Chaos Control in a Nonideal Vibrating Systems

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Abstract This paper presents two control techniques for a non-ideal system with chaotic behavior. To place and maintain the system in a periodic orbit, the techniques of Time Delayed Feedback Control and Continuous-Delay Control with Saturation are considered. The non-ideal system presented is composed of a mass-spring-damper system, with cubic stiffness, and an external excitation from a power limited DC electric motor driven by an unbalanced rotating mass that provides the non-ideal excitation. To suppress the chaotic behavior, additional damping is considered for the mechanical system, and the damping force is estimated by the proposed control strategies. Dynamic analysis of the system is performed by various techniques, including bifurcation diagrams, phase portraits, power spectral densities, and 0–1 test. Numerical simulations demonstrate the effectiveness of the control strategies leading the system to a stable periodical orbit.

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1 Introduction

The study of vibrating systems for the case where the external excitation is influenced by the system response has attracted many researchers, because its problem is a great challenge in both theoretical and practical engineering. When the excitation is influenced by the system response or for which the power supply is limited it is a non-ideal system [1]. Usually, these systems are considered when motors are coupled to structures that need excitation power levels similar to the power capacity of these motors [1–3].

In many cases, the non-ideal nonlinearities of the system can lead the system to the jump effect (Sommerfeld effect) [1, 4, 5], or to chaotic behavior [6], as observed in this work. These behaviors are undesirable in most cases, as they can cause data to the system [7], being necessary to suppress them.

In this paper, control, and dynamic analysis of a nonideal system are applied and investigated. The 0–1 test is used to investigate the chaotic behavior of the system. To suppress the chaotic motion, the Time Delayed Feedback Control (TDFC) and Continuous-Delay Control with Saturation (DCSC) are applied.

The TDFC is originally proposed by Pyragas [8], who considered a continuous control input that stabilizes a chaotic oscillation under the difference between the velocity current output and the previous velocity one [8–10], and successfully applied in nonideal system [9].

The DCSC is proposed in [11], to control a non-ideal system with chaotic behavior, and successfully applied to nonideal systems [12, 13].

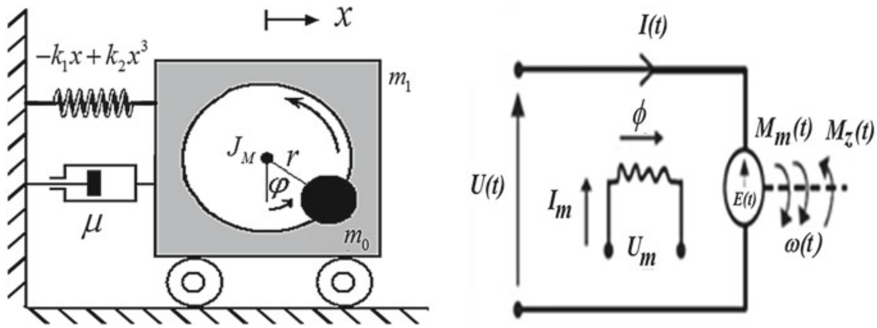
The 0–1 test is originally proposed by [14]. The method consider a time series data, based on the statistical properties of a single variable, and analysing its spectral and statistical properties by considering the asymptotic properties of a Brownian motion [14–18].

2 Mathematical Model

The system presented in Fig. 1, represents a nonideal oscillator, and consists of a structure of mass m_1 connected to a damper and to a nonlinear spring with a nonlinear cubic stiffness. The proposed system is excited by a nonideal DC motor characterized by the moment of inertia J_M and the unbalanced mass m_0 with eccentricity r. The physical schematics of the DC motor is shown in Fig. 1b. [6, 7, 9, 19, 20].

The equations of motion of the non-ideal system are given by [6, 9]

$$
\begin{aligned}
& m_1\ddot{x} + \mu\dot{x} - k_1 x + k_2 x^3 = m_0 r\left(\dot{\phi}^2 \sin\phi - \ddot{\phi}\cos\phi\right) \\
& \left(J + m_o r^2\right)\ddot{\phi} = C_M \Phi I - m_0 r \ddot{x}\cos\phi \\
& \dot{I}_m = -\frac{R_t}{L_t} I_m - \frac{C_E \Phi}{L_t}\dot{\phi} + \frac{U_m}{L_t}
\end{aligned}
\tag{1}
$$



(a) (b)

Fig. 1 **a** Non-ideal oscillator and **b** the DC motor in the electrical schematics

where C_M, C_E are mechanical and electrical constants, respectively. The magnetic flux is represented by Φ and $\omega(t)$ is the angular velocity of the rotor. It is assumed that the external exciting current I_m and voltage U_m are constants and then the magnetic flux Φ is constant.

The dimensionless mathematical model represented in state-space notation, for the system (1) can be expressed by the following system of equations [9]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = A\big(w_1\big(x_4^2 \sin x_3 - p_3 x_5 \cos x_3\big) - \beta x_2 + x_1 - \delta x_1^3\big) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = A\big(p_3 x_5 - w_2 w_1 x_4^2 \sin x_3 \cos x_3 + w_2 \beta w \cos x_3 - w_2 \cos x_3\big(x_1 - \delta x_1^3\big)\big) \\ \dot{x}_5 = -p_1 x_5 - p_2 x_4 + \nu \end{cases} \tag{2}$$

where $x_1 = x$, $x_2 = \dot{x}_1$, $x_3 = \varphi$, $x_4 = \dot{\varphi}$ and $x_5 = I$, and the dimensionless parameters are denoted by
$\omega_0^2 = \frac{k_1}{m_1+m_0}$, $\beta = \frac{\mu}{m_1\omega_0}$, $\delta = \frac{k_2}{k_1}x_0^2$, $w_1 = \frac{m_0 r}{m_1 x_0}$, $w_2 = \frac{m_0 r x_0}{(J+m_0 r^2)}$, $p_1 = \frac{R_t}{L_t I_0 \omega_0}$,$\nu = \frac{U_m}{L_t I_0 \omega_0}$, $p_2 = \frac{C_E \Phi}{L_t I_0}$, $p_3 = \frac{C_M \Phi I_0}{(J+m_0 r^2)\omega_0^2}$ and $A = \frac{1}{1-w_1 w_2 (\cos x_3)^2}$.

3 Numerical Results

For numerical simulation is considered the parameters: $\delta = 0.1$, $w_1 = 0.2$, $w_2 = 0.3$, $p_1 = 0.3$, $p_2 = 3$, $p_3 = 0.15$, $\beta = 0.0337$, $\omega_0 = 46.4$ and $2 \le \nu \le 7$, along with the initial conditions:$x_i(0) = 0$, where i = 1:5 [9]. Where the integration step is considered by $h = 0.001$.

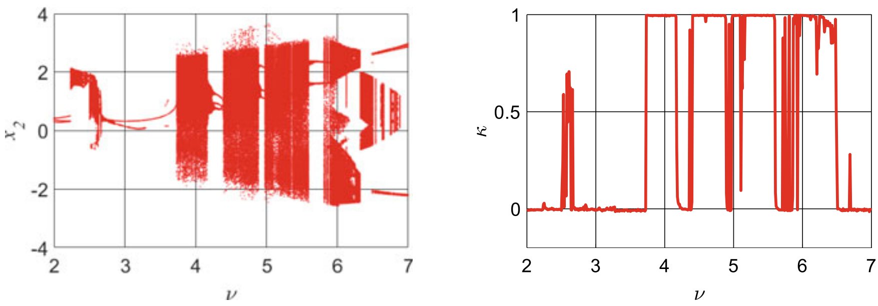


Fig. 2 **a** Bifurcation diagram. **b** 0–1 test

In Fig. 2 one can observe the Bifurcation diagram and 0–1 test, for the system (2) considering:$2 \leq \nu \leq 7$.

As can be seen in Fig. 2a for certain values of (v) the system (2) has periodic or chaotic behavior. Considering the results of the 0–1 test presented in Fig. 2b, one can observe that the system is chaotic for values of (κ) close to 1 and periodic for values of (κ) close to zero [21, 22].

In Fig. 2 one can observe the chaotic behavior of the system (2) for $\nu = 5.4$ ($\kappa = 1$).

As can be seen in Fig. 2 the system (2) without control has a chaotic behavior.

4 Chaos Control

To eliminate the chaotic behavior presented by the system, the proposed control techniques are introduced as a control signal U, given by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = A\big(w_1\big(x_4^2 \sin x_3 - p_3 x_5 \cos x_3\big) - \beta x_2 + x_1 - \delta x_1^3\big) + U \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = A\big(p_3 x_5 - w_2 w_1 x_4^2 \sin x_3 \cos x_3 + w_2 \beta w \cos x_3 - w_2 \cos x_3 \big(x_1 - \delta x_1^3\big)\big) \\ \dot{x}_5 = -p_1 x_5 - p_2 x_4 + \nu \end{cases} \tag{3}$$

4.1 Chaos Control by Time Delayed Feedback Control

The TDF control, was originally suggested by the author of [8], being obtained by the difference between the past and current velocity for a given sampling time [8, 9]. Thus, assuming that the oscillation speed (x_2) can be measured, the TDF control signal U_{TDFc} is given as:

$$U_{TDFc} = k_{TDFc}[x_2(\tau - T) - x_2(\tau)] \quad (4)$$

where T is the time delay and k_{TDFc} the feedback gain.

Figure 4 shows the 0–1 test for different feedback gain intervals of the $0 \leq k_{TDFc} \leq 10$, with fixed $T = 1.133$.

As can be seen in Fig. 4 for gains of ($k_{TDFc} \geq 0.1$), the control (4) takes the system (3) to a periodic behavior, considering that ($U = U_{TDFc}$).

In Fig. 5 we can observe the system (3) with TDF control ($U = U_{TDFc}$) and parameters: $k_{TDFc} = 0.3$ and $T = 1.133$.

As can be seen in Fig. 5 the TDF controlled the system to a chaotic behavior for a periodic with a small control signal (Fig. 3d).

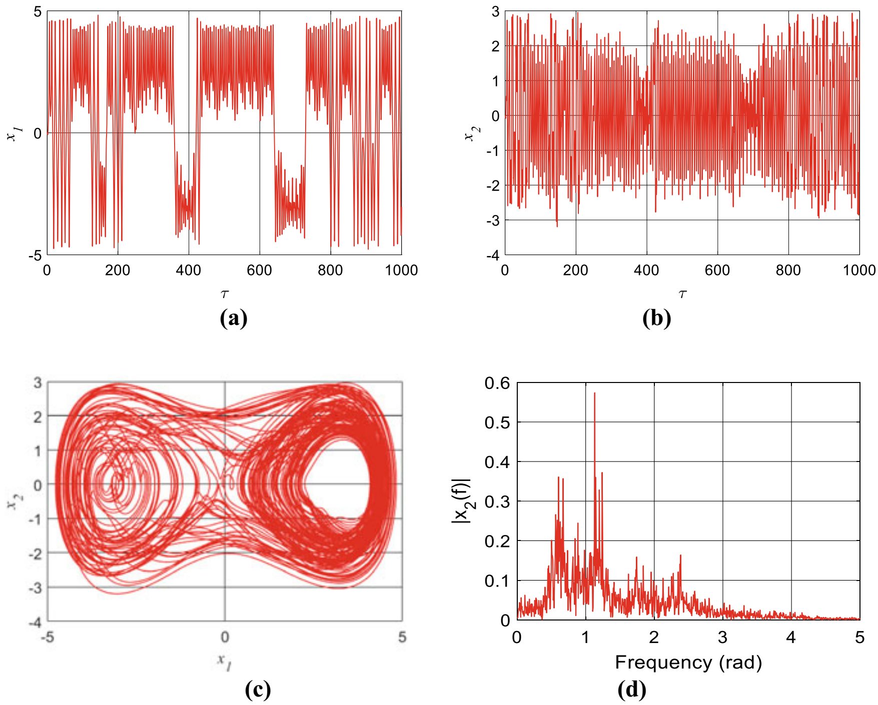


Fig. 3 **a** Time history of the states x_1. **b** Time history of the states x_2. **c** Phase diagram to x_1 versus x_2. **d** Power spectral density (FFT) to x_2

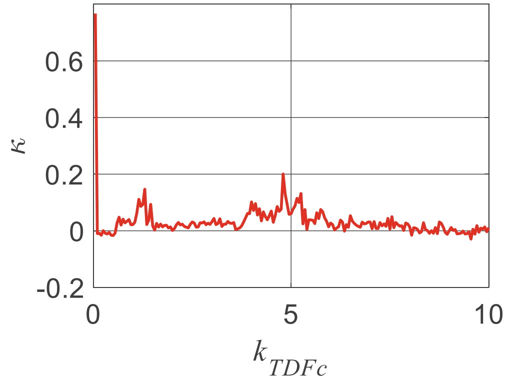


Fig. 4 0–1 test for$0 \leq k_{TDFc} \leq 10$ and $T = 1.133$

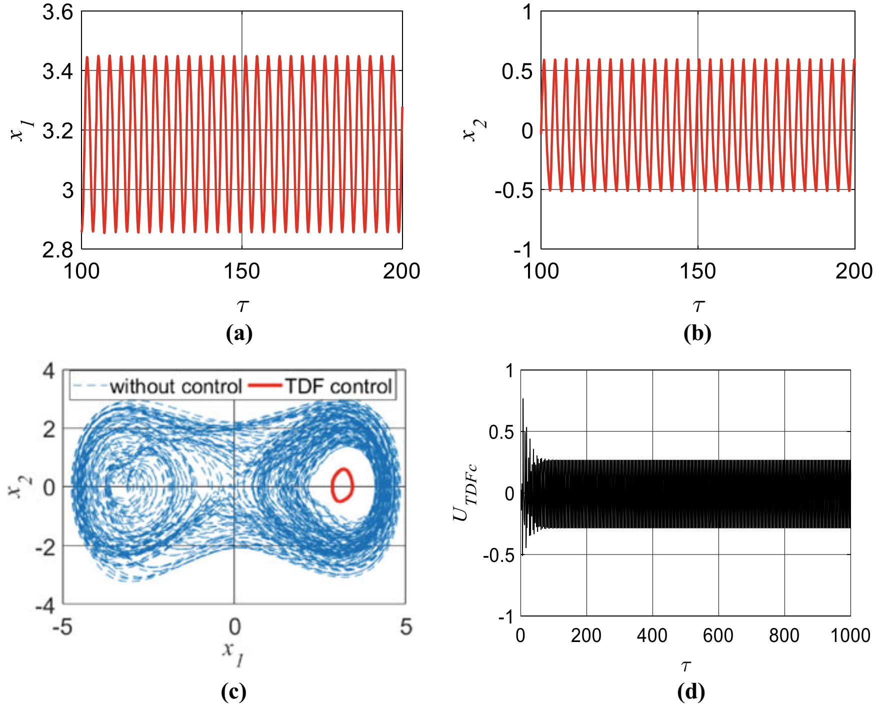


Fig. 5 **a** Time history of the states x_1. **b** Time history of the states x_2. **c** Phase diagram to x_1 versus x_2. **d** Signal control U_{TDFc}

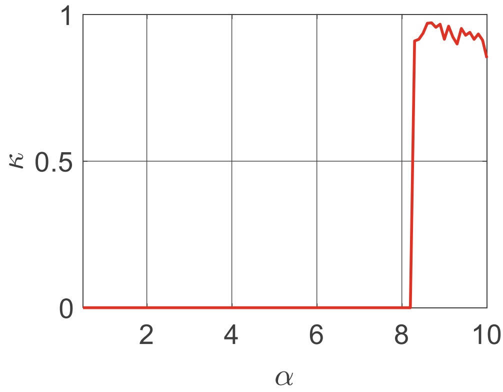


Fig. 6 0–1 test for$0.5 \leq \alpha \leq 10$ and $\beta = 1$

4.2 Suppression of Chaotic Behaviour by Continuous-Delay Control with Saturation

The Continuous-Delay Control with Saturation (CDCS), was proposed by [11] to control a non-ideal system with chaotic behavior, and successfully used in other nonideal mechanical systems [12, 13].

The Continuous-Delay Control with Saturation signal (U_{CDCS}) is given as [12, 13]:

$$U_{CDCS} = \alpha \tanh(\beta x_2) \tag{5}$$

where α and β are positive constant.

Figure 6 shows the 0–1 test for system (3) with ($U = -U_{CDCS}$) and different f gain intervals of the $0.5 \leq \alpha \leq 10$, with fixed $\beta = 1$.

As can be seen in Fig. 4 for gains of ($0.5 \leq \alpha \leq 8.2$), the control (5) takes the system (3) to a periodic behavior, considering that ($U = -U_{CDCS}$).

In Fig. 7 we can observe the system (3) with continuous-delay control with saturation ($U = -U_{CDCS}$) and parameters: $\alpha = 0.5$ and $\beta = 1$.

As can be seen in Fig. 7 the CDCS control drove the system to a chaotic behavior for a periodic with a small control signal (Fig. 7d).

5 Conclusions

To control the chaos of the non-ideal system presented in Eq. (2), Time Delayed Feedback Control and the Continuous-Delay Control with Saturation are considered to be projected and applied. The efficiency of two techniques was demonstrated through numerical simulations in order to eliminate the chaotic behavior of the system, and it was efficient to maintain the amplitude of the non-ideal systems in the periodic

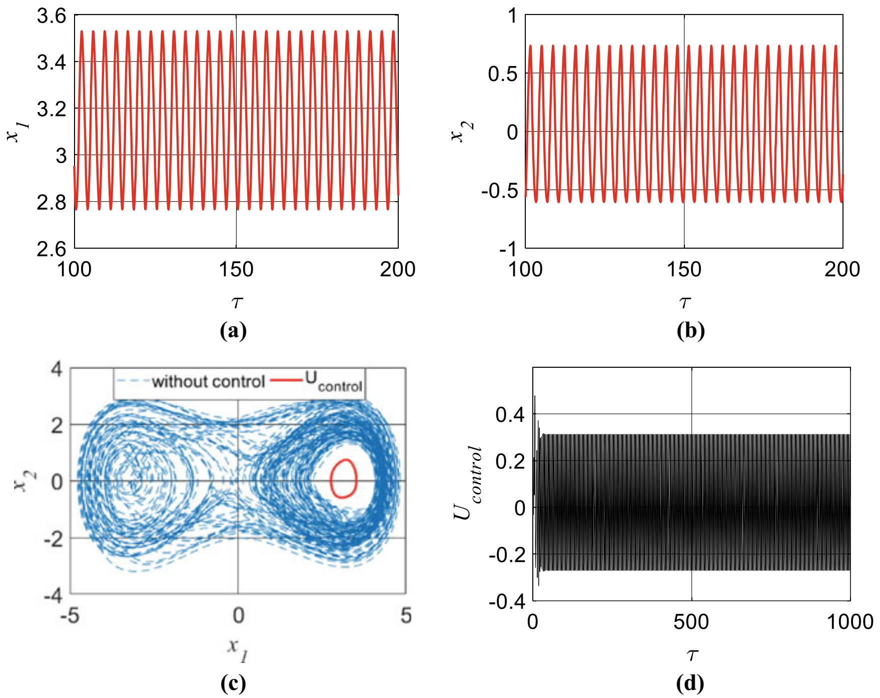


Fig. 7 **a** Time history of the states x_1. **b** Time history of the states x_2. **c** Phase diagram to x_1 versus x_2. **d** Signal control U_{CDCS}

orbit. As can be seen in Figs. 5c and 7c, both controls took the system to the same orbit with practically the same control signal, as can be seen in Figs. 5d and 7d.

Appendix

A.1. The 0–1 Test Method

The 0–1 test consists of estimating a single parameter κ by [17]:

$$\kappa = \frac{\text{cov}(Y, M(\overline{c}))}{\sqrt{\text{var}(Y)\text{var}(M(\overline{c}))}} \tag{A1}$$

where: $\overline{c} \in (0, \pi)$, $M(\overline{c}) = [M(1, \overline{c}), M(2, \overline{c}), \ldots, M(n_{max}\overline{c})]$ and $Y = [1, 2, \ldots, n_{max}]$.

If κ is close to 0 the system is periodic. On the other hand, if κ is close to 1 the system is chaotic. The test utilizes a system variable $x(j)$, where two new coordinates (p,q) are defined as follows [18]:

$$p(n, \bar{c}) = \sum_{j=0}^{n} x(j)\cos(j\bar{c}) \tag{A2}$$

$$q(n, \bar{c}) = \sum_{j=0}^{n} x(j)\sin(j\bar{c}) \tag{A3}$$

The mean square displacement of the new variables $p(n, \bar{c})$ and $q(n, \bar{c})$ is given by [18]:

$$M(n, c) = \lim_{n\to\infty} \frac{1}{N} \sum_{j=1}^{N} \left[(p(j+n, \bar{c}) - p(j, \bar{c}))^2 \ldots + (q(j+n, \bar{c}) - q(j, \bar{c}))^2\right] \tag{A4}$$

where $n = 1, 2, \ldots, N$.

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