To Stochastic Resonance in Homopolar Dynamo

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Abstract This chapter is devoted to discussion of the behavior of one-disk dynamo under the action of harmonic and random signals. Evaluations of separated effects of harmonic and random external voltages in the framework of the linearized Bullard equations have been presented. As random signals with zero average the Gaussian delta-correlated noise and the Langevin stochastic process have been considered. In particular, as physical values characterizing these influences both autocorrelation

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functions of observables and their spectral densities have been calculated. This information is important for design and testing of homopolar dynamo layout to perform analog research of stochastic resonance in this device in nonlinear regime.

Keywords Elecromechanical system · Phase plane · Equilibrium states · Amplitude responses · The Wiener-Khinchin theorem · The Jordan's lemma · Design and testing · The Fourier transform

Abbreviations

GSSP	The Gaussian stationary stochastic process
ACF	The autocorrelation function
SD	The spectral density

1 Introduction

Stochastic resonance is known to be a cooperative effect in nonlinear systems manifesting itself in increasing of the output signal-to-noise ratio under addition of the optimal portion of noise [1].

At present great attention is paid to studying of stochastic resonance in multidimensional systems arising from physics through chemistry to biology and neuroscience [2–5]. However, in our opinion, the most correct path in investigation of stochastic resonance leading to a real understanding of the essence of this phenomenon is choosing of a fairly simple dynamic system with a relatively small dimension and a detailed study of this one. As a rule there are no analytical solutions both the nonstationary Fokker-Plank-Kolmogorov equation for such system and stochastic differential equations describing its behaviour. Numerical solution of these problems is quite hard too [6, 7]. Therefore this system ought to allow experimental investigation.

On the one hand, from the point of view of clarity, preference should be given to mechanical systems. Such systems are easily perceived and interpreted due to our daily experience. On the other hand, electrical systems are characterized by the ease of controlling of external influences. Hence it is convenient to take an electromechanical system as a model system for experimental and theoretical research of stochastic resonance.

In the framework of this approach we study one-disk dynamo (the so-called Bullard dynamo). At first this electromechanical system was suggested in article [8] in order to illustrate a number of astrophysical and geophysical effects concerning motion of electrically conducting fluid in a magnetic field (see [9] and references therein). Contrary to original article [8] we take into consideration both electrical load in parallel with the field coil and friction at the axis of the dynamo. But we restrict ourselves by investigation of the linear response of the Bullard dynamo because of our final aim is design of functioning homopolar dynamo for analog modeling of stochastic resonance in this system. We stress that in our research there is no any magnetohydrodynamic background—compare for instance with work [10].

The rest of the chapter is organized as follows: in Sect. 2, we discuss equations of motion for the Bullard dynamo and their linearization. Section 3 is devoted to calculations of influence of harmonic external voltage on the linearized Bullard system. Section 4 deals with linear responses of the system on random signals with zero average, namely, on the Gaussian delta-correlated noise and the Langevin stochastic process. Final section is devoted to discussion of results elaborated and conclusions.

2 Main Equations

Mathematical model of the homopolar dynamo is given by the following system of stochastic ordinary differential equations:

$$\begin{cases} L \cdot \frac{\mathrm{d}J}{\mathrm{d}t} + R \cdot J = M \cdot J \cdot \Omega + U(t) \\ I \cdot \frac{\mathrm{d}\Omega}{\mathrm{d}t} = K - M \cdot J^2 - 2 \cdot \gamma \cdot \Omega \end{cases}, \tag{1}$$

where

$J(t)$	is electric current via the inductance L on Fig. 1;
$\Omega(t)$	is angular speed of rotation of the disk of dynamo;
R	is value of resistance in the electrical circuit on Fig. 1;
M	is coefficient of mutual inductance:
$U(t)$	is an external voltage;
I	is moment of inertia for the dynamo;
K	is constant mechanical torque on the axis of the dynamo;
$2 \cdot \gamma$	is coefficient of mechanical friction on the dynamo axis.

To study stochastic resonance in the system on Fig. 1 one ought to choose external voltage in (1) as follows:

$$U(t) = U_0 \cdot \cos(\nu \cdot t) + V(t), \tag{2}$$

where

U_0	is amplitude of harmonic signal;
ν	is circular frequency of harmonic signal;
$V(t)$	is the Gaussian stationary stochastic process (GSSP) with zero average:

$$\langle V(t) \rangle = 0, \tag{3}$$

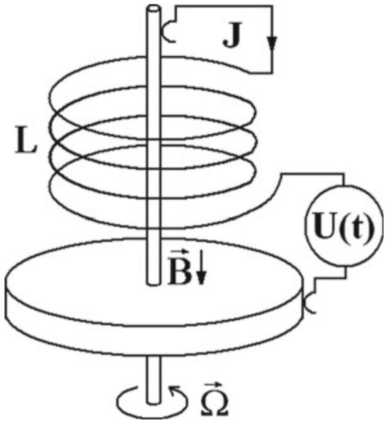

Fig. 1 Structural scheme of the homopolar dynamo

and fixed autocorrelation function (ACF):

$$\langle V(t) \cdot V(t') \rangle = B(t' - t). \tag{4}$$

We underline that our approach in (1) differs sharply from one in paper [11] because of authors of this paper apply separation of the magnetic flux on magnetic flux across disk of the dynamo and magnetic flux across the loops of inductance. This separation of magnetic flux on two parts leads to increasing of dimension of phase space of the system.

For further analysis of system (1) it is convenient to introduce the next dimensionless variables and parameters:

$$\begin{gathered} x_1 = \sqrt{\frac{M}{K}} \cdot J, \quad x_2 = \sqrt{\frac{M \cdot I}{L \cdot K}} \cdot \Omega, \quad \nu_0 = \sqrt{\frac{M \cdot K}{L \cdot I}}, \\ \mu = R \cdot \sqrt{\frac{I}{M \cdot L \cdot K}}, \delta = \gamma \cdot \sqrt{\frac{L}{I \cdot K \cdot M}}, U_m = K \cdot \sqrt{\frac{L}{I}}. \end{gathered} \tag{5}$$

After that one can rewrite system (1) in the following form:

$$\begin{cases} \dot{x}_1 = -\mu \cdot x_1 + x_1 \cdot x_2 + u(\tau) \\ \dot{x}_2 = 1 - x_1^2 - 2 \cdot \delta \cdot x_2 \end{cases}, \tag{6}$$

where

$u(\tau) = U(t) \big/ U_m$	is dimensionless external voltage;
$\dot{x}_{1,2}$	are derivatives of dimensionless variables $x_{1,2}$ with respect to dimensionless time $\tau = \nu_0 \cdot t$.

The system (6) in the absence of external voltage is defined as:

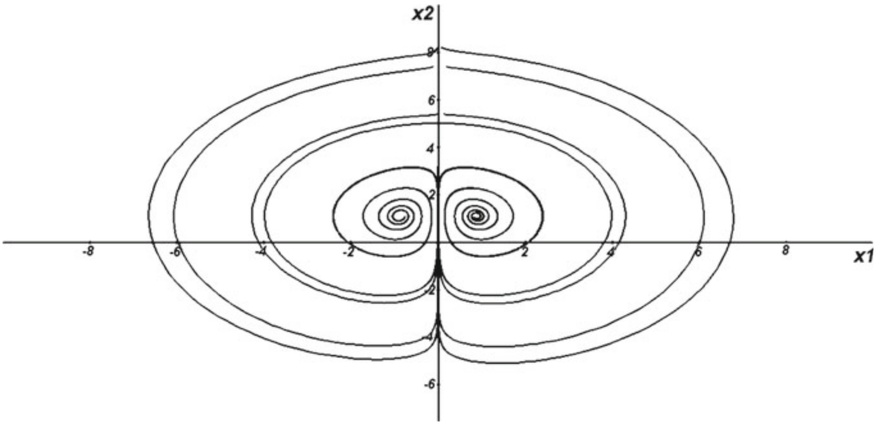

Fig. 2 Phase plane of the homopolar dynamo in the absence of external load

$$\begin{cases} \dot{x}_1 = -\mu \cdot x_1 + x_1 \cdot x_2 \\ \dot{x}_2 = 1 - x_1^2 - 2 \cdot \delta \cdot x_2 \end{cases}. \tag{7}$$

It is easy to see that if $0 < \delta < 1/2\mu$ then system (7) possesses by three equilibrium states: $O^s(0,\ 1/(2 \cdot \delta))$ and $O^{\pm}(\pm\sqrt{1 - 2 \cdot \delta \cdot \mu},\ \mu)$. It is not difficult to check that if $0 < \delta < \sqrt{2 + 4\mu^2} - 2\mu$ then points $O^{\pm}$ are stable focuses and if $\sqrt{2 + 4\mu^2} - 2\mu < \delta < 1/2\mu$ then points $O^{\pm}$ are stable nodes. Point O^s is saddle point in both cases.

We shall suppose that dimensionless damping factor δ is quite small therefore we shall deal with situation when points $O^{\pm}$ are stable focuses. Phase plane of system (7) at $\mu = 1.0$ and $\delta = 0.1$ corresponding to the case under consideration is shown on Fig. 2.

It is obvious that system (7) is invariant under transformation of variables $(x_1, x_2) \rightarrow (-x_1, x_2)$ therefore to calculate linear response of the system (6) it is enough to take into account only vicinity of the point O^+.

Introducing for system (6) new variables $y_{1,2}$ as follows:

$$x_1 = +\sqrt{1 - 2 \cdot \delta \cdot \mu} + y_1, \quad x_2 = \mu + y_2, \tag{8}$$

and rejecting terms with powers of $y_{1,2}$ greater than one we find that system (6) is reduced to this one:

$$\begin{cases} \dot{y}_1 = \sqrt{1 - 2 \cdot \delta \cdot \mu} \cdot y_2 + u(\tau) \\ \dot{y}_2 = -2 \cdot \sqrt{1 - 2 \cdot \delta \cdot \mu} \cdot y_1 - 2 \cdot \delta \cdot y_2 \end{cases}. \tag{9}$$

From system (9) it is easy to observe that variable y_2 obeys to the equation of motion for harmonic oscillator with damping factor δ and fundamental frequency

$\omega_0 = \sqrt{2 \cdot (1 - 2 \cdot \delta \cdot \mu)}$ under the action of external force:

$$\ddot{y}_2 + 2 \cdot \delta \cdot \dot{y}_2 + \omega_0^2 \cdot y_2 = -\sqrt{2} \cdot \omega_0 \cdot u(\tau), \tag{10}$$

and that the behaviour of variable y_1 is governed by the behaviour of variable y_2 as follows:

$$y_1 = -\frac{\dot{y}_2 + 2 \cdot \delta \cdot y_2}{\sqrt{2} \cdot \omega_0}. \tag{11}$$

At last for self-consistency of above presented linearization external dimensionless voltage ought to be weak: $|u(\tau)| \ll 1$.

3 Action of Harmonic Signal on the Linearized Bullard Dynamo

At first let us consider behaviour of the system (9) under the influence of external voltage:

$$u(\tau) = A_0 \cos(\omega\tau), \tag{12}$$

where in accordance with formulae (5) $A_0 = \frac{U_0}{U_m}$; $\omega = \nu/\nu_0$.

Looking at (10) with right hand side (12) one can see that in this case it describes harmonically excited linear oscillator with damping therefore we may solve it in the framework of the well-known complex amplitude method.

Seeking solution of (10) in the following form:

$$y_2(\tau) = \mathrm{Re}[A_2(\omega) \cdot \exp(i\omega\tau)], \tag{13}$$

one can easily find that complex amplitude $A_2(\omega)$ is equal to:

$$A_2(\omega) = -\frac{\sqrt{2} \cdot \omega_0}{\omega_0^2 - \omega^2 + 2i\delta\omega} \cdot A_0. \tag{14}$$

Further substituting expression (13) into (11) and using formula (14) it is not difficult to establish that

$$y_1(\tau) = \mathrm{Re}[A_1(\omega) \cdot \exp(i\omega\tau)], \tag{15}$$

complex amplitude $A_1(\omega)$ in formula (15) being equal to:

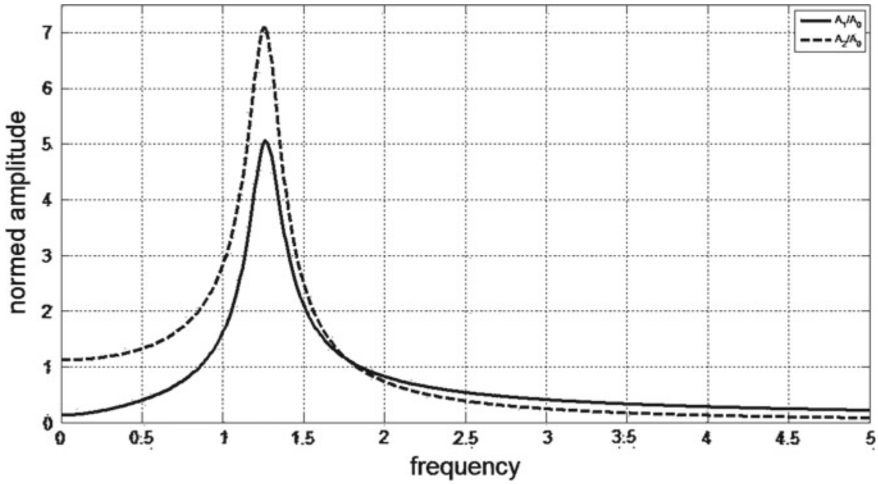

Fig. 3 Amplitude responses of the homopolar dynamo

$$A_1(\omega) = \frac{i\omega + 2\delta}{\omega_0^2 - \omega^2 + 2i\delta\omega} \cdot A_0. \tag{16}$$

Thus from formulas (14) and (16) it is easy to obtain that amplitude responses of dynamical variables of system (9) on voltage (12) are equal to:

$$\begin{aligned} \frac{|A_1(\omega)|}{A_0} &= \sqrt{\frac{\omega^2+4\cdot\delta^2}{(\omega^2-\omega_0^2)^2+4\cdot\delta^2\omega^2}} \\ \frac{|A_2(\omega)|}{A_0} &= \frac{\sqrt{2}\cdot\omega_0}{\sqrt{(\omega^2-\omega_0^2)^2+4\cdot\delta^2\omega^2}} \end{aligned}. \tag{17}$$

Graphs of dependences (17) on dimensionless frequency ω for $\mu = 1.0$ and $\delta = 0.1$ are presented on Fig. 3. On this Figure continuous line corresponds to function $A_1(\omega)$ and dashed line corresponds to function $A_2(\omega)$. Both of them demonstrate typical resonance behavior.

4 Action of the Gaussian Delta-Correlated Noise and the Langevin Stochastic Process on the Linearized Bullard Dynamo

Let us now suppose that external voltage is GSSP purely.

In this case it is interesting to determine the following ACF:

$$B_J(t, t') =< (J(t) - J_a)(J(t') - J_a) >, \tag{18}$$

where

$$J_a = \langle J(t) \rangle \tag{19}$$

is average value of electric current in the circuit on Fig. 1.

Reducing in accordance with formulas (5) input GSSP voltage to dimensionless form:

$$u(\tau) = \frac{V(t)}{U_m} \tag{20}$$

and substituting expression (20) into formula (3) we establish that:

$$\langle u(\tau) \rangle = 0, \tag{21}$$

therefore from formulas (10) and (11) one can immediately obtain that:

$$\langle y_1(\tau) \rangle = \langle y_2(\tau) \rangle = 0. \tag{22}$$

Thus combining formulas (5), (8) and (22) it is easy to find that:

$$J_a = \sqrt{\frac{K}{M}} \cdot \sqrt{1 - \frac{2\gamma \cdot R}{K \cdot M}}, \tag{23}$$

hence

$$B_J(t, t') = \frac{K}{M} \cdot B_1(\tau, \tau'), \tag{24}$$

where

$$B_1(\tau, \tau') = < y_1(\tau) y_1(\tau') > . \tag{25}$$

On the other side in correspondence with formula (11) behavior of value $y_1(\tau)$ is controlled by value $y_2(\tau)$ therefore ACF (25) is expressed via the next ACF:

$$B_2(\tau, \tau') = \langle y_2(\tau) y_2(\tau') \rangle. \tag{26}$$

Inserting expression (11) into definition (25) and using the simplest properties of ACF [12] it is not hard to prove that:

$$B_1(\tau,\tau') = \frac{1}{2\omega_0^2}\left[\frac{\partial^2 B_2(\tau,\tau')}{\partial\tau\partial\tau'} + 2\delta\left(\frac{\partial B_2(\tau,\tau')}{\partial\tau} + \frac{\partial B_2(\tau,\tau')}{\partial\tau'}\right) + 4\delta^2 B_2(\tau,\tau')\right]. \tag{27}$$

Further after looking at formula (4) and comparing it with formula (20) it is obvious that:

$$< u(\tau)\cdot u(\tau') >= b_u(\tau'-\tau), \tag{28}$$

where

$$b_u(\tau'-\tau) = \frac{1}{U_m^2} B(t'-t). \tag{29}$$

It is clear that formulas (28) and (29) demonstrates stationary state of dimensionless input voltage therefore value $y_2(\tau)$ is GSSP too because of it obeys to linear differential equation with constant coefficients (10) [12]. It means that ACF (26) in fact depends only on variable $\theta = \tau' - \tau$:

$$B_2(\tau,\tau') \equiv B_2(\theta). \tag{30}$$

Substituting representation (30) into formula (27) one can easily derive that:

$$B_1(\theta) = \frac{1}{2\omega_0^2}\left[-\frac{d^2 B_2(\theta)}{d\theta^2} + 4\delta^2 B_2(\theta)\right], \tag{31}$$

hence $y_1(\tau)$ is also GSSP.

For further advance it is convenient in accordance with the Wiener-Khinchin theorem [12] to introduce spectral densities (SD) of ACF (30) and (31) as follows:

$$S_{1,2}(\omega) = \int_{-\infty}^{+\infty} B_{1,2}(\theta)\cdot\exp(-i\cdot\omega\cdot\theta)\cdot d\theta. \tag{32}$$

After the Fourier transform relation (31) is reduced to the next one between SD $S_1(\omega)$ and $S_2(\omega)$:

$$S_1(\omega) = \frac{\omega^2 + 4\delta^2}{2\omega_0^2}\cdot S_2(\omega). \tag{33}$$

At last it is well-known that for linear homogeneous system (10) connection between input and output SD is expressed via its amplitude response (17) [12] namely:

$$S_2(\omega) = \left|\frac{A_2(\omega)}{A_0}\right|^2 \cdot S_u(\omega), \tag{34}$$

where

$$S_u(\omega) = \int_{-\infty}^{+\infty} b_u(\theta) \cdot \exp(-i \cdot \omega \cdot \theta) \cdot d\theta \tag{35}$$

is SD for ACF (28).

Thus combining formulas (17), (33) and (34) one can obtain that:

$$S_1(\omega) = \frac{\omega^2 + 4 \cdot \delta^2}{(\omega^2 - \omega_0^2)^2 + 4 \cdot \delta^2 \cdot \omega^2} \cdot S_u(\omega). \tag{36}$$

Inverse Fourier transform of expression (36) is known to represent ACF (25):

$$B_1(\theta) = \int_{-\infty}^{+\infty} \frac{\omega^2 + 4 \cdot \delta^2}{(\omega^2 - \omega_0^2)^2 + 4 \cdot \delta^2 \omega^2} \cdot S_u(\omega) \cdot \exp(i \cdot \omega \cdot \theta) \cdot \frac{d\omega}{2\pi}. \tag{37}$$

If input voltage is the Gaussian delta-correlated noise (the white noise) then ACF (4) is equal to:

$$B(t' - t) = 2 \cdot D_V \cdot \delta(t' - t), \tag{38}$$

therefore

$$b_u(\tau' - \tau) = 2 \cdot D \cdot \delta(\tau - \tau'), \tag{39}$$

where intensity of stochastic process is renormalized in accordance with formula (29) as $D = D_V \cdot \nu_0 \big/ U_m^2$.

Further expression (35) gives us that SD of GSSP with ACF (39) is equal to $S_u(\omega) = 2 \cdot D$. Thus integrand in formula (37) possesses by four simple poles $\pm\sqrt{\omega_0^2 - \delta^2} \pm \mathrm{i} \cdot \delta$ hence using the well-known Jordan's lemma one can calculate explicit representation of ACF (25) in this case:

$$B_1(\theta) = \frac{D \cdot \exp(-\delta|\theta|)}{2\delta\sqrt{\omega_0^2 - \delta^2}} \cdot \mathrm{Re}\left[\frac{\omega_*^2 + 4\delta^2}{\omega_*} \cdot \exp(\sqrt{\omega_0^2 - \delta^2}|\theta|)\right], \tag{40}$$

where $\omega_* = \sqrt{\omega_0^2 - \delta^2} + i \cdot \delta$.

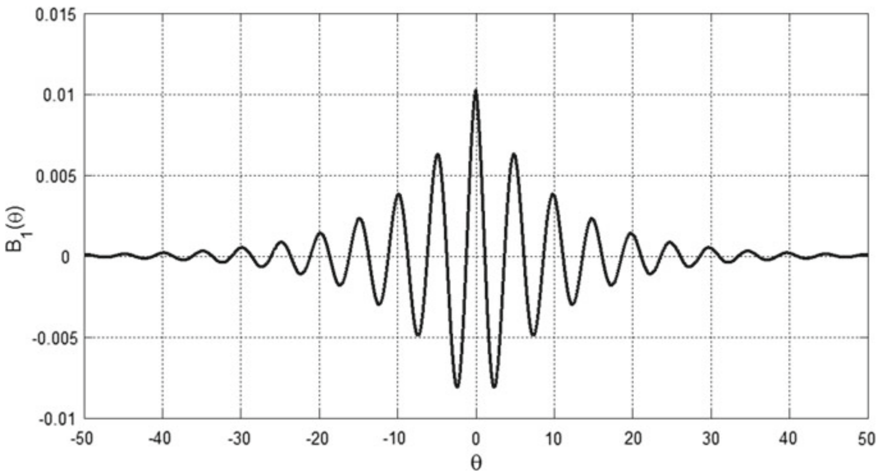

Fig. 4 Reaction of the homopolar dynamo on the Gaussian delta-correlated noise

Graph of the ACF (40) for $\mu = 1.0$, $\delta = 0.1$ and $D = 0.002$ is shown on Fig. 4.

If input voltage is the Langevin sochasic process then dimensionless ACF (29) may be chosen in the following form [13]:

$$b_u(\tau' - \tau) = \sigma^2 \cdot \exp(-\gamma|\tau' - \tau|), \quad \gamma > 0. \tag{41}$$

where σ^2 is dispersion of input GSSP $u(\tau)$.

SD corresponding to ACF (41) is equal to [13]:

$$S_u(\omega) = \frac{2 \cdot \gamma \cdot \sigma^2}{\omega^2 + \gamma^2}. \tag{42}$$

It means that in this case two additional simple poles $\pm i \cdot \gamma$ arise in integrand in formula (37).

In the same manner one can derive that for SD (42) ACF (25) is equal to the next sum:

$$B_1(\theta) = B_1^1(\theta) + B_1^2(\theta), \tag{43}$$

where

$$B_1^1(\theta) = \frac{\gamma \cdot \sigma^2 \cdot \exp(-\delta|\theta|)}{2 \cdot \delta \cdot \sqrt{\omega_0^2 - \delta^2}} \cdot \mathrm{Re}\left[\frac{\omega_*^2 + 4\delta^2}{\omega_* \cdot (\omega_*^2 + \gamma^2)} \cdot \exp(\sqrt{\omega_0^2 - \delta^2}|\theta|)\right] \tag{44}$$

and

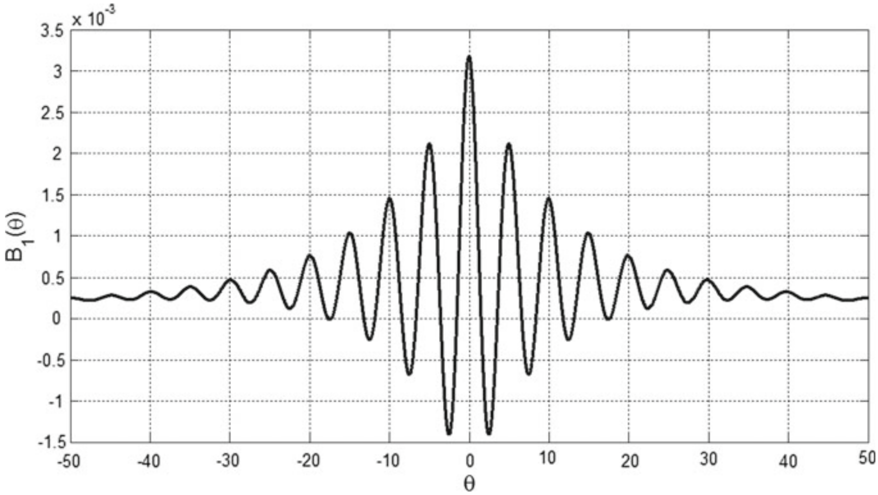

Fig. 5 Reaction of the homopolar dynamo on the Langevin stochastic process

$$B_1^2(\theta) = \sigma^2 \cdot \frac{4 \cdot \delta^2 - \gamma^2}{(\omega_0^2 + \gamma^2)^2 - 4 \cdot \delta^2 \cdot \gamma^2} \cdot \exp(-\gamma|\theta|). \tag{45}$$

Graph of the ACF (43) for $\mu = 1.0$, $\delta = 0.1$, $\gamma = 0.02$ and $\sigma = 0.2$ is presented on Fig. 5. Comparing Fig. 5 with Fig. 4 one can observe that this graph also has oscillatory character stipulated by function (44). But moreover this graph possesses by variable vertical shift caused by contribution of function (45) into expression (43).

5 Conclusion

In the chapter linear responses of the homopolar dynamo both on weak harmonic input voltage and weak GSSP input voltage have been calculated. This preliminary research gives one a possibility of investigation of stochastic resonance in the Bullard dynamo by means of analog modeling.

To realize this research program one ought to evaluate physical parameters of the system on Fig. 1 and then use them to make its layout. After that one can perform a number of tests of the operation of the layout.

The first test is an action of weak ($U_0 \ll U_m$) harmonic signal with very slowly varying circular frequency on the homopolar dynamo layout. If dimensionless circular frequency ω of this input signal gets closer to ω_0 then a sharp increase in amplitude of electric current in the circuit should be observed in accordance with formula (16) (see also Fig. 3).

The second test is an application to the layout of the weak Gaussian delta-correlated noise as an input voltage. In this case measured ACF (18) must correspond to the calculated dependence (40) (see also Fig. 4).

Moreover nonlinearity of a system is known to transform GSSP into non-Gaussian stochastic process [12], therefore, in order to control the role of nonlinearity of system (1) one should measure the following triple ACF [14]:

$$T(t_1, t_2) = \langle (J(t) - J_a)(J(t + t_1) - J_a)(J(t + t_2) - J_a) \rangle \tag{46}$$

and calculate its bispectrum [14]:

$$Q(\omega_1, \omega_2) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} T(t_1, t_2) \cdot \exp(-i\omega_1 t_1 - i\omega_2 t_2) \cdot dt_1 dt_2. \tag{47}$$

If the influence of nonlinearity is small then both value (46) and value (47) must be close to zero due to the Gaussian nature of the input signal.

The third test is an action of the weak Langevin stochastic process as an input voltage. This kind of input voltage can be obtained by means of transferring of the Gaussian delta-correlated noise via four-terminal network with resistance and capacitance [12]. In this case measured ACF (18) must correspond to the calculated dependence (43) (see also Fig. 5). And it is necessary to oversee closeness to zero of values (46) and (47) too.

At last if the layout overcomes these checks successfully then one can proceed to the experimental study of stochastic resonance in the homopolar dynamo under the action of input voltage (2) in nonlinear regime.

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