Chapter 8
State and Attacks Estimation for Nonlinear Takagi–Sugeno Multiple Model Systems with Delayed Measurements

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8.1 Introduction

The present work deals with state and cyber-attacks estimation for nonlinear Takagi–Sugeno systems with variable time-delay measurements. The use of the sector nonlinearity approach with the nonlinear Takagi–Sugeno systems allows us to extend the results to a wide variety of control process. Indeed, fuzzy control systems have been presented as an important tool to represent and implement human heuristic knowledge to control a system. This theory is based on a class of fuzzy models presented by the authors in Takagi and Sugeno (1985), which were designed to describe nonlinear systems as a collection of Linear Time-Invariant (LTI) models blended together with nonlinear functions, known as weighting functions. The Takagi–Sugeno (T–S) fuzzy structure, also called quasi-LPV (linear parameter variable) systems, offers an efficient representation of nonlinear behavior while relatively simple compared to general nonlinear models (Benzaouia and Hajaji 2014). In this contribution, we propose to represent the nonlinear system described by T–S models by an equivalent form extending the result presented in Bezzaoucha and Voos (2019) and Bezzaoucha Rebai and Voos (2019) for state and attacks estimation with delayed measurement. The objective is to obtain sufficient conditions in terms of $LMIs$ formulation for the observer design in order to ensure the asymptotic convergence of the estimation errors with an $\mathscr{L}_2$ attenuation constraint.

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The aim of this chapter is to tackle the state estimation of a nonlinear system subject to data deception attacks and variable time-delay measurements. Based on the same principle of own previous contributions (Gerard et al. 2018; Bezzaoucha Rebaiı et al. 2018) the malicious attacks can be modeled as adversary signals (i.e., like disturbances, unknown inputs, faults,...) introduced via the internal network by hackers and affecting the sensors and/or actuators data (Pajic et al. 2017; Teixeira et al. 2012). The isolation and reconstruction of these cyber-attacks can be seen from a control point of view as uncertain parameter problem.

Indeed, based on Bezzaoucha et al. (2013), we propose to use previously developed approach, applied for joint state and time-varying parameters estimation of Takagi–Sugeno models in order to reconstruct the state and cyber-attack signals for nonlinear LPV systems. In this book chapter, we will consider in addition the delayed measurement constraints.

The considered actuator/sensor attacks are modeled as time-varying parameters with multiplicative effect on the actuator input signal and sensor output signal, respectively. Based on the sector nonlinearity description, and using the convex property, the nonlinear model will be presented in a Linear Parameter-Varying (LPV) form, then an observer allowing both state and attack reconstruction is designed by solving an LMI optimization problem, exactly as detailed in Bezzaoucha and Voos (2019).

So far, to the best of our knowledge, there has been no delay-dependent method reported to study the observer-based H_∞ control for T–S fuzzy systems dealing with the state and attack reconstruction problem. Indeed, in general, practical problems, especially in Networked Control Systems (NCS) , the delayed measurement such as traffic flow in communication networks have to be considered, especially for stability reasons and measurement-based observer design. As it was developed in Orjuela et al. (2007) and Bezzaoucha et al. (2017), the considered approach provides an alternative and attractive path to deal with complex nonlinear systems and to obtain an equivalent representation by bounding the parameters and using the well-known sector nonlinearity transformation (SNT).

8.1.1 Contributions and Outline

Robust control and quadratic stabilization for linear systems with uncertain parameters have been considered in Shaked (2001). For fuzzy systems without uncertainties, Liu and Zhang in Liu and Zhang (2003) have proposed a new design method based on the H_∞ norm. However, their technique is based on a two-step approach which appears to be a drawback. Like in Bezzaoucha and Voos (2019), we proposed a method to simplify and to improve the existing design methods of robust fuzzy state observer design with disturbance attenuation for uncertain T–S fuzzy systems. The developed method gives not only the observer gains (for the state and the attacks) on a single-step analysis.

In practice, time delay often occurs in the transmission of information or material between different parts of a system. Transportation systems, communication systems, chemical systems, and power systems are example of time-delay systems. Also, it has been shown that the existence of time delay usually becomes the source of instability and deteriorates the performances of systems. Therefore, T–S fuzzy systems have been extended to deal with nonlinear systems with time-delay (Benzaouia and Hajaji 2014). The existing results of stabilization and stability criteria for this class of T–S fuzzy systems can be classified into two types: delay independent, which is applicable to delays of arbitrary sizes, and delay dependent, which includes information on the size of delays.

Although it is well known that delay-dependent results are less conservative than delay-independent ones, there are few delay-dependent results which study the problem of observer-based H_∞ control for T–S fuzzy systems with varying time delay. This motivates the research in this work to study this problem, i.e., the state and attacks reconstruction problem for nonlinear Takagi–Sugeno systems with delayed measurements. In this chapter, the asymptotic stabilization of uncertain (attacked) T–S observer systems with variable time-delay measurement is studied. Different from the methods currently found in the literature (Yue and Han 2005; Tian and Peng 2003), the proposed method does not need any transformation in the LKF (Lyapunov–Krasovskii functional), and thus avoids the restriction resulting from any used transformation. It improves the presented results in Bezzaoucha and Voos (2019) and Bezzaoucha et al. (2013) for two main aspects. The first one concerns the polytopic rewriting of the time-varying data deception attacks, and the second one is the time-delay measurement consideration and the delay-dependent stabilization conditions. Based on previous results, published in Bezzaoucha and Voos (2019), and on the sector nonlinearity approach, sufficient conditions in term of $LMIs$ formulation are given for the observer design. We will show that, despite the presence of cyber-attack (i.e., data deception attacks on both actuators and sensors) and the delayed measurements, the proposed observer is efficient and ensures the asymptotic convergence of the estimation errors with an $\mathscr{L}_2$ attenuation constraint.

8.1.2 Chapter Organization

The present contribution is organized as follows. After a brief introduction and a short overview of related works in Sect. 8.1, the problem statement is detailed in Sect. 8.2 by the presentation of the polytopic modeling of time-varying nonlinear systems and time-varying parameters (malicious attacks) with an LPV model of physical plant under data deception attacks. In Sect. 8.3, the main result/contribution of this work is given in terms of a general theorem for the observer design strategy and time-delay-dependent stability conditions. In Sect. 8.5, an illustrative example is given. From a basic nonlinear model of a biological wastewater treatment plant, the proposed approach is applied and illustrated with simulations. Conclusion will be given in the last section.

8.2 Problem Statement

The problem of state reconstruction in the presence of faults and attacks, also denoted as secure state estimation, has recently attracted considerable attention from the control community. The problem of reconstructing the state under actuator/sensor attacks is closely related to fault-detection and fault-tolerant state reconstruction. Based on the approach presented in previous works Bezzaoucha et al. (2013), Bezzaoucha et al. (2013) and adapted to the cyber-security problem, as presented in Bezzaoucha and Voos (2019) we address the design of observers that can accurately reconstruct the state and attacks of a cyber-physical system under actuator/sensor attacks with delayed measurements.

For that, we propose a simultaneous state and time-varying (attacks) observers for nonlinear systems in the presence of corrupted inputs and measurements, more specifically, the so-called false data injection attacks. In the spirit of a Luenberger observer, a state and attacks reconstruction algorithm is proposed based on the LMI approach and convex optimization problem. The second point of the problem statement will be about the variable time-delay measurements, which will be considered in the observer analysis, as shown in Orjuela et al. (2007).

8.2.1 False Data Injection Attacks on Actuators/Sensors

Based on results presented in Bezzaoucha and Voos (2019) and Orjuela et al. (2007), we assume that the attacker modifies the gain/s of the sensor and/or the actuator of the control system, which represent the injection of false information from sensors or controllers. This chapter is also dealing with a problem characterizing dynamical systems, which is the variable time-delay measurements. Mathematically speaking, explicit equations of both sensor and actuator signal attacks are derived and represented as time-varying multiplicative actuator/sensor faults/attacks. The Polytopic T–S approach is then used to reconstruct these signals in real time.

In this section, we assume that a malicious third party wants to compromise the integrity of the system. The attacker is assumed to have the following capabilities:

- He/she knows the system model, i.e., we assume that the hacker knows the system model and matrices.
- He/she can control the readings of the sensors and the actuators, i.e., modifies their values.
- The intrusions are represented as time-varying multiplicative actuator—sensor faults—attacks. The attacks signals are, of course, unknown, but bounded. Their min and max values are supposed to be known. Indeed, this assumption is not conservative since we suppose that if the boundaries are exceeded the attacks effect will be too obvious and easily detectable. Meaning, the hacker should respect the min and max values to a certain extent if he/she wants to remain undetectable.

- The nonlinear system is subject to time-variable delayed measurements. The time delay $\tau(t)$ is assumed perfectly known and satisfies the following conditions:

$$\begin{cases} 0 \leq \tau(t) \leq \tau \\ \dot{\tau}(t) \leq \gamma < 1. \end{cases} \tag{8.1}$$

8.2.2 *Polytopic Modeling of Time-Varying Nonlinear Systems with Delayed Measurements*

Let us consider the nonlinear system represented by the following equations:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(x(t))(A_i x(t) + B_i(t)u(t)) \\ y(t) = C(t)x(t), \end{cases} \tag{8.2}$$

s.t. A_i, B_i, and $C(t)$ are constant matrices with appropriate dimensions.

With the time-varying matrices $B_i(t)$ and $C(t)$ defined by the following:

$$\begin{cases} B_i(t) = B_i + \sum_{j=1}^{n_{\theta_u}} \theta_j^u(t)\overline{B}_{ij} \\ C(t) = (I_m + F(t))C, \end{cases} \tag{8.3}$$

s.t. B_i, $\overline{B}_{ij}$ are constant matrices with appropriate dimensions and $\theta_j^u(t)$ time-varying unknown parameters and correspond to the multiplicative actuator attacks.

The matrix $F(t) \in \mathbb{R}^{m \times m}$ is defined by

$$F(t) = \text{diag}(\theta^y(t)), \tag{8.4}$$

s.t. $\text{diag}(\theta^y(t))$ corresponds to a diagonal matrix with the terms $\theta_j^y(t)$ (sensor attacks) on its diagonal.

The time-varying parameter vector $\theta(t), \theta(t) \in \mathbb{R}^n$ is defined by $\theta(t) = \begin{pmatrix} \theta^u(t) \\ \theta^y(t) \end{pmatrix}$ with $\theta^u(t) \in \mathbb{R}^{n_{\theta_u}}$ and $\theta^y(t) \in \mathbb{R}^{n_{\theta_y}}$ correspond, respectively, to the actuator and sensor attacks ($n = n_{\theta_u} + n_{\theta_y}$). $x(t) \in \mathbb{R}^{n_x}$, $y(t) \in \mathbb{R}^m$ and $u(t) \in \mathbb{R}^{n_u}$ correspond, respectively, to the system state, output, and control. The nonlinear system is modeled thanks to a polytopic representation with r sub-models. This representation may be obtained in a straightforward way by applying the Sector Nonlinearity Transformation (SNT). The interested readers can refer to Bezzaoucha et al. (2013) and Tanaka and Wang (2001) for more development details.

$F(t)$ may be expressed as

$$F(t) = \sum_{j=1}^{n_{\theta_y}} \theta_j^y(t) F_j, \tag{8.5}$$

with $n_{\theta_y} = m$, F_j are matrices of dimension $\mathbb{R}^{m \times m}$ and where the element of coordinate (j, j) is equal to 1 and 0 elsewhere. The coordinate j corresponds to the number of the attacked sensor. The terms $\theta_j^y(t)$ are time-varying unknown parameters and represent the multiplicative sensor attacks.

8.2.3 *Polytopic Modeling of Time-Varying Parameters (Malicious Attacks)*

As presented in Bezzaoucha and Voos (2019), the actuator data deception or false data injection is modeled thanks to the time-varying parameters $\theta_j^u(t)$. These attacks are of course unknown but bounded $\theta_j^u(t) \in [\theta_j^{2u}, \theta_j^{1u}]$, with known bounds. Applying the SNT transformation, each parameter $\theta_j^u(t)$ can always be expressed as

$$\theta_j^u(t) = \widetilde{\mu}_j^1(\theta_j^u(t))\theta_j^{1u} + \widetilde{\mu}_j^2(\theta_j^u(t))\theta_j^{2u}, \tag{8.6}$$

with

$$\widetilde{\mu}_j^1(\theta_j^u(t)) = \frac{\theta_j^u(t) - \theta_j^{2u}}{\theta_j^{1u} - \theta_j^{2u}}, \widetilde{\mu}_j^2(\theta_j^u(t)) = \frac{\theta_j^{1u} - \theta_j(t)}{\theta_j^{1u} - \theta_j^{2u}} \tag{8.7}$$

$$\widetilde{\mu}_j^1(\theta_j^u(t)) + \widetilde{\mu}_j^2(\theta_j^u(t)) = 1, \ \forall t.$$

Based on the same way, the sensor data deception or false data injection is modeled thanks to the time-varying parameters $\theta_j^y(t)$, such that

$$\theta_j^y(t) = \overline{\mu}_j^1(\theta_j^y(t))\theta_j^{1y} + \overline{\mu}_j^2(\theta_j^y(t))\theta_j^{2y} \tag{8.8}$$

with

$$\overline{\mu}_j^1(\theta_j^y(t)) = \frac{\theta_j^y(t) - \theta_j^{2y}}{\theta_j^{1y} - \theta_j^{2y}}, \overline{\mu}_j^2(\theta_j^y(t)) = \frac{\theta_j^{1y} - \theta_j(t)}{\theta_j^{1y} - \theta_j^{2y}} \tag{8.9}$$

$$\overline{\mu}_j^1(\theta_j^y(t)) + \overline{\mu}_j^2(\theta_j^y(t)) = 1, \ \forall t.$$

Replacing (8.6) and (8.8) into (8.3), we obtain

$$\begin{cases} B_i(t) = B_i + \sum_{j=1}^{n_{\theta_u}} \sum_{k=1}^{2} \widetilde{\mu}_j^k(\theta_j(t))\theta_j^{k^u}\overline{B}_{ij} \\ C(t) = \left(I + \sum_{j=1}^{n_{\theta_y}} \sum_{k=1}^{2} \overline{\mu}_j^k(\theta_j^y(t))\theta_j^{k^y} F_j \right) C. \end{cases} \quad (8.10)$$

8.2.4 LPV Model of Physical Plant Under Data Deception Attacks and Delayed Measurements

In order to have the same weighting functions for all the time-varying matrices $B_i(t)$ and write $C(t)$ as a simple polytopic matrix, exploiting the convex sum property of the weighting functions $\widetilde{\mu}_j(\theta_j^u(t))$ and $\overline{\mu}_j(\theta_j^y(t))$ of each parameter $\theta_j^u(t)$ and $\theta_j^y(t)$ (see Bezzaoucha et al. 2013 for computation details), (8.10) is written as

$$\begin{cases} B_i(t) = \sum_{j=1}^{n_{\theta_u}} \left[\left[(\widetilde{\mu}_j^1(\theta_j^u(t))\theta_j^{1^u} + \widetilde{\mu}_j^2(\theta_j^u(t))\theta_j^{2^u})\overline{B}_{ij}\right]\right] \times \\ \qquad \left[\prod_{\substack{k=1 \\ k \neq j}}^{n_{\theta_u}} \sum_{m=1}^{2} \widetilde{\mu}_k^m(\theta_k^u(t)) \right] + B_i \\ \qquad = B_i + \sum_{j=1}^{2^{n_{\theta_u}}} \widetilde{\mu_j}(\theta^u(t))\overline{\mathscr{B}}_{ij} \\ C(t) = \left(I + \sum_{j=1}^{2^{n_{\theta_y}}} \overline{\mu_j}(\theta^y(t))\overline{F}_j \right) C \end{cases} \quad (8.11)$$

with

$$\widetilde{\mu_j}(\theta^u(t)) = \prod_{k=1}^{n_{\theta_u}} \widetilde{\mu}_k^{\sigma_j^k}(\theta_k^u(t)), \overline{\mathscr{B}}_{ij} = \sum_{k=1}^{n_{\theta_u}} \theta_k^{u\sigma_j^k} \overline{B}_{ik} \quad (8.12)$$

and

$$\overline{\mu_j}(\theta^y(t)) = \prod_{k=1}^{n_{\theta_y}} \overline{\mu}_k^{\sigma_j^k}(\theta_k^y(t)), \overline{F}_j = \sum_{k=1}^{n_{\theta_y}} \theta_k^{y\sigma_j^k} F_j, \quad (8.13)$$

where the global weighting functions $\widetilde{\mu_j}(\theta^u(t))$ and $\overline{\mu_j}(\theta^y(t))$ satisfy the convex sum property. The index σ_j^k is either equal to 1 or 2 and indicates which partition of the k^{th} parameter ($\widetilde{\mu_k}^1$ or $\widetilde{\mu_k}^2$, i.e., $\overline{\mu_k}^1$ or $\overline{\mu_k}^2$) is involved in the j^{th} sub-model. The relation between the sub-model number j and the σ_j^k indices is given by the following equation:

$$j = 2^{n_{\theta_u}-1}\sigma_j^1 + 2^{n_{\theta_u}-2}\sigma_j^2 + \cdots + 2^0\sigma_j^{n_{\theta_u}} - (2^1 + 2^2 + \cdots + 2^{n_{\theta_u}-1}) \tag{8.14}$$

for the actuator, and in the same way for the sensor:

$$j = 2^{n_{\theta_y}-1}\sigma_j^1 + 2^{n_{\theta_y}-2}\sigma_j^2 + \cdots + 2^0\sigma_j^{n_{\theta_y}} - (2^1 + 2^2 + \cdots + 2^{n_{\theta_y}-1}). \tag{8.15}$$

Finally, using Eq. (8.11), the nonlinear LPV system (8.2) becomes

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{2^{n_{\theta_u}}} \mu_i(x(t))\widetilde{\mu_j}(\theta^u(t))(A_i x(t) + \mathscr{B}_{ij}u(t)) \\ y(t) = \sum_{k=1}^{2^{n_{\theta_y}}} \overline{\mu_k}(\theta^y(t))\widetilde{C}_k x(t), \end{cases} \tag{8.16}$$

$$\mathscr{B}_{ij} = B_i + \overline{\mathscr{B}}_{ij},\ \widetilde{C}_k = C + \overline{F}_k C. \tag{8.17}$$

Now, if we consider some time-varying delay $\tau(t)$ in the output measurements, the nonlinear LPV system (8.16) becomes

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{2^{n_{\theta_u}}} \mu_i(x(t))\widetilde{\mu_j}(\theta^u(t))(A_i x(t) + \mathscr{B}_{ij}u(t)) \\ y(t) = \sum_{k=1}^{2^{n_{\theta_y}}} \overline{\mu_k}(\theta^y(t-\tau(t)))\widetilde{C}_k x(t-\tau(t)). \end{cases} \tag{8.18}$$

8.3 Main Result: Observer Design

From the system equations (8.18), the aim of this chapter is to tackle the state and actuator/sensor data deception estimation of a nonlinear system subject to delayed measurements, and represented in a polytopic form. An $\mathscr{L}_2$ attenuation approach is applied in order to minimize the attacks effect on the state and malicious input estimation error.

The state and actuator/sensor data deception observer is given by the following equations:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{2^{n_{\theta_u}}}\mu_i(\hat{x}(t))\widetilde{\mu_j}(\hat{\theta^u}(t)) \\ \qquad \big(A_i x(t) + \mathscr{B}_{ij}u(t) + L_{ij}(y(t) - \hat{y}(t))\big) \\ \dot{\hat{\theta}}^u(t) = \sum_{i=1}^{r}\sum_{j=1}^{2^{n_{\theta_u}}}\mu_i(\hat{x}(t))\widetilde{\mu_j}(\hat{\theta^u}(t)) \\ \qquad (K^u_{ij}(y(t) - \hat{y}(t)) - \alpha^u_{ij}\hat{\theta}^u(t)) \\ \dot{\hat{\theta}}^y(t) = \sum_{i=1}^{r}\sum_{k=1}^{2^{n_{\theta_y}}}\mu_i(\hat{x}(t))\overline{\mu_k}(\hat{\theta^y}(t-\tau(t))) \\ \qquad (K^y_{ik}(y(t) - \hat{y}(t)) - \alpha^y_{ik}\hat{\theta}^y(t)) \\ \hat{y}(t) = \sum_{k=1}^{2^{n_{\theta_y}}}\overline{\mu_k}(\hat{\theta^y}(t-\tau(t)))\widetilde{C}_k\hat{x}(t-\tau(t)), \end{cases} \tag{8.19}$$

where $L_{ij} \in \mathbb{R}^{n_x \times m}$, $K^u_{ij} \in \mathbb{R}^{n \times m}$, $\alpha^u_{ij} \in \mathbb{R}^{n \times n}$, $K^y_{ik} \in \mathbb{R}^{m \times m}$, and $\alpha^y_{ik} \in \mathbb{R}^{m \times m}$ are parameter matrices to be determined s.t. the estimated state and malicious input parameters converge to the real system state and attacks (i.e., the estimation errors for both state and malicious input parameters converge to zero).

Let us define the state and data deception estimation errors $e_x(t)$, $e_{\theta^u}(t)$ and $e_{\theta^y}(t)$ as

$$\begin{aligned} e_x(t) &= x(t) - \hat{x}(t) \\ e_{\theta^u}(t) &= \theta^u(t) - \hat{\theta^u}(t) \\ e_{\theta^y}(t) &= \theta^y(t) - \hat{\theta^y}(t). \end{aligned} \tag{8.20}$$

Based on the convex sum property of the weighting functions, from the results presented in Bezzaoucha et al. (2013) and in order to be able to calculate the estimation error dynamics, the system equations (8.16) are rewritten as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{2^{n_{\theta_u}}}[\mu_i(\hat{x}(t))\widetilde{\mu_j}(\hat{\theta^u}(t))(A_i x(t) + \mathscr{B}_{ij}u(t)) + \\ \qquad \delta_{ij}(t)(A_i x(t) + \mathscr{B}_{ij}u(t))] \\ y(t) = \sum_{k=1}^{2^{n_{\theta_y}}}\Big[\overline{\mu_k}(\hat{\theta^y}(t-\tau(t)))\widetilde{C}_k x(t-\tau(t)) \\ \quad + \overline{\delta_k}(t-\tau(t))\widetilde{C}_k x(t-\tau(t))\Big], \end{cases} \tag{8.21}$$

where $\delta_{ij}(t)$ and $\overline{\delta_k}(t)$ are defined by the following equations:

$$\delta_{ij}(t) = \mu_i(x(t))\widetilde{\mu_j}(\theta^u(t)) - \mu_i(\hat{x}(t))\widetilde{\mu_j}(\hat{\theta^u}(t)) \tag{8.22}$$

$$\overline{\delta_k}(t-\tau(t)) = \overline{\mu_k}(\theta^y(t-\tau(t))) - \overline{\mu_k}(\hat{\theta^y}(t-\tau(t))) \tag{8.23}$$

and satisfy the inequalities:

$$-1 \leq \delta_{ij}(t) \leq 1, -1 \leq \overline{\delta_k}(t) \leq 1. \tag{8.24}$$

Equation (8.21) allows to deduce the state and data deception estimation error dynamics in a straightforward way, since the state and output are written now only depending on the weighting functions of the estimate $\mu_i(\hat{x}(t))$, $\widetilde{\mu_j}(\hat{\theta^u}(t))$, and $\overline{\mu_k}(\hat{\theta^y}(t))$.

Now, let us define the following matrices:

$$\Delta A(t) = \sum_{i=1}^{r}\sum_{j=1}^{2^{n_{\theta_u}}} \delta ij(t) A_i = \mathscr{A}\Sigma(t)E_A \tag{8.25}$$

$$\Delta B(t) = \sum_{i=1}^{r}\sum_{j=1}^{2^{n_{\theta_u}}} \delta_{ij}(t)\mathscr{B}_{ij} = \mathscr{B}\Sigma(t)E_B \tag{8.26}$$

$$\widetilde{C}(\nabla) = \left[\, \overline{\delta_1}(\nabla)\widetilde{C}_1 \ \ldots \ \overline{\delta_{2^{n_{\theta_y}}}}(\nabla)\widetilde{C}_{2^{n_{\theta_y}}} \,\right] \tag{8.27}$$

with

$$\mathscr{A} = \left[\underbrace{A_1 \ \ldots \ A_1}_{2^{n_{\theta_u}} times} \ \ldots \ \underbrace{A_r \ \ldots \ A_r}_{2^{n_{\theta_u}} times} \right] \tag{8.28}$$

$$\mathscr{B} = \left[\, \mathscr{B}_{11} \ \ldots \ \mathscr{B}_{r2^n} \,\right] \tag{8.29}$$

$$\Sigma(t) = \text{diag}(\delta_{11}(t), \ldots, \delta_{r2^n}(t)) \tag{8.30}$$

$$E_A = \left[\, I_{n_x} \ \ldots \ I_{n_x} \,\right]^T, \ E_B = \left[\, I_{n_u} \ \ldots \ I_{n_u} \,\right]^T. \tag{8.31}$$

From (8.24) to (8.30), we have

$$\Sigma^T(t)\Sigma(t) \leq I. \tag{8.32}$$

By using (8.25)–(8.31) and the notation $\nabla = t - \tau(t)$, system (8.21) can be written as an uncertain system given by

$$\begin{cases} \dot{x}(t) = \displaystyle\sum_{i=1}^{r}\sum_{j=1}^{2^{n_{\theta_u}}} \mu_i(\hat{x}(t))\widetilde{\mu_j}(\hat{\theta^u}(t)) \\ \qquad ((A_i + \Delta A(t))x(t) + (\mathscr{B}_{ij} + \Delta B(t))u(t)) \\ y(t) = \displaystyle\sum_{k=1}^{2^{n_{\theta_y}}} \overline{\mu_k}(\hat{\theta^y}(\nabla))(\widetilde{C}_k + \widetilde{C}(\nabla))x(\nabla). \end{cases} \tag{8.33}$$

From Eqs. (8.33) and (8.20), the estimation error dynamics are then given by

$$\begin{cases} \dot{e}_x(t) = \sum_{i=1}^{r} \sum_{j=1}^{2^{n_{\theta_u}}} \sum_{k=1}^{2^{n_{\theta_y}}} \mu_i(\hat{x}(t)) \widetilde{\mu_j}(\hat{\theta^u}(t)) \overline{\mu_k}(\hat{\theta^y}(\nabla)) \\ \quad (A_i e_x(t) - L_{ij} \widetilde{C}_k e_x(\nabla) \\ \quad + \Delta A(t) x(t) - L_{ij} \widetilde{C}(\nabla) x(\nabla) + \Delta B(t) u(t)) \\ \dot{e}_{\theta^u}(t) = \sum_{i=1}^{r} \sum_{j=1}^{2^{n_{\theta_u}}} \sum_{k=1}^{2^{n_{\theta_y}}} \mu_i(\hat{x}(t)) \widetilde{\mu_j}(\hat{\theta^u}(t)) \overline{\mu_k}(\hat{\theta^y}(\nabla)) \\ \quad (-K^u_{ij} \widetilde{C}_k e_x(\nabla) - \alpha^u_{ij} e_{\theta^u}(t) \\ \quad - K^u_{ij} \widetilde{C}(\nabla) x(\nabla) + \alpha^u_{ij} \theta^u(t) + \dot{\theta^u}(t)) \\ \dot{e}_{\theta^y}(t) = \sum_{i=1}^{r} \sum_{k=1}^{2^{n_{\theta_y}}} \mu_i(\hat{x}(t)) \overline{\mu_k}(\hat{\theta^y}(\nabla)) \\ \quad (-K^y_{ik} \widetilde{C}_k e_x(\nabla) - \alpha^y_{ik} e_{\theta^y}(t) \\ \quad - K^y_{ik} \widetilde{C}(\nabla) x(\nabla) + \alpha^y_{ik} \theta^y(t) + \dot{\theta^y}(t)). \end{cases} \tag{8.34}$$

Let us now consider the augmented vectors $e_a(t)$ and $\omega(t)$, such that

$$e_a(t) = \begin{pmatrix} x(t) \\ e_x(t) \\ e^u_\theta(t) \\ e^y_\theta(t) \end{pmatrix}, \ \omega(t) = \begin{pmatrix} \theta^u(t) \\ \theta^y(t) \\ \dot{\theta}^u(t) \\ \dot{\theta}^y(t) \\ u(t) \end{pmatrix}. \tag{8.35}$$

From (8.34) and (8.35), it follows that

$$\dot{e}_a(t) = \sum_{i=1}^{r} \sum_{j=1}^{2^{n_{\theta_u}}} \sum_{k=1}^{2^{n_{\theta_y}}} \mu_i(\hat{x}(t)) \widetilde{\mu_j}(\hat{\theta^u}(t)) \overline{\mu_k}(\hat{\theta^y}(\nabla)) \\ \big(\Phi_{ijk}(t) e_a(t) + \Psi_{ijk}(t)\omega(t) - R_{ijk}(\nabla) e_a(\nabla)\big) \tag{8.36}$$

with

$$\Phi_{ijk}(t) = \begin{pmatrix} A_i & 0 & 0 & 0 \\ \Delta A(t) & A_i & 0 & 0 \\ 0 & 0 & -\alpha^u_{ij} & 0 \\ 0 & 0 & 0 & -\alpha^y_{ik} \end{pmatrix} \tag{8.37}$$

$$\Psi_{ijk}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 & \mathscr{B}_{ij} + \Delta B(t) \\ 0 & 0 & 0 & 0 & \Delta B(t) \\ \alpha^u_{ij} & 0 & I & 0 & 0 \\ 0 & \alpha^y_{ik} & 0 & I & 0 \end{pmatrix} \tag{8.38}$$

$$R_{ijk}(\nabla) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ L_{ij}\widetilde{C}(\nabla) & L_{ij}\widetilde{C} & 0 & 0 \\ K_{ij}^{u}\widetilde{C}(\nabla) & K_{ij}^{u}\widetilde{C} & 0 & 0 \\ K_{ik}^{y}\widetilde{C}(\nabla) & K_{ik}^{y}\widetilde{C} & 0 & 0. \end{pmatrix}. \tag{8.39}$$

Now, the objective is to find the observer parameter matrices such that the transfer from $\omega(t)$ to $e_a(t)$ is minimized. This approach assumes that the disturbance, i.e., the external input $\omega(t)$ belongs to a set of norm bounded functions, i.e., is of finite energy. For the considered problem, knowing that the attacks do not appear all time (stealthy attacks), the assumption is realized.

Let us define the following Lyapunov–Krasovskii functional candidate Mondié and Kharitonov (2005):

$$V(t) = e_a^T(t)Pe_a(t) + \int_{-\tau(t)}^{0} e_a^T(t+\theta)e^{2\alpha\theta}Qe_a(t+\theta)d\theta, \tag{8.40}$$

where P and Q are symmetric, positive definite matrices. The convergence with L_2 attenuation is then guaranteed if the following conditions are satisfied:

$$V(t) > 0 \tag{8.41}$$

$$\dot{V}(t) + e_a^T(t)e_a(t) - \omega^T(t)\Gamma\omega(t) < -2\alpha V(t) \tag{8.42}$$

with

$$\Gamma = \text{diag}(\Gamma_l),\ \Gamma_l < \beta\ I,\ \text{for } l = 1, \ldots, 6. \tag{8.43}$$

An appropriate choice of Γ enables to attenuate the transfer from $\omega(t)$ to $e_a(t)$.

The time derivative of $V(t)$ along the trajectory of (8.36) is given by

$$\begin{array}{l} \dot{V}(t) = \dot{e_a}^T(t)Pe_a(t) + e_a(t)P\dot{e_a}^T(t) + e_a^T(t)Qe_at(t) \\ -(1-\dot{\tau}(t))e^{-2\alpha\tau(t)}e_a^T(\nabla)Qe_a(\nabla) \\ -2\alpha\int_{-\tau(t)}^{0} e_a^T(t+\theta)e^{2\alpha\theta}Qe_a(t+\theta)d\theta, \end{array} \tag{8.44}$$

which is upper bounded thanks to the time-delay condition (8.1) by

$$\begin{array}{l} \dot{V}(t) \leq \dot{e_a}^T(t)Pe_a(t) + e_a(t)P\dot{e_a}^T(t) + e_a^T(t)Qe_at(t) \\ -(1-\gamma)e^{-2\alpha\tau}e_a^T(\nabla)Qe_a(\nabla) \\ -2\alpha\int_{-\tau(t)}^{0} e_a^T(t+\theta)e^{2\alpha\theta}Qe_a(t+\theta)d\theta. \end{array} \tag{8.45}$$

By considering (8.36), we also have

$$\dot{V}(t) + e_a^T(t)e_a(t) - \omega^T(t)\Gamma\omega(t) = \sum_{i=1}^{r}\sum_{j=1}^{2^{n_{\theta_u}}}\sum_{k=1}^{2^{n_{\theta_y}}} \mu_i(\hat{x}(t))\widetilde{\mu_j}(\hat{\theta^u}(t))\overline{\mu_k}(\hat{\theta^y}(\nabla))$$

$$\begin{pmatrix} e_a(t) \\ \omega(t) \\ e_a(\nabla) \end{pmatrix}^T \left(\begin{array}{c|c|c} \Phi_{ij}^T(t)P + P\Phi_{ij}(t) + I & P\Psi_i(t) & -PR_{ijk}(\nabla) \\ \hline \Psi_i^T(t)P & -\Gamma & 0 \\ \hline * & * & -(1-\gamma)e^{2\alpha\tau}Q \end{array}\right) \begin{pmatrix} e_a(t) \\ \omega(t) \\ e_a(\nabla) \end{pmatrix}$$

$$-2\alpha \int_{-\tau(t)}^{0} e_a^T(t+\theta)e^{2\alpha\theta}Qe_a(t+\theta)d\theta \tag{8.46}$$

and

$$\dot{V}(t) + e_a^T(t)e_a(t) - \omega^T(t)\Gamma\omega(t) + 2\alpha V(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{2^{n_{\theta_u}}}\sum_{k=1}^{2^{n_{\theta_y}}} \mu_i(\hat{x}(t))\widetilde{\mu_j}(\hat{\theta^u}(t))\overline{\mu_k}(\hat{\theta^y}(\nabla)) \begin{pmatrix} e_a(t) \\ \omega(t) \\ e_a(\nabla) \end{pmatrix}^T \left[\left(\begin{array}{c|c|c} \Phi_{ij}^T(t)P + P\Phi_{ij}(t) + I & P\Psi_i(t) & -PR_{ijk}(\nabla) \\ \hline \Psi_i^T(t)P & -\Gamma & 0 \\ \hline * & * & -(1-\gamma)e^{2\alpha\tau}Q \end{array}\right) + 2\alpha \left(\begin{array}{c|c|c} P & 0 & 0) \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right)\right] \begin{pmatrix} e_a(t) \\ \omega(t) \\ e_a(\nabla) \end{pmatrix}. \tag{8.47}$$

The negativity of condition (8.47) due to the convex sum property of the weighting functions and the quadratic form of the vector $\begin{pmatrix} e_a(t) \\ \omega(t) \\ e_a(\nabla) \end{pmatrix}^T$ is therefore guaranteed if:

$$\left(\begin{array}{c|c|c} C_1 & P\Psi_i(t) & -PR_{ijk}(\nabla) \\ \hline \Psi_i^T(t)P & -\Gamma & 0 \\ \hline * & * & -(1-\gamma)e^{2\alpha\tau}Q \end{array}\right) < 0, \tag{8.48}$$

where $C_1 = (\Phi_{ij} + \alpha I)^T(t)P + P(\Phi_{ij}(t) + \alpha I) + I$. It is also important to highlight that the matrices $\widetilde{C}(\nabla)$ can be written as

$$\widetilde{C}(\nabla) = \sum_{l=1}^{2^{n_{\theta_y}}} \overline{\delta_l}(\nabla)\widetilde{C}_l. \tag{8.49}$$

From (8.49), and based on the convex sum property of $\overline{\delta_l}(t)$, the matrix inequalities (8.48) become

$$\sum_{l=1}^{2^{n_{\theta_y}}} \overline{\delta_l}(\nabla) \left(\begin{array}{c|c|c} C_1 & P\Psi_i(t) & -PR_{ijk} \\ \hline \Psi_i^T(t)P & -\Gamma & 0 \\ \hline * & * & -(1-\gamma)e^{2\alpha\tau}Q \end{array}\right) < 0, \tag{8.50}$$

where

$$R_{ijk} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ L_{ij}\widetilde{C} & L_{ij}\widetilde{C} & 0 & 0 \\ K^u_{ij}\widetilde{C} & K^u_{ij}\widetilde{C} & 0 & 0 \\ K^y_{ik}\widetilde{C} & K^y_{ik}\widetilde{C} & 0 & 0 \end{pmatrix}, \tag{8.51}$$

which is equivalent to solve

$$\left(\begin{array}{c|c|c} C_1 & P\Psi_i(t) & -PR_{ijk} \\ \hline \Psi_i^T(t)P & -\Gamma & 0 \\ \hline * & * & -(1-\gamma)e^{2\alpha\tau}Q \end{array}\right) < 0. \tag{8.52}$$

The observer gains are then obtained by solving the above constraints with the sufficient condition inequality (8.52) for $i = 1, \ldots, r$, $j = 1, \ldots, 2^{n_{\theta_u}}$, $k = 1, \ldots, 2^{n_{\theta_y}}$, and $l = 1, \ldots, 2^{n_{\theta_y}}$.

The results may be summarized by the following theorem:

Theorem 8.1 *There exists a state and actuator/sensor date deception attack observer (8.19) for a nonlinear system (8.2) with delayed measurements and an $\mathscr{L}_2$ gain from $\omega(t)$ to $e_a(t)$ bounded by β ($\beta > 0$) if there exist positive symmetric matrices $P_1 = P_1^T > 0$, $P_2 = P_2^T > 0$, $P_3 = P_3^T > 0$, $P_4 = P_4^T > 0$ and $Q_1 = Q_1^T > 0$, $Q_2 = Q_2^T > 0$, $Q_3 = Q_3^T > 0$, $Q_4 = Q_4^T > 0$; positive matrices Γ_l, $l = 1, \ldots, 5$; matrices $\overline{\alpha}^u_{ij}$, $\overline{\alpha}^y_{ik}$, F^u_{ij}, F^y_{ik}, $\overline{R}_{ij}$; and scalars positive β, λ_A λ_{1B}, λ_{2B}, and α solution of the following optimization problem under LMI constraints (8.54) and (8.57) (see next page)*

$$\min_{\{P_1,P_2,P_3,\overline{R}_{ij},F^u_{ij},F^y_{ik},\overline{\alpha}^u_{ij},\overline{\alpha}^y_{ik},\Gamma_l,\lambda_A,\lambda_{1B},\lambda_{2B},\}} \beta, \tag{8.53}$$

for $i = 1, \ldots, r$, $j = 1, \ldots, 2^{n_{\theta_u}}$, $k = 1, \ldots, 2^{n_{\theta_y}}$, and $l = 1, \ldots, 2^{n_{\theta_y}}$, where the scalar α is called the delay rate.

$$\Gamma_l < \beta I \text{ for } l = 1, \ldots, 5 \tag{8.54}$$

with

$$\begin{aligned} Q_i^{11} &= P_1(A_i + \alpha I) + (A_i + \alpha I)^T P_1 + I_{n_x} \\ Q_5 &= -\Gamma_1 + \lambda_A E_A^T E_A \\ Q_8 &= -\Gamma_4 + \lambda_{1B} E_B^T E_B \\ Q_9 &= -\Gamma_5 + \lambda_{2B} E_B^T E_B \\ Q_{10} &= -(1-\gamma)e^{2\alpha\tau} Q_1 \\ Q_{11} &= -(1-\gamma)e^{2\alpha\tau} Q_2 \\ Q_{12} &= -(1-\gamma)e^{2\alpha\tau} Q_3 \\ Q_{13} &= -(1-\gamma)e^{2\alpha\tau} Q_4, \end{aligned} \tag{8.55}$$

where the observer gains are given by

$$\begin{cases} L_{ij} = P_2^{-1}\overline{R}_{ij} \\ K^u_{ij} = P_3^{-1}F^u_{ij} \\ K^y_{ik} = P_4^{-1}F^y_{ik} \\ \alpha^u_{ij} = P_3^{-1}\overline{\alpha}^u_{ij} \\ \alpha^y_{ik} = P_4^{-1}\overline{\alpha}^y_{ik}. \end{cases} \tag{8.56}$$

$$\begin{pmatrix}
Q_i^{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_1\mathscr{B}_{ij} & 0 & 0 & 0 & 0 & 0 & P_1\mathscr{B} & 0 \\
* & P_2A_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\overline{R}_{ij}\widetilde{C} & -\overline{R}_{ij}\widetilde{C} & 0 & 0 & P_2\mathscr{A} & 0 & P_2\mathscr{B} \\
* & * & -\overline{\alpha}^u_{ij} & 0 & \overline{\alpha}^u_{ij} & 0 & P_3 & 0 & 0 & F^u_{ij}\widetilde{C} & F^u_{ij}\widetilde{C} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\overline{\alpha}^y_{ik} & 0 & \overline{\alpha}^y_{ik} & 0 & P_4 & 0 & F^y_{ik}\widetilde{C} & F^y_{ik}\widetilde{C} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & Q_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\Gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -\Gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & Q_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & Q_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & & * & * & -Q_{10} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -Q_{11} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & & * & * & -Q_{12} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & -Q_{13} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & -\lambda_A I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & -\lambda_{1B} I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -\lambda_{2B} I
\end{pmatrix} < 0. \tag{8.57}$$

Proof Based on condition (8.52), with (8.37) and the variable change (8.56), with the decomposition (8.25) and (8.26), properties (8.32), Schur's complement, and the following lemma:

Lemma 8.1 *Consider (Zhou and Khargonekar 1988) two matrices X and Y with appropriate dimensions, a time-varying matrix $\Delta(t)$ and a positive scalar ε. The following property is verified*

$$X^T\Delta^T(t)Y + Y^T\Delta(t)X \leq \varepsilon X^T X + \varepsilon^{-1}Y^T Y, \tag{8.58}$$

for $\Delta^T(t)\Delta(t) \leq I$

following the same development as the work presented in Bezzaoucha et al. (2013), Bezzaoucha et al. (2013), the Lyapunov stability with an $\mathscr{L}_2$ transfer from $\omega(t)$ to $e_a(t)$ is obtained by solving the optimization problem (8.53) under the LMI constraints (8.54) and (8.57), which ends the proof. □

8.4 Numerical Simulation

In the following, the proposed approach for state and attacks estimation is applied to a basic model of a biological wastewater treatment plant (Bezzaoucha et al. 2013).

The mathematical model is represented thanks to two state variables $x_1(t)$ and $x_2(t)$, corresponding to the biomass and substrate concentration, respectively, the input $u(t)$, which represents the dwell time in the treatment plant and the measured output which is the biomass concentration ($y(t) = x_1(t)$). The time delay that appears in the output of the system has the form $\tau(t) = 0.5 + 0.45\sin(0.5t)$. The upper bound of its derivative is then equal to $\gamma = 0.225$.

8.4.1 LPV Representation of The Process

First step, let us write the nonlinear system equations (8.59) in a polytopic form. As it was developed in Bezzaoucha et al. (2013), and under specific assumptions, some simplifications can be made and the nonlinear model may be given by

$$\begin{cases} \dot{x}_1(t) = \frac{ax_1(t)x_2(t)}{x_2(t)+b} - x_1(t)u(t) \\ \dot{x}_2(t) = -\frac{cax_1(t)x_2(t)}{x_2(t)+b} + (d - x_2(t))u(t), \end{cases} \tag{8.59}$$

where a, b, c, and d are known parameters.

From the system nonlinearities, applying the sector nonlinearity approach with the premise variables $\rho_1(t)$ and $\rho_2(t)$ chosen as follows:

$$\rho_1(t) = -u(t), \quad \rho_2(t) = \frac{ax_1(t)}{x_2(t) + b}. \tag{8.60}$$

From (8.59) to (8.60), the quasi-LPV system (8.61) is deduced as

$$\dot{x}(t) = \begin{pmatrix} \rho_1(t) & \rho_2(t) \\ 0 & -c\rho_2(t) + \rho_1(t) \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ d \end{pmatrix} u(t). \tag{8.61}$$

Since an LPV representation is deduced in a compact set of the state space, the max and min values of the terms $\rho_1(t)$ and $\rho_2(t)$ may be calculated using the knowledge of the domain of variation of $u(t)$, i.e., $\rho_1(t) \in [-1, -0.2]$ and $\rho_2(t) \in [0.004, 15]$.

Applying the convex polytopic transformation, two partitions for each premise variable are defined as

$$\begin{cases} \rho_1(t) = \varrho_{11}(\rho_1)\rho_1^2 + \varrho_{12}(\rho_1)\rho_1^1 \\ \rho_2(t) = \varrho_{21}(\rho_2)\rho_2^2 + \varrho_{22}(\rho_2)\rho_2^1 \end{cases} \tag{8.62}$$

$$\text{with} \quad \varrho_{11}(\rho_1) = \frac{\rho_1(t) - \rho_1^2}{\rho_1^1 - \rho_1^2}, \ \varrho_{12}(\rho_1) = \frac{\rho_1^1 - \rho_1(t)}{\rho_1^1 - \rho_1^2}$$
$$\varrho_{21}(\rho_2) = \frac{\rho_2(t) - \rho_2^2}{\rho_2^1 - \rho_2^2}, \ \varrho_{22}(\rho_2) = \frac{\rho_2^1 - \rho_2(t)}{\rho_2^1 - \rho_2^2}, \tag{8.63}$$

where the scalars ρ_1^1, ρ_1^2, ρ_2^1, and ρ_2^2 are defined as

$$\begin{aligned}\rho_1^1 &= \max_u \rho_1(t),\ \ \rho_1^2 = \min_u \rho_1(t)\\ \rho_2^1 &= \max_x \rho_2(t),\ \ \rho_2^2 = \min_x \rho_2(t).\end{aligned} \tag{8.64}$$

The sub-models are defined by the sets (A_i, B_i, C) with $i = 1, 2, 3, 4$. Based on ρ_1 and ρ_2 definitions, all the B_i matrices are set to $B = \begin{bmatrix} 0 \ d \end{bmatrix}^T$. The output matrix $C = \begin{bmatrix} 1 \ 0 \end{bmatrix}$ and the matrices A_i are given by

$$A_1 = \begin{pmatrix} \rho_1^1 & \rho_2^1 \\ 0 & -c\rho_2^1 + \rho_1^1 \end{pmatrix},\ A_2 = \begin{pmatrix} \rho_1^1 & \rho_2^2 \\ 0 & -c\rho_2^2 + \rho_1^1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} \rho_1^2 & \rho_2^1 \\ 0 & -c\rho_2^1 + \rho_1^2 \end{pmatrix},\ A_4 = \begin{pmatrix} \rho_1^2 & \rho_2^2 \\ 0 & -c\rho_2^2 + \rho_1^2. \end{pmatrix}.$$

The weighting functions $\mu_i(t)$ are defined by the following equations:

$$\begin{aligned}&\mu_1(t) = \rho_{11}(\rho_1(t))\rho_{21}(\rho_2(t)), \mu_2(t) = \rho_{11}(\rho_1(t))\rho_{22}(\rho_2(t))\\ &\mu_3(t) = \rho_{12}(\rho_1(t))\rho_{21}(\rho_2(t)), \mu_4(t) = \rho_{12}(\rho_1(t))\rho_{22}(\rho_2(t)).\end{aligned} \tag{8.65}$$

Since the polytopic representation is obtained in a compact set of the state space, maximum and minimum values that occur in $\rho_1(t)$ and $\rho_2(t)$ may be calculated using the knowledge of the domain of variation of $u(t)$: $\rho_1(t) \in [-1, -0.2]$ and $\rho_2(t) \in [0.004, 15]$.

8.4.2 *Date Deception Attacks Representation on The Actuator/Sensor*

Two types of data deception attacks are considered, i.e., attacks on actuators and sensors. It is assumed that, mathematically speaking, these attacks are modeled as bounded multiplicative actuator and sensor time-varying faults.

For the considered example, it is assumed that parameter d may be hacked. This actuator attack is represented by $d(t)$, such that

$$d(t) = d + \Delta d(t). \tag{8.66}$$

It can also be written as

$$d(t) = d + \theta^u(t)\overline{d}, \quad \theta^u(t) \in [\theta^{u2}, \theta^{u1}] \tag{8.67}$$

with $d = 2.5$, $\overline{d} = 2.1$ and $\theta^{u2} = -0.1958$, $\theta^{u1} = 0.1979$. Parameters a, b, and c have been identified and set to $a = 0.5$, $b = 0.07$, and $c = 0.7$.

Considering the attack on the actuator, the polytopic representation of the input matrix B is then given by two sub-models, such that

$$B_1 = B + \theta^{u1}\overline{B}, \quad B_2 = B + \theta^{u2}\overline{B}, \tag{8.68}$$

where it is defined by $\overline{B} := \begin{bmatrix} 0 \ \overline{d} \end{bmatrix}^T$. The weighting functions $\widetilde{\mu_j}(\theta^u(t))$ are defined as given in (8.7) and (8.12).

Now, for the sensor attack, it is assumed that a bounded multiplicative sensor fault $\theta^y(t)$ affects the output $y(t)$ such that

$$y(t) = (1 + \theta^y(t - \tau))x_1(t - \tau). \tag{8.69}$$

As previously explained, $\theta^y(t)$ can also be written as

$$\theta^y(t) = \overline{\mu}_1{}^1(\theta^y(t))\theta^{y1} + \overline{\mu}_1^2(\theta^y(t))\theta^{y2}, \ \theta^y(t) \in [\theta^{y2}, \theta^{y1}] \tag{8.70}$$

with $\theta^{y2} = 0.125$, $\theta^{y1} = 0.625$, $\overline{\mu}_1{}^1(\theta^y(t))$, and $\overline{\mu}_1^2(\theta^y(t))$ are defined by (8.9) and (8.13).

The polytopic form of the output is then given by

$$y(t) = \sum_{k=1}^{2} \overline{\mu}_k(\theta^y(t - \tau(t)))\widetilde{C}_k x(t - \tau(t)) \tag{8.71}$$

with $\widetilde{C}_1 = \begin{pmatrix} 1 + \theta^{y2} \ 0 \end{pmatrix}$, $\widetilde{C}_2 = \begin{pmatrix} 1 + \theta^{y1} \ 0 \end{pmatrix}$.

8.4.3 Simulation Results

From the considered example, with both attacks on the actuator/sensor, applying the proposed approach by solving Theorem 8.1, a simultaneous state and attacks observer is designed such that the system initial conditions are taken as $x(0) = \begin{pmatrix} 0.1 \ 1.5 \end{pmatrix}$ and $\hat{x}(0) = \begin{pmatrix} 0.09 \ 2.3 \end{pmatrix}$ for its observer. For both attacks, the initial conditions are set to zero, i.e., $\hat{\theta}^u(0) = 0$ and $\hat{\theta}^y(0) = 0$.

The state vector, its estimate as well as the data deception attacks with their estimates are depicted in Figs. 8.1, and 8.2, respectively. From the obtained plots, the efficiency of the proposed observer is highlighted; indeed, both system states and the time-varying multiplicative actuator/sensor attacks are well estimated.

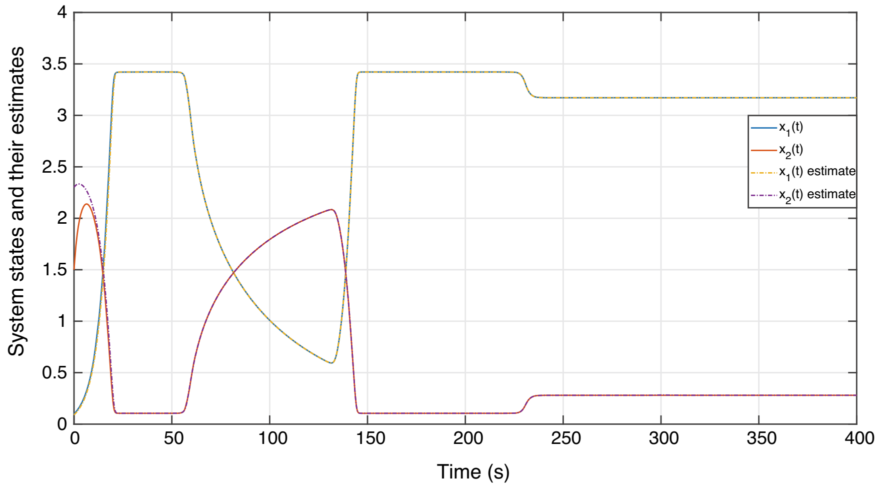

Fig. 8.1 System states and their estimates

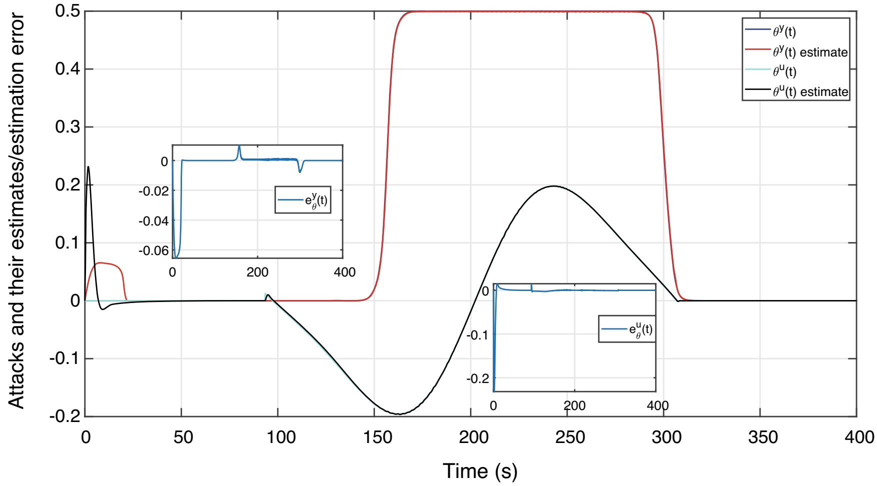

Fig. 8.2 Data deception attacks and their estimates

8.5 Conclusions

In the present book chapter, a polytopic approach was applied to cope with the system state and data deception attacks estimation and delayed measurements. Based on previous work, both attacks on actuator and sensor are modeled as multiplicative time-varying faults and written in a convex set, based only on their min and max bound. A simultaneous state and attack observer is designed by minimizing the $\mathscr{L}_2$

gain from the augmented input to the different estimation errors. The chosen application example is an activated sludge reactor with attacks represented by unknown time-varying parameters on the parameter d and the output. From the nonlinear equations of the system, an LPV model is derived. The proposed observer is designed and the obtained results illustrate its performance.

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