The Interaction of Memristor in Cellular Nonlinear Network for Image and Signal Processing

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Abstract In this paper, we describe the application of memristor in the neighborhood connections of 2D cellular nonlinear networks (CNN) essentially for image and signal processing. We focused particularly on the interaction of memristors between two cells allowing us to study the contribution of the memristor qualitatively and quantitatively. The dynamics and the steady state response of each cell is described. The resistance of a memristor is not fixed, hence the study takes into account the initial state of the memristance characterized by the previous amount of charge passed through the memristor. We show that the system transition and the steady state response depend strongly on the history of the memristor.

Keywords Memristor · 2D cellular nonlinear networks · System of two cells · System dynamic · Steady state response

1 Introduction

Memristor is a nanoscale two terminals solid state device whose conductivity is controlled by the time-integral of the current flowing through it or the time-integral of the voltage across its terminals [1, 2]. The dynamic conductance modulation and

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connection compatibility with complementary metal-oxide semiconductor neurons, are essential features of memristor affirming its potentiality as synaptic function and memristive gird network [3–5].

This paper describes a memristor based 2D cellular nonlinear network using memristor in the coupling mode. The network is essentially for signal and image processing applications. Figure 1 shows the conventional 2D cellular nonlinear network with each cell constituting one linear capacitor in parallel with one nonlinear resistance, and a series resistance coupling [6]. Figure 2 shows the equivalent network using memristors in the coupling mode.

For any cell at a node n and voltage V_n, the nonlinear current function through the nonlinear resistance is given by:

$$I_{NL_n} = \frac{V_n(V_n - V_a)(V_n - V_b)}{R_o V_a V_b}, \tag{1}$$

and the nonlinear resistance, R_{NL_n}:

$$R_{NL_n} = \frac{R_o V_a V_b}{V_n(V_n - V_a)(V_n - V_b)}.$$

The characteristic roots of the cubic resistance are 0, V_a and V_b, meanwhile R_o is its linear approximation. The corresponding potential energy $W(V_n)$ is obtained from (1) as:

$$W(V_n) = \frac{1}{4}V_n^4 - \frac{V_a + V_b}{3}V_n^3 + \frac{V_a V_b}{2}V_n^2 + \kappa,$$

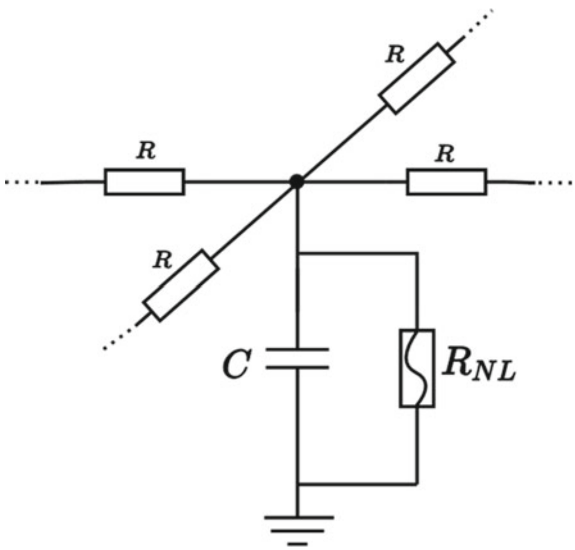

Fig. 1 2D CNN using series resistance coupling [6]

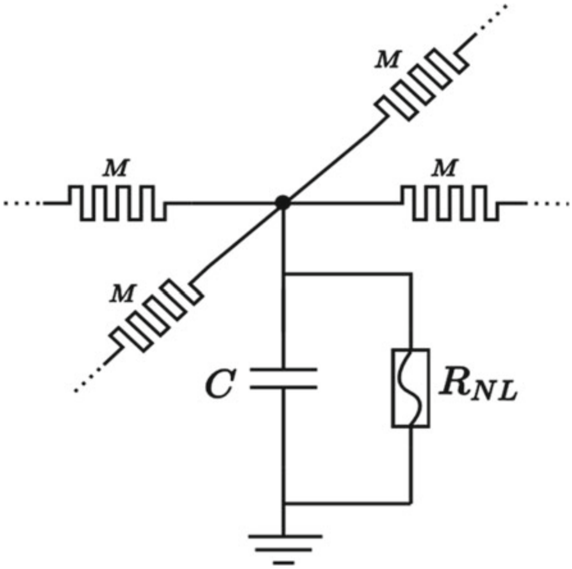

Fig. 2 2D CNN using memristors in the coupling mode

where κ is a constant. Figure 3 shows the response of the cubic resistance and the corresponding potential energy showing the possible equilibrium state. The lower potential energy state are at 0 and V_b marked by numbers 1 and 2 respectively, meanwhile V_a is the unstable state.

We focus on the study of memristor response based on the interaction of one cell with its neighbouring cells. Therefore, using the system of two cells coupled by a memristor, allows us to observe the quantitative and qualitative interaction of memristors in the network.

2 System of Two Cells

Figure 4 shows the network of two cells coupled by a memristor, thus the cells communicate together bidirectionally. One of the cells acts as the master while the other one as slave so that the direction of the flowing charge through the memristor becomes specific. The switchs s_1 and s_2 are closed simultaneously and the network gives the following set of bidirectional coupled equations:

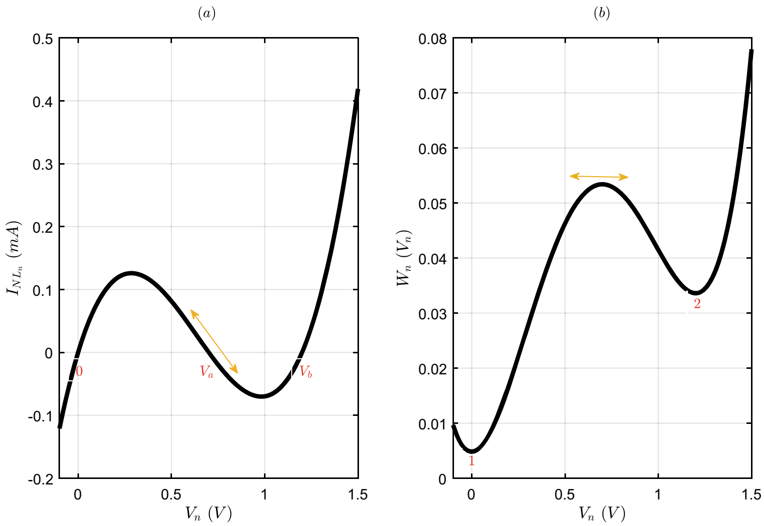

Fig. 3 Response of the nonlinear resistance in the cells. $V_a = 0.7$ V, $V_b = 1.2$ V and $R_o = 1023\,\Omega$. **a** I-V characteristic and **b** the corresponding potential energy. Labels 1 and 2 show the two possible equilibrium states corresponding to $V_n = 0$ and $V_n = V_b$

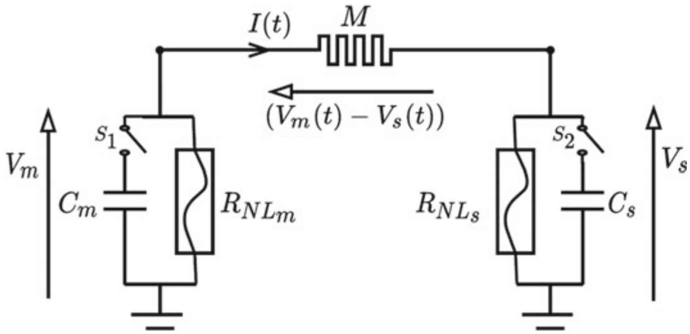

Fig. 4 System of two cells coupled by a memristor. The master and slave cells with their elements given by the subscripts letters m and s respectively

$$\frac{dq}{dt} = -C\frac{dV_m}{dt} - \frac{V_m(V_m - V_a)(V_m - V_b)}{R_o V_a V_b}, \tag{2a}$$

$$\frac{dq}{dt} = C\frac{dV_s}{dt} + \frac{V_s(V_s - V_a)(V_s - V_b)}{R_o V_a V_b}, \tag{2b}$$

$$\frac{dq}{dt} = \frac{V_m - V_s}{M(q)}, \tag{2c}$$

where: $I(t) = \dfrac{dq}{dt}$ is the flowing current through the memristor and $M(q)$ is the memristance function that has a desirable continuity for all the flowing charge [7]:

$$M(q) = \begin{cases} R_{off}, & \text{if } q(t) \le 0 \\ R_{off} - \frac{3\,\delta R}{q_d^2}\, q^2 + \frac{2\,\delta R}{q_d^3}\, q^3, & \text{if } 0 \le q(t) \le q_d \\ R_{on}, & \text{if } q(t) \ge q_d \end{cases} \tag{3}$$

$q_d = \dfrac{D^2}{\mu_v R_{on}}$ is a charge scaling factor depending on the technology parameters [2, 8] and $\delta R = R_{off} - R_{on}$ is the difference between the two limiting resistance values of the memristor, that is, the ON and OFF states, represented here by R_{on} and R_{off} respectively. Charge flows from the master cell to the slave one through the memristor until $V_m(t) = V_s(t)$ and that is when the network is stabilized. Equation (2) is reformulated as:

$$\frac{dV_m}{dt} = -\frac{V_m - V_s}{C.M(q)} - \frac{V_m(V_m - V_a)(V_m - V_b)}{R_o V_a V_b C}, \tag{4a}$$

$$\frac{dV_s}{dt} = \frac{V_m - V_s}{C.M(q)} - \frac{V_s(V_s - V_a)(V_s - V_b)}{R_o V_a V_b C}, \tag{4b}$$

$$\frac{dq}{dt} = \frac{V_m - V_s}{M(q)}. \tag{4c}$$

As illustrated in Fig. 3b, 0 and V_b are the only two possible equilibrium states. The stability of the cells at 0 or V_b is determined by V_a. It can be seen that if $V_b - 2V_a > 0$ the cell stabilizes at V_b and if $V_b - 2V_a < 0$ the cell stabilizes at 0.

The initial conditions of V_m, V_s and q are V_{m_0}, V_{s_0} and q_0 respectively. Figure 5 shows the time evolution of the 2 cells network for $V_{m_0} = 2\,\text{V}$, $V_{s_0} = 0\,\text{V}$, $V_b = 1.5\,\text{V}$, $V_a = 0.7\,\text{V}$, $R_o = 10\,\text{k}\Omega$ and $C_m = C_s = 1\,\mu\text{F}$, hence $V_b - 2V_a > 0$. Initially, the voltage $V_m(t)$ and $V_s(t)$ evolve in the differential mode and thereafter the common mode when $V_m(t) = V_s(t)$ which continues to evolve until the steady state V_b. The charge $q(t)$ flows through the memristor until $V_m(t) = V_s(t)$. When $V_m(t) = V_s(t)$, the voltage across the memristor is 0 (i.e $V_d(t) = 0\,\text{V}$).

3 Discussion

Variation of the system parameters, such as V_a, R_o and q_0 affects the steady state response of the system. The results of Fig. 6 show the variation of V_a with respect to V_b, for example $V_a = \Upsilon V_b$. The results are obtained for $R_0 = 2833\,\Omega$, $V_b = 1.5\,\text{V}$, $V_{m_0} = 2\,\text{V}$, $V_{s_0} = 0\,\text{V}$ and $C_m = C_s = 1\,\mu\text{F}$. Hence, V_a varies according to $\Upsilon =$ [0.25, 0.45, 0.49, 0.5, 0.51, 0.55, 0.75, 0.9] with the corresponding results given by Fig. 6a–h, respectively. Furthermore, the difference $V_b - 2V_a$ is calculated and

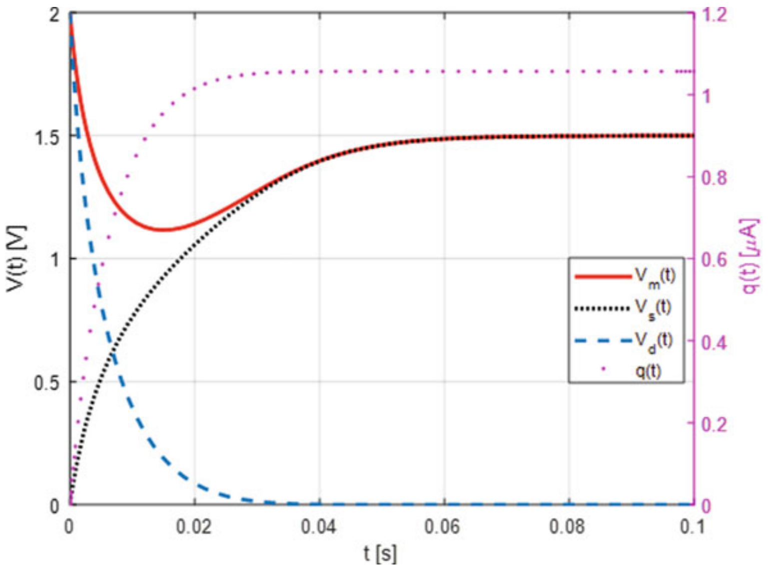

Fig. 5 The evolution of $V_m(t)$, $V_s(t)$, $V_d(t) = V_m(t) - V_s(t)$ and $q(t)$, for $V_{m_0} = 2\,\text{V}$, $V_{s_0} = 0\,\text{V}$, $V_b = 1.5\,\text{V}$, $V_a = 0.7\,\text{V}$, $R_o = 10\,\text{k}\Omega$ and $C_m = C_s = 1\,\mu\text{F}$. The charge $q(t)$ flows through the memristor until $V_d(t) = 0$ and at this time, the combined evolution of $V_m(t)$ and $V_s(t)$ is the common mode which evolves further to stabilizes at the steady state V_b

tabulated in Table 1. The results show two noticeable effects on the evolution of $V_m(t)$ and $V_s(t)$ based on the variation of V_a. The results show different timing at which $V_m(t) = V_s(t)$ and the change in the steady state at V_b or 0 for $V_a < \frac{V_b}{2}$ or $V_a > \frac{V_b}{2}$ respectively.

The initial charge q_0 characterizes the initial memristance of a memristor. The initial condition of a memristor has strong effect on its circuit functionality [9]. Figure 7 shows the effect of changing initial condition of the memristor on the system evolution and stability. It also takes into account the variations of R_0. The initial memristance of the memristor is given by the initial charge q_0. Four different initial charges are considered as: $q_{0_1} = 20\,\mu\text{C}$, $q_{0_2} = 40\,\mu\text{C}$, $q_{0_3} = 60\,\mu\text{C}$ and $q_{0_4} = 80\,\mu\text{C}$, as indicated respectively, by the subscripts numbers **1–4** in Fig. 7a, b. Notice that only one parameter is varied at a time. Figure 7a: $R_0 = 2833\Omega$ while q_0 varied and Fig. 7b: $R_0 = 10\,\text{k}\Omega$ while q_0 varied. In each case, $V_a = 0.7\,\text{V}$, $V_b = 1.3\,\text{V}$, $V_{m_0} = 1.5\,\text{V}$ and $V_{s_0} = 0\,\text{V}$. Even though V_a is the main parameter that plays significant role on the system steady state, the results show that other parameters (e.g. R_o and q_0) affect the dynamics and steady state of the system.

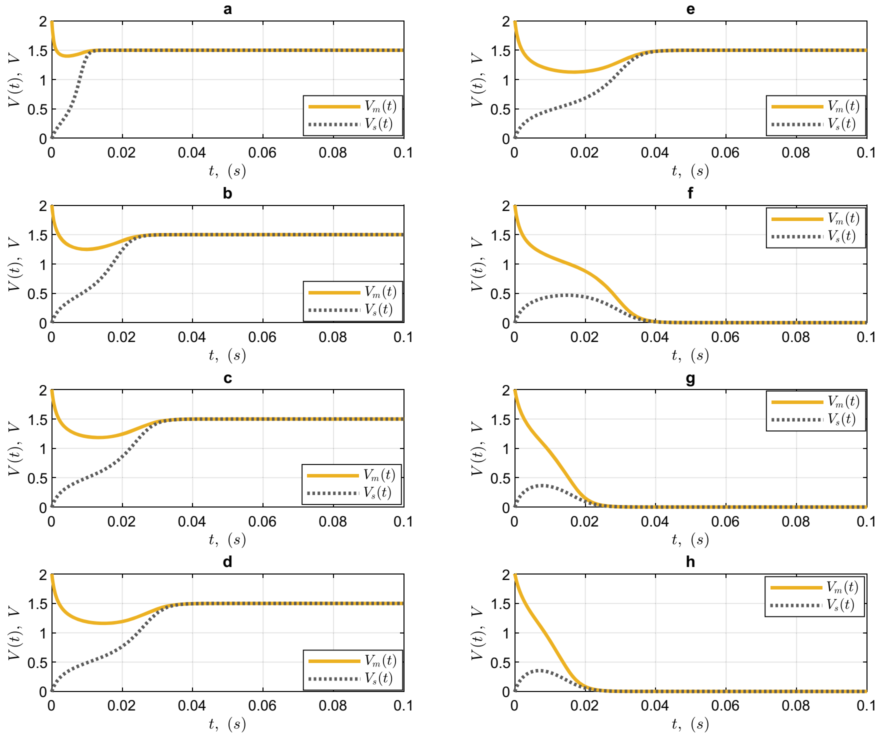

Fig. 6 Evolution of $V_m(t)$ and $V_s(t)$ showing the variations of $V_a \in [0, V_b]$: $R_0 = 2833\,\Omega$, $V_{s_0} = 0\,\text{V}$, $V_{m_0} = 2\,\text{V}$, $V_b = 1.5\,\text{V}$ and $\Upsilon = [0.25, 0.45, 0.49, 0.5, 0.51, 0.55, 0.75, 0.9]$ as shown respectively by figures (**a**–**h**). Variation of V_a affects the time at which $V_m(t) = V_s(t)$ and the steady state at V_b or 0 depending on whether $V_a < \frac{V_b}{2}$ or $V_a > \frac{V_b}{2}$ respectively

Table 1 Table of $V_b - 2V_a$ for Fig. 6. $V_{s_0} = 0\,\text{V}$, $V_a = \Upsilon V_b$, $V_b = 1.5\,\text{V}$ and $V_{m_0} = 2\,\text{V}$

Fig. 6	a	b	c	d	e	f	g	h
Υ	0.25	0.45	0.49	0.5	0.51	0.55	0.75	0.9
$V_b - 2V_a$ (V)	0.75	0.15	0.03	0	−0.03	−0.15	−0.75	−1.20

4 Conclusion

Memristor based 2D cellular nonlinear network is introduced using memristors in the coupling mode. The cells correspond to pixels in image processing applications. Each elemental cell consists of one linear capacitor in parallel with one nonlinear resistance such as Fitzhugh Nagumo. Using the system of two cells coupled by a memristor, the dynamics and the steady state of each cell are observed, with mainly a competition between the role of cubic resistance on one hand, and the role of

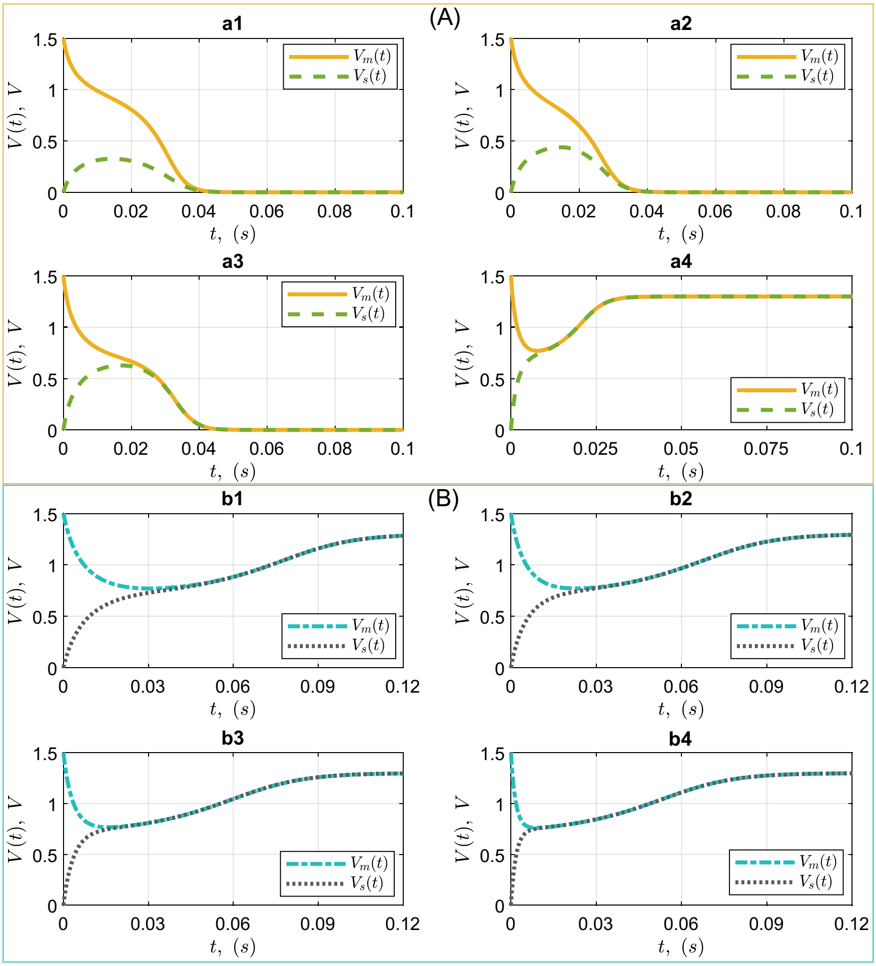

Fig. 7 Results showing the variation effect of q_0 and R_o on the system evolution and the steady state. Four different initial charges are considered as: $q_{0_1} = 20\,\mu\text{C}$, $q_{0_2} = 40\,\mu\text{C}$, $q_{0_3} = 60\,\mu\text{C}$ and $q_{0_4} = 80\,\mu\text{C}$, as indicated respectively by the subscripts numbers **1**–**4** in figures (**a** and **b**). In each case, $V_a = 0.7\,\text{V}$, $V_b = 1.3\,\text{V}$, $V_{m_0} = 1.5\,\text{V}$, $V_{s_0} = 0\,\text{V}$ and $C_m = C_s = 1\,\mu\text{F}$. **a** $R_0 = 2833\,\Omega$ and **b** $R_0 = 10\,\text{k}\Omega$. It shows that values of q_0 and R_0 have an effect on the evolution and steady state of the system

memristor on the other hand. The parameter V_a predominantly decides the system steady state, however other parameters (e.g R_o, q_0 etc...) affect the system steady state. The results show that the network can be used to realize a binarization circuit, for example, to generate different gray scaling. The ongoing study focuses on the implementation of the generalized 2D network for processing any number cells.

References

1. L. Chua, Memristor-the missing circuit element. IEEE Trans. Circuit Theory **18**(5), 507–519 (1971)
2. D.B. Strukov, G.S. Snider, D.R. Stewart, R.S. Williams, The missing memristor found. Nature **453**(7191), 80–83 (2008)
3. S.H. Jo, T. Chang, I. Ebong, B.B. Bhadviya, P. Mazumder, W. Lu, Nanoscale memristor device as synapse in neuromorphic systems. Nano Lett. **10**(4), 1297–1301 (2010)
4. A. Chowdhury, A. Ayman, S. Dey, M. Sarkar, A.I. Arka, Simulations of threshold logic unit problems using memristor based synapses and CMOS neuron. In: *2017 3rd International Conference on Electrical Information and Communication Technology (EICT)* (IEEE, 2017), pp. 1–4
5. S.P. Adhikari, C. Yang, H. Kim, L.O. Chua, Memristor bridge synapse-based neural network and its learning. IEEE Trans. Neural Netw. Learn. Syst. **23**(9), 1426–1435 (2012)
6. J.C. Comte, P. Marquié, J.M. Bilbault, Contour detection based on nonlinear discrete diffusion in a cellular nonlinear network. Int. J. Bifurcat. Chaos **11**(01), 179–183 (2001)
7. A. Isah, A.S.T. Nguetcho, S. Binczak, J.M. Bilbault, Memristor dynamics involved in cells communication for a 2D non-linear network. IET Signal Process. **14**(7), 427–434 (2020)
8. Y.N. Joglekar, S.J. Wolf, The elusive memristor: properties of basic electrical circuits. Eur. J. Phys. **30**(4), 661 (2009)
9. L. Chua, Everything you wish to know about memristors but are afraid to ask. Radioengineering **24**(2), 319 (2015)