On Jamming Game in Multi-user MIMO Downlink AF Relay System

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Abstract. Wireless devices may threaten urban security if they are maliciously used. Some Internet-of-things (IoT) devices have already troubled the city management even hurt citizens. Jamming technique is one of the effective approaches to prevent these malicious activities. In addition, some IoT devices may locate far from base station and the power of received signals may be low. In this paper, we discuss a jamming power allocation problem in a multi-user multiple-input multiple-output (MU-MIMO) downlink relay system when the communication and jamming signals are relatively low due to the long distance between them, in which jammer tries to minimize the sum rate of IoT devices while these devices try to communicate at the fastest rate. Besides, jammer transmits jamming signals after detecting the communication request of devices so base station should decide how to transmit first, which is modeled by a two-person zero-sum Stackelberg game. However, the sum rate in such a scenario is non-convex. Finding the equilibrium strategy is general N-P hard. To address this complex problem, we decompose it into several easy-solved sub-problems and adopt an alternating optimization over them. Further, we simulate the proposed method numerically and results suggest the superiority.

Keywords: Urban security · Relay · MIMO · Jamming · Sum rate · Zero-sum game · Stackelberg game

1 Introduction

The widely used wireless devices facilitate daily life but it threatens urban security. For example, terrorists may use wireless Internet of Things (IoT) devices to destroy public facilities even endanger life safety. The characteristic of exposure to the open-access environment of wireless communication systems makes it susceptible to deliberate jamming attacks. In this way, jammer limits the communication among opponents to protect itself. Traditional adversarial jamming

methods against wireless communication systems include tone jamming, barrage jamming and narrow-band jamming. These methods are widely applied but they are inefficient. On the other hand, jamming techniques are facing the challenges of anti-jamming techniques. For example, cognitive radio (CR) [8] techniques have the inherent merit of evading the jamming signal intelligently. Jamming techniques need to be upgraded.

One of the effective theories referred to by much intelligent jamming research about countering the CR system is game theory. Most related research can be divided into two types: correlated and uncorrelated jamming. Correlated jamming [1,2,7,13,14] means jamming signal correlates to communication signal, while uncorrelated jamming [3–6,9,15–17,19] means they are independent. Obviously, uncorrelated jamming is easier to be implemented. In [4], a Bayesian game between transceiver and jammer was formulated, where jammer did not know the user type (smart or regular one) clearly. In [17], the best strategy (Nash equilibrium) for both authorized users and jammer is to uniformly allocate all power to all the subchannels in AWGN channel in Orthogonal Frequency Division Multiplexing (OFDM) system. Some researchers concentrated on the multi-user system. In the uplink of multi-tone interference single-input-single-output (SISO) channel, the jamming-transmitting capacity game has a Nash equilibrium and it can be achieved by the generalized iterative water-filling algorithm [6]. In [18], a Stackelberg game of a base station and a jammer in nonorthogonal multiple access (NOMA) downlink system was investigated and a Q-learning-based power allocation algorithm was proposed. Furthermore, some researchers were interested in the relay system. In a single-transmitter, single-user and multi-relay communication system, in which some relays are non-cooperative to others and behave maliciously to transceiver like a jammer, it proved that the Nash equilibrium is transmitting Gaussian white noise [3], and [15] extended this research in a two-way relay channel.

Previous research gives much insight into the jamming strategy, however, little research investigates the optimal jamming strategy in the time-invariant multi-user multiple-input multiple-output (MU-MIMO) downlink relay scenario which is common in reality. Besides, the distances between base station and IoT devices may sometimes be long and jammer may be far away from them so signal-to-noise ratio (SNR) and jamming-to-noise ratio (JNR) are relatively low. To jam these devices in such a scenario, we formulate a new static zero-sum Stackelberg game of intelligently jamming MIMO downlink relay system, in which base station tries to maximize the sum rate of the whole communication system while jammer tries to minimize it. However, it is hard to find this equilibrium since the sum rate is non-convex [10]. We propose a feasible algorithm to address this complex problem. We first approximate the sum rate by an inequality. Subsequently, we decompose it into several easy-solved sub-problems and adopt an alternating optimization over them. Finally, we solve every sub-problem by the Lagrange method or majorization-minimization (MM) algorithm.

The rest of this paper is organized as follows. The system model and the jamming-transmitting zero-sum game model are formulated in Sect. 2. In Sect. 3,
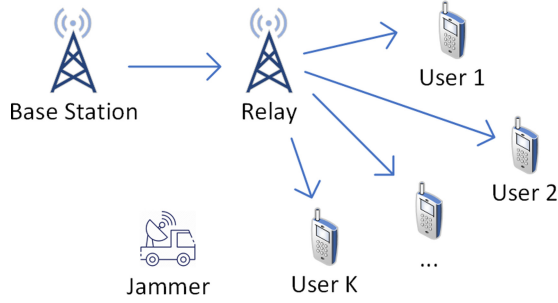
Fig. 1. Downlink model with relay.

we show how to find an approximate equilibrium strategy of the transmitting-jamming zero-sum game in time-invariant channel. Then we simulate those algorithms by numerical method in Sect. 4 and conclude this paper in Sect. 5 (Table 1).

Table 1. Notation table

Notation	Discription
$\mathbf{H}_{(*)(\cdot)}$	channel matrix of $(*)$ to $(\cdot)$
μ	power amplification coefficient of relay
P_S, P_J	power of base station and jammer
k	k-user, $k = 0, 1, ..., K$, 0 means relay
n_k	white Gaussian noise at k-user
$\mathbf{x}_k$	transmitting symbols of base station to k-user
$\mathbf{j}$	jamming symbols
$\mathbf{K}_{Xk}, \mathbf{K}_J$	covariance matrices of $\sqrt{\alpha_k Ps}\mathbf{x}_k, \sqrt{\beta Ps}\mathbf{j}$
$\mathbf{y}_R, \mathbf{y}_k$	receiving symbols at relay and k-user
$\mathbf{H}_{SRk}$	$\mu\mathbf{H}_{Rk}\mathbf{H}_{SR}$
$\mathbf{H}_{JRk}$	$\mu\mathbf{H}_{Rk}\mathbf{H}_{JR}$
$\tilde{\mathbf{N}}_k$	$\mathbf{H}_{RK}N_0\mathbf{H}_{RK}^H + N_k\mathbf{I}$

2 System Model

2.1 Wireless Communication and Jamming Model

As shown in Fig. 1, we consider the downlink model of a MIMO communication system with one amplify-and-forward (AF) relay and K devices. A jammer tries to disrupt this system by transmitting jamming signals to relay and devices. The power of jammer is limited. We assume that the wireless signals from base

station attenuate seriously due to the long distance between base station and devices so we can neglect them. For simplicity, all the radios are equipped with M antennas. The maximum powers of jammer and base station are P_J and P_S separately and the amplification factor of AF relay is μ. In addition, for consistency with other related research, we call k-device k-user.

The received signal at relay is given by

$$\mathbf{y}_R = \mathbf{H}_{SR} \sum_{k=1}^{K} \sqrt{P_S \alpha_k} \mathbf{x}_k + \mathbf{H}_{JR} \sqrt{P_J \beta} \mathbf{j} + \mathbf{n}_0, \quad k = 1, 2, ..., K \qquad (1)$$

and the received signal at k-th user is given by

$$\begin{aligned} \mathbf{y}_k &= \mu \mathbf{H}_{Rk} \mathbf{y}_R + \mathbf{H}_{Jk} \sqrt{P_J \beta} \mathbf{j} + \mathbf{n}_k \\ &= \mathbf{H}_{Rk} \mathbf{H}_{SR} \sum_{l=1}^{K} \sqrt{\mu P_S \alpha_l} \mathbf{x}_l + \mathbf{H}_{Rk} \sqrt{\mu} \mathbf{n}_0 + \mathbf{n}_k \qquad (2) \\ &\quad + \mathbf{H}_{Rk} \mathbf{H}_{JR} \sqrt{\mu P_J \beta} \mathbf{j} + \mathbf{H}_{Jk} \sqrt{P_J \beta} \mathbf{j}, \qquad k = 1, 2, ..., K \end{aligned}$$

where $\mathbf{H}_{(\cdot)(*)}$ denotes the $M \times M$ channel matrix from $(\cdot)$ to $(*)$ and the subscript "S", "R", "J", "k" means base station, relay, jammer and k-user respectively. Each element of $\mathbf{H}_{(\cdot)(*)}$ is supposed to be identically independently distributed (i.i.d.), i.e., $\mathbf{H}_{(\cdot)(*)} = [g_{ij(\cdot)(*)}]_{1 \leq i,j \leq M}$ and $g_{ij(\cdot)(*)} \sim \mathcal{CN}(0, \sigma^2_{(\cdot)(*)})$ where $\mathcal{CN}$ is circular symmetric complex Gaussian distribution. $\mathbf{y}_{(\cdot)}$ is the received signal at $(\cdot)$. $\mathbf{x}_{(\cdot)}$ and $\mathbf{j}_{(\cdot)}$ are the transmitting and jamming symbol to $(\cdot)$ and the subscript "0" means "to relay". Every symbol of $\mathbf{x}_{(\cdot)}$ and $\mathbf{j}_{(\cdot)}$ is normalized and i.i.d. so $\mathbb{E}(x_{(\cdot)i}) = 0$, $\mathbb{E}(j_{(\cdot)i}) = 0$ and $\mathbb{E}(x_{(\cdot)i} x^H_{(\cdot)l}) = 0$, $\mathbb{E}(j_{(\cdot)i} j^H_{(\cdot)l}) = 0$, $l \neq i$. $\alpha_{(\cdot)}$ and $\beta_{(\cdot)}$ are the power proportion allocated to $(\cdot)$ by base station and jammer. $\mathbf{n}_o$ and $\mathbf{n}_k$ are white Gaussian noise and $\mathbf{n}_{(\cdot)} \sim \mathcal{CN}(0, N_{(\cdot)}\mathbf{I})$. SNR is defined by the power of base station P_S divided by the power of noise, P_S/N, and JNR is P_J/N. The covariance matrix of transmitting signal to user k denotes as $\mathbf{K}_{Xk}$ and that of jamming signal is $\mathbf{K}_J$.

2.2   Stackelberg Game Formulation

In this section, we introduce the jamming game model with respect to sum rate. A standard strategic game consists of a finite set of players $\mathcal{N}$ for each play $i \in \mathcal{N}$, a non-empty action set $\mathcal{A}_i$ and for each player a preference relation on $\mathcal{A} = \times_{i \in \mathcal{N}} \mathcal{A}_i$ [11]. We define $\mathcal{N} = \{\text{base station, jammer}\}$. The action spaces of them are $\mathcal{A}_1 = \{\mathbf{K}_{Xk} \in \mathbb{C}^{M \times M} | \mathbf{K}_{Xk} \succeq 0, \sum_{k=1}^{K} \text{Tr}(\mathbf{K}_{Xk}) \leq P_S, k = 1, 2, ..., K\}$ and $\mathcal{A}_2 = \{\mathbf{K}_{Jk} \in \mathbb{C}^{M \times M} | \mathbf{K}_{Jk} \succeq 0, \sum_{k=0}^{K} \text{Tr}(\mathbf{K}_{Jk}) \leq P_J, k = 0, 1, ..., K\}$ respectively. The preference relations are defined by positive associated utility functions: $u_1(\mathbf{K}_{Xk}; \mathbf{K}_{Jk}) = R(\mathbf{K}_{Xk}, \mathbf{K}_{Jk}) = \sum_{k=1}^{K} R_k(\mathbf{K}_{Xk}, \mathbf{K}_{Jk})$ for base station and $u_2(\mathbf{K}_{Jk}; \mathbf{K}_{Xk}) = -R(\mathbf{K}_{Xk}, \mathbf{K}_{Jk}) = -\sum_{k=1}^{K} R_k(\mathbf{K}_{Jk}; \mathbf{K}_{Xk})$ for jammer, where $R_k(*, \cdot)$ is the data rate of k-th user

$$R_k = \log \det(\mathbf{I} + \mu\mathbf{H}_{Rk}\mathbf{H}_{SR}\mathbf{K}_{Xk}\mathbf{H}_{SR}^H\mathbf{H}_{Rk}^H(\tilde{\mathbf{N}}_k + \sum_{l=1,l\neq k}^{K} \mu\mathbf{H}_{Rk}\mathbf{H}_{SR}\mathbf{K}_{Xl}\mathbf{H}_{SR}^H\mathbf{H}_{Rk}^H$$

$$+ \mu\mathbf{H}_{Rk}\mathbf{H}_{JR}\mathbf{K}_J\mathbf{H}_{JR}^H\mathbf{H}_{Rk}^H + \mathbf{H}_{Jk}\mathbf{K}_J\mathbf{H}_{Jk}^H)^{-1}) \tag{3}$$

and $\tilde{\mathbf{N}}_k = \mu\mathbf{H}_{Rk}N_0\mathbf{H}_{Rk}^H + N_k\mathbf{I}$. We define $u(*) = u_1(*) = -u_2(*)$. In such case, the jamming game is denoted by $G_1 = <\mathcal{N}, (\mathcal{A}_1, \mathcal{A}_2), (u_1, u_2)>$ in brief.

Moreover, in a practical reactive jamming regime, jammer usually sends jamming signals after detecting communication signals, and base station is aware of this, which means that base station and jammer must decide in order. On the other hand, base station and jammer are both intelligent enough to choose an optimal transmitting strategy (covariance matrices). Thus, we model game G_1 as a Stackelberg game, which is given by

$$\max_{\mathbf{K}_{X1},...,\mathbf{K}_{XK}} . \min_{\mathbf{K}_J} . \quad R(\mathbf{K}_{X1},...,\mathbf{K}_{XK},\mathbf{K}_J)$$

$$s.t. \quad \begin{cases} \sum_{k=1}^K \text{Tr}(\mathbf{K}_{Xk}) \leq P_S \\ \text{Tr}(\mathbf{K}_J) \leq P_J \\ \mathbf{K}_{Xk} \succeq 0, \qquad k = 1,2,...,K \\ \mathbf{K}_J \succeq 0 \end{cases} \tag{4}$$

3 Stackelberg Equilibrium of Jamming Game in Time-Invariant Channel

In this section, we will address the optimal jamming power allocation problem in the MU-MIMO downlink system in the time-invariant channel. However, problem (4) is quite complex. We first simplify the sum rate function in the regime of low SNR and JNR. Then, we decouple it into two simple sub-problems to find the jamming strategy.

Due to the low SNR and JNR, the covariance matrices of communication and jamming signals are smaller than those of noise. For relatively smaller matrix $\mathbf{X}, \mathbf{Y}$,

$$\log \det(\mathbf{I} + \mathbf{X}(\mathbf{N} + \mathbf{Y})^{-1}) \approx \text{Tr}(\mathbf{X}\mathbf{N}^{-1}(\mathbf{I} - \mathbf{Y}\mathbf{N}^{-1})) \tag{5}$$

It is easy to prove it.

$$\begin{aligned}
&\log \det(\mathbf{I} + \mathbf{X}(\mathbf{N} + \mathbf{Y})^{-1}) \\
&\approx \text{Tr}(\mathbf{X}(\mathbf{N} + \mathbf{Y})^{-1}) \\
&= \text{Tr}(\mathbf{X}\mathbf{N}^{-1}(\mathbf{I} + \mathbf{Y}\mathbf{N}^{-1})^{-1}(\mathbf{I} - \mathbf{Y}\mathbf{N}^{-1})^{-1}(\mathbf{I} - \mathbf{Y}\mathbf{N}^{-1})) \\
&\approx \text{Tr}(\mathbf{X}\mathbf{N}^{-1}(\mathbf{I} - \mathbf{Y}\mathbf{N}^{-1}))
\end{aligned} \tag{6}$$

Substituting formula (5) into (3), R_k can be approximated by

$$R_k \approx \mathrm{Tr}(\mathbf{H}_{SRk}\mathbf{K}_{Xk}\mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1} - \mathbf{H}_{SRk}\mathbf{K}_{Xk}\mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{Jk}\mathbf{K}_J\mathbf{H}_J^H\tilde{\mathbf{N}}_k^{-1}$$
$$- \mathbf{H}_{SRk}\mathbf{K}_{Xk}\mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{JRk}\mathbf{K}_J\mathbf{H}_{JRk}^H\tilde{\mathbf{N}}_k^{-1}$$
$$- \mathbf{H}_{SRk}\mathbf{K}_{Xk}\mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1} \sum_{l=1,l\neq k}^{K} \mathbf{H}_{SRk}\mathbf{K}_{Xl}\mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1})$$

Suppose that

$$\mathbf{A} = \sum_{k=0}^{K} -\mathbf{H}_J^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{SRk}\mathbf{K}_{Xk}\mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{Jk} - \mathbf{H}_{JRk}^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{SRk}\mathbf{K}_{Xk}\mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{JRk} \tag{7}$$

and

$$\mathbf{B}_{1k} = \mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{SRk} - \mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{Jk}\mathbf{K}_J\mathbf{H}_J^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{SRk}$$
$$- \mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{JRk}\mathbf{K}_J\mathbf{H}_{JRk}^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{SRk} \tag{8}$$

$$\mathbf{B}_{2k} = \mathbf{H}_{SRk}^H\tilde{\mathbf{N}}_k^{-1}\mathbf{H}_{SRk} \tag{9}$$

Given $\mathbf{K}_{Xk}$, $k = 1, 2, ..., K$, $R(\mathbf{K}_J; \mathbf{K}_{X1}, ..., \mathbf{K}_{XK}) = \mathrm{Tr}(\mathbf{A}\mathbf{K}_J)$, and given $\mathbf{K}_J$ $R(\mathbf{K}_{X1}, ..., \mathbf{K}_{XK}; \mathbf{K}_J) = \sum_{k=0}^{K} \mathrm{Tr}(\mathbf{B}_{1k}\mathbf{K}_{Xk} - \mathbf{K}_{Xk}\mathbf{B}_{2k}\sum_{l\neq k}\mathbf{K}_{Xl}\mathbf{B}_{2k})$. The original problem (4) is decoupled into two sub-problems:

$$\min_{\mathbf{K}_J} . \quad \mathrm{Tr}(\mathbf{A}\mathbf{K}_J)$$
$$s.t. \quad \begin{cases} \mathrm{Tr}(\mathbf{K}_J) \leq P_J \\ \mathbf{K}_J \succeq 0 \end{cases} \tag{10}$$

and

$$\max_{\mathbf{K}_{X1},...,\mathbf{K}_{XK}} . \quad \sum_{k=0}^{K} \mathrm{Tr}(\mathbf{B}_{1k}\mathbf{K}_{Xk} - \mathbf{K}_{Xk}\mathbf{B}_{2k}\sum_{l\neq k}\mathbf{K}_{Xl}\mathbf{B}_{2k})$$
$$s.t. \quad \begin{cases} \sum_{k=1}^{K}\mathrm{Tr}(\mathbf{K}_{Xk}) \leq P_S \\ \mathbf{K}_{Xk} \succeq 0, \quad k = 1, 2, ..., K \end{cases} \tag{11}$$

3.1   Jamming Power Allocation Strategy Design

Jamming power allocation strategy is acquired by solving problem (10). It is a convex problem. The Lagrangian of it reads

$$L(\mathbf{K}_J, \lambda, \mathbf{Q}) = \mathrm{Tr}(\mathbf{A}\mathbf{K}_J) + \lambda(\mathrm{Tr}(\mathbf{K}_J) - P_j) - \mathrm{Tr}(\mathbf{Q}\mathbf{K}_J) \tag{12}$$

and the Karush-Kuhn-Tucker (KKT) Conditions of it are given by

$$\mathbf{Q} = \mathbf{A} + \lambda\mathbf{I}$$
$$\text{Tr}(\mathbf{K}_J) \leq P_J, \quad \mathbf{K}_J \succeq 0 \tag{13}$$
$$\lambda \geq 0, \quad \mathbf{Q} \succeq 0$$
$$\lambda(\text{Tr}(\mathbf{K}_J) - P_J) = 0, \quad \mathbf{Q}\mathbf{K}_J = 0$$

The solution is discussed as follows: (a) When $\lambda = 0$, $\mathbf{Q} = \mathbf{A}$, leading to $\mathbf{A} \succeq 0$. The optimal jamming covariance matrix $\mathbf{K}_J^*$ satisfies $\text{Tr}(\mathbf{K}_J) \leq P_J, \mathbf{K}_J \succeq 0, \mathbf{A}\mathbf{K}_J = 0$. Obviously, $\mathbf{K}_J = \mathbf{0}$ is one of the solutions. (b) When $\lambda > 0$, $\mathbf{Q} = \mathbf{A} + \lambda\mathbf{I}$. If $\mathbf{Q} \succ 0$, $\mathbf{K}_J = 0$ since $\mathbf{Q}\mathbf{K}_J = 0$. It contradicts with $\text{Tr}(\mathbf{K}_J) = P_J$. Moreover, $\mathbf{A} \succeq 0$ is impossible since $\mathbf{Q}$ can not be strictly positive definite. Let $\mathbf{Q} = \mathbf{A} + \lambda_{Amin}\mathbf{I}$ where λ_{Amin} is the minimum eigenvalue of matrix $\mathbf{A}$. $\mathbf{K}_J^*$ should satisfy $\mathbf{Q}\mathbf{K}_J^* = 0$, $\text{Tr}(\mathbf{K}_J^*) = P_J$ and $\mathbf{K}_J^* \succeq 0$. If $\mathbf{u}$ is one of the bases of the null space of $\mathbf{Q}$, then $\mathbf{Q}\mathbf{u}\mathbf{u}^H = \mathbf{0}$. Consequently, $P_J\mathbf{u}\mathbf{u}^H$ is one of the solutions of $\mathbf{K}_J$.

3.2 Optimizing Base Station Transmitting Strategy

The transmitting power allocation strategy of base station is acquired by solving problem (11). However, it is non-convex. It can be rewritten as

$$\min_{\mathbf{K}_{X1},...,\mathbf{K}_{XK}} . \quad \text{Tr}(\mathbf{K}_X\mathbf{B}_2\mathbf{I}_M\mathbf{K}_X\mathbf{I}_v\mathbf{I}_v^H\mathbf{B}_2 - \mathbf{B}_1\mathbf{K}_X) \triangleq f(\mathbf{K}_X)$$
$$s.t. \quad \begin{cases} \text{Tr}(\mathbf{K}_X) \leq P_S \\ \mathbf{K}_X = \text{blkdiag}(\mathbf{K}_{X1}, \mathbf{K}_{X2}, ..., \mathbf{K}_{XK}) \\ \mathbf{K}_{Xk} \succeq 0, \quad k = 1, 2, ..., K \end{cases} \tag{14}$$

where $\mathbf{B}_1$, $\mathbf{B}_2$ are the block diagonal concatenation of $\mathbf{B}_{1k}$ and $\mathbf{B}_{2k}$ respectively. $\mathbf{I}_v = (\mathbf{I}, \mathbf{I}, ..., \mathbf{I})^T$.

$$\mathbf{I}_m = \begin{bmatrix} \mathbf{0} & \mathbf{I} & ... & \mathbf{I} \\ \mathbf{I} & \mathbf{0} & ... & \mathbf{I} \\ ... & ... & ... & ... \\ \mathbf{I} & \mathbf{I} & ... & \mathbf{0} \end{bmatrix} \tag{15}$$

$\mathbf{I}$ is the $M \times M$ identity matrix. $\mathbf{B}_2\mathbf{I}_M$ may not be positive semidefinite.

To solve problem (14), we use the majorization-minimization (MM) method. MM method consists of two steps: (a) find a surrogate function which is the tight upper bound of original objective. (b) minimize surrogate function. The first critical thing is to find the tight upper bound $\check{f}(\mathbf{K}_X)$ of the objective $f(\mathbf{K}_X)$.

Definition 1 (Lipschitz Continuous [12]). *Given two metric spaces (X, d_X) and (Y, d_Y) where d_* denotes the metric on set $(*)$. A function $f : X \mapsto Y$ is called Lipschitz continuous if there exists a real constant $L \geq 0$ such that for all $x_1, x_2 \in X$, $d_Y(f(x_1), f(x_2)) \leq Ld_X(x_1, x_2)$.*

The second-order Taylor expansion of the quadratic matrix function $g(\mathbf{X}) = \mathrm{Tr}(\mathbf{X}\check{\mathbf{A}}\mathbf{X}\check{\mathbf{B}})$ is convex. Thus, if the gradient of $g(\mathbf{X})$ is Lipschitz continuous, it can be bounded by a convex quadratic function

$$\check{g}(\mathbf{X}) = g(\mathbf{X}_0) + \nabla g(\mathbf{X}_0)(\mathbf{X} - \mathbf{X}_0) + \frac{L}{2}||\mathbf{X} - \mathbf{X}_0||^2 \tag{16}$$

where $||\nabla g(\mathbf{X}) - \nabla g(\mathbf{X}_0)|| \leq L||\mathbf{X} - \mathbf{X}_0||$ and $\nabla g(\mathbf{X}) = (\check{\mathbf{A}}\mathbf{X}\check{\mathbf{B}} + \check{\mathbf{B}}\mathbf{X}\check{\mathbf{A}})^T$. Define

$$\mathbf{V} \triangleq \check{\mathbf{A}}^T \otimes \check{\mathbf{B}} + \check{\mathbf{B}}^T \otimes \check{\mathbf{A}} \tag{17}$$

so

$$
\begin{aligned}
L &\triangleq \sup_{\mathbf{X}} \frac{||\nabla g(\mathbf{X}) - \nabla g(\mathbf{X}_0)||_F}{||\mathbf{X} - \mathbf{X}_0||_F} \\
&= \sup_{\mathbf{X}} \frac{||(\check{\mathbf{A}}(\mathbf{X} - \mathbf{X}_0)\check{\mathbf{B}} + \check{\mathbf{B}}(\mathbf{X} - \mathbf{X}_0)\check{\mathbf{A}})^T||_F}{||(\mathbf{X} - \mathbf{X}_0)||_F} \\
&\overset{\mathbf{Y} \triangleq \mathbf{X} - \mathbf{X}_0}{=} \sup_{\mathbf{Y}} \frac{||\mathrm{vec}(\check{\mathbf{A}}\mathbf{Y}\check{\mathbf{B}} + \check{\mathbf{B}}\mathbf{Y}\check{\mathbf{A}})||}{||\mathrm{vec}(\mathbf{Y})||} \\
&= \sup_{\mathbf{Y}} \frac{||(\check{\mathbf{A}}^T \otimes \check{\mathbf{B}} + \check{\mathbf{B}}^T \otimes \check{\mathbf{A}})\mathrm{vec}(\mathbf{Y})||}{||\mathrm{vec}(\mathbf{Y})||} \\
&= \sup_{\mathbf{Y}} \frac{||\mathbf{V}\mathrm{vec}(\mathbf{Y})||}{||\mathrm{vec}(\mathbf{Y})||}
\end{aligned}
$$

There is a natural bound of L: the spectral norm of $\mathbf{V}$. Let $\check{\mathbf{A}} = \mathbf{B}_2\mathbf{I}_M$, $\check{\mathbf{B}} = \mathbf{I}_v\mathbf{I}_v^H\mathbf{B}_2$, so the upper bound is expressed as

$$\check{f}(\mathbf{K}_X) = \check{g}(\mathbf{K}_X) - \mathrm{Tr}(\mathbf{B}_1\mathbf{K}_X) \tag{18}$$

$$
\begin{aligned}
&\min_{\mathbf{K}_{X1},...,\mathbf{K}_{XK}} \quad \check{f}(\mathbf{K}_X; \mathbf{K}_{X0}) \\
&s.t. \quad \begin{cases} \mathrm{Tr}(\mathbf{K}_X) \leq P_S \\ \mathbf{K}_X = \mathrm{blkdiag}(\mathbf{K}_{Xk}), \quad k = 1, 2, ..., K \\ \mathbf{K}_{Xk} \succeq 0, \quad k = 1, 2, ..., K \end{cases}
\end{aligned} \tag{19}
$$

Thus, the problem (11) can be solved by successively solving convex problem (19).

In summary, the original optimal jamming power allocation Stackelberg game (4) of the MU-MIMO downlink system in the time-invariant channel can be solved by alternatively optimizing problem (10) and problem (11). Furthermore, problem (11) can be converted into sub-problem (19). The procedures to find the jamming strategy are in Algorithm 1.

Algorithm 1. Jamming Power Allocation Strategy of Stackelberg game

Input: Action Spaces $\mathcal{A}_1$ and $\mathcal{A}_2$, CSI of all channel
Output: $\mathbf{K}_J$
1: **repeat**
2:     calculate $\mathbf{A}$ in (7) and its minimum eigenvalue λ_{Amin}
3:     **if** $\mathbf{A} \succeq 0$ **then**
4:         $\mathbf{K}_J = 0$
5:     **else**
6:         find a base vector $\mathbf{u}$ of null space of $\mathbf{K}$
7:         $\mathbf{K}_J = P_J \mathbf{u} \mathbf{u}^H$
8:     **end if**
9:     init $\mathbf{K}_X^0$ and $t := 0$
10:     **repeat**
11:         $t \leftarrow t + 1$
12:         $\mathbf{K}_X^t$: solve problem (19) at $\mathbf{K}_{X0} = \mathbf{K}_X^{t-1}$ by some convex solvers
13:     **until** convergence
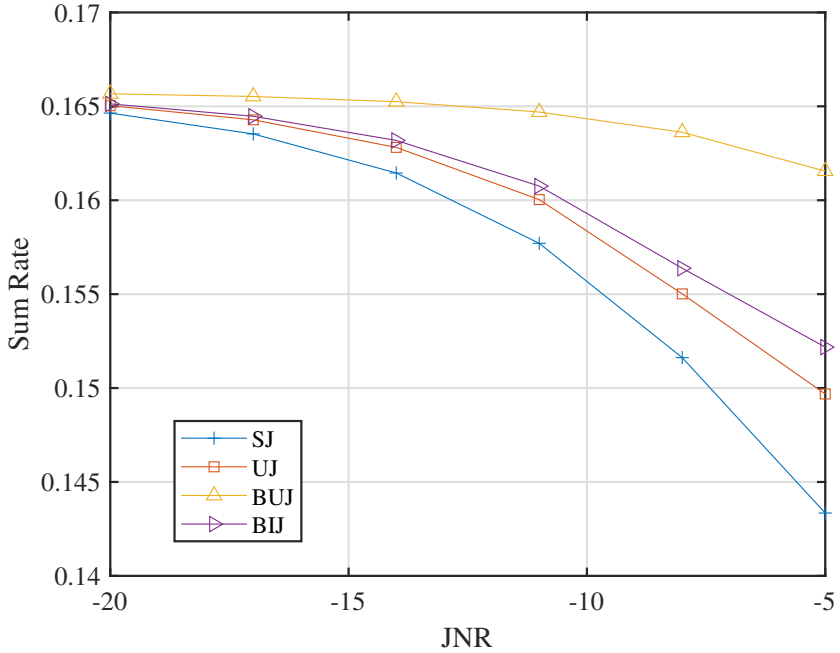14: **until** convergence
15: Return $\mathbf{K}_J$.

Fig. 2. Sum rate in time-invariant channel against JNR when $M = 2$, $K = 5$, SNR $= -10$ dB, $\mu = 2$.

4   Numerical Results

In this section, we will verify the performance of the proposed jamming power allocation algorithm. We compare the proposed method with several widely-used jamming methods numerically. Results indicate that the proposed jamming method is superior to other methods. We present the performance of four methods: Stackelberg jamming (SJ) strategy, the uniformly jamming (UJ) strategy, the strategy of matching the jamming channel to base station but uniformly allocating power ($\mathbf{K}_J = \frac{P_J}{n}\mathbf{V}\mathbf{V}^H$, marked as 'BUJ'), the strategy of inverting the channel between jammer and base station (marked as 'BJI'). Without loss of generality, the channel matrices $\mathbf{H}_{(\cdot)(*)} = [g_{ij(\cdot)(*)}]_{1\leq i,j\leq M}$ and $g_{ij(\cdot)(*)} \sim \mathcal{CN}(0,1)$ and $N_k = 10$ for all $k = 0, 1, ..., K$.

We first focus on Fig. 2. Obviously, all the sum rates under different jamming and transmitting strategies decrease with SNR. The sum rate of the proposed method (marked as 'SJ') is the lowest compared to other methods. Then we turn to reveal the performances with different SNR in Fig. 3. With SNR increases, the sum rate of all users increases with no doubt for all methods. So proposed equilibrium jamming strategy is still the lowest one. These two results manifest that jammer can jam adversaries more effectively by using the proposed method.
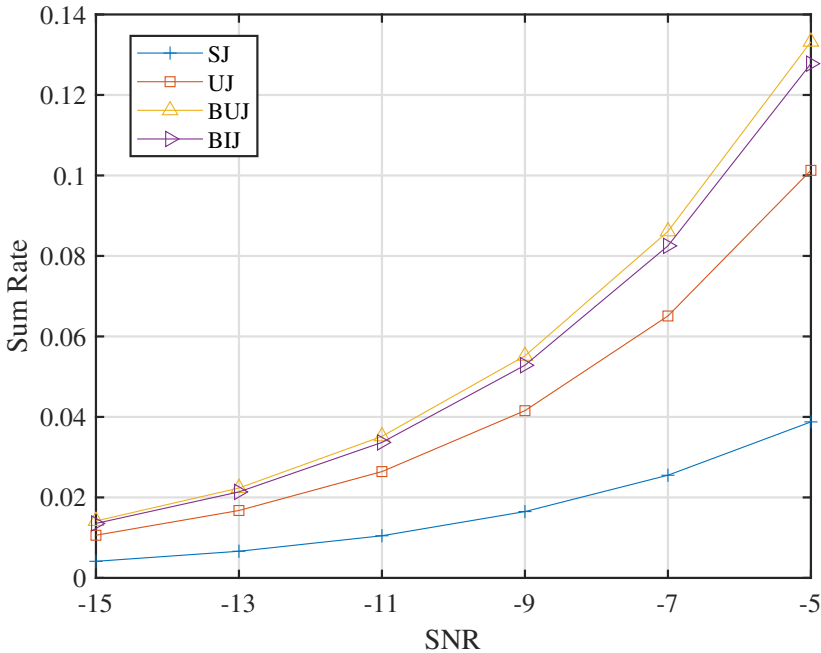

Fig. 3. Sum rate in time-invariant channel against SNR when $M = 2$, $K = 5$, JNR $= -10$ dB, $\mu = 2$.

5   Conclusion

In this paper, we formulate a two-person zero-sum Stackelberg game about jamming power allocation problem in MU-MIMO downlink system with smart jammers in low SNR and JNR regime. We derive the approximated form of the original problem and solve the approximated equilibrium strategy by an iterative method. Numerical results show that the proposed jamming power allocation method can successfully jam such a system. In the future, we plan to extend this work in a non-orthogonal multiple-access (NOMA) regime.

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