The Tapestry of Mathematics—Connecting Threads: A Case Study Incorporating Ecologies, Languages and Mathematical Systems of Papua New Guinea

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Abstract Tapestries interweave strands and colours to create beauty. Mathematics is created through time and place to provide beautiful systems of patterns and purposes. One such place of diverse ecologies and languages that spans thousands of years of intact cultures is Papua New Guinea (PNG). Its diversity of places and peoples with ancient cultures has created a mathematical tapestry. This chapter expands on a small section of this tapestry to challenge most Indo-European views of the beginnings of mathematics such as number systems and mathematical reasoning. Archaeological linguistics, local environments and ecology, sociology of economics, sociopolitical expectations, and mathematical ways of reasoning form threads to create sections of the tapestry. These pieces provide examples of ethnomathematics valuing Indigenous knowledges as mathematical and as important for all societies both within PNG and around the world. Alternative ways of understanding mathematics assist in creating new pieces of the tapestry of mathematics.

Keywords Papua New Guinea · Mathematics and languages · Kula trade and art · Time, place and economy · Ethnomathematics

1 Introduction

Papua New Guinea is situated in the western Pacific Ocean just below the equator and consists of the eastern half of the large island of New Guinea and many other islands, large and small. There are high mountain ranges with steep valleys and fast flowing rivers as well as large fertile valleys. There are coastal areas with large and small river systems, sago swamps, and beautiful coral reefs and beaches. There is often dense rainforest but also areas covered with tall blade-like kunai grass. Papua New Guinea's Indigenous peoples are Melanesian. Indigenous landowners in Timor Leste, West Papua, Ambon and other places in the eastern half of Indonesia, and

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in Island Melanesia from the Solomon Islands to Fiji are also Melanesian although some Polynesian groups have come to these islands.

Although people from south Asia may have visited over the latter half of the 40 000 years of its known inhabitancy, European travellers often passed it by until the mid-1800s. By the end of that century, the colonial power of Germany began colonising the north east of New Guinea and neighbouring islands[1] while Britain colonised the south eastern area and islands (British New Guinea with administration from another colony, Queensland in Australia) with a border agreed in 1884. After Australian Federation in 1901, Australia administered the southern section as a Territory. The First World War saw the north become a Mandated Territory of New Guinea also administered by Australia. Much of the country's north and east were occupied by the Japanese Army during the Second World War, and after the war both Territories were jointly administered by Australia until the country became Independent in 1975. Thus its colonial influence was relatively short especially given that much of the interior of the island was not regarded as safe for travel or colonisation until the late 1940s when patrols were first made into the interior ranges (Gammage, 1998). Air travel was the main form of transport before Independence and is still needed to travel from the capital, Port Moresby to Provincial capitals unless going by sea. History relevant to mathematics education is provided by Paraide, Owens, Muke, Clarkson, and Owens (forthcoming 2022).

In this chapter, the idea of threads of tapestry creating a colourful image is used as a metaphor for the diversity in mathematics in Papua New Guinea creating intriguing, intricate, holistic sections of mathematics. The reason for such diversity is based on language diversity but not solely as place (ecology) and time also impact on mathematical practices. The language details are presented and explicated by looking at the diversity of counting systems. Although all Papua New Guineans are Melanesian, recognition of the differences in the languages and cultures is important as this impacts on the diversity of mathematics, evident firstly in the variety of counting systems but in many other activities requiring systematic reasoning especially those adapting to their ecologies. The chapter provides an overview of how the diversity developed by considering also migrations, cultural relationships, and some of the evidence for this diversity. Language, culture, position in PNG and relationships of different groups provides the backdrop for recognising the diversity of counting systems which have been collated, analysed, and evaluated by Lean (1992), Owens, Lean, with Paraide and Muke (2018) providing a large, textured, and colourful part of the tapestry. The current study further considers culture and ecology for they are intertwined and impact on movement on water and land, land tenure and land use over time which is itself another dimension of life. A few specific cultures are explored in this chapter with sufficient depth to illustrate the mathematics within activities. Diversity provides the texture and colour of the tapestry.

[1] The Netherlands colonised the western half of New Guinea (West Papua, a name used by the Indigenous population) and much of Indonesia.

2 Threads of the Tapestry

The varied colours and textures of the tapestry of mathematics provides a picture of mathematics in this country prior to colonisation but continuing on today. To make this picture intertwining threads from the following disciplines and sources are explored:

- Language and linguistics,
- Archaeology,
- Sociology, culture, and economics,
- Migration, trade, geography and ecology,
- Oral and written histories, and
- Mathematical systems and reasoning.

Some threads form sections of the picture and then putting these together gives a larger picture. Thus these categories intertwine. The result is a fascinating journey into the past with a currency today of culturally different mathematical systems and ways of thinking.

From these disciplines, the threads of time, place, language, culture, thought, and mathematics are woven together to form a rich tapestry of beauty. The creativity of the Indigenous cultures has resulted in beauty but also the tapestry of interwoven knowledge creates mathematical beauty. The chapter focuses on the counting systems to illustrate the interweaving of this knowledge summarising earlier research (Owens, Lean, with Paraide, & Muke, 2018a, 2018b) before extending the arguments by studying time and work, economy, trade, travel, and art.

With limited early European colonisation beginning in the age of enlightenment (second half of the 1800s) when new knowledges were respected and with the continuity of the cultures to this day with little change, the tapestry's threads are strong. The western influence has been limited so the cultural understandings are generally intact despite the impacts of Christian ideals and European languages, especially with the relatively cloudy versions that sometimes occurred for example in churches, and the impacts of other colonialists which went from appalling by today's standards to patronising at best. It seemed ubiquitous that students at government schools were often forbidden to speak in their own language although many church schools allowed it such as the LMS schools in Port Moresby if they had teachers who knew the language or if the linguist working with the language provided some early readers and programs in language (Paraide et al., forthcoming 2022). Language is likely to maintain the abstract concepts and ways of thinking that are often associated with mathematical reasoning. However, with good teaching especially with practical examples and materials, students would still think in their own language (Muke in Paraide et al., forthcoming, 2022). Today teachers who know one of the languages of the children will use those in various forms of codeswitching between languages (Paraide et al., forthcoming, 2022).

2.1 Caveat on This Perspective of the Tapestry

Much, but not all, of the expression of archaeological and historical development of knowledge has been influenced by western approaches to knowledge. In some ways this is fortuitous as this knowledge at least identifies difference and values the cultural ways of knowing and being. Much of the recording may have been western but it was by necessity informed by Papua New Guineans albeit dependent on the quality of the anthropology, linguistics, and the non-Papua New Guineans' relationships with the local community. Further the anthropologists, linguists, and educated members of the community may have known the language of the group to varying degrees. When Papua New Guineans themselves have been researchers and translators working with their people and perhaps with others, mathematics may be noticed by both the Indigenous and non-Indigenous persons as mathematical reasoning and knowledge.

Other areas of knowledge have developed through transcultural linguistic and cross-disciplinary work. This was the case for establishing proto languages in the region and in establishing the diversity of counting systems. Using a mathematical lens on anthropological work has also been utilised. In these cases, the research is dependent on the quality of the initial disciplinary work and its detail. Both these areas are discussed below.

However, it is also acknowledged that these knowledge developments and indeed this chapter could well be viewed differently by people of the cultural group whose relationship with the artefacts, each other, the land and time may not be expressed in western terms (McConaghy, 2000; Nakata, 1998; Report from a seminar in Kárášjohka Norway, 2008). An effort is made to reflect the original expressions even if it is recorded by non-Indigenous people.

3 The Diversity of Cultures

Papua New Guinea has 850 different language or cultural groups who are classified as Melanesian. The languages of these different groups are both Austronesian and non-Austronesian. The diversity of the non-Austronesian languages is exceptional coming from a range of language phyla but also diverse within the one phylum. Some, increasingly many, are endangered languages while others have over 10 000 speakers. Most languages are changing rapidly. The mountainous landscape and fast flowing rivers have not by themselves led to diversity but diversity seems to come from trade, marriage and other relationships, the sense of identity that a language provides, and the pride of people in having a language of their own and being able to speak the languages of others. Neighbours in the same valley or island can speak different languages while other languages, albeit with dialects, might cover hundreds of mountainous square kilometres. On the other hand, across vast waters, the Austronesian Oceanic languages have some similarities with words, grammars and counting systems although there are still differences by which they are separate languages.

Capell (1969) classified the languages, broadly as verbal-oriented or noun-oriented and this affects the way in which plurality is expressed. For example, a numeric adjectival would be likely in a noun-oriented language whereas a suffix, infix, or prefix on a verb root is likely in a verb-oriented language. Furthermore, counting systems have developed patterns for extending the basic counting words into higher levels of counting. This pattern of relationships and combinations of numbers is part of the tapestry on number explored later in this chapter.

However, the landscape is important in the way people, who are mostly subsistence farmers, gain shelter from the available natural materials, decide where to live, how to obtain food, how to maintain ongoing community relations, relate to the land and waters, enjoy life and believe how life begins, continues, and moves to another state. Each of these aspects of life have a mathematical implication. For each, people reason to make decisions and build understanding, some of which are mathematical reasoning in a broad sense. The purpose of this chapter is to express a view of mathematics that focusses on understanding pattern, classification and relationships. The description may not be expressed in algebraic terms but in recognition of these mathematical ways of thinking. In this sense, as an example of ethnomathematics, this chapter values the Indigenous knowledges as mathematical and as important for the societies and for societies beyond, both within PNG and around the world.

3.1 Time and Place

Archaeological evidence covers land usages tens of thousands of years old; DNA evidence of travel, changes in climate, seismic activity, and tools such as obsidian, pottery, and importantly single-outrigger canoes for sea voyaging. Without these, the suggestions for longevity of the languages would be limited. People inhabited the land at least 40 000 years ago and there is understood to be more than one migration across the land. Archaeological sites are spread across the mainland and on the islands. The Australian Indigenous population probably passed through during the ice-age sometime after 150 000 years ago when Sahul (New Guinea, Australia, Tasmania, and Melanesian Islands) was mostly connected.

Excavations at Kuk in the Waghi valley indicated agricultural drainage as far back as 10 000 BP or earlier (Muke et al., 2007). Drainage systems require a mathematical eye for slope and straightness assisted by tools like wooden spades and string tied to pegs. Gardens are formed to cater for cold and water management with various forms of mounding and mulching.

There was an ice-age finishing around 50 000 years ago and a mini ice-age about 15 000 years ago and those together with volcanic activity have changed the nature of the environment in different places. The various changes in the environment have impacted on how cultures survived, moved and interacted with each other. Some of these are recorded in the archaeological linguistic literature (e.g., Swadling, 1997, 2010).

3.2 Time and Language

Linguistic work has also been linked with archaeology. As an example, work by Swadling (1997, 2010) summarised and analysed the archaeology of the Ramu-Sepik inland sea that has appeared at times at different levels since 150 000 years ago with significant movement between 6 000 BP and 2 000 BP but with some continuing changes. These changes had affects on migrations and languages. For example, the related Sawos, Abelam, and Boiken languages moved north and the Boiken Austronesian language having characteristics of a Non-Austronesian language. The sea going inland allowed for marine travel and changes in diets although the water was not salty enough in some areas for marine shellfish. It meant that there was a thriving agricultural centre similar to those found in the highland valleys. The wooden artefacts (paddles or spades) survive and are similar to those used in the highlands and found in mud of the Kuk and Waghi valleys. Trade between coast and highlands was affected by the changes in the level of the inland sea. Again this affects the languages of the area with influences both ways on the Austronesian and non-Austronesian languages. Pottery remains have been found not only in the highland areas but also in the Sepik-Ramu (some with beautiful incision) and further west in West Papua predating the Lapita pottery. Pigs and other items suggesting trade with the highlands e.g. of bird of paradise plumes and possum also appear at the Akari (Eastern Ramu) site from 5 000 BP. Evidence of the trochus shell trade with findings at Akari and at Kutepa rock shelter (near Pogera, Enga by Jo Mangi) suggest that trade also encouraged loan words from Austronesian languages into the Engan family (and also in Southern Highlands and East Sepik areas).

This linguistic archaeology indicates that the Sepik-Ramu Phylum is the oldest, followed by the West Papuan, East Papuan and Torricelli Phyla, and later came the Trans New Guinea Phylum (Ross, 2005; Usher, 2018; Wurm, Laycock, Voorhoeve, & Dutton, 1975). While there may be some differences in the identified languages and phyla, the overall suggestion is that these languages are quite ancient (up to 60 000 years ago for the Sepik-Ramu Phylum in particular) based on archaeological evidence and language differences. This is one reason for claiming that the counting systems are old (Foley, 2005). Furthermore, there are six minor phyla and 7 phylum-level isolates with other isolates at the subphylum level in the New Guinea languages suggesting an existence a very long time ago.

3.3 Migrations and Languages

The Sepik-Ramu Phylum was widely spread by the inland sea in the Sepik area and it provided an easy way of trading between coast and the inland. A significant date was 4 000 BP with the inland lake between the Sepik and Ramu quickly filling in becoming swampy and the alluvial plains being lost requiring migrations (this area is still sparsely populated) (Swadling & Hide, 2005, 2010). At the same time a volcano in West New Britain erupted dispersing people to other islands together with their culture, Lapita pottery, and language. Lapita pottery trade in the New Guinea islands was associated with shell monetary trade. Obsidian trade was also becoming widespread (Spriggs, 2011).

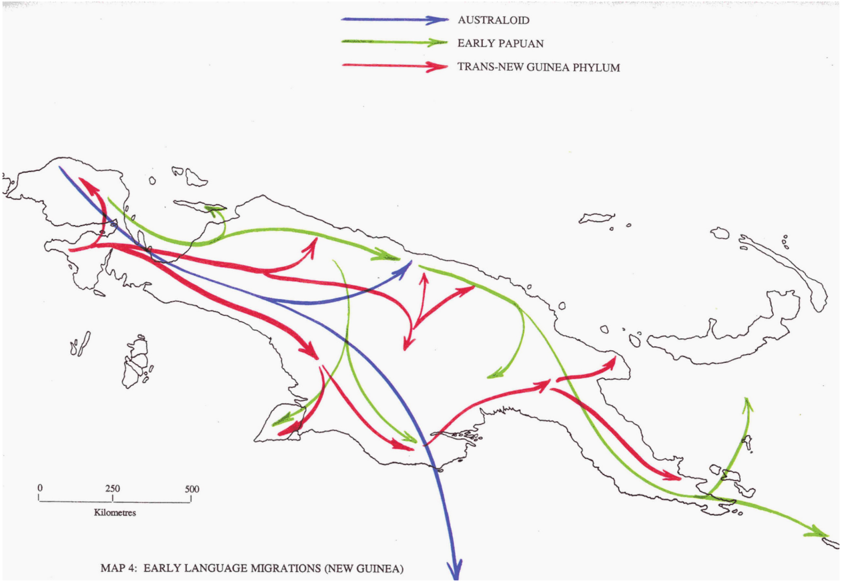
Connections in linguistic archaeology suggest migrations have been in several waves (Ross, 1988) as shown in Fig. 1. Nevertheless, migration of languages has not meant languages have been overridden but rather that the languages may have been modified including the counting systems and thus providing evidence for the history of number.

Bowdler (1993, p. 66) has suggested "mainland southeast Asia, Wallacea (west of New Guinea), Melanesia and Australia colonised virtually simultaneously by modern human beings some 40 000–50 000 years ago". Terrell et al. (1997) in fact reported that there were 9 sites in the Bismarck area (New Ireland and New Britain) with evidence of occupation 40 000–50 000 years ago including inland sites so not just for chance or brief visits from elsewhere. Trade occurred between groups based on pottery and obsidian throughout the Pleistocene period across the north coast and into island Melanesia based on intergenerationally inherited friendships (Terrell et al., 1997).

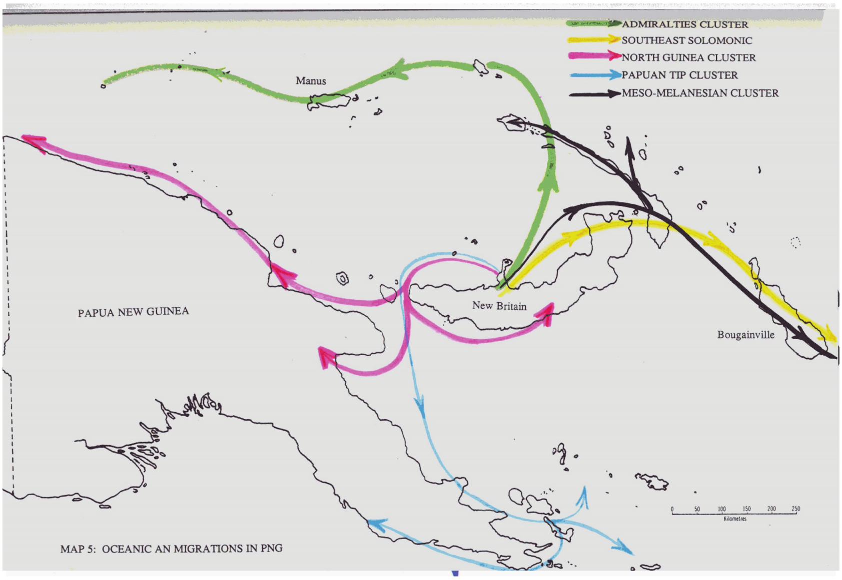
3.4 DNA Assessments

Study of DNA suggests that the Polynesians have mtDNA[2] of Melanesians. The Melanesians may have provided an origin but they are most likely influenced by considerable travel. Within the island Melanesian area travel was frequent over relatively short distances with currents and winds that returned the sailors to their home. For the Pacific, they also show exceptional skills for long travel across the Pacific with evidence of travel both ways (Terrell & Welsch, 1997). There is then evidence from DNA assessments that this occurred even if there were other Asian influences. JCVirus studies (Czarnecki, 2003) would support that Polynesians had a Melanesian origin although later migrations from Asia occurred. The study also suggested that the Australian Aboriginal groups were not directly derived from highland New Guinea groups and that in fact the Trans New Guinea (TNG) phylum was a later

[2] mtDNA is the DNA in the mitochondria of cells and it comes down the line from mothers to sons and daughters so it can be indicative of a long genealogy.

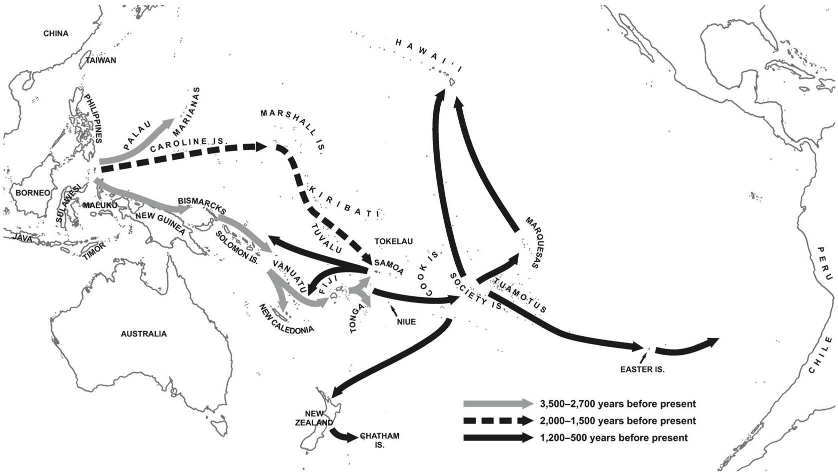


(Source. Lean, 1992 available from Owens, et al., 2018, p. 15).
a. Early language migrations



(Source. Lean, 1992, available from Owens, et al. 2018, p. 16)
b. Austronesian Oceanic migrations.

Fig. 1 Early language migrations



(Source. Addison and Matisoo-Smith (2010, p. 8)

c. Austronesian Oceanic and Polynesian language migrations.

Fig. 1 (continued)

group. Also the Oceanic groups did not penetrate inland on the north coast of New Guinea confirming that the origin of these groups was from the Bismarck area.

It is interesting to consider the degree of influence of Non-Austronesian (NAN) and Austronesian Oceanic (AN) cultures and languages on the counting systems of the languages of the alternate group near the Bismarck Archipelago. Close by, Bougainville in terms of language shows considerable influence between the AN and NAN groups in terms of base 10 counting systems. A distinguishing feature of many AN languages is the use of classifiers in which a class of objects are identified by a morpheme which is combined with the counting system morphemes. Thus counting morphemes are always associated with a classifier morpheme, for example one fish, two fish, three fish, and not free-standing as in English one, two, three etc. Furthermore, some of the neighbouring NAN languages such as Nasioi have classifiers. Interestingly from the counting system point of view, the neighbouring language, Uisai, has the pattern of Manus[3] Province languages with numbers 6, 7 and 8 transcribed as needing 4, 3, and 2 more respectively to make the full group (Kaleva, personal communication, 2006; North Solomons data in Appendix in Lean (1992)).[4] Furthermore, islands such as Mortlock Island, off the coast of Bougainville have Polynesian similarity which is not unexpected given that there are groups in the Solomon Islands of Polynesian background. It does support the view that the Polynesians and Melanesians were linked in terms of their sea-faring capacities and relationships and their on-going traversal back and forth across the Pacific.

[3] See examples on Manus in Fisher (2010), Owens et al. (2018a, 2018b).

[4] There are a couple of languages such as Fore that have 9 as needing one more to make 10.

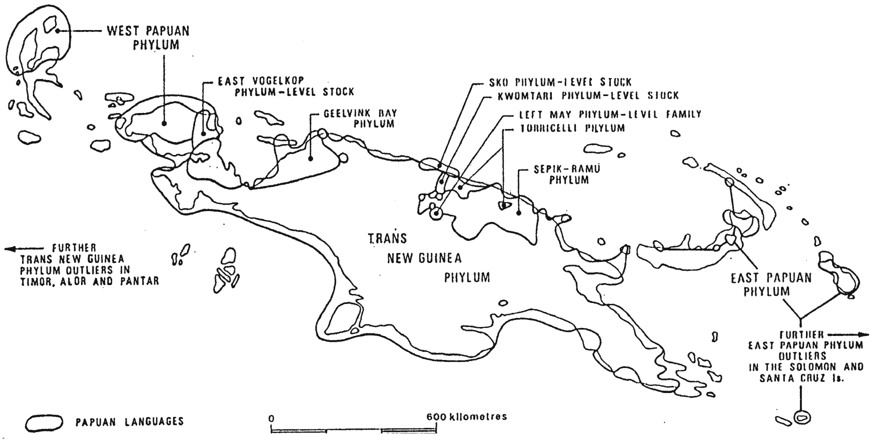
3.5 Displacement and Migration

Although there is not a lot known about small groups being displaced, there is evidence from oral history of displacements. The Aitape people, in East Sepik Province, occupying a sandy land strip when the tsunami hit in 1998 were descended from a group who had migrated from West Papua. Daino (personal communication, 2014) talks of a group in the 1940s moving into her valley near Chuave in Simbu Province with their own language but accepted by her language group to stay. In Mid-Wahgi, the community accepted a displaced group and they now have their own role to play within the clan groupings (Muke, personal communication, 2014; Muke, 2000). One language that was recorded before it died out by Holzknecht (2001) was also indicative of the impact of families having the remaining children taken in by neighbours. There is evidence of people on Manus island migrating there a couple of centuries ago and around Madang in the last century. Displacements may account for different pockets of related languages.

3.6 Proto Languages

The extensive work of Pawley, Ross and their colleagues (e.g., Bowden, Himmelmann, & Ross, 2010; Pawley & Green, 1985; Pawley & Ross, 2006; Pawley, Attenborough, Golson, & Hide, 2005; Ross, 2005) has added over many decades to the extraordinary knowledge of these languages and relationships between languages. It has been their work on Proto Oceanic and on Proto Trans New Guinea Phylum that has been one of the threads used by Lean and myself and colleagues to discuss the longevity of the counting systems (Owens & Paraide, 2019). The linguistic evidence establishes Proto Trans New Guinea Phylum (TNGP) for the hundreds of languages on the mainland through associated archaeological evidence of people residing in places still using languages of this phylum. Languages of older phyla are on the verges of the TNGP (see Fig. 2). A Proto language must have existed at the start of the phylum. That is 10 000 to 20 000 or more years ago in the case of this phylum based on archaeological evidence and language comparison and classification.

By contrast, Proto Oceanic languages are believed to have begun in the Bismarck Archipelago (near Willaumez Peninsula, New Britain, a north east New Guinea island) about 4 200 years ago and spread north west to New Ireland and Manus, west along the mainland coast, east to Island Melanesia, and south around the southern coast. There is evidence of movement through both the pottery known as Lapita and obsidian rock from New Britain, albeit in different waves. Figure 1b illustrates the spread. The phonology of Proto Oceanic is fairly well determined (Pawley & Ross, 2006; Ross, 1989, 2010; Ross, Pawley, & Osmond, 2003). In general, the Oceanic languages are subject-verb-object languages but the subject-object-verb languages may have been in the original Proto Oceanic language or it has been a Non-Austronesian (Papuan) influence as it tends to only occur in New Guinea and



(Source. Lean, 1992, available from Owens, et al., 2018, p. 12)

Fig. 2 Language phyla in New Guinea

in some Solomon Island languages. However, the Polynesian languages have verb-subject-object order as do a few of the Oceanic languages in New Caledonia. This variation is not yet fully explained in terms of archaeological linguistics but it does suggest that this very basic structure of a language can be influenced by surrounding languages over time and through relationships.

While there are some similarities of language across the wide ocean expanses for Austronesian Oceanic languages, there is surprising diversity among the non-Austronesian languages including within some phyla. However, there is some evidence regarding the similarities and the differences of neighbouring languages and why they might have occurred through trade, relationship building and warfare (Holzknecht, 1989; Owens, Lean, with Paraide et al., 2018a, 2018b; Pawley et al., 2005). One important point to raise is that the commonalities between the languages was not as a result of a lingua franca for trade purposes as some have suggested but given the sense of identity associated with the cultures, the Proto languages and the other associated evidence provided below, a result of intergenerational relationships being passed on to future generations for centuries (Terrell et al., 1997). These continue to this day with a strong sense of identity with the family, their customs and beliefs, and their language(s).

3.7 Social Values, Sociopolitics, and Language

Each of the group of villages is autonomous and may be situated within a larger language group with considerable ties. There were, however, many times when food was needed either due to the seasonal weather, climatic changes, catastrophic changes like volcanoes, warfare or poor land quality or lack of area. Since each village and each language remained autonomous, allegiances were often formed through gifts and trade. These were especially important for warfare against other tribes or even within tribes. Shortages of food and goods together with negotiations about land and women were often sources of travel and trade. Trade routes occurred via the ocean and from valley to valley and from the highlands to the coast. Western Highland green stone and coastal shells are found across the highlands. There has been trade of food, pottery, wood, canoes, other implements, signs of importance, culture, designs, and practical subsistence knowledge such as growing food, making food, and building shelters. There was trade between the older non-Austronesian languages and local Austronesian languages and vice versa. Further details of these languages in terms of impact of one on another and resultant changes in counting systems are given in Owens and Lean with Paraide and Muke (2018).

Foley (2010) noted, supported by my personal communications, that some languages have tended to reduce the use of other languages. This occurs in the Sepik and Madang coastlines in particular but also in the Eastern Highlands with the Usarufa men speaking the Kamano-Kanite and Fore languages (Kravia, personal communication, 2015) but not vice-versa while the Siane men and women also speak Chuave (Daino, personal communication, 2014). The Bel language tended to dominate the Madang and Rai Coast area of Madang Province due to long-term trading and a large number of languages, mainly Austronesian but also Non-Austronesia (Sondo, personal communication, 2018 and Leo, 2014). In Papua, Magi, a non-Austronesian language, has influenced the decline in use of Magori, an Austronesian language although there is also widespread use of Motu and English (Onagi, personal communication, 2014).

Furthermore, with the numerous exogamous marriages, women were bilingual for their home and partners' languages but men who traded or travelled were also likely to be multilingual:

> The effects of this intense language contact over many millennia have been profound: the languages show borrowing and diffusion of traits at all levels: lexical items, phonological patterns, bound morphology, word order, syntactic constructions, discourse styles, genres. (Foley, 2010: 797)

As a result, the counting systems, along with other aspects of culture and language, did change and it is necessary to decipher what might be the proto language counting system and what may have been borrowed from a neighbouring language.

A counting system has a structure that is at a deep level in a language as it involves not only words or morphemes (frame words) but also how these morphemes follow an operative pattern. For example, in a base 10 system in English, forty five indicates the number is four tens plus five so the operative pattern is 1 to $9 \times 10 + 1$ to 9. One way of establishing knowledge about the counting systems and their interrelatedness is to analyse and classify them. Using frame words, and operative patterns for combining these words, e.g., it is possible to establish cycles that form the basis of the counting system. For example, a (5, 20) cycle system uses frame words 1 to 5 then 20 by which all other numbers are formed for example 6 to 9 are $5 + 1$ to $5 + 4$ or $n + 1$ to $n + 4$

where n is another morpheme. Only when the cycle system is repeated in a regular way with a new frame word with a new power (e.g., $10 \times 10 = 10^2 = 100$ like the Hindu-Arabic base 10 system) is the word 'base' used to describe the system.

Clearly time has been a factor in the development of these cultures but the environment has also been significant. House styles, garden foods, gathered foods, and the sources of protein vary from the coast to the ridges of the coastal provinces and to the highland ridges and valleys. The interweaving of time, place, culture and language created the mathematical tapestry.

4 The Tapestry Section on Number Systems—A Window into Diversity

Culture impacts on when people count, how much they value counting, how much they value difference between languages, and how much counting is a part of relationship building within the language group and beyond. Our counting (Lean, 1992; Owens et al., 2018a, 2018b) and measurement data (Owens, 2015; Owens & Kaleva, 2008a, 2008b) were derived from first contact records with people who were first colonised in the late 1800s and early 1900s, linguistic records, from people who knew their languages and mathematical practices, through field trips, interviews, University of Goroka mathematics students' reports, focus groups, anthropological studies, and both counting and measurement questionnaires completed by university students, staff and teachers. Examples of these systems and mathematical ways of thinking are provided and variation is explained by considering time and place, culture and language (Owens, Lean, with Paraide et al., 2018a, 2018b). Among the systems are those of different composite units, both of number cycles and for measurement. For example, while most Austronesian languages have a base 10 or decimal counting system, most of the non-Austronesian languages do not. Owens et al. (2018a, 2018b) provides many examples of counting systems with some analysis. A few examples are provided in Tables 1 and 3.

Thus a language with an operative pattern of $6 = 5 + 1$, $7 = 5 + 2$ etc., indicates that the person has a group of 5 and 1 then a group of 5 and 2 units more. This is typical in a digit tally system where 5 is often the word for hand. This is a fairly entrenched system and common across many of the Trans New Guinea Phylum languages. However, this is not the only system within the phylum. Some also have groupings of 4. In order to explore the systems and possible longevity of systems and thus a genealogy, Lean collected about two-thirds of the languages of Papua New Guinea, and West Papua and most of the systems of the rest of Oceania. This diversity of personal oral knowledge and written records provide further colour hues for our tapestry. The ways of analysing or sorting the languages by their cycles, operative pattern and frame words become another important section of the tapestry.

Furthermore, the grouping of the various systems tells of the diversity and that these languages do have significant differences but it is the comparison of neighbouring counting systems that begins to explain some of the ways in which number systems developed. First there have been spontaneous development of counting systems, mostly influenced by the use of hand gestures to tally but also from counting specific kinds of objects in the society. In fact, numerous systems, mostly Oceanic but not all, have classifiers attached to counting morphemes in order to express the counting words. The classifiers range from very few to over 100. There are pockets of systems using classifiers in Non-Austronesian languages that neighbour the Oceanic languages suggesting some kind of influence. The 4-cycle and 6-cycle systems also tend to be in specific areas (examples in Tables 1 and 2). The body-part tally systems are generally across the Sepik, Western, Hela and Southern Highland areas. It should be noted that the early investigators from groups such as the Cambridge Anthropological Expedition to the Torres Strait or more recent philosophical anthropological understandings may indicate quite controversial suggestions, especially the studies of the late 1800s and early 1900s.

Owens et al. (2018a, 2018b) also provided evidence of longevity of counting systems in some cases for tens of thousands of years and of the way systems might develop spontaneously since body digits and body parts were useful for tallying, and pairs and other groupings were significant or naturally practiced in various societies. In some cases, system structures were borrowed from neighbours as well as modifications from the proto language's system with evidence from both proto Trans New Guinea Phylum and Austronesian Oceanic (see Malcolm Ross and colleagues work) (e.g., Ross, Pawley, & Osmond, 2003).

There are exceptions for both the AN and NAN groups, usually associated with neighbouring languages and trade. Some languages have systems with only 1 and 2 combined for other numbers. There are systems that have numerals for 1–5 but others combine 1 and 2 for 3, 2 and 2 for 4 with a numeral for five (with various variations on this) (Owens et al. 2018a, 2018b). These have cycles of 5 or (2, 5) respectively. Many of these then continue on to have combinations such as $5 + 1$, $5 + 2$, up to two 5s then $+1$, $+2$ up to three 5s or more likely these are called two hands and then two hands and a foot, the latter called digit-tally systems. Then there are variations of the base 10 systems. Some have pairs for numbers between 6 and 10 while of the base 10 systems have classifiers so that a different morpheme is used for different categories that are counted. Some of these also have a numeral classifier to distinguish hundreds and thousands. There are also systems with composite groups of 4 or 6. Some of the languages with groups of 4 also have composite groups of 8. The final kind of system, considered an early type of system (Lean, 1992; Owens et al., 2018a, 2018b) are the body-part tally systems in which not only the fingers (hands) and feet are used but others have the number to make 10 so 7 needs 3 more to complete the group of 10. Some other body parts usually up the arm from the small finger, across the head to the other side and down again. There are some languages that have more than one system of counting. Table 2 summarises the pattern of diversity of the counting systems for the NAN languages and the vitality of this section on counting within the tapestry of mathematics.

Table 1 Examples of different counting systems

Language system	Iqwaye (2, 5, 20) cycles	Fasu *or* Namo Me Body part tally	Hagen (4, 8)-cycles plus 10 cycle	Enga–Mai dialect (4, 60) cycles
1	ungwonangi (1)	mēno (little finger)	tendta,tikpa (1)	me(n)dai
2	huwlaqu (2)	tetá (ring finger)	ralg (2)	lapo
3	huwlaqungwa *or* huwlaqanga (2 1)	isiá (middle finger)	raltika (3)	tepo
4	hyaqu-hyaqu (2 2)	kitafá (index finger)	timbikak (4)	kitome(n)de
5	hwolyempu (hand)	kakórea (thumb)	timbikak pumb ti pip (4 to next 1)	
6	hwolye indeumoni ungwonangi (hand to the next 1)	namá (palm of hand)	timbikak pumb ralg pip (4 to next 2)	yungi (yugi) (time)
7	hwolye indeumoni huwlaqu (hand to the next 2)	yatipinu (inside wrist)	timbikak gul ragltiki (4 to next 3)	kalage
8	hwolye indeumoni huwlaqungwa (hand to the next 3)	kāri (forearm)	engag *or* ki tendta (hands complete)	tukulapo (two arrows)
9	hwolye indeumoni hyaqu-hyaqu (hand to the next 4)	tōkona (inside elbow)	engag pumb to gugl (hands to next 1)	**tukutepon(ya) me(n) dai (1 of three arrows)**
10	hwolye kaplaqu (two hands)	kaeyako (upper arm)	engag pumb ralg pip (hands to next 2)	**tukutepon(ya) lapo (2 of three arrows)**
11	hyule yengwonye ungwonangi (down-to leg 1 (two hands implied))	kinu (shoulder)	engag pumb ralg pip to pentipa (hands = 8 to next 2 to next 1)	**tukutepon(ya) tepo (3 of 3 arrows group)**
12	hyule yengwonye huwlaqu (down-to leg two)	kenó (collar bone)	engag pumb ralg pip gulg ragl (hands and 2 to next 2)	**tukutepon (three arrows)**
15	hyule umance hyelaq (leg half that-all)	pari (cheek bone)	engag pumb ralg pip timbikak pukit pumb ti *or* pumb rakltiki (hands 2 3)	mapun(ya) tepo (3 of sweet potato group)
20	hwolye kaplaqu hyule kaplaqu *or* amnye ungwonangi (two hands and two feet *or* one person)	tāku hi (other eye)	engag pump ragl pip wote engag pump ragl pip *or* ki ralg timbikak pukit (8 + 2 8 + 2 *or* 8 × 2 + 4)	yupun(ya) gato (ground earth group complete)

(continued)

Table 1 (continued)

Language system	Iqwaye (2, 5, 20) cycles	Fasu *or* Namo Me Body part tally	Hagen (4, 8)-cycles plus 10 cycle	Enga–Mai dialect (4, 60) cycles
30	anmye ungwonangi amnye ungwoli …. amnye indeumoni hwolye kaplaqu (one, one person to the next two hands)	tāku nama (other palm of hand)	ki raltika *or* engag pump ragl pip engag pumb ragl pip wote engag gul ragl *or* ki ragltiki wote timikak pukit gul ragl (three hands *or* 8 + 2and 8 + 2 and 8 + 2 *or* 3 hands with 4 fingers with 4 fingers and 2 included)	yanapun(ya) lapo (2 of dog)
40	[amnye hyule hwolye hyepu], amnye huwlaqu (two persons)		ki timbikak pumb tip pip *or* engag pumb ragl pip engag pumb ragl pip engag pumb ragl pip engag pumb ragl pip (8x(4 + 1) or (8 + 2) + (8 + 2) + (8 + 2) + (8 + 2)	kujupun(ya) gato (I cut complete)
100	amnye hwolyempu kokoleoule hwolye hyelaqapu (five persons)		kolg mong kiki ki tenta *or* ki engag pumb rag pip gul ragl wote timbikak pukit *or* engag pumb ragl pip engag (8 × 8 8 × 4 2)	

Note The Fasu system has 35 body points with the central one (18) being the nose ridge. Not all parts (neck and ear parts) are given here for brevity. Hagen has many variations so only one source is provided here. The unusual groups of four for the Mai dialect have a pattern shown in bold and applied to all groups of four, so 30 is part of the group, in English 32, in Mai dialect 'dog' and 40 completes the group (I cut complete)

Source Owens, Lean, with Paraide and Muke (2018) based on work by Mimica (1988), May and Loeweke (1981), Mark (2003), and Lean (1992) respectively

Table 2 Showing the distribution of counting system and tally types among the Phyla of the non-Austronesian languages

Types Cycles	West Papuan	East Papuan	Torricelli	Sepik-Ramu	Trans N. G	Minor Phyla	Total
(2)	0	0	0	3	39	0	42
(2, 5)	0	1	16	5	86	1	109
(2’, 5)	0	1	3	5	17	1	27
(2”, 5)	0	0	5	3	31	1	40
(5, 20)	0	1	2	17	52	7	79
(4)	0	0	0	1	6	2	9
(6)	0	0	0	0	5	0	5
Body-parts	0	0	0	8	58	4?	70?
(5, 10)	2	12	0	3	4	0	22
(5, 10, 20)	5	0	0	0	4	3	13
(10)	1	8	0	1	2	0	13
(10, 20)	2	0	0	0	1	0	3

Note Trans N.G. = Trans New Guinea Phylum. These are numbers from collected data, which are most languages but not all languages in Papua New Guinea and Oceania. They exclude 11 West Papuan languages in North Halmahera. From data, it was difficult to classify some languages so the symbol ‘?’ is used to identify that this number may not be correct at the time that the data was collected in the 1970s and 1980s (today, too, more languages have been identified). 2’ are 2-cycle systems with numerals for 1, 2, and 4, all NAN and usually occurring with body-tally systems; 2” are 2-cycle systems with numerals for 1, 2, and 3 with many found also in AN languages—Buang family in Morobe and in Milne Bay usually as (2”, 5, 20) cycle systems
Source Owens et al. (2018a, 2018b: 196)

From Table 2, the rich tapestry of diverse “colours” of patterns in the systems are evident. While the various hues of the (2, 5) systems dominate together with the (5, 20) systems, there is a surprisingly large number of body-part tally systems. These systems fall mainly on the western side of the mainland and are associated with the TNGP but have similarities to the Sepik-Ramu Engan languages or these are a subgroup of TNGP (Pawley, 2012; Ross, 2005). Interestingly these may have very old origins and may be used more for ceremonial and cultural purposes for specific groups (Owens, 2001). This type of system seems to be found only in PNG and Australia. Lean (1992) had provided a number of systems from East Sepik, Sandaun, Enga, Southern Highlands, Hela and Western Provinces, there was a possible system with unlikely features far to the east in Madang Province known only by an older man (Wassmann & Dasen, 1994) but not supported by my oral communications with younger men of the village area interested in their ethnomathematics. Dwyer and Minnegal (2016) have shown that one of the languages that Lean had considered was indeed several different languages and they provided even more data on these systems. These body-tally systems often occurred with 2-cycle systems. Many are truncated for various reasons such as introduced modern currency or similar tonal words being unmentionable in public (Owens et al., 2018a, 2018b; Saxe, 2012, nd).

Details about the 6-cycle systems (see Table 3) have been confirmed to relate to trade from the Ndom Island to the mainland (Hammarström, 2009). Interestingly, in Kanum, it seems that there was a reasonably well developed base system but that different orders of words, and to some extent different morphemes (Donahue, 2008) depended on the relative size of the numbers. Donahue classified these as simple, moderate, and complex number systems. However, given the intricacies of counting in PNG there may be cultural reasons for these variations. It is also worth noting that in other language groups, I found people within the one language group would use the morphemes in different orders and ways to represent numbers suggesting a much more fluid approach to number than a western base 10 system might indicate (Owens et al., 2018a, 2018b). Without the detailed work of linguists like Hammarström, Donahue, Dwyer and Minnegal, it might not be possible to "see" the beauty and hues of these systems in our tapestry.

In the examples, the numbers from 7 to 11 follow the pattern of 6 or another word +1, 2, 3, 4, and 5 (*cf*. Table 3). Various ways of showing 12 include double six and six plus group (probably *hand*). The second power of 6 has a new frame word[5] like hundred is a new frame word for the second power of ten. The words in bold help to focus on the 6-cycle. The Kanum data differ in spelling from earlier data but are otherwise similar in pattern.

Table 3 Three examples of 6-cycle numeral systems of the non-Austronesian Papuan languages

	Kimaghama	Ndom	Riantana	Kanum
1	növere, nubella	sas	mebö	aempy
2	kave	thef	enava	ynaoaempy
3	pendji	ithin	pendö	ylla
4	jando	thonith	wendö	eser
5	mado	meregh	mata	tamp
6	turo, ibolo-nubella	mer	törwa	ptae
7	**iburo**-növere	**mer abo** sas	mebö-**me**	aempy **ptae** 1 + 6
8	iburo-kave	mer abo thef	enava-me	ynaoaempy ptae 2 + 6
9	iburo-pendji	mer abo ithin	pendö-me	ylla ptae 3 + 6
10	iburo-jando	mer abo thonith	wendö-me	eser ptae 4 + 6
11	iburo-mado	mer abo meregh	mata-me	tamp ptae 5 + 6
12	-	**mer an** thef	törwa-me	tarwmpao 12
18		**mer an** ithin		nimpe
36				(ntaop) ptae (big) 6
216				tarwmpao 216

Note This data was sourced from Galis (1960, p. 148) and Drabbe (1926, pp. 6–7) with agreement from Boelaars (1950, p. 34) for Kimaghama, Donahue (2008) for Kanum complex numbers
Source Owens et al. (2018a, 2018b: 120, 122)

[5] Frame words are the numeral words on which other words are built; e.g. 1, 2, 3, 4, 5, 6, 36, 216, etc.

The 6- and 4-cycle counting systems are not common but they are noticeable colours on the tapestry. One reason for this is the way in which the systems have higher powers. There are higher power, larger numbers and a sense of infinity in other systems including the digit-tally systems and those with powers of 10.

An interesting confusion arose with the recording of numbers where reduplication of the morpheme was used for doubling. Some Gahuku speakers recorded the numbers which had *logosi* ('five and five') repeated for addition as *logosi*^{2} and when repeated 4 times (five, five, five, five) as *logosi*^{4} (Owens et al., 2018a, 2018b). Although this might suggest poorly understood school concepts inadvertently used to express traditional counting systems, it also reflects transcultural humour used for reduplication of words like *kaukau* as *kau*^{2} or *kaukau paua* ('sweet potato power').

In summary, the tapestry threads of location of counting systems and the classification of counting systems are intertwined to present a larger part of the picture. That is, counting systems can develop spontaneously, be taken on by another group through relationships such as trade or reciprocity in intergenerational relationships or for privilege. The system may be highly valued such as non-Austronesian (NAN) Ekagi in West Papua with a base 10 (Lean, 1992) or the Iqwaye with a digit tally system of powers of 20 linked to mythology (Mimica, 1988) and oneness (counting to 20 completes the one, the whole, oneness) (Pickles, 2009). Counting systems may be valued for specific cultural activities.

Counting systems of a particular category tend to be found in specific phyla for NAN languages and of clusters of Oceanic languages. The interpretation of the closeness of counting with body parts or objects does not reduce the notion of mathematical reasoning as the early writers suggested, nor can the wholeness and oneness as found in most of the digit tally systems (Iqwaye being one of them), be interpreted by or superimposed on western mathematical ways of thinking about number (Pickle, 2009).

5 Mathematics in Cultural Activities

Mathematics in culture requires reading the knowledges critically in order to establish the impact of culture and the ways in which it has created the mathematics of the culture. In the cases of PNG, the tapestry of mathematics is influenced by several key aspects of the culture. These are:

- trade,
- group decision making and displays,
- valuing culture in mathematical language,
- intergenerational relationships, and
- a need for large numbers for cultural reasons.

5.1 Trade

In the discussion of the archaeological sites, mention has been made of trade and of the impact on languages through loan words but also major changes in the languages. Besides Boiken, Adzera is another language impacted by Non-Austronesian languages in terms of the counting systems. They adapted a 2-cycle system rather than continue with the 10-cycle system as they were building relationships with a number of non-Austronesian language groups on both sides of the Markham valley. Trading routes from the highlands to the coast and between coastal and island communities and between seaboard and inland coastal villages were very common. To carry out trade, relationships between neighbouring villages occurred and a person able to speak multiple languages was highly regarded. However, much of the trade was more like reciprocity exchanges and the quality of the goods in a basket as well as the number of baskets or containers were taken into consideration.

Interestingly, the trade of shells, pottery, obsidian, and designs in tortoise shell attached to hard shell symbolising leadership and male–female relationships together with the trade routes for the ocean-going canoes seem to have encouraged the Oceanic languages to spread widely. They carried some similarities in the languages that developed from Proto Oceanic. By comparison, for the Non-Austronesian languages, it seems that identity was closely linked with the individual language or even differences in dialects or villages. To maintain independent identity meant that the languages developed and remained quite different.

5.2 Group Decision Making and Displays

Most decisions are made by the group whether it be for marriage and bride price or for comparing piles of food items. In counting systems with a cycle of 5, there was ready access to hands and "borrowing" hands of another indicated usually by a nod to the other person and then the next person in the group and so on. This occurs with the Yabiyufa[6] system. Thus the counting in cycles of 5 would reach quite large numbers. This is extended to the digit-tally (5, 20) systems where another person represents each group of 20. It should be noted that group displays often accompanied counting or replaced the need to count as in Dobu-speaking groups on the Papuan islands. It also led to multiple counting systems as in Hagen where the counting system is basically a (4, 8) cycle system but when men counted the line of pigs, one counted up to ten and another keeping track of the tens. Ten involved bending down the four fingers of one hand, then the other (as they would for the (4, 8) cycle system) and then counting the thumbs and marking the full ten; for example, in Gawigl which is a dialect or similar to Hagen, Western Highlands, with the hands together and the

[6] Many languages in the Eastern Highlands Province seem to have a 5-cycle system that may have been recently modified to incorporate a 10 or 20 cycle but Tok Pisin is so common that these languages are becoming endangered.

thumbs were wiped down the lips. In recent times with large compensation claims, 100s and 1000s are marked by objects such as a coil of rope. In general Hagen has a (4, 8) cycle system with multiples of 8 used in counting as shown in Table 1 (Mark, 2003; Strathern, 1977; Vicedom & Tischner, 1943). Both systems have their places and are well recognised in cultural contexts.

5.3 Valuing Culture in the Mathematical Language

Specific ways of counting occurred for different food items. For example, in many coastal areas, coconuts were tied together in pairs and a child might carry one pair in each hand, then as they got older they could carry two pairs and then three pairs in each hand. This is a total of 12 and so counting in 12s occurred in the Tolai or Tinatatuna language of East New Britain (Paraide, 2018). It seems that some of the 6-cycle systems were associated with tying taros in threes and then carrying three in each hand, a total of six. These were placed down in a display pattern for six groups of six. In the Sepik, balls of sago were often sold as a group of 10 and in Motu small fish were strung together in tens. Sometimes this led to the group being given different names for the different items as occurs in Motu, the significance is the recognised group of a specific item.

Motu also has a counting system affected by pairs. Roro, the language to the west in the same Family, clearly has $6 = 2 \times 3$, $7 = 2 \times 3 + 1$, $8 = 2 \times 4$, and $9 = 2 \times 4 + 1$ while Motu has a specific numeral for 7. Some, but not all, Polynesian languages have cyclic patterns at 2×10^2 and 2×10^3 (not a base system at 20^2 and 20^3). These counting systems developed through cultural practices of counting in pairs, often to large numbers (Bender & Beller, 2006; Best, 1907). When counting in pairs using the words used for counting single items, it was understood that the counting of those objects was of pairs. In Yu Wooi or Mid-Wahgi, Jiwaka Province counting of the fingers in pairs was common without actually using ordered number words. As the fingers were bent two at a time, they counted two, two, two, two, two (i.e. hands).

5.4 Intergenerational Relationships

Paraide (2018), a Tolai, provides details of how she learnt her Tinatatuna counting systems and arithmetic from collecting coconuts and taros with her mother with groups of 2, 4 and 12 as well as 3, 6, and 12 respectively, and 120 (ten groups of 12). Muke shares how his father would remember large numbers of pigs or exchange items using his body parts (Owens, Lean, & Muke, 2018a, 2018b). The Kaveve village reports, field trips and personal communications and interviews, and student projects in our study were often a result of how knowledge was part of village living and sharing (Owens et al., 2018a, 2018b).

When certain practices were no longer carried out as a family group then these needed to be revived. However, families like Sondo's from Malalamai, Madang Province, were deliberately involved in everyday and specific cultural activities by which the children learnt their families' (mother's and father's) mathematics, science and language. Children eagerly watch as Elders display their knowledge as in Muke's village on our field trip.

6 The Need for Large Numbers for Cultural Reasons

"Counting does not exist in isolation. It quantifies and qualifies relations between people, objects and other entities" (Bowers & Lepi, 1975, p. 322) for many of the foundational societies of PNG. Goods were for prestige, power, and privilege so the display was important whether or not the goods were counted precisely as some valued counting whereas for others the display itself with approximate amounts was sufficient. For the Tolai counting fathoms of shell money, the generously looped length between the arms was then displayed in groups of a hundred and when there was 500 or 1000 they were bundled and tied into an annulus (Paraide, 2018).

The Tolai and other Austronesian Oceanic groups use shells as money, usually sewn into a strip. There have even been banking systems using shell money and there was a resurgence in using during the COVID-19 crisis. Thus certain items are known to have certain values in terms of the shell money. The impact on counting and measuring as a result is significant. First there can be more than one composite unit or cycle for counting. Thus the composite units of 10, 12, 100, 1000 are used together with lengths such as fathom and half fathom. Second, large numbers are distributed; and the need for counting but also for display is reinforced. Bundles of 10 and 100 fathoms are made so there can be 300 displayed as three bundles. Furthermore, the money is distributed in small lengths providing a sense of division and parts of a whole. This places the counting numbers, whole numbers, into the continuous number system (Paraide, 2018).

Muke (2000; Owens, Lean, & Muke, 2018a, 2018b) and Strathern (1971, 1977) and Mimica (1988) have all shown how highlands communities used large numbers and displays for cultural reasons with interesting and varied complexity.

7 The Tapestry Section on Time and Work Patterns

Having provided the extraordinary section on counting systems as a window into diversity and a review of the cultural aspects that impact on the mathematical tapestry, we take a look at other sections of the tapestry. In Owens (2015), various cultural practices such as making houses and canoes were described and analysed in terms of the profound use of visuospatial reasoning such as having memory for shapes and amounts for different situations, trialling the impact of different heights of the stays for the cross-bars to the outrigger of a canoe and assessing balance, and the embodied memory in which time could be assessed whether asleep, sailing or walking. Each of these had an ecocultural aspect such as the location of the village and the need for food gardens or fish and travel. Movement patterns between places occurred on a regular basis. This section exemplifies further understanding of patterns when thinking differently from western modern cultures in terms of time and activity within the cultural context.

Time was particularly related to everyday activity of the society in Kragur village and across the Kairiru language group (a Western Oceanic, Shouten Cluster (SIL, nd) spoken on Kairiru, Karesau and You Islands off the north coast of PNG, 10 km north west of Wewak (Smith, 1994)). Kragur is on the rough and higher northern end of the island. Patterns of behaviour are influenced by the worldview of cause and effect linked to the supernatural and the people of the present connecting with the supernatural. All accidents and illness are considered as a result of a failed relationship with someone. Certain people have the knowledge of magic to assist with gardens and fishing. People are expected to meet together to discuss and plan for a fishing trip for the *konan*[7] fish to bring the fish *together* for catching. If someone is not present then the fish will not come together. The meeting times and lengths of time seem dependent on who comes, what is happening at sea, suggestions made by the group, and many other things. Groups meet according to relationships between individuals and across the group. Features of the group are identified. Groups vary. Family groups may be identified for the purpose of gathering together sufficient for a gift of, for example, taro. Points for discussion now arise around the gift, its nature and quality, who contributes, and whether some items should now be commodified and paid for. The latter is more and more expected today but that brings some conflict in establishing relationships upon which you can trust in times of difficulty.

However, it is in the comparison of how time and work are understood in the subsistence community and in western workforces that the mathematical patterns emerge more clearly. Previously we have noted that time was embodied so people could arise in the middle of the night to participate in activities before the dawn or they knew how far they had walked or travelled on a canoe taking account of terrain and swells and tides (Owens, 2015). Smith (1994) provides several examples of time and work. In one, he goes with a villager to cut down a tree for a house that the community are helping to build. Through the day, they need to wait for various people to join them and although the man complains about the delay, it is expected. There are several side trips to do other things but they also work together to turn the log into a rectangular beam using adze, axes and knives. The time and work interplay with one informing the other in terms of meaning. Furthermore, the people talk about the moon or sun moving fast when they are busy in their subsistence life style which is always active, always out and about compared to doing a task for which they

[7] *Konan* is the Leing Tau (Kairiru's language) for these long-nosed small fish. They come in schools during the rainy season in the middle of the year close to the beach when the water is calm but now trees have been cut down and the water is too rough so they do not come often so they are hard to catch. When they grow big, the people call them *Wurmak* or *smol tuna* (Tok Pisin) (personal communication, Monika Sikas, 2021).

are paid like being a driver. Some of the paid work was not regarded as work and bored them. They craved the active life of the village. Some village work was sitting around talking, it was still seen as valuable work unlike some paid work where you sit. Some paid work was even considered as being in a prison cell especially during the Japanese occupation where they were not allowed to move around at all but had to work for the Japanese. Time passed too slowly or rather the sun did not move quickly. Some even talked of making magic when working on a plantation to make the sun set more quickly. The sun can go rapidly or slowly. However, their contact with the western use of time such as school, church or employment meant they now had a seven-day week. Thus, as Hallowell (1955) suggested, time is different depending on the experience. There is a different temporal orientation reflected in the language of time for the Kragur. A year is denoted by the passing of Pleiades called *abil* and the moon cycle as *kareo*. The fact that 13 moon cycles matches 12 calendar day months is explained by the days not ending when the moon dies—quoting a villager, Smith reports "the sky goes fast but the days go slowly". The task is valued over the passage of time. Time is not commodified as in the western view of time.

8 The Tapestry of Transactions

Foundational (traditional) mathematics, from generations past to present, is associated with trade of objects and some of the trade routes that help to identify the longevity of cultures (Paraide et al., forthcoming 2022) and interactions of languages and counting systems (Owens et al., 2018a, 2018b). The transactions that occur in the Was valley among the Wola, not far from Mendi and Lake Kutubu in the Southern Highlands are discussed by Sillitoe (2010). The language is a variant of Angal or Mendi, of the Angal-Kewa Family, probably of the Engan Stock, of the Trans New Guinea Phylum (SIL, nd). Sillitoe (2010) provides a comparative analysis of the "economy" of the valley indicating the role land and food play in this subsistent community as well as the kind of transactions and goods. He also argues against the idea of scarcity of time and food as universal driving forces in transactions and against the notion of work for increasing wealth. Rather he considers the sociopolitical aspects of the societies for driving the economy. The establishment and maintenance of relationships is the main force in these societies. Politically it is related to status, not necessarily power to control or demand. Status is related to wealth and to actions involving wealth.

Time such as five days earlier or seven days later can be named together with the moon cycles and two seasons but there is no sense of a period of time being a composite of another unit of time or that using extra time to work will create excess products or wealth. Their efforts contribute to their well-being but it is not work, there is no word for work per se. Transactions and effort for these transactions maintain social order. Thus the economy is for sociopolitical reasons and subsistence. However, there are patterns and Sillitoe was able to classify transactions according to their features and thus an ethnomathematical way of thinking.

Sillitoe divides objects of transactions into categories in order to explain the transactions that take place and the importance of sociopolitical roles in transactions in various ways. Local consumables used in transactions in general include pigs, but taro, pandanus, marsupials, and cassowary are used in a few restricted transactions. Other objects obtained from elsewhere include cosmetic oil and salt in general transactions or cassowary in restricted transactions. More durable local materials for general transactions include feather headdresses (also from elsewhere), possum-teeth beard pins and cassowary eggshell headbands while products originating elsewhere that are durable for all general transactions include pearl shells, cowrie shell necklaces, *nassa* shell headbands, stone axes, black palm bows, steel tools and cash. Equivalence of items is not really an issue here. These are transactions for sociopolitical reasons rather than for amassing wealth. Some are enjoyed, others are practical, some are adornment, some commodities like the shells have lost their value since they are no longer scarce.

While 'men of the clearing' (a meeting place) discuss and make decisions for the community or for exchanges and involve others in projects, there is not a stratified society based on wealth, family, land, or control of other people's time and wealth. These men are leaders and in sociopolitical terms show their wealth and influence in exchanges. While exchange of time and effort does occur, usually at different times and not directly (e.g., helping to make a garden might be repaid by similar action sometime later), and food payments are made together with other small exchanges for time and effort, these transactions are for sociopolitical or subsistence purposes rather than for accumulating wealth. However, there may be economic transactions particularly for outsiders to pay or for a wide range of purchases such as a pig or to receive payment for assistance with a task, or a crop in the garden. These are specific transactions rather than lumped together as sales and commodities. Imported items may be sold on further perhaps in smaller quantities like the cosmetic oil and so some wealth might be made. Products were made like the possum teeth combs which were made from fortuitously capturing a possum and saving up or trading to get enough teeth to make the comb but without specific intention to gain wealth. Men will expect to "receive exchangeables in transactions, not work to produce them" (Sillitoe, 2010: 433) so they do not work or produce to make wealth. There are spheres of transactions—the sociopolitical sphere and that of household subsistence. Sharing taro, preparing a pig kill, or displaying shell money are all ways of noting one's worth as part of the society and perhaps as a man of the clearing. Wealth needs to become part of the exchange system to be used sociopolitically and vice versa and until recently there was little possibility for these to occur due to a lack of cash-paying work. Durable exchange items tend to circulate and be collective property and other items like pigs and cosmetic oil are often divided up among many and passed on. Transactions support people's social standing rather than their social standing being dependent on accumulation of wealth. The individual maintains personal rights within community.

The valuing of these items is fuzzy, as Sillitoe (2010) says. There is considerable tacit knowledge, or knowledge as practice, and a general embodiment of values rather than precise values given to choices about land, food items, pigs, shell, cosmetic oil, wood or other items. People look after themselves and their immediate family so they are individuals but there are community expectations including assisting with tasks that may take more than one person to do, sharing land or assisting with exchanges. Reciprocal relations are important. Most garden work is done alone (roughly six times more likely to be done alone although one other person is also common) despite the hard work involved with clearing and fencing. Fencing is carried out to stop pigs from damaging the ground before or after planting. Steel axes probably halve the time taken compared to a stone axe but this does not seem to have resulted in more gardens being made, at least while there is no access to a market for selling produce. The shape of the garden depends on the landscape and it is tacit knowledge again that is used to decide if it is large enough for the family's needs. Making the fences involves appropriate tools such as axes and wedges to split the logs. It also requires some mathematical tacit skills like use of tools, binding, spacing, sharpening both ends of the posts (to reverse in future when one rots in the ground), or making use of ditches. The ownership of land with rights to others' land assists families when there is a need. The land holdings and gardens tend to be widely spread across the landscape with cultivable land available in pockets throughout so people can expand their holdings if they wished (to gain wealth) without exhausting their resources but they tend not to do this (Sillitoe, 2010).

There is a well-accepted situation in terms of inherited land and land use that is flexible, taking into account family situations and needs. Men may make gardens on their inherited land, their wife's land, his relatives' land and occasionally on his wife's family's land. Efforts are made to keep land used by others recognised as one's inherited land and sometimes there are disputes that need to be resolved. However, land is not sold or used to build up wealth. Except for disasters such as frosts and fighting, the Wola are able to plant sufficient sweet potato to feed families using available plots. The plots used will be negotiated. Most gardens are within an hour walk of the settlement. Some gardens for mixed vegetables are usually closer while taro gardens are often further afield and somewhat hidden—taros are distributed as part of the sociopolitical status. The work of the garden is divided by gender although occasionally this is not followed. Men usually start the garden cutting down rainforest, trees, and kunai grass if planting on an earlier garden that has been left fallow. Women keep the garden clear of weeds and harvest. They are likely to spend four to six hours a day in these activities. However, like the neighbouring Kailila and most other PNG groups, time is not considered as needing to be used today so that more time on work will gain more garden or wealth. Gardens are prepared for a purpose such as feeding the family or preparing for a payment or gift to another family. Sweet potato can last in the ground for some time but taro once picked needs to be eaten fairly quickly. The corms are given to relatives to maintain relationships. Size is important to impress in giving. The family themselves might end up with small corms (Sillitoe, 2010; personal observations).

It is possible to consider the various variables that impact on the various transactions but it is in a fuzzy tacit agreement rather than in numerical values. Exact equivalence of goods is not possible. Goods may remain in one category or move between categories as discussed above. Importantly, the mathematical ways of thinking are different to western views of equivalence by using numbers. Patterns of exchange are reciprocal but not necessarily linear or direct. Sociopolitical reasons influence decisions on transactions which are discussed and open to others to see as a way of maintaining order and memory.

9 The Tapestry of Mathematics in Art

Ethnomathematical studies often refer to art such as the sand drawings of the Pacific and Africa (Ascher, 1994) or wall art with each block having interesting symmetries and transformations (Gerdes, 1998). In PNG, mathematics occurs in the art of many cultural groups including the *kapkap* leadership designs of New Ireland and other Austronesian groups (Owens, 2015). The various designs themselves were traded. In the art of Sepik communities like the Abelam, the line is significant in each panel of artwork and connected through panels (Owens, 2016). The art on shields of the Mid-Wahgi of Jiwaka Province (Muke, 1993; Paraide et al., forthcoming, 2022) involves three zones representative of head, torso and legs with differing colours and shapes depicted on them but connecting to spiritual supports in the battle. These areas of art indicate that shapes, lines and position were indicative of relationships and more than just a shape. Vandendriessche (2014, 2015 discussed the mathematics, particularly the operations, subprocedures and procedures of the string art commonly played in the Trobriands (as elsewhere but with local meaning and story, see also Haddon (1930, reprinted in 1979) and discussions in Owens 2015).

The art of the Trobriands in particular in the Kula trade also display a section of the mathematical tapestry created by patterns that are followed in travel and in design. Kula was the trading between various groups in the islands of Milne Bay Province. Over considerable distance, the sailing canoes travelled to exchange shell-based objects of importance. There is considerable meaning behind the art of the bow (seen by all and resisting splash) and stern boards (a neat trim for the line of the boat and its movement). Both generally occurred at each end because the canoes can in fact go in both directions. The side splashboards were also decorated. The art of the boards forms part of the Massim style of these Milne Bay islands (or Louisiade Islands). It has been highly regarded and sort after since the mid-1800s and pieces can be found in museums. Importantly, symbols or procedures have cultural meanings. They can be positioned, connected and related.

A study of the symbols used in the Kula art is productive in terms of patterns and mathematics. (Fig. 3a, b is a canoe on Kiriwina, Trobriands Islands). This analysis is based on the exceptional, detailed work of Campbell (2002) who initially developed it from museum pieces before an extensive period on Kiriwina to explore the meaning of shape and position, particularly of the Vakuan style (her book shows other canoes). Local people can distinguish between different island art and more local variations and that of the master artisan in the area.

Euclidian shapes, especially those with straight sides, are limiting in terms of understanding shape from many Indigenous cultures whose emphases are on curves

a. Canoe prow, 1973

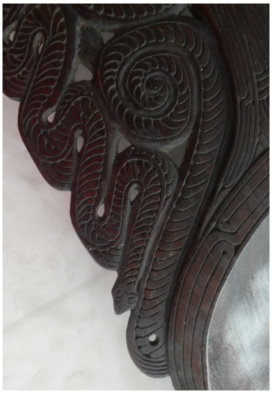
d. End of bowl similar to prow head and body, C curve, tight S with curved end

g. Egret, prow-like carving, flower, circles, modified C curve (above brow of head)

b. Trobriands Canoe, 1973

e. C curve, horizontal opening downwards, repeated

h. Wave curve on head



c. Curve with tight single end and curved long section (like apostrophe; wave, snake, filled in apostrophe markings

f. Lower body section, single end of S with straight section and S-shapes flowing together and intertwined

i. Leader symbol on head similar to some prows at top – this is the head of small ebony statue

Fig. 3 Trobriand art on different objects (*Source* Owens collection)

(Owens, 2015). In the case of Kula art, Campbell has classified a range of curves. Different places and carvers created their own styles. Figure 3 also shows carving undertaken by a follower but not the main empowered carver on wooden artefacts sold to tourists in 1973 (Owens collection) when their art was highly regarded. Tourist art was allowed and not seen as usurping the main carver's role or power. However, you can see a number of the common curves (described in detail below) and that curves did not intertwine (compare with, for example, Celtic knots that intertwine).

The boards themselves have different sections given anthropomorphic names, and like the art of the Mid-Wahgi, specific shapes are likely to be kept within them. Kula art also impressed to engage the spiritual for the journey and trade. Certain parts of the stern board are called head and nose, body or chest, throat, and tail and for the prow board, they are head, body or chest, wing or arm and sometimes a tail to one side. Certain curved shapes represent so-called animals (not necessarily real animals). Position, shape and orientation were important. The orientation often resulted from the position on the board (Campbell, 2002).

Often to support the carved outline, further lines are carved parallel or concentric or in rotated position. Other carving is to fill in the spaces although if painted these would all be of the one colour. This is noticeable on the snake in Fig. 3c. Figure 3g is the representation of a bird, probably an egret and artistic license is taken to curve the beak. Campbell's classification of curves had several subcategories for each category. One of these I will call the S curve. The curve may bend at both ends like an S but they can vary in their tightness, the equality or not of the ends, and the amount of curve with one variation being made of two Ss, one of which is in the rotated position so forming an enclosed twist (Fig. 3f). Another subcategory has one end curved and a straight section (more like an open P) (Fig. 3f middle) and another subcategory has one curve but a very slightly curved section more like an apostrophe (Fig. 3c, g). The curve with a straight section is often duplicated with slide or reflected symmetry and joined or unjoined ends for the curve.

An important category features as the main curve on the head and body section of the prow board. It has the curve forming a C shape with more of a curve on both ends than a C, usually horizontally positioned with the opening up or down (Fig. 3d, g) or obliquely. Another category of curves is like a U with ends curved outwardly or repeated like a wave and either reflected or repeated beneath (Fig. 3c, h).

Another category of shapes include the circles (several in Fig. 3), the parabola shapes with horizontal axes (Fig. 3f), and various enclosed shapes with concave curved ends and slightly curved sides that may be duplicated (see Fig. 4 where similar shapes are used on tapa cloth). These shapes often had another common feature of carvings which was to repeat the end of shapes for emphasis (below the circular hole on Fig. 3c). Some enclosed shapes resemble stylised whales or fish, bats, or other simple beautiful shapes with 3 or 4 curved sides.

Each classification, especially the first group have multiple variations of sub-categories. This is similar to classifying a variety of triangles in Indo-European school geometry. The categories could be seen as similar to classifying polygons, each type of polygon (named by the number of sides and angles) having its own properties. However, each curve and shape is used in specific sections of the boards.

Fig. 4 Tapa designs

While the classifications and positions were recorded by Campbell from artefacts, the follow-up discussions in the villages gave meaning to the shapes and arrangements and confirmed the classification groupings that were made.

The boards in Fig. 3a illustrate, however, the classificatory nature of the curves, and the impact of their position. The basic S curve is repeated around the key C curve on the prow board as a chain of basic Ss. However, when different sized curves in the S are used in the central position of the board, they are at the end of the chain. Wavy curves are used across the top for emphasis of the stylised figures in the centre top of the boards.

Pigments are also used with different meanings and linked to their position in a carving. White is placed in grooves and across the surface of the carved wood with red used in oblique edges that drop to a deeper level of carving marked in black.

From this discussion, the tapestry of patterns is evident by the categorising of shapes and by the associated positions in which they are placed on the board for different meanings and effects. Intergenerational knowledge is passed on to other carvers but only one person will be the new major carver of the group, an inherited position. For the other carvers, careful variations are made as in the tourist pieces, building on some basic curves, procedures and relationships between curves.

Interestingly, paintings on tapa cloths from Oro Province next to Milne Bay Province have similar but unique non-intertwining curves and lines (Fig. 4).

10 Discussion

Fuzzy mathematics is an important aspect of learning mathematics for students (Owens, 2009). It is a way of visualising mathematical concepts. It is often a result of embodied learning as, for example, a child placing counters, stones or other objects in a group for different numbers and noting the name and perhaps symbol associated with the number. This action leads to a sense of size for each of the numbers and their associated numerals. Children learn that 8 and 9 are bigger than 7, for example. This embodied learning and reckoning is an important way of thinking mathematically. In the examples given above, mathematical thinking is associated with doing, with actions. Sillitoe (2010) points out the Wola speakers have no word for work but there are various words for different "doings" for garden making, negotiating, and exchanging.

From the cultural mathematics discussed here there are several important principles of mathematics evolving. These include the notion of patterns of activities including everyday village life. There is considerable mathematical knowledge as part of the tacit knowledge of the environment upon which decisions are made for gardening, fishing, or other activities. There is a memory of the past and these memories are used for comparison and evaluation of current practices albeit gardening or carving or negotiating. Sizes are tacitly and visually known and compared. Discussion is frequently a key aspect of making decisions and the group is important for knowledge to be accepted and valued in various discussions. In using known shapes or curves in art, these classifications and their use and permitted use, their position and variation is known from observation, practice and intergenerational knowledge sharing. None of the activities or symbols of communication are without cultural import.

Counting as illustrated in Tables 1, 2, and 3 might seem clear cut. The system of counting for any language can be classified in terms of its cycles or tally system. However, in practice, the sense of size might not readily be explained in terms of the counting system. For example, there is no measuring of lengths or ways of assessing area of gardens for the Wola. They just look and decide on whether the garden will be large enough based on their previous experiences of the ground and soil, position in the valley, previous use etc. Similar approaches are found across PNG from my conversations, observations, interviews and students' reports. Our classifying of systems is also based on extrapolating from data that may not be quite so straight forward as different groups within the same language group may have recorded different words. These systems may be relatively flexible, especially the 2-cycle system and as we found in different villages and with different informants of the Gahuku-Asaro Alekano system. Systems might occur together with another system or ways of making assessment of size usually in displays or gestures. Displays and gestures with counting in themselves are rituals of cultural significance as in the case of the Iqwaye (Mimica, 1988) and Tolai (Paraide, 2018). Finally counting is generally part of an open discussion in which various other aspects of the situation are also considered especially "oneness" or being one (Pickles, 2009). Thus counting is part of the mathematical tapestry created and used for decision-making.

11 Conclusion

The rapidly changing cultures of today beg for knowledge of their past for the purposes of identity and understanding of their cultural ways of thinking that have mathematical connotations (Owens, 2015). Importantly they are recognising the patterns of actions, curves and other relations, classifications, and measures of some kind. Measures might relate to time or effort or value for transactions or sociopolitical position. However, there is a need also to see if there is a link with school mathematical connotations. In school, numbers might predominate but they are best understood in terms of relations between numbers if the whole of counting is to be applicable to other facets of life or mathematics. Establishing the idea of relationships is a significant way of understanding the essence of mathematics. Detail of relationships such as in art, transactions, and understanding of time are negotiated mathematically. How might there be trajectories in these mathematical understandings as we establish their incorporation into school mathematics? The learning of cultural knowledge is usually over time and with tacit intention. It may be however important to identify these learnings to recognise and strengthen the cultural mathematical ways of thinking and extending these to new areas of reasoning mathematically. Will these be linked then to school mathematics that has entered from a western world?

Real life problems are fuzzy. Sometimes there is information that is not presented but either known from previous experience or needing to be sought. Looking for similarities in earlier problems may help but sometimes it is a matter of looking for all the key issues involved in the situation. Having the disposition to think of alternative approaches when faced with a problem is developed by students who have not learnt that mathematics is a set of procedures learnt from the teacher. Rather they have been presented with problems and situations throughout their life and schooling that encourages them to think of alternative approaches to problems. They have learnt to assess other students' ideas on solving the problem. They do not have a mindset to following set procedures in problem solving. They know useful processes and strategies, and they know how to inquire.

One of the important parts of our learning new concepts revolves around getting started. These preliminary ideas begin with "the stored memories and information processing strategies of the brain interact(ing) with the sensory information received from the environment to actively select and attend to the information and to actively construct meaning" (Osborne & Wittrock, 1983, p. 4). The beginnings of conceptualising are "primitive knowing, making and having images" (Pirie & Kieren, 1994). From these properties are noticed assisting students to learn new concepts and structures (Towers & Martin, 2014). Students will develop concepts that take them beyond the physical object that they are seeing to understanding the concepts embodied in the object or action on the object. Students begin to form relationships between ideas and the concepts develop. At this time, a word from the teacher or fellow student will help students to place the work in the schema stored in the mind, to tag the ideas and images, and to focus attention on the features of the object or experience. Fuzzy beginnings in students constructing their own concepts rather than following a set of procedures results in strong personal and flexible ownership of the concepts. That means they will be able to modify, extend, develop these ideas themselves and relate to other ideas in a mathematically thinking way. Furthermore this thinking often involves patterns and relationships and is the beginning of mathematical modelling so ethnomathematics is a beginning of this whole area of mathematics.

The PNG student familiar with their cultural ways of reasoning mathematically has a head start on fuzzy mathematical beginnings that will permit stronger relationships between their cultural knowledge and mathematical modelling presented from a western or hybrid perspective. They have the capacity to tackle the new mathematical concepts. With creativity and emphasis on inquiry where details are established, sorted and related, and then applied to further areas of learning (Murdoch, 2019; Owens et al., 2015), mathematics may be established to link and grow school mathematics in ways that are meaningful to the society and those living within the society in its subsistence and new commodified arena.

Furthermore, Indigenous knowledges are significant for the whole community and indeed world especially in changing times. Issues of climate change in particular highlight the significance of Indigenous knowledges relevant to science and mathematics that links to the sciences related to the use of land, sea, and air resources. Alternative worldviews are particularly important in respecting and relating to these spaces. Ways of thinking mathematically that are apparent in the Indigenous cultures of Papua New Guinea indicate that there are alternatives in terms of data, patterns and relations and decision-making that may create a stronger world in which to live, one that respects the Indigenous mathematical knowledges and provides for creative mathematical solutions. This chapter analyzes some of the mathematics of Indigenous peoples of PNG and provides a means by which formal schooling can bring about a balanced reconciliation of different epistemologies and worldviews in comparison to the dominant educational model of assimilation. The chapter highlights the importance of political, cultural and educational rights of Indigenous peoples worldwide.

12 PostScript

PNG cultures do not weave cloth from fibrous string. They use the inner bark of trees to make the string but they also beat the bark flat to make tapa cloth for coverings. The string is used to make continuous string bags called bilums using figure-8 loops but can also be made into coverings for body parts by using tight figure-8 loops and variants. Other coverings consist of a group of large leaves strapped over private parts of the body, “grass skirts” made from shredded leaves or grasses, and some use penis goulds. Thus there was no need to weave cloth. However, the various bamboos, pitpits, grasses, pandanus and coconut leaves were used for weaving baskets of various kinds such as fish traps, large nut holders, or food-carrying baskets as well as mats, wall coverings and decorations such as armulets (see Owens, 2015 for illustrations of these artefacts).

However, one art centre in PNG is Kainantu Pottery in the Eastern Highlands. It introduced weaving because there was a good source of carded wool from an agricultural farm in the same province (sheep would not survive in most areas and this farm does not appear to have continued for some years although the rugs are still made, perhaps from imported wool). Its tapestry weaving was that of pictures and abstract design rather than regular "zig-zag" or "diamond" patterns so often found in bilums, woven walls, and baskets (Owens, 2015). It is this kind of tapestry that provides the metaphor (Lakoff, 1987) for the fuzzy mathematics that visually provides fluid mathematical systems and ways of reasoning in PNG without strict single-answer representations. Nevertheless, all tapestries have threads and materials that weave together to form the creative art. Ethnomathematics, language studies, cultural practices, mathematical language, archaeological evidence, oral histories and stories, activities in practice, and field study observations have all joined together to lead myself and others to notice, represent, analyse and understand PNG mathematics. PNG mathematics has the usual mathematical features of representation, analysis, patterns, and relations but they are embedded in PNG cultures. Using language, creating with language, cultural and spiritual practices and associated reasoning, knowledge of the past, of ancestors and of places, continuing oral stories and discussions, practical mathematics in many cultural activities, and observations in the place of belonging have all been part of the crafting of mathematical reasoning in Papua New Guinean societies.

References

Addison, D. J., & Matisoo-Smith, E. (2010). Rethinking Polynesians origins: A West-Polynesia triple-I model. *Archaeology in Oceania, 45*(1), 1–12.

Ascher, M. (1994). *Ethnomathematics: A multicultural view of mathematical ideas*. New York, NY: Chapman & Hall.

Bender, A., & Beller, S. (2006). Numeral classifiers and counting systems in polynesian and micronesian languages: Common roots and cultural adaptations. *Oceanic Linguistics, 45*(2), 380–403.

Best, E. (1907). Maori numeration, the vigesimal system. *Journal of the Polynesian Society, 16*, 94–98.

Boelaars, J. (1950). *The linguistic position of south-western New Guinea*. Leiden, The Netherlands: Brill.

Bowden, J., Himmelmann, N., & Ross, M. (Eds.). (2010). *A journey through Austronesian and Papuan linguistic and cultural space: Papers in honour of Andrew Pawley* (Vol. 615). Canberra, Australia: Pacific Linguistics, Australian National University.

Bowdler, S. (1993). Sunda and Sahul: 30 KYR BP culture area? In M. Smith, M. Spriggs, & B. Franhauser (Eds.), *Sahul in review: Pleistocene archaeology in Australia, New Guinea and island Melanesia: Occasional papers in prehistory, 24* (pp. 60–70). Pacific School of Pacific Studies, Australian National University.

Bowers, N., & Lepi, P. (1975). Kaugel Valley systems of reckoning. *Journal of the Polynesian Society, 84*(3), 309–324.

Campbell, S. (2002). *The art of Kula*. Berg, Oxford International Publishers.

Capell, A. (1969). *A survey of New Guinea languages*. Sydney University Press.

Czarnecki, M. (2003). *JCVirus genotypes in Papua New Guinea: Implications for the Peopling of Oceania*. (PhD). Temple University, USA.

Donahue, M. (2008). Complexities with restricted numeral systems. *Linguistic Typology, 12*, 423–429. https://doi.org/10.1515/LITY.2008.043

Drabbe, P. (1926). Spraakkunst der lamdeensche taal. *Verhandelingen van het Koninklijk Bataviaasch Genootschap van Kunsten en Wetenschappen, 67*(2).

Dwyer, P., & Minnegal, M. (2016). Counting systems of the Strickland-Bosavi languages, Papua New Guinea. *Language and Linguistics in Melanesia, 34*(1), 1–36.

Fisher, J. (2010). *Enriching students' learning through ethnomathematics in Kuruti elementary schools in Papua New Guinea.* Lae, Papua New Guinea: ResearchGate.

Foley, W. (2005). Linguistic prehistory in the Sepik - Ramu basin. In A. Pawley, R. Attenborough, J. Golson, & R. Hide (Eds.), *Papuan pasts: Cultural, linguistic and biological histories of Papuan-speaking peoples* (pp. 109–144). Pacific Linguistics, Australian National University.

Foley, W. (2010). Language contact in the New Guinea region. In R. Hickey (Ed.), *The handbook of language contact* (pp. 795–813). Wiley & Sons.

Galis, K. (1960). Telsystemen in Nederlands-Nieuw-Guinea. *Nieuw Guinea Studien*, *4*(2), 131–150.

Gammage, B. (1998). *The sky travellers: Journeys in New Guinea 1938–1939.* Miegunyah Press, Melbourne University Press.

Gerdes, P. (1998). *Women, art and geometry in southern Africa.* African World Press.

Haddon, K. (1930, reprinted in 1979). *Artists in strings*. New York, NY: AMS.

Hallowell, A. I. (1955). *Culture and experience*. Schocken Books.

Hammarström, H. (2009). Whence the Kanum base-6 numeral system? *Linguistic Typology, 13*(2), 305–319. https://doi.org/10.1515/LITY.2009.014

Holzknecht, S. (1989). The Markham languages of Papua New Guinea. *Pacific Linguistics, C-115*.

Holzknecht, S. (2001). Aribwatsa: A "lost" language of the Markham Family, Papua New Guinea? *Pacific Linguistics D-89: Materials on languages in danger of disappearing in the Asia-Pacific region No. 1: Some endangered languages of Papua New Guinea: Kaki Ae, Musom, and Aribwatsa, D-89*, 145–164.

Lakoff, G. (1987). *Women, fire, and dangerous things: What categories reveal about the mind*. University of Chicago Press.

Lean, G. (1992). *Counting systems of Papua New Guinea and Oceania.* (Unpublished PhD Thesis). PNG University of Technology, Lae, Papua New Guinea.

Mark, K. (2003). *Ethno-mathematics: Is there any mathematics in my culture? Deriving mathematics from culture.* University of Goroka, Papua New Guinea.

May, J., & Loeweke, E. (1981). *Fasu namo me: Fasu lexicon and grammar*. Ukurumpa, Papua New Guinea: SIL.

McConaghy, C. (2000). *Rethinking indigenous education: Culturalism, colonialism and the politics of knowing*. Post Press.

Mimica, J. (1988). *Intimations of infinity: The mythopoeia (cultural meanings) of the Iqwaye counting and number systems.* Berg Publishers.

Muke, C. (2000). *Ethnomathematics: Mid-Wahgi counting practices in Papua New Guinea.* (Unpublished MEd thesis). University of Waikato, Waikato, New Zealand.

Muke, J. (1993). *The Wahgi opo kumbo: An account of warfare in the central highlands of New Guinea.* (PhD). University of Cambridge, Cambridge, UK.

Muke, J., Denham, T., & Genorupa, V. (2007). Nominating and managing a World Heritage Site in the highlands of Papua New Guinea. *World Archaeology, 39*(3), 324–338. https://doi.org/10.1080/00438240701464947

Murdoch, K. (2019). A model for designing a journey of inquiry. Retrieved from https://static1.squarespace.com/static/55c7efeae4b0f5d2463be2d1/t/5dcb82551bdcf03f365b0a6f/1573618265386/A+MODEL+FOR+DESIGNING+A+JOURNEY+OF+INQUIRY.pdf

Nakata, M. (1998). Anthropological texts and Indigenous standpoints. *Australian Aboriginal Studies, 2*(2), 3–12.

Osborne, R., & Wittrock, M. (1983). Learning science: A generative process. *Science Education, 67*, 489–508.

Owens, K. (2001). The work of Glendon Lean on the counting systems of Papua New Guinea and Oceania. *Mathematics Education Research Journal, 13*(1), 47–71.

Owens, K. (2009). The importance of fuzzy mathematics. *Reflections, 34*(1), 20–22.

Owens, K. (2015). *Visuospatial reasoning: An ecocultural perspective for space, geometry and measurement education.* Springer.

Owens, K. (2016). The line and the number are not naked in Papua New Guinea. *International Journal for Research in Mathematics Education. Special issue: Ethnomathematics: Walking the mystical path with practical feet, 6*(1), 244–260.

Owens, K., Cherinda, M., & Jawahir, R. (2015). The importance of an ecocultural perspective for Indigenous and transcultural education. In K. Owens (Ed.), *Visuospatial reasoning: An ecocultural perspective for space, geometry and measurement education* (pp. 245–273). Springer.

Owens, K., & Kaleva, W. (2008a). Case studies of mathematical thinking about area in Papua New Guinea. In O. Figueras, J. Cortina, S. Alatorre, T. Rojano, & A. Sepúlveda (Eds.), *Annual Conference of the International Group for the Psychology of Mathematics Education (PME) and North America chapter of PME, PME32 - PMENAXXX* (Vol. 4, pp. 73–80). Morelia, Mexico: PME.

Owens, K., & Kaleva, W. (2008b). Indigenous Papua New Guinea knowledges related to volume and mass. In *Paper Presented at the International Congress on Mathematics Education ICME 11, Discussion Group 11 on The Role of Ethnomathematics in Mathematics Education.* Monterray, Mexico. Retrieved 2020 https://researchoutput.csu.edu.au/en/publications/indigenous-papua-new-guinea-knowledges-related-to-volume-and-mass

Owens, K., Lean, G., & Muke, C. (2018a). Chapter 3: 2-cycle systems including some digit-tally systems. In K. Owens, G. Lean, with P. Paraide, & C. Muke (Eds.), *History of number: Evidence from Papua New Guinea and Oceania.* Springer.

Owens, K., Lean, G., Paraide, P., & Muke, C. (2018b). *The history of number: Evidence from Papua New Guinea and Oceania.* Springer.

Owens, K., & Paraide, P. (2019). The jigsaw for rewriting the history of number from the Indigenous knowledges of the Pacific. Open Panel 106 Indigenous mathematical knowledge and practices: (crossed-) perspectives from anthropology and ethnomathematics. In A. Arante et al. (Eds.), *Annals of 18th World Congress of United Anthropological and Ethnological Societies IUAES18 Worlds (of) encounters: The past, present and future of anthropological knowledge, Anais 18 Congresso Mundial de Antropologia* (pp. 3468–3487). Florianópolis, Brazil: IUAES18.

Paraide, P. (2018). Chapter 11: Indigenous and western knowledge. In K. Owens, G. Lean, with, P. Paraide, & C. Muke (Eds.), *History of number: Evidence from Papua New Guinea and Oceania.* Springer.

Paraide, P., Owens, K., Clarkson, P., Muke, C., & Owens, C. (2022). *Mathematics education in a neo-colonial country: The case of Papua New Guinea.* Springer.

Pawley, A. (2012). How reconstructable is Trans New Guinea: Problems, progress, prospects. *Language and Linguistics in Melanesia, 12*, 88–169.

Pawley, A., Attenborough, R., Golson, J., & Hide, R. (Eds.). (2005). *Papuan pasts: Cultural, linguistic and biological histories of Papuan-speaking peoples.* Pacific Linguistics, Australian National University.

Pawley, A., & Green, R. (1985). The Proto-Oceanic language community. In R. Kirk & E. Szathmary (Eds.), *Out of Asia: Peopling the Americas and the Pacific* (pp. 161–184). Journal of Pacific History, Australian National University.

Pawley, A., & Ross, M. (2006). The prehistory of Oceanic languages: Current view. In P. Bellwood, J. Fox, & D. Tryon (Eds.), *The Austronesians: Historic and cultural perspectives.* Pacific Linguistics, Australian National University.

Pickles, A. (2009). Part and whole numbers: An "enumerative" reinterpretation of the Cambridge Anthropological Expedition to Torres Straits and its subjects. *Oceania, 79*(3), 293–315.

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics, 26*(2 and 3), 165–190.

Report from a seminar in Kárášjohka Norway. (2008). *Ethics in Sámi and Indigenous research: Report of Kárášjohka, 23–24 October, 2006*. Bredbuktnesv, Norway: Author.

Ross, M. (1988). Proto Oceanic and the Austronesian languages of western Melanesia. *Pacific Linguistics, C-98.*

Ross, M. (1989). Early Oceanic linguistic prehistory. A Reassessment. *Journal of Pacific History, 24*(2), 135–149.

Ross, M. (2005). Pronouns as a preliminary diagnostic for grouping Papuan languages. In A. Pawley, R. Attenborough, J. Golson, & R. Hide (Eds.), *Papuan pasts: Cultural, linguistic and biological histories of Papuan-speaking peoples* (pp. 15–65). Pacific Linguistics, Australian National University.

Ross, M. (2010). Lexical history in the Northwest Solomonic languages: Evidence for two waves of Oceanic settlement in Bougainville and the northwest Solomons. In J. Bowden, N. Himmelmann, & M. Ross (Eds.), *A Journey through Austronesian and Papuan linguistic and cultural space: Papers in honour of Andrew Pawley* (Vol. 615, pp. 245–270). Pacific Linguistics, Australian National University.

Ross, M., Pawley, A., & Osmond, M. (2003). *The lexicon of Proto Oceanic: The culture and environment of ancestral Oceanic society. 2: The physical environment*. Pacific Linguistics, Australian National University.

Saxe, G. (2012). *Cultural development of mathematical ideas: Papua New Guinea studies*. Cambridge University Press.

Saxe, G. (nd). Cultural development of mathematical ideas. Retrieved from http://www.culturecognition.com/.

SIL. (nd). *Ethnologue: Languages of the world*. Retrieved from http://www.ethnologue.com/.

Sillitoe, P. (2010). *From land to mouth*. Yale University Press.

Smith, M. F. (1994). *Hard times on Kairiru Island: Poverty, development, and morality in a Papua New Guinea village*. University of Hawaii Press.

Spriggs, M. (2011). Archaeology and the Austronesian expansion: Where are we now? *Antiquity, 85*(328), 510–528.

Strathern, A. (1971). *The rope of moka: Big-men and ceremonial exchange in Mount Hagen, New Guinea*. Cambridge University Press.

Strathern, A. (1977). Mathematics in the *moka. Papua New Guinea Journal of Education, Special Edition the Indigenous Mathematics Project, 13*(1), 16–20.

Swadling, P. (1997). Changing shorelines and cultural orientations in the Sepik-Ramu, Papua New Guinea: Implications for Pacific prehistory. *World Archaeology, 29*(1), 1–14. https://doi.org/10.1080/00438243.1997.9980360

Swadling, P., & Hide, R. (2005). Changing landscape and social interaction: Looking at agricultural history from a Sepik-Ramu perspective. In A. Pawley, R. Attenborough, J. Golson, & R. Hide (Eds.), *Papuan pasts: Cultural, linguistic and biological histories of Papuan-speaking peoples* (Vol. 572, pp. 289–327). Pacific Linguistics, Australian National University.

Swadling, P. (2010). The impact of a dynamic environmental past on trade routes and language distributions in the lower-middle Sepik. In J. Bowden, N. Himmelmann, & M. Ross (Eds.), *A journey through Austronesian and Papuan linguistic and cultural space: Papers in honour of Andrew Pawley* (Vol. 615, pp. 141–159). Pacific Linguistics, Australian National University.

Terrell, J., Hunt, T., & Gosden, C. (1997). Human diversity and the myth of the primitive isolate. *Current Anthropology, 38*(2), 155–195. https://doi.org/10.1086/204604

Terrell, J., & Welsch, R. (1997). Lapita and the temporal geography of prehistory. *Antiquity, 71*, 548–572.

Towers, J., & Martin, L. (2014). Building understanding through collective property noticing. *Canadian Journal of Science, Mathematics, and Technology Education, 14*(1), 58–75.

Usher, T. (2018). Newguineaworld. Retrieved from https://sites.google.com/site/newguineaworld/about-newguineaworld.

Vandendriessche, É. (2014). Cultural and cognitive aspects of string figure-making in the Trobriand Islands. *Journal De La Société Des Océanistes, 209–224*,. https://doi.org/10.4000/jso.7182

Vandendriessche, E. (2015). *String figures as mathematics: An anthropological approach to string figure-making in oral traditional societies*. Springer.

Vicedom, G., & Tischner, H. (1943). *Die Mbowamb; die Kultur der Hagenberg-Stämme im östlichen Zentral-Neuguinea*. Hamburg, Cram: de Gruyter.

Wassmann, J., & Dasen, P. (1994). Yupno number system and counting. *Journal of Cross-Cultural Psychology, 25*, 78–94.

Wurm, S., Laycock, D., Voorhoeve, C., & Dutton, T. (1975). Papuan linguistic prehistory, and past language migrations in the New Guinea area. In S. Wurm (Ed.), *New Guinea area languages and language study. Vol. 1: Papuan languages and the New Guinea linguistic scene, Pacific Linguistics (C-38)* (pp. 935–960).