# A Genetic Algorithm to Optimize Dynamics of Supply Chains

**Luigi Rarità**

**Abstract** This paper focuses on a model for supply chains, based on partial and ordinary differential equations, that model, respectively, densities of parts on suppliers and queues between consecutive arcs. An optimization approach is discussed via a cost functional that, in consideration of a wished outflow, weights queues of materials by variations of processing velocities for suppliers. The minimization of the cost functional is achieved via a genetic algorithm that, as for the processing velocities, considers mechanisms of selection, crossover and mutation. A simulation example is discussed for the optimization procedure.

**Keywords** Genetic algorithms · Supply chains · Simulations

## 1 Introduction

Managing supply systems is an important issue, as particular phenomena, such as dead times and bottlenecks, represent serious matters within industrial contexts. Various mathematical approaches are useful in this regard. Some of them are based on Discrete Event Simulations (DES) [1], while others refer to Ordinary and/or Partial Differential Equations (ODEs, PDEs) [2–4]. In this paper we consider a continuous model that, based on differential equations for the dynamics of goods on arcs and queues among them, is introduced in [5], further analyzed in [6, 7], and solved numerically in [8]. Notice that the proposed model is different from others based on mixed integer linear programming with possible issues about combinatorial optimization, see for instance [9]. On the other hand, the used numerical approach is similar to ones described, for instance, in [10–12].

L. Rarità (✉)
Department of Management and Innovation Systems, University of Salerno, Fisciano, Italy

Department of Science and Technology, University of Sannio, Benevento, Italy
e-mail: lrarita@unisa.it

 
L. Amorosi et al. (eds.), *Optimization in Artificial Intelligence and Data Sciences*,
AIRO Springer Series 8, https://doi.org/10.1007/978-3-030-95380-5_10

In the proposed research, once the PDE-ODE model is solved numerically by an upwind scheme for the density of parts over suppliers, and by an explicit Euler method for queues of goods between consecutive arcs, an optimization procedure is described. A correct definition of the optimal performances is useful to improve the productivity and often involves different questions. For instance, in [6], two different optimal control problems arise: the first considers the minimization of queues in terms of a pre-defined outflow; the second focuses on possible values of distributions rates that minimize queues for a supply network with vertices of dispersing type. In [13], the authors describe a procedure that, by adjusting a piecewise constant input flow, aims to minimize queues and to approximate the wished supply chain outflow. In particular, [6] focuses on a rich numerical investigation, based on the software Matlab; [13], on the other hand, provides a correct analysis for an analytical optimal solution, but only for particular cases of input flows.

In this paper, a procedure, based on a genetic algorithm (GA), allows, from one side, to compute optimal solutions for a generic supply system that could have an input flow of various shapes, unlike the case presented in [13]. On the other hand, the adoption of a GA ensures a suitable theoretical basis for optimization issues, that are considered only in terms of simulations in [6]. Finally, various analysis of the approach proposed in this paper confirmed the results of [6] and [13], thus showing a robust approach of resolution. Such features provide the key elements of novelty for the following paper: the possibility of adapting classical numerical schemes in order to simulate networks of medium/big dimensions with reduced computational times; the definition of a robust optimization approach, based on a GA, for a supply system modelled by ODEs and PDEs. Notice that the adoption of GAs is only a starting point as other possible simulation schemes, based for instance on particle swarm optimization as well as ant/bee colony dynamics, could be proposed. Indeed, unlike other suitable optimization procedures, that are still under investigation, GAs already present a complete analysis of various properties, see [14–18], while applications of GAs in the context of supply systems are reported in [19] and [20]: the former describes an integrated model for a supplier selection model of both multi-item and multi-supplier frameworks via a two-level GA that decides about selections of suppliers and splitting of demands; in the latter, a GA works as a decision support system for dynamics of an integrated inventory control in case of backlogged shortage. Here, a GA is used in a different way. Precisely, a cost functional (see [6]), that weights the amount of queues and a wished outflow, is minimized in terms of processing velocities of suppliers. The different iterations of the GA allow variations of the velocities of suppliers by mechanisms of selection, crossover and mutation.

Some numerical simulations are also discussed. In particular, a possible supply chain with twenty arcs is considered. The queues show an evident dependence on processing velocities and maximal capacities of suppliers. A further investigation allows a possible optimization. Different iterations of the genetic algorithm are considered and it is shown the queue decrease in successive steps.

The paper is structured as follows. Section 2 focuses on the ODE–PDE model and numerical approaches. Section 3 describes a possible optimal control problem for the chosen model. Section 4 focuses on a test case and its optimization. The paper ends with conclusions and future research activities in Sect. 5.

## 2 Model and Numerics for Supply Chains

We describe a model for supply chains, characterized by ODEs and PDEs ([5, 6]), on the basis of a reformulated approach proposed in [2].

A supply chain has a set of vertices $\mathscr{V} = \{1, \ldots, M-1\}$ and a set of arcs $\mathscr{A} = \{1, \ldots, M\}$. Each arc $m \in \mathscr{A}$ is a supplier, indicated by an interval $[\alpha_m, \beta_m]$. For each vertex, one incoming arc is connected to one outgoing arc and the various arcs are consecutively labelled, namely arc $m$ connects arc $m+1$ with $\beta_m = \alpha_{m+1}$. For the first and the last arc, $\alpha_1 = -\infty$ and $\beta_M = +\infty$, respectively, with suitable boundary data.

For each supplier $m \in \mathscr{A}$, we have: length $L_m > 0$; processing time $T_m > 0$, and hence a processing velocity $V_m := L_m/T_m$; the highest processing capacity $\mu_m > 0$; the density of parts at point $x$ and time $t$, represented by the continuous function $D_m(t, x) \in \left[0, D_m^{\max}\right]$. Finally, for each supplier $m \in \mathscr{A} \setminus \{1\}$, at $x = \alpha_m$ the function $Q_m(t)$ represents a time dependent queue of goods, that travel between consecutive arcs.

Then, for densities $D_m(t, x)$ and queues $Q_m(t)$, the model obeys the equations:

$$\frac{\partial D_m(t,x)}{\partial t} + \frac{\partial \phi_m(D_m(t,x))}{\partial x} = 0, \quad \forall\, x \in [\alpha_m, \beta_m], \quad t > 0, \tag{1}$$

$$D_m(0,x) = D_{m,0}(x) \geq 0, \quad D_m(t,\alpha_m) = \frac{\phi_{m,inc}(t)}{V_m}, \tag{2}$$

$$\frac{d}{dt} Q_m(t) = \phi_{m-1}(D_{m-1}(\beta_{m-1},t)) - \phi_{m,inc}(t), \quad m \in \mathscr{A} \setminus \{1\}, \tag{3}$$

$$Q_m(0) = Q_{m,0} \geq 0, \tag{4}$$

where: $\phi_m(D_m(t,x)) := \min\{\mu_m, V_m D_m(t,x)\}$ is the flux function; $D_{m,0}(x)$ is the initial datum (to assign); $\phi_{m,inc}(t)$ is the flux on the outgoing arc $m$, namely:

$$\phi_{m,inc}(t) := \begin{cases} F(t), & m = 1, \\ \min\{\phi_{m-1}(D_{m-1}(\beta_{m-1},t)), \mu_m\}, & Q_m(t) = 0, m \in \mathscr{A} \setminus \{1\}, \\ \mu_m, & Q_m(t) > 0, m \in \mathscr{A} \setminus \{1\}, \end{cases} \tag{5}$$

whose interpretation is as follows: if $m = 1$ (first arc of the supply chain), $\phi_{m,inc}(t)$ is $F(t)$, assigned input profile on the left boundary $\{(\alpha_1, t) : t \in \mathbb{R}\}$. If $m \in \mathscr{A} \setminus \{1\}$,

$\phi_{m,inc}(t)$ is dependent on the queue: if $Q_m(t) = 0$, inflow to supplier $m$ and outflow from supplier $m-1$ are equal; otherwise, we get the maximal inflow.

*Remark 1* Notice that $D_m(t,x) \geq 0$, $Q_m(t) \geq 0$ for every $m \in \mathscr{A}$, $t \geq 0$ and $x$, see [8] for details.

Now, consider suitable numerical schemes to approximate $D_m(t,x)$, $m \in \mathscr{A}$, and $Q_m(t)$, $m \in \mathscr{A} \setminus \{1\}$.

For an arc $m \in \mathscr{A}$, denote by $N_m$ and $\eta_m$, respectively, the number of grid points for a partition of $[0, L_m] \times [0, T]$. Consider a fixed time mesh $\Delta t$ and varying space meshes $\Delta x_m = V_m \Delta t$. Then, the grid points are $(x_i, t^n)_m = (i\Delta x_m,\ n\Delta t_m)$, $i = 0, \ldots, N_m$, $n = 0, \ldots, \eta_m$.

The upwind scheme, useful to define the parts density of arc $m$, reads as:

$$\frac{{}^mD_i^{n+1} - {}^mD_i^n}{{}^mD_{i-1}^n - {}^mD_i^n} \Delta x_m = \Delta t V_j, \tag{6}$$

where ${}^mD_i^n$ is the approximation of $D_m$ at $(x_i, t^n)_m$, see (1), $\forall\ m \in \mathscr{A}$, $i = 0, \ldots, N_m$, $n = 0, \ldots, \eta_m$, while the Courant-Friedrich-Levy (CFL) condition is satisfied since:

$$\Delta t = \min\left\{ \frac{\Delta x_m}{V_m} : m \in \mathscr{A} \right\}. \tag{7}$$

The proposed numerical approach allows advantageous computational times, as well as properties of convergence and stability, as described carefully in [8].

If $\alpha_j < -\infty$, the explicit Euler method, that allows to construct queues, reads as:

$$Q_m^{n+1} - Q_m^n + \Delta t\ \phi_{m,inc}^n = \Delta t\ \phi_{m-1}^n({}^mD_{N_m}^n), \quad n = 0, \ldots, \eta_m, \tag{8}$$

where $\phi_{m,inc}^n$ is defined by using (5) while details for numerical corrections are in [8]. Notice that, if $\alpha_j = -\infty$, boundary data are used by ghost cells.

## 3 Optimization

Now, we consider a possible optimal control problem for the model of Sect. 2. Fix a time horizon $[0, T]$ and define the cost functional:

$$G(V_1, V_2, \ldots, V_M) = \sum_{k=2}^{M} \int_0^T Q_k(t)\, dt + \int_0^T [V_M D_M(\beta_M, t) - \delta(t)]^2\, dt, \tag{9}$$

where $Q_k$ refers to (3), $V_M D_M(\beta_M, t)$ is the outflow of the supply chain with density level lower than $\mu_M$, while $\delta(t) \in L^{\infty}((0, T), [0, +\infty[)$ is a pre-assigned flow. The second integral of (9) represents a sort of measure between the effective outflow of the supply chain and a reference output $\delta(t)$. Notice that the solution of (1), $D_m$, is implicitly part of (9), hence the numerical solution of (1) and (3) represents a priority for the optimization issue.

We analyze the minimization problem:

$$\min_{(V_1,\ldots,V_M)} G(V_1, V_2, \ldots, V_M), \tag{10}$$

with $V_m^{\min} \leq V_m \leq V_m^{\max}$, $m = 1, \ldots, M$. Hence, the aim is the minimization of the queues and the distance between the effective outflow and $\delta(t)$ by referring to the velocities $V_m$, $m = 1, \ldots, M$.

A solution to problem (10) is sought via a Genetic Algorithm (GA). Such an approach is deeply considered [14] for numerical optimization, while convergence details are widely analyzed in [17, 18].

For a maximal number of iterations $\Lambda$, the algorithm works as follows: at the iteration 0, generate an initial population $V^0 = \left(V_1^0, V_2^0, \ldots, V_M^0\right)$ and compute the value $\Gamma_0 := G\left(V_1^0, V_2^0, \ldots, V_M^0\right)$ of the fitness function (9).

In general, indicating by $\Gamma_k := G\left(V_1^k, V_2^k, \ldots, V_M^k\right)$ the value of (9) at the iteration $k$, $k \geq 1$, the steps are:

Step 1 Via selection, crossover and mutation, get $\overline{V^k} = \left(\overline{V_1^k}, \overline{V_2^k}, \ldots, \overline{V_M^k}\right)$ from $V^{k-1} = \left(V_1^{k-1}, V_2^{k-1}, \ldots, V_M^{k-1}\right)$;

Step 2 compute $\overline{\Gamma_k} := G\left(\overline{V_1^k}, \overline{V_2^k}, \ldots, \overline{V_M^k}\right)$ of (9);

Step 3 if $\overline{\Gamma_k} < \Gamma_{k-1}$, set $V^k := \overline{V^k}$ and go to step 4; otherwise, come back to step 1;

Step 4 set $k := k + 1$ and come back to step 1 if $k \leq \Lambda$; otherwise, stop.

*Remark 2* The just described procedure uses a maximal number of iterations $\Lambda$ as stop criterion. Indeed, further optimization schemes could be addressed, also considering different ways to stop iterations, see for instance [14, 15].

## 4 Simulations

For simulations, we deal with a test supply chain of twenty arcs, see Fig. 1 for a possible structure. The number of arcs is purely indicative as the aim is to simulate a network of medium dimensions, also considering the possibilities due to the used numerical approaches. For the analysis of different supply networks, as well as for computational times, see [8]. The supply chain has the following characteristics: for arcs, $L_k = T_k = 1$, $m = 1, \ldots, 20$; $\mu_1 = 550$; $\mu_{20} = 10$; $\mu_m = 50 - 2m$,

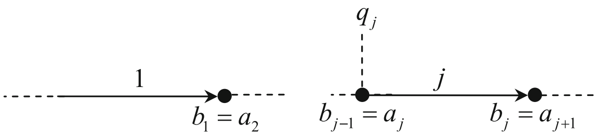


**Fig. 1** An example of supply chain, see [8]

$m = 2, \ldots, 19$; $D_m(0, x) = 0$, $m = 1, \ldots, 20$; $Q_m(0) = 0$, $m = 2, \ldots, 20$; total simulation time $\overline{T} = 1800$; input profile given by:

$$F(t) = \begin{cases} t, & 0 \le t \le 60, \\ 60, & 60 < t \le 130, \\ 216 - \frac{6}{5}t, & 130 < t \le 180, \\ 0, & t > 180. \end{cases} \tag{11}$$

As for the initial conditions, the supply chain is simulated in case of empty arcs and queues. The processes velocities are all equal to one in order to simulate a homogeneous starting situation when optimization criteria are used. The processing capacities of all arcs $m = 2, \ldots, 20$, are chosen in order to create queues among arcs. In fact, following the model described in Sect. 2, in case of equal processing velocities among consecutive arcs, queues occur if $\mu_{m+1} > \mu_m$, $m = 1, \ldots, 19$. The processing capacity of the first arc is chosen so that the inflow $F(t)$ is not cut and totally directed to the first arc, as foreseen from the model of Sect. 2. In order to simulate dynamics that are typical of industrial realities, function $F(t)$ is provided in order to simulate an inflow of this type: strong injection (increasing profile), constant injection, light injection (decreasing profile). Finally, the input of the system equals zero, and phenomena on the test supply chain are only due to possible dynamics on the last arcs.

According to the numerical schemes described in Sect. 2, we used $\Delta t = 0.025$. Figure 2 presents various queues.The behaviour of queues is a direct consequence of the choice of $F(t)$, considering that conservation laws have flux functions that could, in some cases, be constant. Richer phenomena, that deal with further profiles for queues, are widely described in [6, 8, 13]. In the case of the presented paper, the slopes of $Q_m(t)$, $m = 17, 18, 19$, are quite different due to the values of $\mu_m$, $m = 1, \ldots, 20$. Moreover, although (11) is zero $\forall\, t > 180$, queues dynamics is very slow. This is confirmed by $Q_{19}(t)$ that vanishes at $t \simeq 350 > 180$.

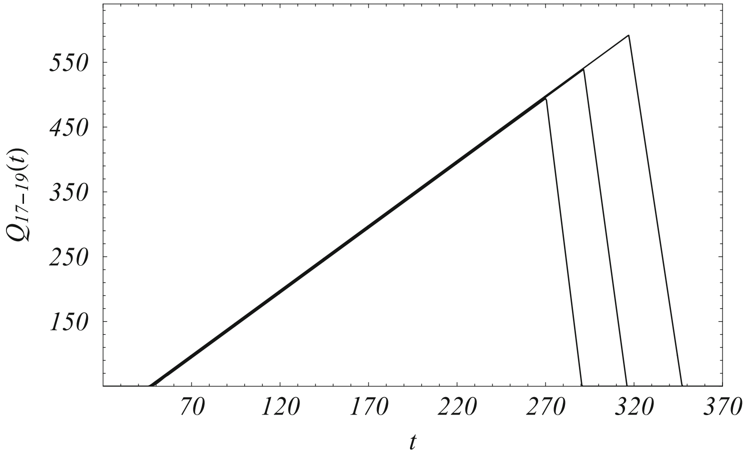


**Fig. 2** Queues $Q_m(t)$, $m = 17, 18, 19$; $Q_{17}(t)$ is the first on the left, $Q_{18}(t)$ the second on the left, and so on

**Table 1** Iterations, values of some velocities at various iterations and corresponding values of (9)

| Iteration $k$ | $(V_5^k, V_6^k, V_7^k)$ | $\Gamma_i$ | Iteration $k$ | $(V_5^k, V_6^k, V_7^k)$ | $\Gamma_i$ |
|---|---|---|---|---|---|
| 1 | (1.05, 1.08, 1.21) | 356603 | 10 | (1.55, 1.73, 1.54) | 315021 |
| 2 | (1.07, 3.11, 1.71) | 350021 | 11 | (1.61, 1.74, 1.22) | 312212 |
| 3 | (1.12, 2.45, 1.54) | 349121 | 12 | (1.71, 1.87, 1.17) | 309989 |
| 4 | (1.24, 2.31, 1.13) | 348112 | 13 | (1.61, 1.85, 1.89) | 308874 |
| 5 | (1.44, 2.18, 1.29) | 339212 | 14 | (1.58, 1.88, 2.17) | 306721 |
| 6 | (1.52, 2.14, 1.17) | 334121 | 15 | (1.59, 1.81, 2.19) | 305361 |
| 7 | (1.78, 1.79, 1.16) | 317719 | 16 | (1.59, 1.77, 2.21) | 303218 |

For the optimization, we fix $V_m^{\min} = 0.35$, $V_m^{\max} = 2.75$, $m = 1, \ldots, 20$, $\delta(t) = 155$ and $\Delta t = 0.05$. The initial population is defined by $V^0$ with entries $V_m^0 = 1$, $m = 1, \ldots, 20$. In this case, the fitness function (9) equals $\Gamma_0 = 356603$. Fixing $\Lambda = 16$ iterations, Table 1 reports the various values of (9) and some processing velocities.

All queues decrease at the various iterations. Figure 3 presents $Q_{19}(t)$ for the first and the last iteration. For iteration 0, $Q_{19}(t)$ has a maximum $M \simeq 580$, and vanishes at $t_v \simeq 350$; for the last iteration, $M \simeq 400$, and vanishes at $t_v \simeq 230$.

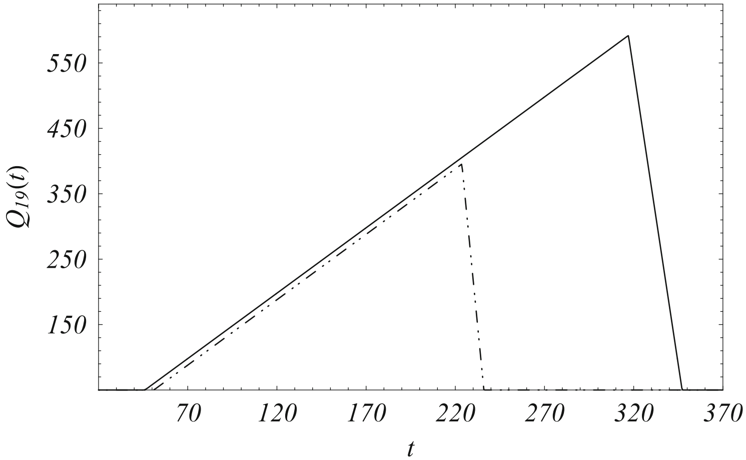


**Fig. 3** Evolution of $Q_{19}(t)$ for iteration 0 (continuous line) and iteration 16 (dot dashed line)

# 5 Conclusions

This paper has described possible dynamics of supply chains modeled by PDEs and ODEs, and solved numerically by an upwind scheme for densities and an explicit Euler method for queues. A genetic algorithm has been described and tested to optimize the performances of a supply system via minimization of a cost functional that considers either queues or a pre-defined outflow. The aim of future research issues is to study different evolutionary algorithms for the optimization.

# References


1. Daganzo, C.: A Theory of Supply Chains. Springer, New York (2003)
2. Armbruster, D., Degond, P., Ringhofer, C.: A model for the dynamics of large queueing networks and supply chains. SIAM J. Appl. Math. **66**, 896–920 (2006)
3. Armbruster, D., Degond, P., Ringhofer, C.: Kinetic and fluid models for supply chains supporting policy attributes. Bull. Inst. Math. Acad. Sin. **2**, 433–460 (2007)
4. Armbruster, D., Marthaler, D., Ringhofer, C., Kinetic and fluid model hierarchies for supply chains. Multiscale Model. Simul. **2**, 43–61 (2003)
5. Göttlich, S., Herty, M., Klar, A.: Network models for supply chains. Commun. Math. Sci. **3**, 545–559 (2005)
6. Göttlich, S., Herty, M., Klar, A.: Modelling and optimization of supply chains on complex networks. Commun. Math. Sci. **4**, 315–330 (2006)
7. Herty, M., Klar, A., Piccoli, B.: Existence of solutions for supply chain models based on partial differential equations. SIAM J. Math. An. **39**, 160–173 (2007)
8. Cutolo, A., Piccoli, B., Rarità, L.: An Upwind-Euler scheme for an ODE-PDE model of supply chains. SIAM J. Sci. Comput. **33**(4), 1669–1688 (2011)

9. Merchant, D.K., Nemhauser, G.L.: A model and an algorithm for the dynamic traffic assignment problem. Transp. Sci. **12**(3), 183–199 (1978)
10. Tomasiello, S.: Q based methods: theory and application to engineering and physical sciences. In: Leng, J., Sharrock, W. (eds.) Handbook of Research on Computational Science and Engineering: Theory and Practice, pp. 316–346. Hershey, IGI Global (2012). https://doi.org/10.4018/978-1-61350-116-0.ch014
11. Macías-Díaz, J.E., Tomasiello, S.: A differential quadrature-based approach àla Picard for systems of partial differential equations associated to fuzzy differential equations. J. Comput. Appl. Math. **299**, 15–23 (2016)
12. Tomasiello, S.: A functional network to predict fresh and hardened properties of self-compacting concretes. Int. J. Numer. Methods Biomed. Eng. **27**(6), 840–847 (2011)
13. D'Apice, C., Manzo, R., Piccoli, B.: Optimal input flows for a PDE-ODE model of supply chains. Commun. Math. Sci. **10**(4), 1225–1240 (2012)
14. Michalewicz, Z., Janikow, C.Z.: Genetic algorithms for numerical optimization. Stat. Comput. **1**(2), 75–91 (1991)
15. Berthiau, G., Siarry, P.: Etat de l'art des methodes d' "optimisation globale". RAIRO Oper. Res. **35**(3), 329–365 (2001)
16. Kar, A.K.: Bio inspired computing - a review of algorithms and scope of applications. Exp. Syst. Appl. **59**, 20–32 (2016)
17. Barrios, D., Malumbres, L., Rios, J.: Convergence conditions of genetic algorithms. Int. J. Comput. Math. **68**(3–4), 231–241 (1998)
18. Cerf, R.: Asymptotic convergence of genetic algorithms. Adv. Appl. Prob. **30**(2), 521–550 (1998)
19. Sana, S.S., Chedid, J.A., Navarro, K.S.: A three layer supply chain model with multiple supplier, manufacturers and retailers for multiple items. Appl. Math. Comput. **229**, 139–150 (2014)
20. Pourakbar, M., Farahani, R.Z., Asgari, N.: A joint economic lot-size model for an integrated supply network using genetic algorithm. Appl. Math. Comput. **189**, 583–596 (2007)