 2015. Thereby, the simple valuation approach described in Sect. 11.3 is used. The data used for the study as well as the corresponding spreadsheet *HPFC_Optvalue.xlsx* contained in the electronic appendix to this chapter.

For the construction of the HPFC, we apply the typical day method, with one typical day for each weekday. Yet as consumption and price patterns on Tuesdays to Thursdays are rather similar, they are aggregated to one typical day. For reasons of simplicity, we use only 2015 data to construct the HPFC. Following the procedure described in Sect. 11.2 above, we get for the corresponding steps (cf. also Table 11.2):

Table 11.2 Key elements for an HPFC for Q3 2016 based on price data of 2015

Row no.	Typical days s	Historical values Average prices p_s		Future frequencies in Q3 2016
		Base	Peak	Number of days
(1)	Monday	35.28	40.10	13
(2)	Tuesday–Thursday	35.74	39.07	39
(3)	Friday	35.92	38.31	14
(4)	Saturday	28.83		13
(5)	Sunday	22.33		13
		Future values for Q3 2016		
		Base	Peak	Off-peak
(6)	Number of hours	2208	792	1416
(7)	Weighted historical average $p_H(\tilde{T})$	32.83	39.11	29.32
(8)	Futures $\tilde{F}(t,\tilde{T})$ on Dec 30, 2015	27.94	33.80	24.69 (computed)
(9)	Calibration factor $g(t,\tilde{T})$	0.851	0.863	0.842

1. **Definition of the typical time segments:**

$$S = \{\text{'Mon } h1\text{'}, \text{'Mon } h2\text{'}, \ldots, \text{'Mon } h24\text{'}, \text{'Tue}-\text{Thu } h1\text{'}, \text{'Tue}-\text{Thu } h2\text{'}, \ldots, \text{'Fri } h1\text{'}, \ldots, \text{'Sun } h24\text{'}\}$$

   Hence, there are 5 typical days and 120 different typical time segments.

2. **Selection of the historical observation period $\tilde{T}_H$:** As proposed above, we only use 2015 data as historical observations, i.e., limiting ourselves to the summer months, we get

$$\tilde{T}_H = \{\text{'Jul 1 2015}, h1\text{'}, \text{'Jul 1 2015}, h2\text{'}, \ldots, \text{'Sep 30 2015}, h24\text{'}\}$$

3. **Definition of the mapping function $s = m(t)$:** we map each observation in the historical period $\tilde{T}_H$ onto the typical time segment with the corresponding weekday (respectively, the weekday aggregation Tue–Thu) and the same hour. The same is done for the future time period $\tilde{T}$.
   There is no concise mathematical description of the mapping function, yet it may be easily implemented in software code (cf. electronic supplement).

4. **Computation of the average historical prices p_s for each time segment:** The average price in hour 8 on Mondays over Q3 2015 is found to be 44.86 €/MWh. Prices averaged over base and peak periods and typical days are also indicated in Table 11.2, rows labelled (1) to (5).
5. **Computation of the average price $p_H(\tilde{T})$ for the considered future period $\tilde{T}$ based on historical prices:** The results are given in row (7) of Table 11.2, using the frequencies indicated in rows (1)–(5) in the right-hand column. Besides the average base and peak price, also an off-peak price is computed.
6. **Determination of the calibration factor $g(t, \tilde{T})$:** Based on the prices $p_H(\tilde{T})$ (row (7)) and the current future price $\tilde{F}(t, \tilde{T})$ (row (8)), the calibration factors $g(t, \tilde{T})$ are determined as indicated in row (9) of Table 11.2.
7. **Use of the calibration factors $g(t, \tilde{T})$:** to have a unique calibration factor for each time segment, we use the calibration factor obtained for peak hours for hours $h9$ to $h20$ on Mondays to Fridays. For all other time segments, the off-peak calibration factor is used. The base calibration factor is hence only given for information purposes.

The resulting prices for the typical time segments are shown graphically in Fig. 11.6. It is thereby evident that prices on Saturdays and especially Sundays are on average lower than during the week. In addition, the early Monday morning hours are more similar to weekend hours than to other weekdays.

With the hourly price forward curve, we may compute the intrinsic value for a thermal power plant. To determine the total option value along the approach developed in Sect. 11.3, including the time value, we have to estimate the standard deviation for the spot prices. A straightforward way to do so is to use the same data as for the estimation of the price forward curve.

We, therefore, compute for each hour of each typical day the standard deviation of the prices around the observed mean. They are then calibrated using the same factors as for the HPFC. The resulting standard deviations and expected prices (HPFC) are plotted for the typical day Tuesday–Thursday in Fig. 11.7.

In the same graph, we show the results from applying the option valuation formula derived in Sect. 11.3, namely Eq. (11.15). It becomes evident that the option value of the stylized power plant is close to zero during night hours when expected prices are far below variable cost and that the value increases to about 10 €/MWh during morning and evening hours. The option is said to be “deep in the money”, i.e. it is very unlikely that it is not used, and the value is close to the (positive) difference between expected price and variable cost, which is the **intrinsic value**. Comparing the option value for hours 9 and 20, the impact of time-varying volatility becomes obvious. Although the expected price (and correspondingly the intrinsic value) is slightly higher in hour 20, the option value is

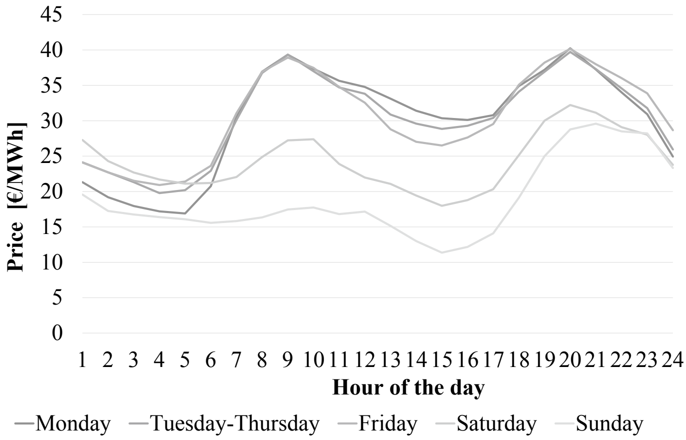

Fig. 11.6 Hourly price forward price curve (HPFC) for summer (Q3) 2016 based on price data of 2015

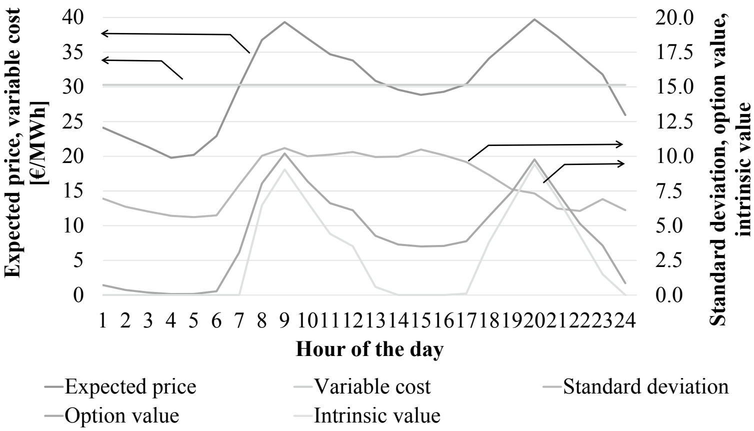

Fig. 11.7 Prices and option value for a power plant on an hourly basis

higher in hour 9 due to the higher price uncertainty. The highest difference between the total option value and the intrinsic value, i.e. the highest **extrinsic value**, occurs when the expected price is close to the variable cost, i.e. in hours 13–17.

The obtained values may be compared to the actual realisations of spot prices and option values during the period Q3 2016. For single hours, stochastic deviations may strongly influence the result. Therefore, we focus on the average values over the 2216 h of the period under question. The results are summarised in Table 11.3. It turns out that the ex-ante option value (left column) exceeds the realised option value if the variable cost as of the end of 2015 (30.29 €/MWh) is

Table 11.3 Backtesting of option values for a gas plant in summer (Q3) 2016

Ex-ante value end 2015	Ex-post value at constant variable cost	Ex-post value at actual variable cost
3.08 €/MW/h	2.09 €/MW/h	4.70 €/MW/h

used (middle column), cf. Fig. 11.7. In that comparison, the realised option value is lower by roughly one third. On the other hand, when taking the actual gas and CO_2 spot prices as a basis for the variable cost, the realised value (right column) exceeds the option value by roughly 50%. Hence, the model provides a first rough approximation, yet it needs to be enhanced to cope with fuel and CO_2 prices uncertainties for more accurate results.

11.8 Challenge: From Asset to System Perspective

We now come back to the question that served as a starting point of our discussion of flexibilities in the electricity system: What is the value of flexible assets in a future sustainable electricity system? One key issue has to be tackled: the endogeneity of market prices in bottom-up electricity system models. Put differently: the methods described in the previous sections, be it the Black–Scholes model or the Bachelier model, treat prices as exogenous (stochastic) input factors. From a system perspective, prices result from the interplay between supply and demand, including their respective rigidities and flexibilities. Therefore, prices and quantities are determined simultaneously in a stochastic equilibrium. And whenever some kind of storage is part of the flexibilities under consideration, this stochastic equilibrium will be one interlinking multiple periods in the year. Solving such an equilibrium in a detailed system modelling approach is challenging.

If we want to evaluate a single flexibility in the context of a prespecified electricity system, there is yet a possible way out: we can start with a stochastic process describing the fluctuations in residual load and then make use of a simple supply-stack model as described in Sect. 7.1.1 to transform the demand fluctuations into price variations.[10] Then, the flexibility may be valued against these prices using standard numerical approaches for option valuation, notably the **least-squares Monte Carlo** approach (cf. Longstaff and Schwartz 2001; Nadarajah et al. 2017, see also Sect. 8.6). Yet one must be aware that this approach breaks down as soon as larger quantities of this flexibility are introduced in the market – because then, the flexibility will start to influence prices in the market. And also the valuation of one flexibility (e.g. batteries) in the presence of another (e.g. pumped hydro storage) is only possible if the latter's operation and pricing strategy are approximated.

[10] The so-called ParFuM-model used by Kallabis et al. (2016) and Beran et al. (2019) is a somewhat more sophisticated version of a merit-order type model that may be applied in that context, cf. Pape (2018) for an application with more long-term focus.

Even more challenging would such an undertaking become if investments into the technologies are to be treated endogenously. In the context of fuel price uncertainty, a corresponding approach has been proposed in Weber (2005), yet this does not cover the full challenge of uncertain renewable power infeed. Hence, important research challenges are still ahead in that field.

11.9 Further Reading

Hull, J. (2021). Options, Futures and other Derivatives. 11th edition. Harlow et al.: Pearson.

This seminal textbook discusses the derivative markets and the various methods to value options on financial markets. It provides an introduction to the world of stochastic calculus applied in finance. Beyond that, it also includes a small chapter on energy and other commodity derivatives.

Burger, M., Schindlmayr, G., & Graeber, B. (2014). Managing Energy Risk. A Practical Guide for Risk Management in Power, Gas and other Energy Markets. 2nd edition. Chichester: Wiley.

The book provides an accessible mathematical treatment of energy trading and the corresponding risks, including the valuation of optionalities.

11.10 Self-check of Knowledge and Exercises

Self-check of Knowledge

1. What is the simple stochastic process in continuous time that serves as the basis for defining other, more complex stochastic processes? What are the key properties of this process?
2. Give the formulas of the following stochastic processes: generalised Wiener process, geometric Brownian motion and mean-reversion process. Indicate also key application areas for these processes.
3. What is an hourly price forward curve and what is it used for?
4. Why are power plants called real options?
5. Explain the basic principles that are used to derive the Black–Scholes option pricing formulas.
6. When is the time value of an option highest? What are the implications for the value of a flexible power plant – especially, when the difference between the expected price (from an hourly forward curve) and variable costs changes?

Exercise 11.1: Mean-Reversion Process

A mean-reversion process according to Eq. (11.6) applied to electricity spot prices p leads to the equation:

$$\mathrm{d}p = \kappa \cdot (\mu - p) \cdot \mathrm{d}t + \sigma \cdot \mathrm{d}z. \tag{11.23}$$

It can be shown that with given price $p(t_0)$, a solution of the stochastic differential equation may be written as

$$p(t) = \left(1 - e^{-\kappa(t-t_0)}\right) \cdot \mu + e^{-\kappa(t-t_0)} \cdot p(t_0) + \sigma\sqrt{\frac{1 - e^{-2\kappa(t-t_0)}}{2\kappa}}\varepsilon \tag{11.24}$$

with ε distributed according to a standard normal distribution, i.e. $\varepsilon \sim N(0,1)$. This may also be rewritten using the notation $\Delta p = p(t) - p(t_0)$ and $\Delta t = t - t_0$:

$$\Delta p = \left(1 - e^{-\kappa\Delta t}\right) \cdot \mu - \left(1 - e^{-\kappa\Delta t}\right) \cdot p(t_0) + \sigma\sqrt{\frac{1 - e^{-2\kappa\Delta t}}{2\kappa\Delta t}}\varepsilon. \tag{11.25}$$

1. Use the time series of daily average spot prices given below to estimate the parameters of the linear regression:

$$\Delta p_t = a + b \cdot p_{t-1} + \tilde{\varepsilon}. \tag{11.26}$$

2. Compare the terms in Eqs. (11.25) and (11.26) to derive formulas to compute the parameters κ, μ and σ of the mean-reversion process from the regression results.
3. Compute the estimated values $\hat{\kappa}$, $\hat{\mu}$ and $\hat{\sigma}$ from the regression parameters $\hat{a}$, $\hat{b}$ and $\hat{\sigma}_{\tilde{\varepsilon}}$, where $\hat{\sigma}_{\tilde{\varepsilon}}$ corresponds to the estimated standard deviation of $\tilde{\varepsilon}$.
   In case, you have not solved part (2) of the exercise, you may use the relationships:

$$\hat{\kappa} = -\frac{1}{\Delta t}\ln(1 + \hat{b}) \quad \hat{\mu} = -\frac{\hat{a}}{\hat{b}} \quad \hat{\sigma} = \hat{\sigma}_{\tilde{\varepsilon}}\sqrt{\frac{2\ln(1 + \hat{b})}{(1 + \hat{b})^2 - 1}}. \tag{11.27}$$

4. Compare the terms in Eq. (11.24) to those of a naïve discretisation of Eq. (11.23) obtained by simply replacing the infinitesimal differences d by discrete differences and using the property given in Eq. (11.3). Using a Taylor series expansion, you may demonstrate that the two converge when Δt tends towards zero.

Exercise 11.2: Hourly Price Forward Curve

The objective is to compute an hourly price forward curve for spot prices on Mondays in February 2021 based on historical observations from preceding years. Collect the historical data for all days in Februarys between, e.g., 2011 and 2020.

Table 11.4 Computation scheme for an HPFC for Feb 2021 based on information available on Nov. 19, 2020

Line no.	Typical days s	Historical values Average prices p_s		Future frequencies in Feb 2021
		Base	Peak	Number of days
1.	Monday			
2.	Tuesday–Thursday			
3.	Friday			
4.	Saturday			
5.	Sunday			
		Future values for February 2021		
		Base	Peak	Off-peak
6.	Number of hours			
7.	Weighted historical average $p_H(\tilde{T})$			
8.	Futures $\tilde{F}(t,\tilde{T})$ on Nov. 19, 2020			
9.	Calibration factor $g(t,\tilde{T})$			

The computation is to be based on the information available on November 19, 2020. On that day, the price quote for Germany at the EEX was 40.39 €/MWh for the product base Feb 2021 and the quote for the product peak Feb 2021 49.86 €/MWh.

1. You may perform the necessary computations using Excel and insert the intermediate results step-by-step into Table 11.4 (cf. also the similar Table 11.2).
2. Make a diagram showing both the average hourly historical prices for Mondays in February and the obtained HPFC for 2021. What are your key observations?
3. February 2021 is still amidst the COVID-19 pandemics that started to swipe over Europe in March 2020. What adjustments, if any, are advisable on the HPFC to reflect the ongoing pandemic situation?
4. Do you expect a lignite power plant with variable costs of 21 €/MWh will be in the money during all hours in February 2021? Why?

Exercise 11.3: Valuation of Financial Options

Evaluate a European call option on a financial stock using the Black–Scholes option pricing model.

The current underlying price is 41.72 €, and the annual volatility σ is estimated at 50%. The risk-free rate is assumed to be 3%. There are 262 trading days per year.

1. Evaluate the option with a time to maturity of 53 (trading) days and a strike price of 44 €. Thereby, you may use Excel and implement the Black–Scholes formulas for option pricing given in Eqs. (11.20–11.22).

2. Now evaluate the value of the corresponding put option with the same expiry date and same strike price. You may use the formulas again from above or make use of the so-called put-call parity:

$$V^{\mathrm{Call}}(S,t) - V^{\mathrm{Put}}(S,t) = S_t - Xe^{-rT} \tag{11.28}$$

3. What happens to the option values when you double the time to maturity? And what if the volatility is doubled?
4. Why is this valuation approach not appropriate when assessing the flexibility value of a power plant?

Exercise 11.4: Valuation of a Power Plant as a Real Option

We aim to determine the hourly value of a power plant with variable costs of 44 €/MWh for a Monday in February 2021 based on the information available on Nov. 19, 2020.

1. Use the HPFC determined in Exercise 11.2 and compute the intrinsic value of the power plant for each hour of this Monday in February.
2. Assume the price volatility for all Monday hours in February is 9.38 €/MWh. What is then the total option value in each hour based on the Bachelier model? You may use Eq. (11.15) to compute this value.
3. Compare the total option values obtained for the different hours of the day – both among themselves and with the corresponding intrinsic values computed in the previous step.
4. Compare the average of the hourly option values with the option value obtained for a financial option with rather similar parameters in Exercise 11.3. What drives the difference? You may also compute the option value using the Bachelier model for the average daily price to support your analysis.

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