A Tutorial on the Simplification of Electromechanical Dynamic Models

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Abstract DC motors are electromechanical systems designed to convert electric power to mechanical power. Their dynamics depend on the features of the motor and the load which they move. The dynamics of the motor is influenced by the external loads acting during the rotation. The electromagnetic forces interact according to the mechanical and electric characteristics of the motor. A common procedure is to neglect the effect of inductance in the steady-state speed and constant current. However, a recent analysis in literature claimed the inductance may be highly relevant in some cases in a steady-state regime. However, including the inductance in computer simulations causes highly time-consuming. Therefore, the intention in the present text is to investigate when it is mandatory to consider the motor inductance in the numerical simulation. The conclusion is that the inductance is relevant when the external loads are relatively high and vary in time.

1 Introduction

This chapter focuses on a discussion on a relatively classic theme in dynamical systems, the modeling of nonlinear dynamics of electromechanical systems involving DC motors [1–5]. In addition to contributing to the formation of new researchers on the subject, the text also aims to clarify some points that still confuse the literature. In

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this sense, the text presents a discussion of the most important aspects of modeling a system where a cart is coupled to a DC motor via a scotch yoke mechanism.

A DC motor is an electric machine that converts direct current electrical energy into mechanical energy. These motors have a permanent magnet stator with the poles N and S, where the electromagnetic forces arise from the current subject to the magnetic field of the stator [2]. The main goal of this analysis is to understand the influence of the inductance in the dynamics of the DC motors. The inductance may be defined as a tendency of an electrical conductor to oppose a change in the electric current flowing through it. In DC motors, the current flows in coils, what causes a higher effect in the magnetic field. Therefore, the consequence is that in some cases the inductance interferes in the dynamics, but in others, it could be neglected. A DC motor is governed by a set of differential equations. An electric equation considering current and voltage. The other is a mechanic differential equation based on the equilibrium of torque. In literature, a useful approach does not consider the inductance of the motor after the transient condition [5, 6]. However, recent analysis claimed that this approach is not reasonable in all cases. An analysis with a motor with a mass and scotch yoke was performed in [7], neglecting the inductance in this case caused a discrepancy when compared with a complete model which considers the inductance. In the present text, some simulations are worked out where is possible to verify that under some conditions there is a divergence in results when the inductance is considered or not. On the other hand, in some situations, the inductance does not interfere significantly in the dynamics, and the act of disregarding it does not have great losses, with the advantage of simplifying the mathematical model.

2 Electromechanical System

The mechanism comprehends a cart moved to left and right through a scotch yoke mechanism. This mechanism is powered by a DC motor and it is illustrated in (Fig. 1). This type of motor has its electric equation written in as

$$L\,Q'' + R\,Q' + G\,\Theta' = \mathcal{V}, \tag{1}$$

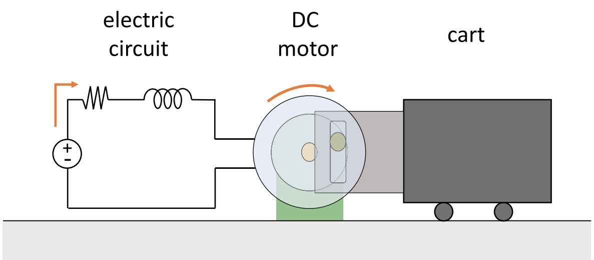

Fig. 1 Illustration of the electromechanical system, which consists of a cart coupled to a DC motor by a scotch yoke mechanism

while the Newton Laws of rotation applied over the rotor in the motor results in the mechanical equation

$$J\,\Theta^{''} + B\,\Theta^{'} - G\,Q^{'} = \mathcal{T}. \tag{2}$$

In the last two equations we have $Q^{'} = Q^{'}(T)$ and $\Theta = \Theta(T)$ respectively denote the electrical current and angular displacement of rotor in time T; the upper prime is an abbreviations for physical time derivative, i.e., $\square^{'} = d\,\square/dT$; L represents an electrical inductance of the motor armature; R stands for the internal electrical resistance of the motor; J the rotational inertia of the rotor; B describes a damping coefficient related to a viscous friction; while G is an electromechanical coupling coefficient, it is equal to the torque constant and speed constant. The voltage source $\mathcal{V} = \mathcal{V}(T)$ and the external torque $\mathcal{T} = \mathcal{T}(T)$ acting over the motor correspond to (possibly) time-dependent external excitations. The torque may also be a function of the electromechanical system coordinates and their derivatives, i.e., $\mathcal{T} = \mathcal{T}(\Theta, \Theta^{'}, \Theta^{''}, T)$. In the present problem the external torque $\mathcal{T}$ is a consequence of the cart reaction and is written as

$$\mathcal{T} = F\,D\,\sin\Theta. \tag{3}$$

The position of the cart is represented in Eq. (4), where the cart depends on the angular position of the rotor

$$X = D\,\cos\Theta. \tag{4}$$

The force of the motor acting over the cart is

$$M\,X^{''} = F, \tag{5}$$

and the resultant torque over the motor is given by

$$\mathcal{T} = -M\,D^2\,\sin\Theta\,\left(\sin\Theta\,\Theta^{''} + \cos\Theta\,\Theta^{'2}\right). \tag{6}$$

The initial conditions for the dynamic system are represented by

$$Q^{'}(0) = Q_0^{'}, \quad \Theta^{'}(0) = \Theta_0^{'}, \quad \text{and} \quad \Theta(0) = \Theta_0, \tag{7}$$

The dynamic system described by Eqs. (1) and (2) is considered the full-order dynamic model. In the next section we will introduce and comment the reduced model.

3 Reduced-Order Dynamic System

The *electrical characteristic time* of the problem is defined by

$$T_Q = \frac{L}{R}, \tag{8}$$

which in a Resistor-Inductor circuit means 63.2% of the time necessary to reach a steady-state current. Additionally, the *mechanical time constant*

$$T_\Theta = \frac{J\,R}{G^2}, \tag{9}$$

represents 63.2% of the time used to reach the maximum speed without external loads on the DC motor.

In literature, it is common authors [5, 6, 8–14] affirm the inductance could be neglected when the electrical characteristic time T_Q is much smaller than the T_Θ mechanical characteristic time. Although this approach is very useful, [7, 15] demonstrated an example where the models highly diverge in results. The reduced-order dynamic model considers the inductance multiplied by the derivative of the current is irrelevant in steady-state conditions. Therefore, a set of equations may be resumed in a single equation.

When inductance is neglected the Eq. (1) turns into

$$Q^{'} = \frac{\mathcal{V}}{R} - \frac{G}{R},\ \Theta^{'} \tag{10}$$

which, after isolating the term $Q^{'}$ and substituting in the mechanical equation Eq. (2) we obtain the reduced-order equation including both electrical and mechanical aspects

$$J\,\Theta^{''} + B\,\Theta^{'} - \frac{G\mathcal{V}}{R}\mathcal{V} + \frac{G^2\Theta^{'}}{R} = \mathcal{T}. \tag{11}$$

The initial conditions now do not consider the current. The electrical part depends on the voltage set. Therefore the initial conditions are simply

$$\Theta^{'}(0) = \Theta_0^{'}, \quad \text{and} \quad \Theta(0) = \Theta_0. \tag{12}$$

4 Dimensionless Formulation

In this sections the dimensionless parameters are introduced. A method commonly applied is the Buckingham Π theorem. The theorem states one can combine parameters from a physical problem and find dimensionless parameters. Some of the most known are the Reynolds number and Mach number in fluid dynamics. In the present problem the dimensionless parameters considered are

$$t = \frac{T}{JR/G^2}\,, \;\; \theta = \Theta\,, \;\; \dot{\theta} = \frac{\Theta^{'}}{G^2/JR}\,,$$
$$\dot{q} = \frac{Q^{'}}{G^3/JR^2}\,, \;\; \ell = \frac{L}{JR^2/G^2}\,, \tag{13}$$
$$\nu = \frac{\mathcal{V}}{G^3/JR}\,, \;\; b = \frac{B}{G^2/R}\,, \;\; d = \frac{D}{\sqrt{J/M}}\,.$$

Here the lower case letters represent the dimensionless parameters and the upper case letters are physical parameters previously mentioned. The electric equation of the DC motor represented in Eq. (1) turns into the dimensionless electric equation

$$\ell\,\ddot{q} + \dot{q} + \dot{\theta} = \nu, \tag{14}$$

while the mechanical equation in Eq. (2) become the dimensionless equation

$$\ddot{\theta} + b\,\dot{\theta} - \dot{q} = \tau, \tag{15}$$

which the right-hand-side is given by the dimensionless external torque

$$\tau = -d^2\,\sin\theta\,\left(\sin\theta\,\ddot{\theta} + \cos\theta\,\dot{\theta}^{\,2}\right). \tag{16}$$

The dimensionless initial conditions are

$$\dot{q}(0) = \dot{q}_0, \;\; \dot{\theta}(0) = \dot{\theta}_0, \;\; \text{and} \;\; \theta(0) = \theta_0. \tag{17}$$

In the reduced model we consider the dimensionless electric equations as

$$\dot{q} + \dot{\theta} = \nu, \tag{18}$$

so that when the dimensionless current $\dot{q}$ is replaced in Eq. (15), results in the reduced dimensionless mechanical equation

$$\ddot{\theta} + (b+1)\,\dot{\theta} - \nu = \tau. \tag{19}$$

In consequence, the initial conditions for the reduced model does not contain the electric current

$$\dot{\theta}(0) = \dot{\theta}_0, \quad \text{and} \quad \theta(0) = \theta_0. \tag{20}$$

5 Results and Discussion

In figures from Fig. 2, 3, 4, 5, 6, 7, 8 and 9 we have in each one the graphics of electric current $\dot{q}$ versus angular velocity $\dot{\theta}$, the time history of the motor angular speed $\dot{\theta}$ and the time history of the motor angular position θ. The intention is to vary some parameters and find some conclusion about the influence of the different parameters. There are many parameters in the present problem and all of them may have its own relevance. However, for practical purposes we considered that three of them are more important. A dimensionless parameter for inductance represented through the parameter ℓ which is varied from Figs. 2, 3 and 4, while the others parameters are maintained constant. In Fig. 2, the value of ℓ is 0.02, a relatively small inductance

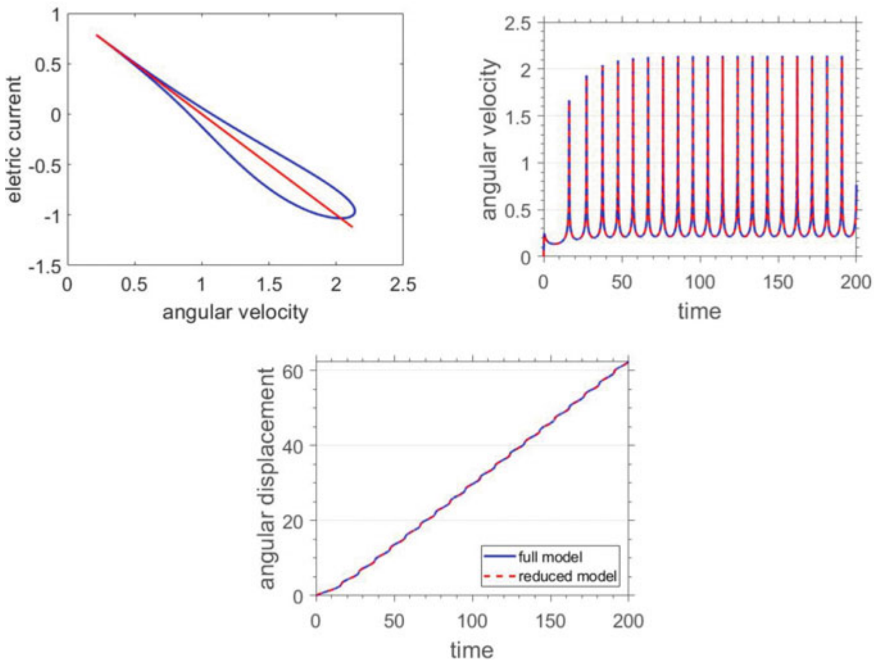

Fig. 2 Phase-space of electric current $\dot{q}$ versus angular velocity $\dot{\theta}$, angular velocity $\dot{\theta}$ versus t and angular displacement θ versus t. Dimensionless parameters: $\ell = 0.02$, $\nu = 1$, $b = 1$, $d = 10$. Initial conditions: $(\theta_0, \dot{\theta}_0, \dot{q}_0) = (0, 0, 3\,\nu)$

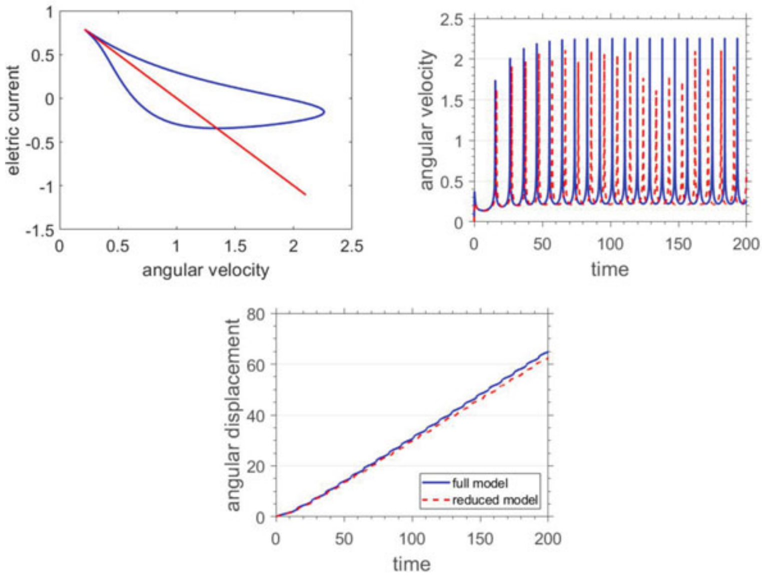

Fig. 3 Phase-space of electric current $\dot{q}$ versus angular velocity $\dot{\theta}$, angular velocity $\dot{\theta}$ versus t and angular displacement θ versus t. Dimensionless parameters: $\ell = 0.2$,$\nu = 1$, $b = 1$, $d = 10$. Initial conditions: $(\theta_0, \dot{\theta}_0, \dot{q}_0) = (0, 0, 3\,\nu)$

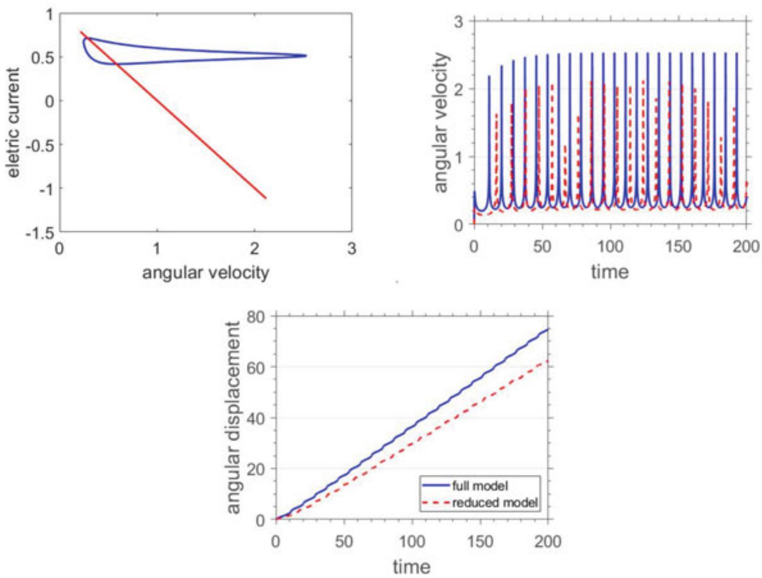

Fig. 4 Phase-space of electric current $\dot{q}$ versus angular velocity $\dot{\theta}$, angular velocity $\dot{\theta}$ versus t and angular displacement θ versus t. Dimensionless parameters: $\ell = 2$,$\nu = 1$, $b = 1$, $d = 10$. Initial conditions: $(\theta_0, \dot{\theta}_0, \dot{q}_0) = (0, 0, 3\,\nu)$

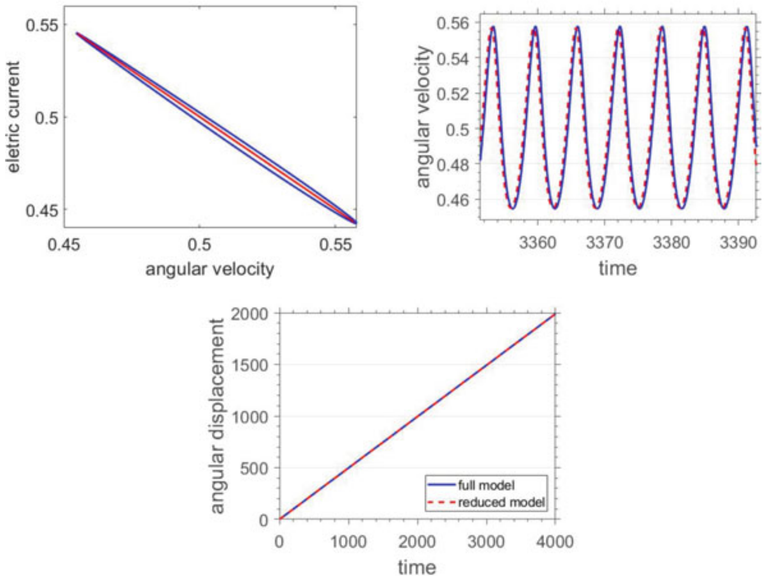

Fig. 5 Phase-space of electric current $\dot{q}$ versus angular velocity $\dot{\theta}$, angular velocity $\dot{\theta}$ versus t and angular displacement θ versus t. Dimensionless parameters:$d = 1$, $\ell = 0.05$,$\nu = 1$, $b = 1$. Initial conditions: $(\theta_0, \dot{\theta}_0, \dot{q}_0) = (0, 0, 3\,\nu)$

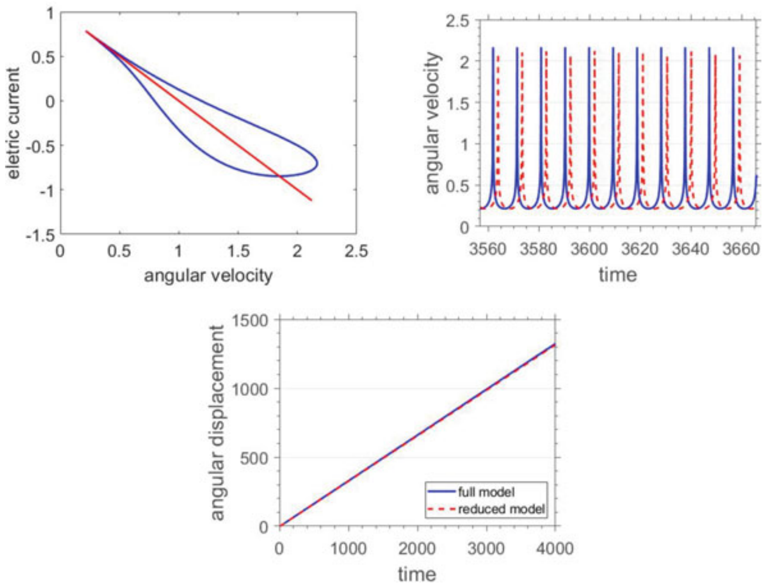

Fig. 6 Phase-space of electric current $\dot{q}$ versus angular velocity $\dot{\theta}$, angular velocity $\dot{\theta}$ versus t and angular displacement θ versus t. Dimensionless parameters:$d = 10$, $\ell = 0.05$,$\nu = 1$, $b = 1$. Initial conditions: $(\theta_0, \dot{\theta}_0, \dot{q}_0) = (0, 0, 3\,\nu)$

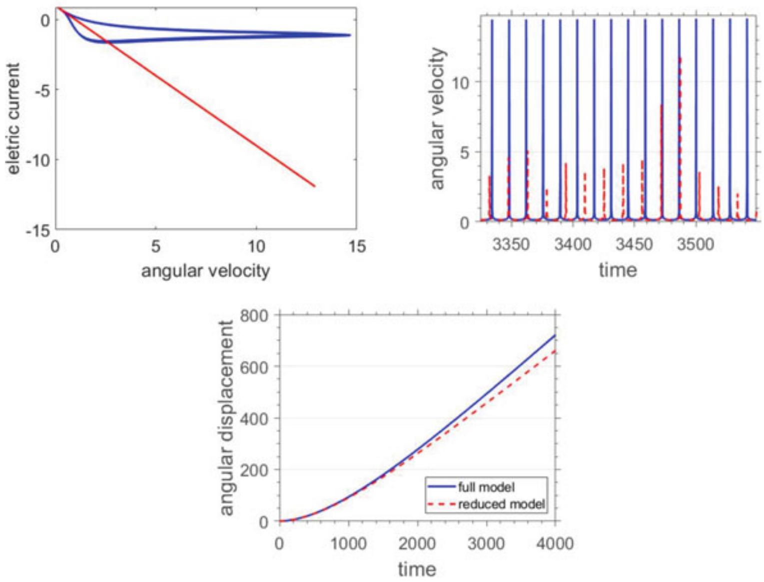

Fig. 7 Phase-space of electric current $\dot{q}$ versus angular velocity $\dot{\theta}$, angular velocity $\dot{\theta}$ versus t and angular displacement θ versus t. Dimensionless parameters:$d = 100$, $\ell = 0.05$,$\nu = 1$, $b = 1$. Initial conditions: $(\theta_0, \dot{\theta}_0, \dot{q}_0) = (0, 0, 3\,\nu)$

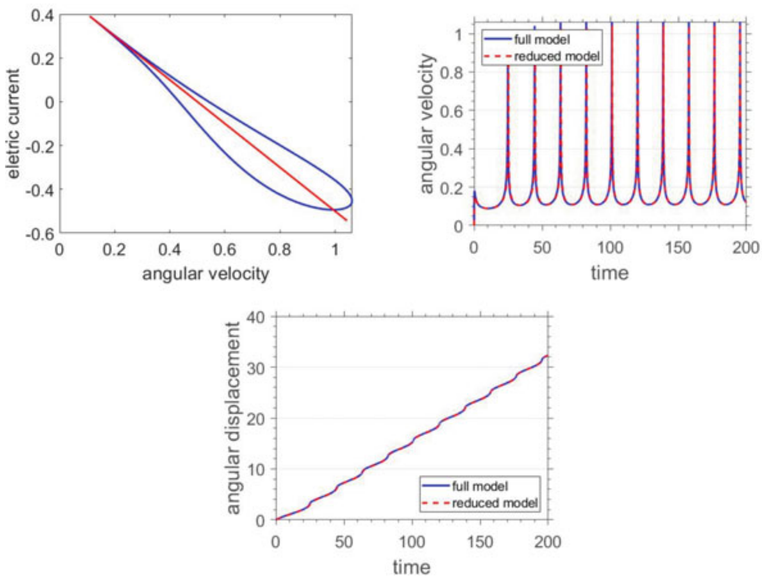

Fig. 8 Phase-space of electric current $\dot{q}$ versus angular velocity $\dot{\theta}$, angular velocity $\dot{\theta}$ versus t and angular displacement θ versus t. Dimensionless parameters:$\nu = 0.5$, $d = 10$, $\ell = 0.05$, $b = 1$. Initial conditions: $(\theta_0, \dot{\theta}_0, \dot{q}_0) = (0, 0, 3\,\nu)$

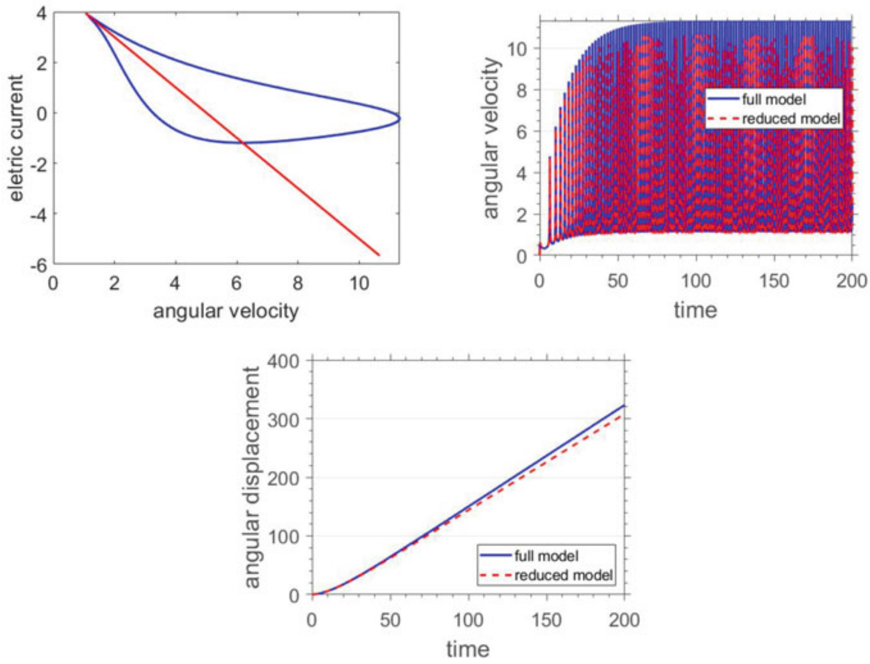

Fig. 9 Phase-space of electric current $\dot{q}$ versus angular velocity $\dot{\theta}$, angular velocity $\dot{\theta}$ versus t and angular displacement θ versus t. Dimensionless parameters:$\nu = 5$, $d = 10$, $\ell = 0.05$, $b = 1$. Initial conditions: $(\theta_0, \dot{\theta}_0, \dot{q}_0) = (0, 0, 3\,\nu)$

where the results demonstrated low divergence when the full and reduced model are compared. An intermediate condition of divergence may be found in Fig. 3 when the value of ℓ is 0.2. Finally, in Fig. 4 which the highest value of inductance with ℓ equal to 2, the diverge appears clearly. This comparisons among these three figures evidence the relevance of the inductance on the present eletromechanical system. Larger values for the inductance have more impact in the divergence between the two models.

Another parameter that is took into account is the d. According the Eq. 13, the term d depends on the mass displaced, the moment of inertia of the rotor and the crank radius. From Figures 5, 7 and 7 the parameter d is varied while the inductance is fixed with ℓ equal to 0.05.The results show good accordance between reduced and full model in Fig. 5. Differences begin to rise in Fig. 6, specially in the graphics for the angular velocity time histories. Lastly, in Fig. 7 the difference are obvious in the three graphics.

The third parameter analyzed is the ν. Higher values for the voltage will exhibit divergence for the two models. In the Fig. 8 where ν equal to 0.5 the two models have good accordance in the results of motion, while in Fig. 9 with ν equal to 5 the reduced and the full models demonstrate a relevant divergence.

6 Conclusion

The conclusion begins pointing the importance to express the system in dimensionless terms. The common reason for this advice is the purpose to obtain a more generic result, suitable for diverse range of conditions. However, in this eletromechanical system when you turn the differential equations into dimensionless differential equations you avoid very small coefficients that oblige the usage of tiny step for integration. In other words, very small values in the coefficients cause difficulties for the computer calculus. The effect of high inductance is easier to understand. It follows the derivative of the current, the consequence is that higher the inductance, higher is the effect to the dynamics of the eletromechanical system. Not so obvious is the influence of the parameter d. This parameter is not composed by electric parts. The parameter d is essentially mechanical and it is responsible for the external loads acting on the motor. A higher value of d means high external forces alternating during time. It is similar to a motor in the transient regime, while it is accelerating. The voltage represented by the parameter ν also provokes divergences between the two models. The explanation is that high voltage causes a higher current and also contributes for a greater forces involved in the mechanism. A mathematical note is that in Eqs. 1 and 14 when the speed of motor is reduced near to stall, the electric part of the equation will have higher influence. It means a higher influence for inductance and electric resistance. Therefore, the main advice for electromechanical systems with DC motor is to test different models including and disregarding the inductance. Operating in high speeds and with constant load are the best conditions to neglect the influence of the inductance in DC motors. For more details on the problem discussed in this chapter the reader is invited to see reference [16].

Code Availability

The simulations of this chapter used a Matlab code dubbed **ElectroM - ElectroMechanical Dynamic Code**. To facilitate the reproduction of the results, this code is available for free on GitHub [17]. Other animations of electromechanical dynamics are available on a YouTube playlist [18].

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