The Role of the Angular Momentum in Shaping Collective Effects

E. Prozorova

Abstract The main goal of the work is to clarify the consequences arising from the disregard of the law of the angular momentum as an independent law. As a result, some of the collective effects in mechanics are not taken into account but they are essential. The main laws in physics and mechanics are the laws of conservation of mass, momentum, energy, angular momentum, charge, and some others. In the article it is shown that the sum of the forces is insufficient for a complete description of the interacting particles. Any redistribution of particles is accompanied by the emergence of collective effects, which is associated with the action of the angular momentum and, consequently, with the action of an additional force. The effect always manifests itself, regardless of the branch of science: the formation of fluctuations, structures, quantum mechanics and some others. When constructing a theory, it is impossible to restrict oneself to potential forces that depend only on the distance between particles, since when the particles move, the center of inertia shifts, forming a moment. In continuum mechanics, for example, the stress tensor loses its symmetry for this reason.

Keywords Conference · CHAOS · Chaotic Modeling · CMSIM style

1 Introduction

Classical mechanics deals with material points and, as a rule, with closed systems. The definition of material points in mathematics and physics is different. The main equation in theoretical mechanics is the Liouville equation, which describes the motion of a system of material points of a closed system. Collective interactions occur through an external force, but the main interaction is the binary interaction of particles. The initial and boundary conditions are not considered, although the impossibility of considering them is stipulated due to the huge number of particles.

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However, Hamilton's formalism is legitimate to use in the case of a no dissipative system, when there is no dependence on the velocity, which is not observed in the presence of disturbing surfaces or under conditions of large gradients of velocities, temperatures, densities, or other characteristics. Using the formalism of Bogolyubov [1], for certain conditions the Boltzmann equation is derived. When deriving the Boltzmann equation and other kinetic equations, the assumption is made that the process is "Markov", that is, there is no dependence on the "past". In reality, however, it is partially manifested through flows at the border. The effect of the boundary is essentially visible in the calculations by the molecular dynamics method and in the numerical solution of the Boltzmann equation [2].

The solutions coincide if a large number of particles are taken and there are no flows across the border. Thus, the Boltzmann equation takes into account the change in state only within the elementary volume. Therefore, it is only suitable for small gradients. In addition, the Boltzmann equation does not fulfill the law of conservation of angular momentum. In the same work, the validity of Hilbert's hypothesis is proved about the dependence of the distribution function on time only through the dependence on macroparameters. The Navier-Stokes (Barnett, etc.) equations are derived from the Boltzmann equation by the Chapman-Enskiy method or by some other method, and the continuity equation is determined, which coincides in form with the Liouville equation. Thus, the consistency of the whole theory is proved. The concepts of "closed" and "open" systems are introduced on the example of systems of "particles", the motion of which is described by the reversible Hamilton equations. These include, for example, the "Boltzmann" gas—a system of "structureless atoms" [3]. However, the "mathematical" and "physical" points are very different. While we are considering a "mathematical point" we are not very interested in whether it rotates or not. For a physical "point", both its rotation and the structure of the "point" under consideration are important. It is known that the moment of force (angular momentum) is responsible for rotation. The role of the angular momentum is manifested in all processes associated with the uneven distribution of particles or their physical parameters. The magnitude of the additional force is determined by the value of the gradient of physical quantities (density, speed, momentum, temperature). The action of the angular momentum, i.e. moment of forces essentially depends on the position of the axis of inertia (center of inertia). The angular momentum is a vector quantity. Additive schemes for calculating intermolecular interactions, in which non-additivity is included in the parameters of atom-atomic potentials, does not take into account the entire variety of conditions.

Analysis of the parameters included in the description of the rarefied gas flow showed that for the equilibrium distribution function the ratio of the gas mean free path l to the characteristic macro length d [4]:

for 37% of trajectories $l/d > 1.0$,
for 90% of trajectories $l/d > 0.1$,
for 99% of trajectories $l/d > 0.01$, etc.

The commonly used criterion $l/d > 1.0$, indicated above, takes the form $Kn > 1$; $Kn = 1.0$ means that $l/d > 1.0$ for only 37% of the trajectories, which does not satisfy

the condition l/d > 1.0, while Kn = 10 satisfies the condition l/d > 1.0 for 90% of the trajectories, and Kn = 100—for 99%.

The theory originally proposed for the solution of relaxation problems is extrapolated to the solution of problems associated with gas dynamics, including for solving problems of gas flow near the surface. Limitations of the scheme N. N. Bogolyubov stipulated by the author himself and is associated with the fulfillment of the conditions for the weakening of correlations, the existence of four characteristic time scales (respectively, spatial scales), a particular class of solutions for the s-particle distribution function as a function that depends on time through a single-particle distribution function $f_s(t) = f_s(f)$, binary central interactions providing the law preserving the angular momentum in collision integral, the potential U rapidly falling with distance, the dominance of volume effects and neglect borders.

Potential of interaction of molecules $\Phi = \Phi|\boldsymbol{r} - \boldsymbol{r}_0|$. An additional implicit assumption is the weak deviation of the distribution function from the equilibrium state. For relaxation problems and elastic collisions, all assumptions are satisfied. The case is excluded when the characteristic relaxation times of the single-particle F_1 and the two-particle F_2 are commensurable. It should be noted that for molecules with a more complex interaction potential depending on the angle, averaging over the angle is performed before calculating the collision cross section (potential averaging). There are no studies concerning the influence of the permutation of the operations of averaging the collision cross section and the potential. Here f is the distribution function in the phase γ-space. When deriving the modified equation, the designations will remain generally accepted, that is, r is the radius vector; x—point coordinate; ξ is the velocity of the point, m is the molecular weight, and, according to the definition of the distribution function f_N, the probability of finding the system at the points (x_i, ξ_i) in the intervals $dx_i d\xi_i$ is $f_N(t, x_1, \ldots, x_N, \xi_1, \ldots, \xi_N)dx_1, \ldots, d\xi_1, \ldots, d\xi_N)$.

When calculating macroparameters through the distribution function and projecting values on the coordinate axis, the symmetry of some quantities may be violated. This can happen when calculating the pressure and the pressure tensor:

$P_{ij} = m \int \mathbf{c_i} c_j f(\mathrm{t}, \mathbf{x}, \boldsymbol{\xi})\mathrm{d}\boldsymbol{\xi}$. The symmetry of the stress tensor is postulated on the basis of this form.

In aeromechanics, the projections of the calculated values are used, and not the indices of the velocities included in the formula. Therefore, there is no way to speak unambiguously about symmetry. Symmetry will be observed provided that the rotation of the elementary volume is canceled. The Navier-Stokes equations are obtained under the indicated condition.

An important difference between the interaction of gas and plasma molecules is the long-range nature of the interaction of plasma molecules. A distinctive feature of plasma is a combination of properties characteristic of both a continuous medium (long-range nature of the Coulomb interaction) and systems of individual particles. Therefore, the kinetic theory of plasma differs from the kinetic theory for gas. As we have already noted, there are significant differences in the definitions of mathematical and physical points. Hence, it became necessary to develop a generalized kinetic theory. The need for general definitions of physically infinitesimal scales has matured and is currently given, for example, in [3]. Fluctuations of particles in a liquid

play a separate and important role. Their behavior is also determined by collective interactions. The nature of the interaction differs from the interaction of molecules in a gas and from the interaction in a plasma. It should be noted that the generally accepted kinetic equations, by virtue of considering only the translational motion of the medium, without taking into account rotation and fluxes through the boundary, do not take into account the action of the moment of force and diffusion fluxes through the boundaries. The need to take into account certain effects depends on the specific task. For example, when considering waves in a "cold" isotropic plasma, it is not necessary to take into account the angular momentum and diffusion. In any case, the absence of motion of heavy particles also does not require taking into account the moment and diffusion. The movement of electrons alone does not create a change in the position of the center of inertia (due to the difference in masses) if there is no movement of the ions. When considering the Landau collision integral (the kinetic equation for a weakly interacting gas, including a Coulomb plasma), it is necessary to take into account the influence of the moment. The question of Landau damping, which consists in the damping of a perturbation in a plasma as it propagates from the point of origin, despite the collisionless (without binary collisions) nature of the interaction of molecules, requires additional research. This work is devoted to the study of the influence of the angular momentum in collective interactions.

2 Kinetic Equations

The classical derivation of the Boltzmann equation is to write the particle balance in terms of the relation for the one-particle distribution function

$$f(t+dt, \boldsymbol{r}+\boldsymbol{\xi}_i dt, \boldsymbol{\xi}_i + \boldsymbol{F}_i dt) d\boldsymbol{r} d\boldsymbol{\xi}_i = f\left(t, \boldsymbol{r}, \boldsymbol{\xi}_i\right) \boldsymbol{d r d \xi}_i + \left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{t}}\right)_{\boldsymbol{coll}} dt$$

The latter is often written in the form

$$\boldsymbol{f}(\boldsymbol{t}+\boldsymbol{dt}, \boldsymbol{r}+\boldsymbol{\xi}_i \boldsymbol{dt}, \boldsymbol{\xi}_i + \boldsymbol{F}_i \boldsymbol{dt}) = \boldsymbol{f}\left(\boldsymbol{t}, \boldsymbol{r}, \boldsymbol{\xi}_i\right) + \widetilde{\left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{t}}\right)_{\boldsymbol{coll}}} \boldsymbol{dt}.$$

where $(\frac{\partial f}{\partial t})_{coll}$, $(\frac{\partial \tilde{f}}{\partial t})_{coll}$—are the collision integrals written in different phase spaces. Outwardly, these equalities are identical, however, the second relation is fulfilled at the times of interaction of molecules and all interactions are correlated. For gas-dynamic problems, the characteristic length of the elementary volume, for which equality (2) is written, equal to cm is small and the requirement for a large number of particles in the elementary volume is not fulfilled for altitudes of 120–300 km in the earth's atmosphere. Indeed, the required minimum size is 10^{-3} cm. Since, $N = \pi R^2 \cdot \boldsymbol{\xi} \cdot \boldsymbol{\tau} \cdot \boldsymbol{n}$, here R is the radius of the cylinder of the elementary volume; τ is the mean time of free path, then for statistical independence the number of particles

N must be at least 100. Then, i.e. see. In addition, the possibility of reducing the characteristic size is limited not only by the limited computer memory, but also by the limits of applicability of the model [5, 6], as well as by the growth of computational errors. In this equation, it is assumed that the elementary volume does not rotate and there are no incoming particles through the side surfaces.

When working with a physical elementary volume, it is necessary to take into account the action of the angular momentum responsible for rotation, and due to the finite value of the radius, it is necessary to take into account the arrival of molecules with a selected speed due to diffusion. We consider the hydrodynamic approximation, assuming the definition of a point in terms of the mean free path.

The usual transition to the Boltzmann equation involves expanding the function in a series and obtaining the following equation

$$\left(\frac{\partial}{\partial \boldsymbol{t}} + \boldsymbol{\xi}_i \nabla_{\boldsymbol{r}} + \boldsymbol{F}_i \nabla_i\right) \boldsymbol{f}\left(\boldsymbol{t}, \boldsymbol{r}, \boldsymbol{\xi}_i\right) = \left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{t}}\right)_{\boldsymbol{coll}} \boldsymbol{dt} = \mathrm{I}$$

Taking into account rotation and diffusion, the equation () has the form

$$\begin{aligned} & f(t+dt, \boldsymbol{r}+\boldsymbol{\xi}_i dt + r \times \omega, \boldsymbol{\xi}_i + \boldsymbol{F}_i dt + \frac{\partial \mathrm{M}}{\partial \mathrm{r}} dt) d\boldsymbol{r} d\boldsymbol{\xi}_i \\ & + G_2\left(t+dt, \boldsymbol{r}+\boldsymbol{\xi}_i dt + r \times \omega, \boldsymbol{\xi}_i + \boldsymbol{F}_i dt + \frac{\partial \mathrm{M}}{\partial \mathrm{r}} dt\right) \\ & = f\left(t, \boldsymbol{r}, \boldsymbol{\xi}_i\right) \boldsymbol{d r d \xi}_i + G_1\left(t, \boldsymbol{r}, \boldsymbol{\xi}_i\right) + \left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{t}}\right)_{\boldsymbol{coll}} dt. \end{aligned}$$

M is the moment associated with the collective action of all particles on each other as a result of the displacement of the center of inertia, which is the result of the movement of particles with different speeds. G_1 and G_2—flows through the boundaries of the considered elementary volume. Let's calculate these values.

$G_1 = m\boldsymbol{\xi}_i \frac{\partial f}{\partial \mathbf{r}}$.

Accounting for flows across the border (G_1, G_2) leads to the equations of Vallander [7, 8]

Here E is the internal energy, $E = c_v T$, where c_v is the heat capacity coefficient

$$Q_x = D_1 \frac{\partial \rho}{\partial x} + D_2 \frac{dT}{dx}, \; Q_y = D_1 \frac{d\rho}{dy} + D_2 \frac{dT}{dy}, \; Q_z = D_1 \frac{d\rho}{dz} + D_2 \frac{dT}{dz},$$

$$t_x = k_1 \frac{d\rho}{dx} + k_2 \frac{dT}{dx}, \; t_y = k_1 \frac{d\rho}{dy} + k_2 \frac{dT}{dy}, t_z = k_1 \frac{d\rho}{dz} + k_2 \frac{dT}{dz}$$

$$D_1 = \frac{\mu}{\rho}\alpha_1, \; D_2 = \frac{\mu}{T}\alpha_2, k_1 = \frac{\mu c_v T}{\rho}, k_2 = \mu c_v \beta_2, \lambda = \alpha\mu,$$

where $\alpha_1, \alpha_2, k_1, k_2$ are numerical constants depending on the type of gas.

Q_x, Q_y, Q_z are the mass fluxes across the face perpendicular to the coordinate axes of the moving gas with the velocity V, ρ-density, D_1, D_2 are the coefficients of self-diffusion and thermal diffusion,k_1, k_2 are the thermal conductivity coefficients, and R is the gas constant.

Here I consider it necessary to add to these equations a term related to the velocity gradient (bulk viscosity D_3), so that $Q_x = D_1 \frac{\partial \rho}{\partial x} + D_2 \frac{dT}{dx} + D_3 \frac{\partial u}{\partial x}$. The rest of the values change in the same way. Let us recall the difference between the values obtained through the distribution function and by the molecular dynamics method [9–12].

The general formula for the distribution function (dependence on r).

$$f = f(t, \boldsymbol{r}(t), \boldsymbol{\xi}(t))$$

$$\frac{\partial f}{\partial t}|_{r=const} == \frac{\partial}{\partial t} \frac{\sum_{i=1}^{n} \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\sum_{i=1}^{N} \delta(\boldsymbol{r_i} - \boldsymbol{r})}.$$

By construction $\delta(\boldsymbol{r_i} - \boldsymbol{r})-$ depends on t only through the $\boldsymbol{r_i}(\boldsymbol{t}) - \boldsymbol{r}(\boldsymbol{t})$. Here n is number molecules in elementary volume, N− in full volume.

A more complex option when there are time-dependent flows across the border

1. Without flow across the border

$$\frac{F_1}{F_3} - \frac{F_2}{F_4} = \frac{\sum_{i=1}^{n} \delta(\boldsymbol{r_i} - \boldsymbol{r}) + \sum_{i}^{n} \Delta t \frac{\partial \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\partial t} + \ldots}{\sum_{i=1}^{N} \delta(\boldsymbol{r_i} - \boldsymbol{r}) + \sum_{i}^{N} \Delta t \frac{\partial \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\partial t} + \ldots} - \frac{\sum_{i=1}^{n} \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\sum_{i=1}^{N} \delta(\boldsymbol{r_i} - \boldsymbol{r})}$$
$$\approx \frac{\sum_{i=1}^{n} \delta(\boldsymbol{r_i} - \boldsymbol{r}) + \sum_{i}^{n} \Delta t \frac{\partial \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\partial t} + \ldots}{\sum_{i=1}^{N} \delta(r_i - r)} \left(1 - \frac{\sum_{i}^{N} \Delta t \frac{\partial \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\partial t} + \ldots}{\sum_{i=1}^{N} \delta(\boldsymbol{r_i} - \boldsymbol{r})}\right)$$
$$- \frac{\sum_{i=1}^{n} \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\sum_{i=1}^{N} \delta(r_i - r)}$$
$$\approx \frac{\sum_{i}^{n} \Delta t \frac{\partial \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\partial t} + O((\Delta t)^2}{\sum_{i=1}^{N} \delta(\boldsymbol{r_i} - \boldsymbol{r})}$$

$\frac{\partial \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\partial t}$ – thus, when solving the Boltzmann equation, the time derivative of distribution function will indeed be determined by the dependence through the macro parameters. This approximation, which is made in the theory of rarefied gas in the construction of the Enskog-Chapman solution

2. If there is a flow across the border, depending only on time. The force is not.

$$\frac{F_1}{F_3} - \frac{F_2}{F_4} = \frac{\sum_{i=1}^{n} \delta(\boldsymbol{r_i} - \boldsymbol{r}) + \sum_{i}^{n} \Delta t \frac{\partial \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\partial t} + \sum_{j} \frac{p_j \Delta t}{m} \delta(r_j - r) + \sum_{j} \frac{p_j}{m} \Delta t^2 \frac{\partial \delta(r_j - r)}{\partial t} + \ldots}{\sum_{i=1}^{N} \delta(\boldsymbol{r_i} - \boldsymbol{r}) + \sum_{i}^{N} \Delta t \frac{\partial \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\partial t} + \sum_{j} \frac{p_j}{m} \delta(r_j - r) + \sum_{j} \frac{p_j}{m} \Delta t \frac{\partial \delta(r_j - r)}{\partial t} + \ldots}$$
$$- \frac{\sum_{i=1}^{n} \delta(\boldsymbol{r_i} - \boldsymbol{r})}{\sum_{i=1}^{N} \delta(r_i - r)}.$$

$\sum_j \frac{p_j}{m}\delta(r_j - r) = J_2 - J_1$—is a flow of fast molecules from neighboring cells. The first two terms correspond to the number of molecules in the volume and their motion. Thus, for large gradients the role of flows across the border is increasing. The distribution function can no give a correct contribution to the distribution of molecules. We need in large number particles in elementary volume. There remains the method of molecular dynamics with a very small time step.

Most often, the kinetic Boltzmann equation is taken as the initial one, and one of the variants of the perturbation theory in a small parameter is used to pass to the aeromechanical equations. As we can see, the Boltzmann equation, depending on the problem, requires modifications, since it does not fulfill one of the laws of theoretical mechanics, the conservation law of angular momentum. For the obtained equations, for example, for Navier-Stokes, additional assumptions are made: discarding the rotational velocity component and using Pascal's law obtained for the equilibrium case to nonequilibrium flows. As a result, the pressure becomes a scalar. Using the Boltzmann equation, we obtain an equation for the internal stress tensor. Here, the gas-dynamic functions ρ, u, T are the moments of the velocity v or the deviation of the velocity from its mean value: $\delta v = v - u$.

$$P_{ij}(r,t) = mn \int \delta v_i \delta v_j f(r,p,t)dp,$$

$$\left(\frac{\partial p}{\partial t} + u_k \frac{\partial p}{\partial r_k} + \frac{5}{3}\frac{\partial u_k}{\partial r_k}\right)\delta_{ij} + p\left(\frac{\partial u_i}{\partial r_j} + \frac{\partial u_j}{\partial r_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial r_k}\right)$$
$$= mn \int \delta v_i \delta v_j I_B(r,p,t)dp,$$

$$P_{ij}(r,t) = \delta_{ij}\, p(r,t) + \pi_{ij}$$

Pascal's formula does not follow from the formula and pressure is not defined as 1/3 of the sum of the pressures on the coordinate pads. An interesting feature of the all research is the emphasis on the openness of the considered elementary volumes and, despite the "openness", the use of conservation laws for closed volumes. For example, the law of conservation of energy. We have already shown that the distribution function gives an idea of a probabilistic state in an elementary volume without the influence of boundaries and, therefore, information about flows across the boundary is lost. In addition, information about the "rearrangement" of the arrangement of molecules due to the influence of the motion of the center of inertia is lost. These collective effects should be taken into account when writing kinetic equations and for equations of a continuous medium. "A unified description of kinetic and hydrodynamic processes" [3] requires the same correction. In this case, there is no contradiction between the kinetic equations, the equations for fluctuations, the Fokker-Planck equation, and the Landau damping in plasma. It is essential that these terms are not included in the collision integral. Formally, the equation is without dissipation and is reversible, but in fact the diffusion flows have dissipative properties. It should be recalled that to satisfy Hilbert's hypothesis, one should take the macroparameters

of the modified Navier-Stokes equation in the solution for the locally equilibrium function, but not Euler to match the orders of approximation in the Chapman-Enskiy solution. In addition, the definition of pressure must be changed and a torque gradient must be entered. Then the nonequilibrium.

Chapman-Enskiy solution implies the existence of a vector distribution function, which is observed in numerical calculations when solving the Boltzmann equation [10], the proof of this is the different temperature values along the coordinate axes.

Recall that the stress tensor is not symmetric and the symmetry condition for the stress tensor is one of the conditions for closing the problem; to fulfill the condition, it is required to discard the rotation of the elementary volume. For numerical calculation, the latter simplifies programming only slightly. The classic Chapman-Ensky solution is given below.

$$nf(r,\,p,\,t) = \frac{\rho/m}{(2\pi m k_b T)^{3/2}} exp\left[-\frac{(p-mu)^2}{2\pi m k_b T}\right] \times \left[1 + \frac{\pi_{ij}}{2p}\frac{m\delta v_i \delta v_j}{3k_b T} + \frac{m(\delta v_i q)}{pk_b T}\left(\frac{m(\delta v)^2}{3k_b T} - 1\right)\right].$$

Changes in the values will be in the macroparameters of the local equilibrium distribution function, the collision integral will not change. In kinetic theory, when considering the role of delay for rarefied gas, one must understand the question of what is measured in the experiment: instantaneous values or averages. If the experiment deals with average values, then it is important to choose the time and scale of averaging. At the agreed times, in this case, it is not necessary to take into account the delay, except for the cases of commensurability of the relaxation and retardation times.

3 Damping of Longitudinal Oscillations of an Electron Plasma (Landau Damping), Kinetic Equations of Langevin and Fokker-Planck

Let us consider oscillations in a plasma without collisions, that is, let us proceed to the study of waves propagating in a plasma, the frequency of which is high in comparison with the frequency of pair collisions of electrons and ions. In this case, there are several options to consider. Landau collisional damping for large Knudsen numbers; for small Knudsen numbers in unbounded plasma; for small Knudsen numbers in a confined plasma. They differ from each other. When studying oscillations, we will consider small deviations from equilibrium [3, 13–18].

Since we are interested in wave attenuation, we need to consider the plasma dielectric constant ε, which is determined by the attenuation coefficient γ. First, let's trace the waves in the "cold" isotropic plasma. The variant corresponds to the "collisionless" wave approximation. In this case, the Maxwell distribution functions

$$f_e^{(0)} = \frac{1}{(2\pi m_e k_b T)^{3/2}} exp\left(-\frac{p^2}{2m_e k_b T}\right),\; f_i(p) = \delta(p).\; \frac{r_D}{l} \ll \frac{\lambda}{l} \ll 1,$$

the damping is determined by diffusion, but not by the Landau damping. The influence of the thermal motion of plasma particles on such oscillations is always small [3].

Here $f_e^{(0)}$ is the equilibrium distribution function, l is the mean free path r_D is the Debye radius, λ is the wavelength, the rest of the notation is generally accepted. Consider an unbounded plasma for small Knudsen numbers $l \ll \lambda$. Diffusion works here as well. Let us consider the dispersion and damping of longitudinal oscillations of an electron plasma under the influence of the thermal motion of plasma particles. Let us investigate a variant of a limited plasma, a free-molecular flow with a region of wavelengths (values of wave numbers) for which the contribution corresponding to Landau damping is the main one.

$$l \to L(l \gg L), r_D \ll \gamma \ll \sqrt{r_D L}\ (r_D \ll L \ll l)$$

We must use the Vlasov kinetic equations [18] with a self-consistent field. Since we are interested in high-frequency oscillations, for which $\omega\tau \gg 1$, where τ is the average time between pair collisions of particles, we can ignore the integrals of particle collisions in the kinetic equations. Longitudinal oscillations of an electron plasma in the classical case are described by the following two equations (collisionless case, Vlasov equation)

$$\frac{\partial \delta f}{\partial t} + v\frac{\partial \delta f}{\partial r} + e\delta E\frac{\partial f_0}{\partial p} = 0,$$

$$div\delta E = 4\pi \int dp\delta f.$$

$$\varepsilon_l(\omega, k)e\int dp\delta f(p, k, \omega) = i\int dre^{-ikr}e\int dp\frac{\delta f(p, r, t_0)}{\omega - kv}.$$

Suggested variant is

$$\frac{\partial \delta f}{\partial t} + v\frac{\partial \delta f}{\partial r} + e\delta E\frac{\partial f_0}{\partial p} + \frac{\partial \delta M}{\partial r}\frac{\partial f_0}{\partial p} + \frac{\partial}{\partial r}D\frac{\partial \delta f}{\partial r} = 0,$$

$$div\delta E = 4\pi \int dp\delta f.$$

$$\varepsilon_l(\omega, k)e\int dp\delta f(p, k, \omega) = i\int dre^{-ikr}e\int dp\frac{\delta f(p, r, t_0)}{\omega - kv} + i\int dre^{-ikr}\int dp\frac{\delta M(p, r, t)}{\omega - kv}\frac{\partial f_0}{\partial p}.$$

Qualitatively, we can say that for this case, diffusion plays a small role and, since part of the energy is converted into rotational motions (the action of the moment), the reversible operator will act as a dissipative one. Note that at the initial moment, the distributed moment of force also exists and concentrates a certain amount of energy.

For monochromatic waves of large amplitude, the action can lead to the formation of a vertical velocity component, forming complex plane flows. Despite the collisionless nature of the movement binary collisions exist, as follows from the table of mean free paths presented in the introduction. They create additional dissipation. It should be noted that the generalized equation for a unified description of kinetic and gas-dynamic processes is suitable for "weak" interactions. As before, the contribution of the angular momentums in the motion of molecules is not taken into account. Most likely, the difference between the most probable and average values is due precisely to the lack of taking into account the rotational movements for which the moment is responsible. Similar effects will be essential for Brownian motion. The theory of Brownian motion is one of the main branches of the statistical theory of open systems. Fluctuation (from Latin fluctuation—fluctuation)—any random deviation of any value. In mechanics, a deviation from the mean value of a random variable characterizing a system of a large number of chaotically interacting particles. In the theory of Brownian motion elementary objects are small particles, while in kinetic theory, the main objects are molecules. Both models are macromodels, but the level of description of the structure of the environment is different. Fluctuations exist both in nonequilibrium states and in unsteady processes; in their absence, relaxation would be a "smooth" process and they could be described by single-valued functions of time. The presence of thermal fluctuations causes random deviations of real processes from such a "smooth" flow. The kinetic equation corresponds to a more detailed description. We believe that the environment is in equilibrium. We will consider two approaches to solving problems: the equation for a single particle and for an ensemble of particles (the Fokker-Planck equation) To take into account the atomic structure of a liquid, Langevin introduced an additional force into the equations of motion

$$F_L = My(t), F = -M\gamma v, \gamma = \frac{6\pi a}{M}\eta, \eta = \rho v.$$

Equations
$\frac{dr}{dt} = v$, $\frac{dp}{dt} + \gamma p = F_0 + My(t)$, $F_0 = -gradU$. F_0- external force.
$\langle y_i(t)\rangle, \langle y_i(t), y_i(t')\rangle = 2D\delta_{ij}(t-t')$, the coefficient D was determined by Einstein.

First, about a single particle. Let us repeat the reasoning performed in [3, 19], but replace y (t) with the moment of force M_i calculated for a given period of time. It can be calculated using the operation algorithm. As before, we assume that the characteristic correlation time of the values of the Langevin force is $\tau_{cor}^{L} \ll \tau_{rel} = \frac{1}{\gamma}$. As a result, we arrive at an expression for two time moments:

$$\langle M_i(t)\rangle = 0, \langle M_i(t)M_j(t')\rangle = 2D\delta_{ij}(t-t').$$

$D = \gamma\frac{k_bT}{m}$—Einstein's coefficient, parenthesis means a function from a function (functional).

When using the kinetic description of Brownian motion, it is necessary to introduce an ensemble of noninteracting Brownian particles—he corresponding Gibbs ensemble. In this case, we represent the ensemble of Brownian particles as a continuous medium. However, the difference lies in the use of the "Hamiltonian" formalism for moving particles; for a continuous medium, in this case, the Langevin equation is used. The kinetic classical Fokker-Planck equation has the form [18, 19]

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial r} - \frac{1}{M}\frac{\partial U}{\partial r}\frac{\partial f}{\partial v} = D\frac{\partial^2 y}{\partial v^2} + \frac{\partial}{\partial v}(\gamma v f).$$

The equation of A. Vlasova

$$\left\{\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial r} + e\left(E + \frac{1}{c}[vB]\right)\frac{\partial}{\partial p}\right\}F(r, p, t) = 0.$$

Here E, B are the total electric and magnetic fields, which are composed of external and self-consistent fields generated by plasma particles. They satisfy Maxwell's equations.

In the classical case, equilibrium is possible between Brownian particles and the medium; the particles can be distributed evenly [20]. However, such an assumption can be considered unlikely due to the distribution of particles over velocities and the formation of new moments for individual particles due to the motion of the center of inertia. The fact is that in this case the action of the moment creates a force that distributes the particles not only in terms of velocities, but also in coordinates. The proposed modified Fokker-Planck equation has the form:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial r} - \frac{1}{M}\frac{\partial U}{\partial r}\frac{\partial f}{\partial v} + \frac{1}{M}\frac{\partial M}{\partial r}\frac{\partial f}{\partial v} = D\frac{\partial^2 y}{\partial v^2} + \frac{\partial}{\partial v}(\gamma v f).$$

Thus, in the kinetic theory for a gas, for the Landau damping and the motion of Brownian particles, the nonuniform distribution of particles in velocities and coordinates is supported by the angular momentum and creates fluctuations in physical quantities that must be taken into account. Consider the consequences associated with taking into account the moment in the mechanics of a continuous medium.

4 The Influence of the Angular Momentum in the Equations of Continuum Mechanics

Conservation laws were obtained experimentally and therefore were originally written in integral form. Differential laws are obtained in two ways: using the finite volume method for an elementary volume and using the Ostrogradsky Gauss theorem by replacing the surface integral to the volume integral, that is, taking the integral by parts with further use of the theorems on the conditions Integral turning in zero.

Usually the derivation of conservation laws is analyzed using the Ostrogradsky-Gauss theorem for a fixed volume without moving. The theorem is a consequence of the application of the integration in parts at the spatial case. In reality, in mechanics and physics gas and liquid move and not only progressively, but also rotate. Let us consider the consequences that arise from the generally accepted conservation laws in the mechanics of a continuous medium and which do not correspond to classical theoretical mechanics and mathematics. The speeds of various processes at the time of writing the equations were relatively small compared to modern ones. In further studies, the scope of the theory developed for potential flows to flows with significant gradients of physical parameters was expanded. It was based on the laws of balance of forces, the law of conservation of moment was considered as a consequence of the fulfillment of the law of balance of forces. Allocating the rotational velocity component and ignoring it leads to a symmetric stress tensor. The symmetric tensor is obtained only if the rotational velocity component is neglected. However, this variant of closing the problem is one of the possible variants of solving the system of three equations in the plane case for four unknowns [9–11]. A similar conclusion can be made for the three-dimensional case. For modern computer technology, it is possible to solve the complete equations of fluid mechanics, rather than truncated ones (like Navier-Stokes). From the definition of pressure, both from the classical Boltzmann equation and the modified one, it does not follow that the hydrostatic pressure is one third of the sum of the pressures on the coordinate areas. Using Pascal's law for equilibrium, the pressure is chosen equal to one third of the pressure on the coordinate pads. However, the theory remains the same when determining the different pressure on each of the sites, i.e.p_x, p_y, p_z. The use of one pressure is possible under equilibrium conditions (Pascal's law), but for nonequilibrium conditions the fact is not obvious. Neglecting outside the integral term when taking integrals by parts (the Ostrogradsky-Gauss theorem) is possible only for slow laminar flows. Writing out separately the law of equilibrium for forces and separately for moments of forces without taking into account the mutual influence, although the moment creates an additional force, we come to the conclusion about the symmetry of the stress tensor. If we consider different pressures in different directions, we lose a moment of force, but the pressure gradient is a force. The proposed modified equations of continuum mechanics include the action of the moment and are given in [9–11] and new equations:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho f_1+\frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \sigma_{yx}}{\partial y}+\frac{\partial \sigma_{zx}}{\partial z}+\rho f_{M_x}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho f_2+\frac{\partial \sigma_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \sigma_{zy}}{\partial z}+\rho f_{M_y},$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho f_3+\frac{\partial \sigma}{\partial x}+\frac{\partial P\sigma_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}+\rho f_{M_z},$$

$$y\left(\frac{\partial \sigma_{xz}}{\partial x}+\frac{\partial \sigma_{yz}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}+\rho f_3\right)-z\left(\frac{\partial \sigma_{xy}}{\partial x}+\frac{\partial \sigma_{yy}}{\partial y}+\frac{\partial \sigma_{zy}}{\partial z}+\rho f_2\right)$$

$$+\sigma_{zy}-\sigma_{zy}++M_x=0,$$

$$x(\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{zy}}{\partial z}+\rho f_2)-y\left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{yx}}{\partial y}+\frac{\partial\sigma_{zx}}{\partial z}+\rho f_1\right)$$
$$+\sigma_{yx}-\sigma_{xy}++M_y=0,$$

$$x(\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}+\rho f_1)-z(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{yx}}{\partial y}+\frac{\partial\sigma_{zx}}{\partial z}+\rho f_2)$$
$$+\sigma_{zx}-\sigma_{xz}++M_z=0.$$

Here all designations are standard, f_{M_x}, f_{M_y}, f_{M_z} forces created by the moment, M_x, M_y, M_z are external moments.

5 Conclusion

The paper proposes to take into account the influence of the angular momentum (force) in kinetic equations and in stochastic processes. The definitions of a material point in mathematics and physics are different. As a result, some of the collective effects in mechanics are not taken into account. The main laws in physics and mechanics are the laws of conservation of mass, momentum, energy, angular momentum, charge, and some others. In the article it is shown that not all of the forces are enter for a complete description of the interacting particles. Any redistribution of particles is accompanied by the emergence of collective effects, which is associated with the action of the angular momentum and, consequently, with the action of an additional force. The effect always manifests itself, regardless of the branch of science: the formation of fluctuations, structures, quantum mechanics and some others. When constructing a theory, it is impossible to restrict oneself to potential forces that depend only on the distance between particles, since when the particles move, the center of inertia shifts, forming an angular momentum. In continuum mechanics, for example, the stress tensor loses its symmetry for this reason. Some modification of the theory is suggested.

References

1. N.N. Bogolyubov, Problems of dynamic theory in statistical physics. M.: Gostekhizdat (1946), 146 p
2. K.P. Gurov, Foundations of the kinetic theory. M.: Nauka (1966), 350 p
3. Y.L. Klimontovich, Statistical theory of open systems. T. 1.2. Moscow: Lenand, (2019)
4. F. Goodman, G. Vachmann, Dynamics of gas scattering by a surface. M.: Mir, (1980), 423p
5. E.V. Prozorova, Mathematical modeling of the processes of mechanics with large gradients (St. Petersburg. St. Petersburg University, 339c, 2005)

6. E.V. Prozorova, The influence of dispersion in models of continuum mechanics (St. Petersburg. St. Petersburg University, 2013), 94 p
7. S.V. Vallander, Equations of motion of a viscous gas. Rep. Acad. of Sci. USSR **78**(1), 25–27 (1951)
8. S.V. Vallander, M.P. Elovskih, The theoretical dependence of the heat conductivity of gases on temperature. Rep. ANAS **79**(1), 37–40 (1951)
9. E.V. Prozorova, Features of the rarefied gas description in terms of a distribution function. APhM2018. IOP Conf. Ser. J. Phys. Conf. Ser. **1250**, 012023 (IOP Publishing, 2019). https://doi.org/10.1088/1742-6596/1250/1/0120232
10. E. Prozorova, Influence the form of writing conservation laws in computation. JP J. Heat Mass Transf. 28 Oct 2019. Accepted: 23 Nov 2019
11. E. Prozorova, The effect of angular momentum and Ostrogradsky-Gauss theorem in the equations of mechanics WSEAS TRANSACTIONS on FLUID MECHANICS. https://doi.org/10.37394/232013.2020.15.2
12. E. Prozorova, The influence of the no symmetric stress tensor on the flow separation WSEAS TRANSACTIONS on APPLIED and THEORETICAL MECHANICS. https://doi.org/10.37394/232011.2020.15.9
13. E. Prozorova, Consequences of the Ostrogradsky-Gauss theorem for numerical simulation in aeromechanics. Elsever Granthaalayah, **8**(6) (2020). https://doi.org/10.29121/granthaalayah.v8.i6.2020.549
14. R. Balescu, Equilibrium and nonequilibrium statistic mechanics. A Wiley-Intersciences Publication John Willey and Sons. New-yourk-London (1975)
15. N.G. Van Kampen, Stochastic processes in physics and chemistry. North-Holland (1984)
16. A.I. Akhiezer, I.A. Akhiezer, R.V. Polovin, A.G. Sitenko, K.N. Stepanov, Plasma electrodynamics. M.: Science (1974)
17. P. Silin, Introduction to the kinetic theory of gases. M.: Science (1971)
18. S. Ishimaru, Basic principles of plasma physics. M.: Atomizdat (1975)
19. A.A. Vlasov, Nonlocal statistical mechanics. M.: Science (1978)
20. P. M¨orters, Y.P. Brownian, Notes on Brownian motion and related phenomena Deb Shankar Ray*, Department of Physical Chemistry, Indian association for the cultivation of science, Jadavpur, Calcutta 700032 (2008)