The Study of Fuzzy Quantifiers in Multi-criteria Decision-Making

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Abstract. The paper examines fuzzy quantifiers, which serve to formalize human reasoning. In this paper, quantifiers are considered in relation to the problems of decision-making on a set of alternatives based on a combination of criteria. Using fuzzy quantifiers and OWA aggregation operators, in which quantifiers are used to calculate weights, it is possible to implement basic decision-making strategies. In this paper, we study various quantifiers that are most often used when choosing the best alternative, such as "Most", "The more, the better", "At least k%". As a result of the study of these quantifiers, the boundaries of the values of the parameters of the membership functions were found, at which the OWA operator will have compensatory properties. The paper also points out the disadvantages of the most commonly used quantifiers when they are used in the OWA operator. The presence of "insensitivity zones" of the quantifier with piecewise linear functions of belonging to the change in the values of the components of the criteria vector is established. It is shown that this problem is solved when passing to a continuous membership function in the form of an s-shaped (logistic) curve. A modification of the OWA operator is proposed in the form of a superposition of partial estimates and a membership function of the fuzzy concept of "Good correspondence". This modification ensures that when comparing alternatives, not only the number of private assessments that meet the criteria is taken into account, but also the quality of compliance.

Keywords: OWA operator · Multi-criteria · Fuzzy quantifiers · Decision-making

1 Introduction

In the tasks of making a decision on a multiple alternatives based on a set of criteria, information about the acceptable form of compromise between estimates according to different criteria (private estimates) plays an important role. The tolerance level is a subjective, fuzzy concept that can be defined by a fuzzy quantifier [1, 2]. Fuzzy quantifiers are an extension of the classical quantifiers of generality and existence, they serve to formalize human reasoning. A certain quantifier is fuzzy if it is possible to construct a function of belonging to the corresponding fuzzy set for it [3]. Examples of fuzzy quantifiers are such concepts as "about half", "in general", "most", etc. In the theory of

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decision-making, a fuzzy quantifier is a fuzzy statement about the acceptable form of compromise between particular estimates, reflecting the intuitive idea of the decision-maker about the preference of decisions. In particular, fuzzy quantifiers are used in OWA aggregation operators [4]. A special feature of these operators is to obtain a weighted average ordered by the magnitude of the partial estimates.

When comparing alternatives with each other with the help of OWA, it is necessary to take into account how large the values of partial estimates for one criteria compensate for small values for other criteria. This property of the OWA operator is called compensation [5]. It is important to be able to manage these compensatory properties, which depend on the form of the quantifier. In this paper, we study various quantifiers that are most often used when ordering and choosing the best alternative. Among the quantifiers, it is necessary to choose those that most adequately reflect the form of compromise and take into account the opinion of the decision-maker. The conditions under which the aggregating operator will have compensatory properties are considered. The disadvantages of the most commonly used quantifiers when they are used in the OWA operator are indicated. It is proposed that when comparing alternatives, not only to take a decision on a set of alternatives based on a set of criteria into account the number of private assessments that meet the criteria, but also the quality of compliance. All of the above is illustrated by examples.

2 Formalization of the Decision-Making Problem on a Set of Alternatives

Let's consider the problem of making a decision on a set of alternatives based on a set of criteria $C = \{c_1, c_2, \ldots, c_J\}$. Such a multi-criteria problem can be represented by the following tuple [6]

$$<X, G, P, D>, \tag{1}$$

where $X = \{x_1, x_2, \ldots, x_m\}$ – the set of alternatives; $G = \{g_1(x), g_2(x), \ldots, g_n(x)\}$ - the vector evaluation of the alternative $x \in X$, where the partial estimates $g_j(x) : R \to [0,\ 1]$ determine the degree of compliance $x \in X$ with the criteria $c_j \in C$, $\left(j = \overline{1, n}\right)$; P – the system of preferences of the decision-maker; D - the decisive rule.

With the help of the preference system P, it is possible to determine a strategy for comparing partial estimates of alternatives and build a decisive rule D. It sets the procedure (algorithm) for performing the required action on a set of alternatives. The specified action may consist in ordering alternatives by preference, distributing them by classes of solutions, or choosing the optimal alternative [6].

The system of preferences of the decision-maker P can be represented in the form

$$\left\langle \underset{x \in X}{agg}(g_1(x), g_2(x), \ldots, g_n(x)) \to \max, \quad x \in X; \quad \Lambda,\ Q \right\rangle, \tag{2}$$

where agg is the aggregation operator; Λ is information about the relative importance of criteria, usually given as a set of weights $\lambda_j \geq 0$, giving 1 in total; Q is information about the acceptable form of compromise between estimates for different criteria.

Further, instead of $g_j(x)$, we will write simply g_j.

Let's take a closer look at the concept of an aggregation operator. The aggregation operator is a function of n variables (criteria, partial estimates) $agg : \bigcup_{j \in n} [0, 1]^j \to [0, 1]$ that satisfies a number of mandatory conditions [7, 8] on a set of arbitrary $x, y \in [0; 1]$:

1) $agg(x) = x$;
2) $agg(0, \ldots, 0) = 0$ and $agg(1, \ldots, 1) = 1$;
3) $agg(x_1, \ldots, x_n) \le agg(y_1, \ldots, y_n)$ if $(x_1, ..., x_n) \le (y_1, ..., y_n)$.

The aggregation operator allows us to obtain a generalized (complex) assessment that characterizes the object as a whole according to all criteria [5]. At the same time, three main strategies can be implemented [9]:

a) a conjunctive strategy, according to which a generalized estimate cannot be better than the worst of the partial estimates; in this case, the degree to which the alternative $x \in X$ meets all the criteria at once is defined as

$$agg(g_1, \ldots, g_n) = \min(g_1, \ldots, g_n); \tag{3}$$

b) a disjunctive strategy, according to which the generalized estimate is determined by the best of the partial estimates. The degree to which it meets at least one of the criteria is defined as

$$agg(g_1, \ldots, g_n) = \max(g_1, \ldots, g_n); \tag{4}$$

c) a compromise strategy, according to which the generalized estimate occupies an intermediate position between the private estimates involved in the aggregation:

$$\min(g_1, \ldots, g_n) \le agg(g_1, \ldots, g_n) \le \max(g_1, \ldots, g_n). \tag{5}$$

The disjunctive strategy is characteristic of the optimistic position of the decision-maker, while the pessimistic decision-maker's tends to rely on the worst properties of objects in its judgments, and, consequently, on the conjunctive strategy.

In cases where the importance of the values of particular estimates is primary, the ordinal weighted aggregation operators [5–7, 10, 11] are used, OWA operators that aggregate the components of the vector estimate ordered in a certain way:

$$agg(g_1, \ldots, g_n) = OWA(g_1, \ldots, g_n) = \sum_{j=1}^{n} w_j g_{\sigma(j)}, \tag{6}$$

where σ is the index of ordering by the magnitude of the elements, such $g_{\sigma(1)} \ge g_{\sigma(2)} \ge \ldots \ge g_{\sigma(n)}$, $w = (w_1, \ldots, w_n)^T$ is the vector of weights, such that $\sum_{j=1}^{n} w_j = 1,\ w_j \ge 0,\quad j = \overline{1, n}$.

In this operator w_j, the weight is associated not with a specific element of the vector G, but with its comparative value relative to other objects (the largest element gets the weight w_1, the next one after it w_2, etc.).

At the same time, by assigning certain values of weights, it is possible to implement disjunctive, conjunctive and compromise strategies for aggregation.

For example, when $w = (1, 0, \ldots, 0)^T$, we get

$$OWA* = OWA(g_1, \ldots, g_n) = 1 \cdot g_{\sigma(1)} = \max(g_1, \ldots, g_n), \tag{7}$$

that is, the OWA* operator implements a disjunctive strategy. Therefore, the aggregation takes into account the best property (correspondence) of the object.

When $w = (0, 0, \ldots, 1)^T$,

$$OWA_* = OWA(g_1, \ldots, g_n) = 1 \cdot g_{\sigma(n)} = \min(g_1, \ldots, g_n), \tag{8}$$

that is, the operator OWA_* implements a conjunctive strategy, only the worst property of the object is taken into account. Therefore, a generalized estimate cannot be better than the worst of the partial estimates.

For $w = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$,

$$OWA(g_1, \ldots, g_n) = \frac{1}{n} \sum_{j=1}^{n} g_{\sigma(j)} \tag{9}$$

is the arithmetic mean.

It is assumed that the aggregation operator has a compensatory property if small values of partial estimates for one indicator are compensated by large values of estimates for other indicators. Operator (3) does not have compensatory properties, operator (4) implements full compensation.

The indicator that characterizes the presence of a compensatory property in OWA with a particular set of weights is calculated using the formula

$$orness(w) = \frac{1}{n-1} \sum_{j=1}^{n} (n-j) w_j \tag{10}$$

If, for a given set of weights $orness(w) > 0{,}5$, the OWA operator has compensatory properties and implements a strategy close to disjunctive. If $orness(w) < 0{,}5$, then to the conjunctive.

Another indicator associated with $orness(w)$ and the inverse of it in value

$$andness(w) = \frac{1}{n-1} \sum_{j=1}^{n} (j-1) w_j = 1 - orness(w) \tag{11}$$

For example, for (7), $orness(w) = 1$, $andness(w) = 0$, for (8), $orness(w) = 0$, $andness(w) = 1$, for (9), $orness(w) = 0{,}5$.

3 Investigation of the Properties of Quantifiers in Relation to the Problem of Decision-Making on a Set of Alternatives

Let's return to the system of preferences (2). To formalize the information Q, we use the concept of a fuzzy quantifier [4]. Fuzzy quantifiers are an extension of the classical set of logical quantifiers, which includes the quantifiers ∃ ("exists") and ∀ ("for all"), by. introduction of fuzzy concepts "almost for everyone", "about half", etc.

Let's consider the case when we are talking about ordering alternatives by preference and the criteria are of equal importance, that is $\lambda_j = const$. If we consider the behavior of the decision-maker, then the natural reasoning associated with the choice of the most preferred alternative will be those that are based on the assumption that the more criteria the alternative meets, the better. Another type of reasoning related to determining the quality of an alternative is based on the fact that the alternative must meet the majority of criteria or at least k% of the criteria. Such arguments can be formalized using quantifiers. The quantifier determines an approximate estimate of the number of aggregated values that greatly affect the value of the generalized estimate [5]. The quantifier is a fuzzy variable, the carrier of which is the fraction of partial estimates of r, defined on the segment [0; 1]. The membership function of the quantifier Q (r) corresponds to the degree of preference of an alternative that satisfies the fraction r of the entire set of criteria. For example, for Q(0,6), the specified percentage corresponds to 60%. In the future, we will consider quantifiers whose membership function satisfies the conditions:

$$1.\ Q(0) = 0, \quad Q(1) = 1; \tag{12}$$

$$2.\ Q(r_1) \le Q(r_2),\ \text{at } r_1 < r_2; \tag{13}$$

$$3.\ Q(r)\ -\ \text{piecewise continuous function.} \tag{14}$$

Then, using such a quantifier, we can find the weights w_j of descending-ordered partial estimates in the OWA operator by the formula

$$w_j = Q\left(\frac{j}{n}\right) - Q\left(\frac{j-1}{n}\right). \tag{15}$$

That is, the greater the rise in the value of the quantifier gives an increase in the share of the set of ordered partial estimates due to the j-th estimate, the greater its weight. The geometric meaning of the weights found by the formula (15) is shown in Fig. 1.

Suppose that, according to the information Λ, all partial estimates have the same importance. Then the decisive rule D in the model (1)–(2) will have the form

$$D(x) = \underset{x \in X}{OWA}(g_1(x),\ g_2(x),\ \ldots,\ g_J(x)) \to \max, \tag{16}$$

where the weights of the OWA operator are given by the formula (15).

Let's consider the types of quantifiers that satisfy the conditions (12)–(14) that are most often used in decision-making.

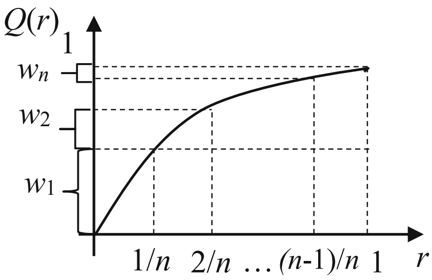

Fig. 1. The geometric meaning of the weights found by the formula (15)

The quantifier "For all" is determined by the formula (Fig. 2)

$$Q_\forall(r) = \begin{cases} 0, & r < 1, \\ 1, & r = 1. \end{cases}$$

The weights obtained by the formula (15) will be equal to

$$w_j = \begin{cases} 0, & j < n, \\ 1, & i = n. \end{cases}$$

The membership function of the quantifier is shown in Fig. 2.

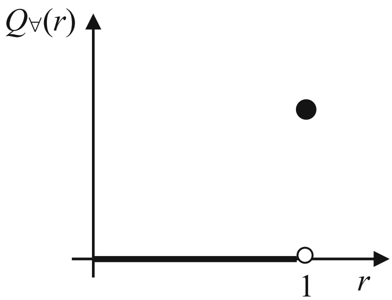

Fig. 2. The "For All" quantifier

When substituting these weights in (6), we get the operator $OWA*$.
The quantifier "Exists" (Fig. 3) is determined by the formula

$$Q_\exists(r) = \begin{cases} 0, & r = 0, \\ 1, & r \leq 1. \end{cases}$$

The weights obtained by the formula (12) will be equal to

$$w_j = \begin{cases} 1, & i = 1, \\ 0, & i < n. \end{cases}$$

In this case, we get the operator $OWA*$.

The quantifier "The more, the better" or "For as many as possible" (Fig. 4) can be determined by the formula

$$Q(r) = r \tag{17}$$

The weights obtained by the formula (12) will be equal to

$$w_j = \frac{1}{n}$$

and in this case we get the operator (9).

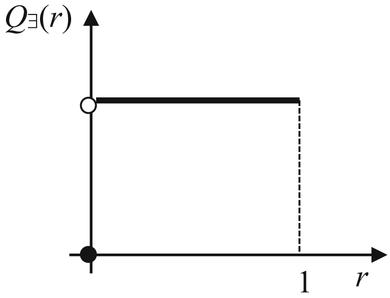

Fig. 3. The quantifier "Exists"

In order to determine to what extent the OWA operator, whose weights are found by formula (15), can implement conjunctive or disjunctive strategies, use the formula

$$orness(Q) = \int_0^1 Q(r)dr.$$

Consider a family of quantifiers whose membership function depends on the parameter $\alpha > 0$

$$Q(r) = r^\alpha \tag{18}$$

We investigate the influence of the parameter on the compensation properties, for this we define

$$orness(Q) = \int_0^1 r^\alpha dr = \frac{1}{\alpha + 1}. \tag{19}$$

The operator will have a compensation property if $\frac{1}{\alpha+1} > 0{,}5$ or $\alpha < 1$.

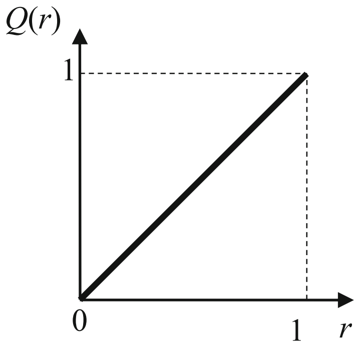

Fig. 4. The quantifier “The more, the better”

Let’s consider a quantifier that can be used to express the concept of “For the majority”, an example of such a quantifier is given in [6] (Fig. 5)

$$Q(r) = \begin{cases} 0, & 0 \leq r \leq a, \\ \frac{x-a}{b-a}, & a < r \leq b, \\ 1, & b < r \leq 1. \end{cases} \tag{20}$$

We’ll find it

$$orness(Q) = \int\limits_0^1 Q(r)dr = \frac{b-a}{2} + (1-b) = 1 - \frac{a+b}{2}.$$

$$orness(Q) > 0.5, \text{ at } a + b < 1$$

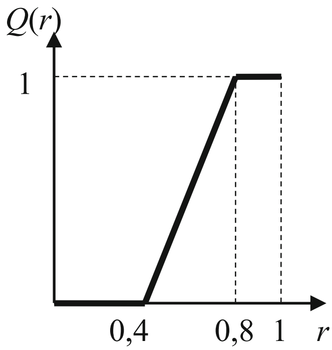

Fig. 5. Graphical representation of the quantifier “For the majority”

Consider a family of quantifiers that depend on two parameters, of the form

$$Q(r) = \begin{cases} 0, & r = 0, \\ \frac{1}{1+e^{-a(r-b)}}, & 0 < r < 1, \\ 1, & r = 1, \end{cases}$$
$$a > 1, \quad 0 < b < 1. \tag{21}$$

This function is continuous, monotonically increasing, $a > 1$ and has an s-shape and one inflection point with coordinates $x = b, \quad y = 0{,}5$. The higher the value of a, the faster the transition from the shape of the curve convex down to convex up.

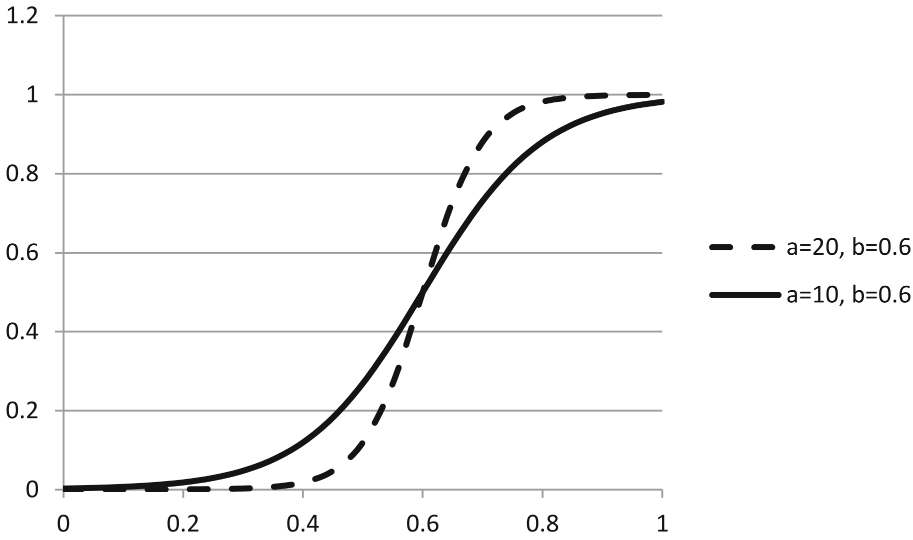

Fig. 6. Dependence of the curve shape of the function (21) on the parameter a

With the help of such a family of quantifiers, it is convenient to express concepts such as "At least for k%". The quantifier "For the majority" can be considered as a special case of this quantifier. Because "the majority" can be interpreted as "at least for 50%". We examine this quantifier for the presence of compensatory properties:

$$orness(Q) = \int_0^1 \frac{1}{1+e^{-a(r-b)}} dr = \frac{1}{a} \ln\left(e^a - 1\right) - b. \tag{22}$$

Let's determine at what values of parameters a and b it will have compensatory properties, that is

$$\frac{1}{a} \ln\left(e^a - 1\right) - b > 0{,}5 \text{ or } \frac{1}{a} \ln\left(e^a - 1\right) > b + 0{,}5. \tag{23}$$

Since $a > 1$, the function $f(a) = \frac{1}{a}\ln(e^a - 1)$ is monotonically increasing and $\lim_{a\to\infty} \frac{1}{a}\ln(e^a - 1) = 1$, then this inequality will be satisfied only when $b < 0{,}5$. That is, the function will have compensatory properties if the abscissa of the inflection point is less than 0,5.

In order for the indicator $orness(Q)$ to reach a certain value, which we denote by α^*, it is necessary to solve the equation

$$\frac{1}{a}\ln\left(e^a - 1\right) - b = \alpha^*. \tag{24}$$

Let's fix a, in this case $b = \frac{1}{a}\ln(e^a - 1) - \alpha^*$.

If b is fixed, then it is necessary to solve the equation with respect to a:

$$e^{a\left(\alpha^* + b\right)} - e^a + 1 = 0, \quad a > 1. \tag{25}$$

This equation has a single root, provided $\alpha^* + b < 1$.

4 Problems that Arise When Using Quantifiers in OWA Operators

We note a number of problems that may arise when using certain quantifiers in the OWA operator.

When using the "for the majority" quantifier in the form (20) with parameters $a = 0{,}4$, $b = 0{,}8$ (Fig. 5), the OWA operator becomes insensitive to the first values $g_{\sigma(j)}$ for which their total share does not exceed 0.4. The weights of these estimates will be zero. For example, if we are talking about comparing two alternatives, whose partial estimates are based on a set of criteria (already ordered in descending order):

(0,9; 0,6; 0,4; 0,3; 0,1) and (0,7; 0,6; 0,4; 0,3; 0,1),

then they turn out to be equivalent. For both options, OWA = 0,38. Although the first alternative is slightly preferable to the second due to the larger value of the first estimate.

Or, for example, for alternatives with private estimates (0,9; 0,6; 0,4; 0,3; 0,1), (0,8; 0,8; 0,4; 0,3; 0,1) and (0,9; 0,7; 0,4; 0,3; 0,1), OWA will also be equal to 0,38. Between the two alternatives with private estimates (0,4; 0,3; 0,3; 0,2; 0,2) and (0,9; 0,8; 0,3; 0,2; 0,2) the OWA operator will also not allow you to choose the best one, for both alternatives it will be equal to 0,29.

This is due to the fact that for the first ranked values of private estimates, their share is small and until it reaches 40%, the values of these estimates will not be taken into account. To avoid this problem, we can use the sigmoid function (21) as the membership function for the "for the majority" quantifier. With the help of the parameter b (the abscissa of the inflection point), it is possible to influence the proportion of criteria reached, as the concept of majority becomes more pronounced, with the value of the membership function greater than 0,5.

For the above cases, the OWA operator with the use of the quantifier "most" in the form of the function (21) will give the following results:

for (0,9; 0,6; 0,4; 0.3; 0,1) OWA = 0,3501 for (0,7; 0,6; 0,4; 0,3; 0,1) OWA = 0,3500, that is, the first alternative is slightly preferred;

for the alternative with private estimates (0,9; 0,6; 0,4; 0,3; 0,1) OWA = 0,3501 for (0,8; 0,8; 0,4; 0,3; 0,1) OWA = 0,3536 and (0,9; 0,7; 0,4; 0,3; 0,1), OWA = 0,3518;

for (0,4; 0,3; 0,3; 0,2; 0,2) OWA = 0,24997 and (0,9; 0,8; 0,3; 0,2; 0,2), OWA = 0,25896.

The OWA operator using the quantifier "The more, the better" in the form (17) may have problems when comparing alternatives if the sums of the partial estimates are the same. This is due to the fact that the weights w_j obtained using this quantifier are the same. As a result,

$$OWA(g_1, \ldots, g_n) = \sum_{j=1}^{n} w_j g_{\sigma(j)} = w \sum_{j=1}^{n} g_{\sigma(j)} \tag{26}$$

and, for example, alternatives with partial estimates (0,4; 0,35; 0,3; 0,03; 0,02) and (0,3; 0,2; 0,2; 0,2; 0,2) they turn out to be equivalent, OWA = 0,23.

5 Modification of the OWA Operator

To fine-tune the mechanism for comparing alternatives with each other using the OWA operator, it is necessary not only to take into account the share of private estimates of r, but also how well this or that private estimate involved in the formation of this share corresponds to the representation of the decision-maker about the degree of achievability of compliance with the criterion. It is proposed to supplement the OWA operator with a fuzzy function $h\big(g_{\sigma(j)}\big) : [0, 1] \rightarrow [0, 1]$ that allows describing the fuzzy concept of "Good matching". In this case, the rule Q for the quantifier Q1 = "for the majority" can be formulated as follows: "A GOOD match must be achieved for most criteria". For the quantifier Q2 = "the more, the better", the rule will take the form: "The more criteria a GOOD match is achieved, the better". The membership function "GOOD match" can be set:

$$h\big(g_{\sigma(j)}\big) = \begin{cases} 0, & g_{\sigma(j)} = 0, \\ \frac{1}{1+e^{-a(g_{\sigma(j)}-0,5)}}, & 0 < g_{\sigma(j)} < 1 \\ 1, & g_{\sigma(j)} = 1. \end{cases},$$
$$a > 1, \tag{27}$$

where the abscissa of the inflection point is 0,5.

Note that each criterion can have its own degree of reachability of a good match, then the function $h\big(g_{\sigma(j)}\big)$ for each criterion must be set separately.

Then the OWA operator will take the form

$$OWA(g_1, \ldots, g_J) = \sum_{j=1}^{J} w_j h\big(g_{\sigma(j)}\big) \tag{28}$$

We will use the operator (28) with the quantifier "for the majority" in the form (21) and the rule "A GOOD match must be achieved for most criteria" with the membership function (27) ($a = 10$), which is the same for all criteria for ranking alternatives with characteristics:

(0,9; 0,6; 0,4; 0,3; 0,1) and (0,7; 0,6; 0,4; 0,3; 0,1). In the first case, OWA = 0,20064, in the second case, OWA = 0,20061;

(0,9; 0,6; 0,4; 0,3; 0,1), (0,8; 0,8; 0,4; 0,3; 0,1) and (0,9; 0,7; 0,4; 0,3; 0,1), in the first case, OWA = 0,20064, in the second, OWA = 0,20454, in the third, OWA = 0,20328;

(0,4; 0,3; 0,3; 0,2; 0,2) and (0,9; 0,8; 0,3; 0,2; 0,2). OWA1 = 0,0833, OWA2 = 0,0983.

When ranking alternatives with characteristics (0,4; 0,35; 0,3; 0,03; 0,02) and (0,3; 0,2; 0,2; 0,2; 0,2) using the quantifier "the more, the better" and the rule "The more criteria a GOOD match is achieved, the better", the OWA operator will allow them to be ranked, in the first case OWA = 0,1175, in the second case OWA = 0,062.

6 Conclusion

As a result of the study of various fuzzy quantifiers, the boundaries of the values of the parameters of the membership functions were found, at which the OWA operator will have compensatory properties. The presence of "insensitivity zones" of the quantifier with piecewise linear functions of belonging to the change in the values of the components of the criteria vector is also established. It is shown that this problem is solved when passing to a continuous membership function in the form of an s-shaped (logistic) curve.

A modification of the OWA operator is proposed in the form of a superposition of partial estimates and a membership function of the fuzzy concept of "Good correspondence". This modification ensures that when comparing alternatives, not only the number of private assessments that meet the criteria is taken into account, but also the quality of compliance. At the same time, for each criterion, the degree of reachability of a good match can be set by its own membership function.

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