Demographic Dynamics of Inhomogeneous Economic Communities as an Institutional Trap

V. G. Zhulego and A. A. Balyakin

Abstract In this article we discuss the demographic dynamics modelling in communities of countries with different levels of economic development. Our approach is based on the stratum model of population growth, proposed by the authors earlier. The observed processes of depopulation of the periphery of such communities were studied within the framework of the model. The phenomenon of institutional trap is considered as an explanatory principle of the functioning of complex socio-economic structures. Its main traits are discussed. Based on the proposed model, the forecasts of population growth in several countries were calculated. Within the proposed model of institutional trap a set of measures to overcome the negative demographic trends were formulated.

Keywords Simulation · Demographics · Institutional trap · Stratum model · Forecast

1 Introduction

Significant progress has been made in the field of creating mathematical models for complex social systems in recent decades [1]. The results of scientific forecast (foresight) of socio-economic processes of countries and the world as a whole by methods of social, humanitarian and natural sciences are used both in the field of public administration and strategic planning, and in large business when developing a growth strategy [2]. Mathematical modeling of social processes, and, in particular, the population growth forecasts should be recognized as an integral element of foresight that enlightens the trends of economic development of the society [3]. The topic of forecasting the population growth of countries and the whole world continues

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to be relevant not only because of the limited life resources and the prospect of overpopulation of the planet, but also because countries tend to build management decisions based on reliable long-term forecasts. The study of this problem leads to the conclusion that such forecasts can be made on the basis of adequate mathematical models.

One of the first and most famous experiences of successful modeling in the field of social and economic sciences was the work of Malthus [4], which caused sharp criticism at the time. The main idea of T. R. Malthus was the point that the difference in the growth of the population and the productive forces (the wealth of society) leads to a complication of the social situation, producing wars, crises and diseases. The discussion of the "overpopulation crisis" predicted by Malthus, which was expected by 2004, led to the correction of the growth model and the creation of several variants of such model. In general, the proposed approaches can be divided into 3 groups:

- models-concepts based on the identification and analysis of general historical patterns and their representation in the form of cognitive schemes describing logical connections between various factors affecting historical processes (ideas of J. Goldstein, I. Wallerstein, L. N. Gumilev, N. S. Rosov, etc.). These models have a high degree of generalization, but they are not mathematical, but purely logical, conceptual in nature;
- pure mathematical models of the simulation type devoted to the description of specific historical events and phenomena (Yu. N. Pavlovsky, L. I. Borodkin, D. Meadows, J. Forrester, etc.). The applicability of such models is usually limited to a fairly narrow space–time interval since they are related to a specific geopolitical situation;
- "intermediate" mathematical models between the two specified types. Their task is to identify the basic patterns that characterize the flow of processes of the type under consideration.

Our work aimed at studying the relationship between socio-economic life and demographic processes, thus relating to the third type of models. This approach involves both conducting mathematical modeling, and taking into account and describing the factors and processes that affect the phenomena under consideration.

One of the first steps in this direction that should be named is the Verhulst logistics model [5] and the concept of "world-systems" [6]. Within the framework of these approaches, the unified system (the world) was divided into subsystems (economic subsystem, and/or social and demographic ones). Later, dynamic models were proposed that go beyond the neoclassical model of economic growth by R. Solow [7], based on equilibrium, when in a stationary state the rate of labor productivity growth is equal to the rate of technological progress, and the rate of economic growth is the sum of the rate of technological progress and the rate of population growth.

At the same time, the Solow model could not explain many problems related to economic growth, which was caused by the fact that many parameters of the model were set exogenously. The next step was the Cobb–Douglas model [8], the Ramsey–Cass–Koopmans model [9] and the Mankiw Romer and Vail model [10].

All the models considered assumed a different format of “combining” social, economic and demographic parameters [6, 11]. In practice, we should mention the Gushchin–Malkov model of macroeconomic dynamics (which describes the economic cycles of US GDP growth, see [12]) and the Korotaev model of great divergence/convergence [13], as well as the Kapitsa population growth model [11]. The last work proposed an exponential model of world population growth, showing the “limits of growth” beyond which a global catastrophe can await the planet. Based on this model, the trend change point or “transition point” (2005) was also predicted, when exponential growth is replaced by a slowdown in growth. Although this model made it possible to predict global trends quite well, it was completely unsuitable for calculating population growth forecasts for individual countries, in particular because migration processes play an important role in these processes, which were not taken into account in the model in any way.

Methodologically, our study continues the approach of dividing a single system into a number of subsystems, and in this sense, the approach can be called hierarchical. The subject of our study will be the population growth in the subsystems of a single “economic community”, taking into account the socio-economic development of this system. It is assumed that in the system under consideration, it is possible to explicitly allocate the Center and the Periphery (i.e., to allocate subsystems). A similar problem was solved earlier in the course of mathematical modeling of the population size based on the stratum model [14].

Our special attention was attracted by the Korotaev’s model [13] of great divergence/convergence, in which an attempt is explicitly made to take into account population growth in the economic model of the development of countries. The authors drew the conclusion about the “inevitable convergence of heterogeneous economic systems of the Periphery-Center type”. Meanwhile, this conclusion contradicts the trends observed in some similar situations, in particular in the EU-Baltic states, where not only convergence is not observed, but rather divergence occurs, accompanied by the process of depopulation of the Periphery.

The obvious disadvantage of mentioned model is the lack of migration in it—and, as it seems to us, this process leads to the opposite effect: to the growth of the economic gap between these parts of the system and, ultimately, leads to the depopulation of the Periphery. The most important resource of economic growth is the labor force leaving depressed regions, which leads to a significant decrease in the economic growth potential of these regions.

The way out of this situation, in our opinion, may be the stratum model of population growth proposed by us in 2014 [14], which can be generalized to the case of a heterogeneous economic system/commonwealth of countries with different levels of development. An additional argument in favor of the attempt to combine the convergence model with the stratum model of population growth was our analysis of population growth forecasts for 2014 for several countries in comparison with statistical data for these countries over the past 4 years, which showed a good agreement of these forecasts with statistics.

The essence of the stratum model is that the population of a country is considered not as homogeneous, but as consisting of several strata. We used following denotations: x(t)—the number of urban population, y(t)—the number of rural population. The parameters that determine the dynamics of changes in each stratum are different, in particular, both the birth rate and mortality in the strata can differ significantly, in addition, there is a significant migration of the population (almost always it is the move of the rural population to the city). Taking into account these circumstances, the model of population growth in a particular country can be presented in this form:

$$\frac{dx}{dt} = a_x x(t) - d_x x^2(t) + c_x \frac{x(t)y(t)}{x^2 + \alpha^2}$$
$$\frac{dy}{dt} = a_y y(t) - d_y y^2(t) - c_y \frac{x(t)y(t)}{x^2 + \alpha^2} \quad (1)$$

The meaning of the parameters a_x and a_y is that they are determined by the balance of instantaneous fertility and mortality in each stratum. Since the economic conditions for the existence of strata and the way of life within strata are different, the characteristic coefficients a_x and a_y can vary greatly. The parameters d_x and d_y conditionally determine the "capacity of the corresponding niche", i.e. they reflect the limited life resources, and the ratio of the parameters (a_x, a_y) and (d_x, d_y) determines the linear "transition point".

The system (1) is written in a symmetric form, the coefficients (a_x, a_y) and (d_x, d_y) determine the internal dynamics of the stratum, and c_x and c_y determine the migration between the strata. The migration flow may depend on many factors, but for each specific country it is a fairly stable parameter, and the last term in the equations should be proportional to the frequency of meetings of residents of the city and village, it is this part of the equation that can significantly accelerate the dynamics of changes in the system.

This is also connected with the possibility of economic growth of the country exceeding the population growth rate, since the migration of the rural population to the city provides additional needs for industrial labor resources. For a particular country, the migration rate is determined by the coefficients $c_{x,y}$ (in the simplest case, these coefficients are equal and opposite in sign, which means that all those who left the "village" ended up in the "city"). If we start from the stratum model of population growth of one country (1), then it is easy to build a model of world population growth based on the principle of hierarchy: for this it is necessary to determine the coefficients (a_x, a_y), (d_x, d_y), (c_x, c_y) for each individual country, moreover, an additional term describing the emigration of the population from one country to other countries should be introduced, this term will be similar to the third in the system (1). The forecast calculated in this order for each country allows us to find the total population of the world.

It should be said that in this approach, the amount of calculations increases significantly, but the accuracy of the forecast also increases. It should be emphasized that it was the forecasts for these countries that led us to the idea of the existence of so-called "institutional traps" that individual countries fall into, i.e. such situations that do not disappear by themselves, but require a purposeful restructuring of the institutional environment. It is for this reason that below we present the results of calculations of forecasts for a group of countries, the choice of which is due to the fact that, using the example of these countries, we will try to construct and test a mathematical model of an "institutional trap", i.e. a situation caused by the institutional characteristics of countries and a way out of which is possible only as a result of serious institutional changes.

In the next section we present the results of numerical simulation for population growth of countries that can be considered as good axample of center–periphery system.

2 The Dynamics of Population Growth as an Institutional Trap. Former USSR Republics Case

Let us discuss the demographics dynamics of former soviet republics, precisely—Russia, Belarus, Ukraine and Baltic states. Here we do not take into account the pandemic influence, hoping its negative consequences will be overcome soon, and the population trend will restore.

A characteristic feature of the demographic dynamics of Russia was a change in the trend from neutral (fluctuation in the population near 147 million) to moderately optimistic growth since 2010 year. The trend change was mainly due to the growth of urban population, while the rural population continued to decline. The graph on the right shows statistics for 4 years from 2014 to 2018. Thus, it can be argued that the managerial decision made in Russia on cash payments at the birth of the second and third child has already influenced the dynamics of population growth (Fig. 1).

Since the collapse of the USSR, Belarus is the only post-Soviet republic where the population began to grow. Urban growth continued after the collapse of the USSR, the rural population continued to decline at a constant rate, while the total population began to grow only in 2014. These changes however are not stable and can be reversed due to political tensions.

Since 1991, Ukraine has shown a steady population decline dynamics: population was decreasing at a constant rate of 1% per year. Moreover, the decrease in the population is taking place both in the countryside and in the city at almost the same rate. This dynamic indicates the ongoing economic crisis in the country. Data for 4 years (from 2014 to 2018) do not provide any indication of a change in trend (Fig. 2).

After the collapse of the USSR in the period up to 2000, the population decline in Estonia was the fastest among the post-Soviet countries, and then it diminished. The shift of the dynamics in recent years (positive growth) should be considered separately, since there is no obvious explanation for such a change.

After 1991, the most dramatic situation developed in Lithuania—depopulation was stable both in the city and in the countryside, and there is no need to talk about

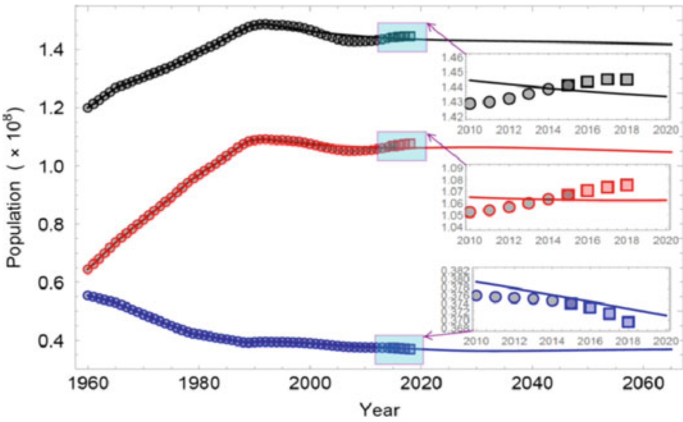

Fig. 1 Russia population forecast and statistical data

a change in trend. In Latvia the situation is almost completely similar to Lithuania: depopulation is going on at a constant pace, and no change in the trend is expected.

The graphs indicate the presence of two groups of countries: these are the post-Soviet countries that remained outside the economic blocs and the post-Soviet countries that entered the EU (or are associated with the EU). The demographic situation in these groups is radically different.

All this suggests that a mathematical model should be created that would take into account migration between countries and at the same time reveal the reasons for such large-scale population migrations. In our opinion stratum model is the best possible starting point on the way to such a socio-demographic model. As an explanatory principle we rely on institutional reasons. Since the ideas to link demographic variables with economic and social variables have appeared for a long time, we decided to make an attempt to create an economic-socio-demographic model that would allow us to consider the problems of demography in connection with economic and social ones. Moreover, one of these attempts raised the problem of convergence of countries with different levels of economic development.

Our explanation of the differences in dynamics boils down to the following causal chain: joining the EU (or association) opens borders for migration of the population, the existing gap in the standard of living and education leads to the flow of migrants from conditionally "poor countries" towards rich ones. Such migration deprives poor countries of human development resources, and then their economic development slows down.

The problem of heterogeneity or uneven development of countries has already been considered in the above mentioned work [13]. Obviously, without taking into account migration, such a model leads to "great convergence"—a completely fair goal, which was set by the countries that joined the EU. However, in fact, the entry of the Baltic countries into the EU led to depopulation and an increase in the income gap. A similar process is underway in the case of Ukraine (although with some delay).

We can expect the same situation with Belarus—Russia case, or Ukraine—Russia (if political tensions declined).

All this allows us to conclude that accounting for migration in the Center–Periphery model is necessary. Otherwise, the model will not adequately describe the dynamics of the system (Figs. 2, 3 and 4).

Of course, we will model a system that is simplified compared to the real one, but we will keep its most important features: the free movement of people (migration) and the heterogeneity of the system. Therefore, we will assume that the system consists of a developed Center and a backward Periphery. From the point of view of the stratum model, the new EU countries represent the same “city” and “rural” in the country, and there is unlimited and practically unregulated migration between

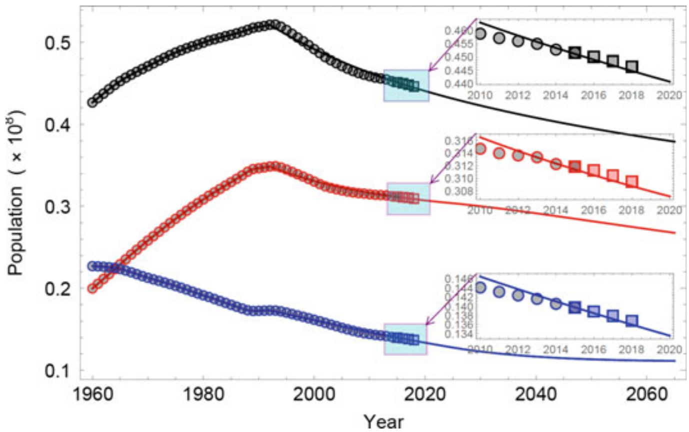

Fig. 2 Ukraine population forecast and statistical data

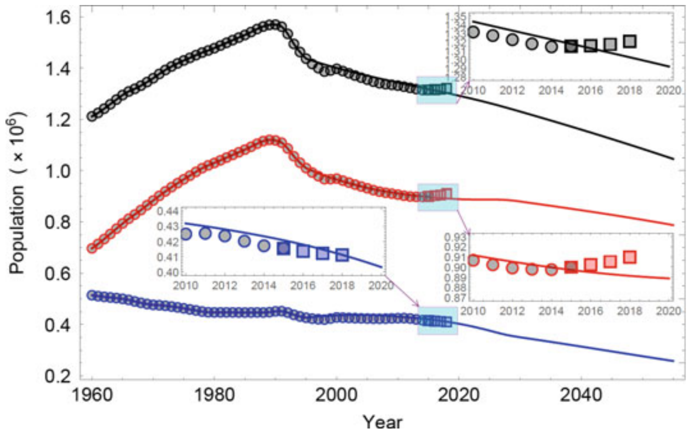

Fig. 3 Estonia population forecast and statistical data

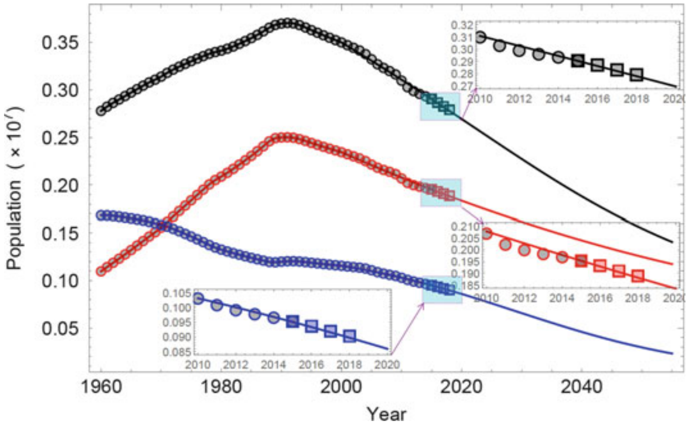

Fig. 4 Lithuania population forecast and statistical data

these strata. We note that the proposed model should transform into previous model of [14] in the case of low migration process.

3 Model Improvement: Inclusion of Social and Economic Factors

For the sake of generality, we will keep the previous designations: x(t)—the population of the Center, y(t)—the population of the Periphery. Let's supplement system (1) by adding equations to take into account the socio-economic development of the regions. By analogy with the work [13], we will introduce the level of "wealth"—$S_{x,y}$ and "education" $E_{x,y}$.

In practice, these factors reflect material wealth and other intangible benefits. The meaning of the parameters a_x and a_y is that they are determined by the balance of instantaneous fertility and mortality in each of the subsystems: in the Center and Periphery. Since the economic conditions of existence in the Center and on the Periphery, as well as the way of life within each subsystem are different, the characteristic coefficients ax and ay can be very different. The parameters d_x and d_y conditionally determine the "capacity of the corresponding niche", i.e. they reflect the limited life resources in the subsystems.

$$\frac{dx}{dt} = a_x x(t) - d_x x^2(t) + \left(A\frac{S_x - S_y}{S_x + S_y} + B\frac{E_x - E_y}{E_x + E_y} \right) \frac{x(t)y(t)}{x^2 + \alpha^2}$$
$$\frac{dy}{dt} = a_y y(t) - d_y y^2(t) - \left(A\frac{S_x - S_y}{S_x + S_y} + B\frac{E_x - E_y}{E_x + E_y} \right) \frac{x(t)y(t)}{x^2 + \alpha^2}$$

$$\frac{dS_x}{dt} = b_x S_x E_x\left(1 - \frac{G}{G_{\text{lim}}}\right)$$
$$\frac{dS_y}{dt} = b_y S_y E_y\left(1 - \frac{G}{G_{\text{lim}}}\right) + \varphi$$
$$\frac{dE_x}{dt} = f_x E_x(1 - E_x)$$
$$\frac{dE_y}{dt} = f_y E_y\left(1 - E_y\right) + \psi \tag{2}$$

The first two equations of the system (2) is written in a symmetric form, the coefficients (a_x, a_y) and (d_x, d_y) determine the internal dynamics of the subsystem Center and the Periphery, here, instead of the coefficients c_x and c_y, which determine the migration of the population from one part of the subsystem to another, some functions of the variables $S_{x,y}$ and $E_{x,y}$ are selected, which characterize the per capita income and the level of education in each part of the system: S_x is a relatively excess product per capita of the Center population, and S_y is a relatively excess product per capita of the Periphery population; E_x is the level of education of the population in the Center, and E_y is the level of education of the population in the Periphery. Differences in income level and in the level of education in the Center and on the Periphery will induce migration of the population to the Center. Additionally, the following notation is introduced in system (2): $G(t) = x(m + S_x) + y(m + S_y)$ is the GDP of the Center–Periphery system, m is the minimum necessary product (estimated as $ 440), G_{lim} is a certain fundamental limitation and a normalization term that defines a fundamental constraint in the system. In the model [14] describing the world-system, G_{lim} = $ 400 trillion, in the model we propose, G_{lim} should coincide in order with the EU GDP, i.e. about $ 100 trillion.

Let us pay attention to the choice of signs in these terms in the first two equations of the system (2) that fixes the direction of migration from the Periphery to the Center. Thus, to describe the dynamics of interaction of the heterogeneous Center–Periphery system, a socio-economic demographic mathematical model (2) of the system is proposed, which takes into account both the dynamics of the population of individual parts of the system, and the migration of the population from one part of the system to another, due to the difference in income in different subsystems and the difference in the level of education. Equation (2) contain φ, ψ which we will call "convergence functions", which show the relationship between the Center and the periphery. Unlike [13], we do not postulate the form of these functions, moreover, in our opinion, their form needs serious refinement.

At the same time, the choice of convergence functions should reflect the main trends in the modern world. Thus, according to the authors [12, 13], the gap between highly and medium-developed countries has been decreasing at a particularly rapid pace in recent years, and the gap between highly and underdeveloped countries is decreasing at a noticeably slower pace, at the same time they show an increase in the gap between medium and underdeveloped countries. In practice, it turns out that advanced economies are "going into isolation", medium-developed countries receive the greatest benefits from globalization, catching up with developed countries, but underdeveloped countries are moving to increasingly worse positions. According to the author [15], we are talking about the reconfiguration of the world-system and the trend towards the concentration of income. There is a discrepancy between the richest and the poorest people in the world, despite the general convergence of average incomes.

These conclusions, in our opinion, are very controversial. The existing practice shows that a country's participation in a successful economic community does not guarantee its automatic convergence, does not automatically raise it to the level of the Center. In our opinion, correct accounting of migration can lead simultaneously to depopulation and economic degradation of the Periphery, and "on average" (or "per capita") there will be an increase in welfare.

In the model we propose, in the simplest case:

$$\varphi = 0$$
$$\psi = -\gamma E_x E_y$$

Note that the function ψ must be non-zero, otherwise the resulting solutions for E_y will tend to 1, i.e. the village becomes fully educated, which will lead to a drop in the birth rate. Practically, in the system (2), the presence of the term ψ ensures, with the simplest choice of the convergence function, the presence of a stable stationary solution of the form

$$(E_x, E_y) = \left(1, 1 - \frac{\gamma}{c_y}\right)$$

It corresponds to full education in the Center and some non-zero (but not 100%) education in the Periphery. The proposed (2) are essentially nonlinear, and may contain complex dynamics, such as periodic oscillations, periodic oscillations with attenuation or increase in amplitude, or, conversely, an asymptotic output to constant values.

Stability analysis for the system (2) reveals that there are always trivial solutions for $E_{x,y} = 0$ or 1 (totally educated or fully uncivilized strata). Assuming two extreme cases $S_x \gg S_y$ (the welfare of the Center significantly exceeds the welfare of the Periphery) and $S_x = S_y$ (the convergence occurred) we obtain following results. For $S_x \gg S_y$ we get $A_{>} A_{(\text{critical value})} = x^{(0)} a_y = a_y\, a_y/d_x$.

For the case $E_x = 1$, $E_y = 0$ (developed center and backward periphery) we get the restriction $(A + B)_{(\text{critical value})} = x(0)\, a_y$. The system loses stability if the sum of the parameters (A + B) exceeds the critical value. Thus interesting realistic solutions system (2) will be played out around the adiabatic values of variables x(t) and y(t).

A similar approach (adiabatic change of parameters) can be applied in the case of convergence functions, assuming that they are a small perturbation that has the greatest impact on the population of the periphery.

It seems that the next step would be reasonable to choose the convergence functions by analogy with the terms in the first two equations in the form

$$\varphi = \delta\left(A\frac{S_x - S_y}{S_x + S_y} + B\frac{E_x - E_y}{E_x + E_y}\right)S_x S_y$$
$$\psi = -\gamma\left(A\frac{S_x - S_y}{S_x + S_y} + B\frac{E_x - E_y}{E_x + E_y}\right)E_x E_y \quad (3)$$

The obtained (2) are characterized by the following properties. Firstly, there are no exogenous variables in them, taking into account external factors is contained only in the parameters (coefficients) of the equations. Secondly, the meaning of the parameters contained in the model follows from the equations themselves, and the values of these parameters can be determined from the analysis of statistical data for a certain period. Third, the type of convergence functions is not defined a priori.

It seems that this system of equations will allow us to study various modes of behavior of the Center–Periphery system depending on the values of the parameters, as well as to predict the behavior of the Center–Periphery system, in the case when the parameters are determined. This model is a development of the stratum model, taking into account the ideas about the functioning of the "world-system" [6]. The authors suggest that the developed approach will allow us to consider complex systems where simplified approaches do not work.

4 Conclusion

The proposed model of the institutional trap is described by a system of (2), which is characterized by the following properties:

Firstly, there are no exogenous variables in them, external factors are taken into account only in the parameters (coefficients) of the equations.
Secondly, the meaning of the parameters contained in the model follows from the equations themselves, and the values of these parameters can be determined from the analysis of statistical data for a certain period.
Third, the type of convergence functions is not defined a priori and may vary depending on the task.

It seems that this system of equations will allow us to study various modes of behavior of the Center–Periphery system depending on the values of the parameters, as well as to predict the behavior of the Center–Periphery system, in the case when the parameters are determined.

We emphasize once again that this model is a development of the stratum model [14], taking into account the ideas about the functioning of the "world-system" [6, 13]. The authors suggest that the developed approach will allow us to consider complex systems where simplified approaches do not work. The results obtained, however, should be used with caution: their applicability to specific situations is limited by both the initial conditions and the current operating conditions of the system under consideration. In terms of the institutional trap, this is equivalent to the destruction of the trap in the course of institutional restructuring (reform). Note that a similar effect can be achieved by a sharp change in the initial parameters of the system (the population in the Periphery, which has changed dramatically, for example, in the results of uncontrolled migration), which is equivalent to the "transfer" of the system to the pool of attraction of another attractor.

Given the almost unlimited labor migration within the EU, it is assumed that the proposed model will adequately describe the case of an institutional trap that occurs in the Center–Periphery system, in which economic integration does not lead to an equalization of the level of per capita income in the subsystems, but leads to the depopulation of the Periphery. The results of computer modeling should make it possible to estimate the characteristic times of the development of unfavorable dynamics (the "half-life" of the Periphery countries). The authors also hope that the study of this model will allow us to formulate recipes for getting out of emerging institutional traps (by controlling the parameters of the system).

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