Construction of Digital Twins of Socio-Economic Systems Using Mathematical Models Based on Status Functions

Irina Veshneva and Alexander A. Bolshakov

Abstract The problem of the development of mathematical models for the construction of digital twins of socio-economic structures associated with certain specific features is shown. The use of complex mathematical models of digital twins of socio-economic systems based on the mathematical apparatus of status functions is proposed. In this regard, the formulation of the problem of developing new mathematical models for determining the state of socio-economic systems has been formulated. The classification of approaches to the digitalization of socio-economic systems is described. It is shown that the integration of digital twins into the classical control system is associated with the traditional implementation of the functions of observation, analysis, and control. Trends in the synthesis of digital twins for use in socio-economic structures are described, which are associated with an increase in the number of states, an increase in productivity, and more complex requirements for the structure and quality of the estimated parameter values. The concept of development and integration of new mathematical models using status functions is proposed. The architecture of an integrated platform of a cyber-physical system has been developed using mathematical modeling for the application of effective mathematical methods based on status functions. An example of the implementation of the stage of the proposed concept associated with the construction of a complex of problem-oriented programs is considered. Expressions for basic status functions are obtained on the basis of the linguistic description of the values of variables and data of the FIRA PRO system, and the corresponding graphs are also built.

Keywords Digital twins · Cyber-physical systems · Socio-economic structures · Mathematical models · Status functions

I. Veshneva (✉)
Saratov National Research University Named after N.G. Chernyshevsky, 83, Astrakhanskaya Street, Saratov 410012, Russia

A. A. Bolshakov
Peter the Great St. Petersburg Polytechnic University, 29, Polytechnicheskaya, St. Petersburg 195251, Russia

A. G. Kravets et al. (eds.), *Society 5.0: Human-Centered Society Challenges and Solutions*, Studies in Systems, Decision and Control 416,
https://doi.org/10.1007/978-3-030-95112-2_11

1 Introduction

The development of Industry 4.0 [1] is associated with the industrial revolution characterized by the following features: new energy technologies are not used, new raw materials are massively used, a significant increase in the energy efficiency of production, the ability to predict the future, a significant acceleration of modernization and the emergence of an exponential increase in labor productivity. In the industrialized countries of Europe and North America, states are the sources of the actors of the ongoing changes. In Russia, a national project for increasing labor productivity has been proposed, which presents the goal of ensuring by 2024 the rate of increase in labor productivity in organizations belonging to the basic, non-resource sectors of at least 5% per year. By 2024, it is planned to increase labor productivity by more than 20%. This will allow the Russian Federation to move to the level of the world's leading economies. The modern fourth industrial revolution has features similar to the previous ones: the modernization of production technology depends on the improvement of the management system. At the same time, for more complex technologies it is required to increase the relative volume of automated control systems. Experts say that more than 40% of enterprises use digital technologies at a critical level, then an exponential growth of industry digitalization is expected [2, 3]. Such digitalization requires the massive implementation of cyber-physical systems capable of self-tuning and adapting to the new needs of manufacturers and consumers.

The structure of these changes includes the following distinctive features: Internet of Things (IoT for short) [4, 5]; digital twins (Digital Twin) [6]; 3-D printing [7–9]; cloud computing and big data [10–12]; cybersecurity [13, 14]; digital platforms [15].

The fourth industrial revolution has begun, its distinctive features are manifested quite clearly and extensively. At the same time, it can be argued with confidence that, as in previous revolutions, the one who is able to create, develop and use technologies of distinctive features earlier than others will gain an advantage over other enterprises and countries, and the gap will grow exponentially.

According to expert estimates, the benefits will be given to organizations that are actively working with innovations and have their own competencies in the aspect of digitalization.

One of the possible obstacles is the lack of innovative technologies and models. Fundamentally new technology for constructing and using mathematical models and automated control systems can be based on mathematical models using the mathematical apparatus of status functions [16].

Moreover, in this regard, the problem is the need to develop possible mathematical models of digital twins of socio-economic structures that differ from the known models in specific features.

Let us present a model for the formation of digital twins of socio-economic structures based on status functions developed and proposed in [17].

2 Statement of the Problem of Creating New Mathematical Models for Assessing the State of Socio-Economic Systems

The process of integrating digital twins (CDs) into the classical control system is actively developing in each of the following functions.

1. Observation function. There is a change in the technological order. Industry 4.0 deals with completely new phenomena that are digital in nature. Digital Twin in our case is a software analogue of the socio-economic system (SES).
2. Control function. synthesis and improvement of CD.

A brief description of the subject area of SES digitalization is given in Table 1.

The analysis performed in Table 1 shows that the development of models of the CSD of socio-economic structures finds application in the analysis of big data and predictive analytics for the formation of forecasts based on new mathematical models. At the same time, communication between socio-economic structures can be carried out using the CD. Then the communication in the system governing body—controlled socio-economic object can be reduced to the model of meta-communications: governing body—CD—managed socio-economic object, managed socio-economic object—CD—the governing body. The new data analysis model will create a virtual integrated environment for big data analysis and predictive analytics. In the new integrated complex, in the implementation of management actions using big data based on the central data center using problem-oriented software tools, the corresponding technological and technical platform is created by cyber-physical systems. For the synthesis of this organizational and technical platform, the technology of building a CD is of decisive importance.

3. Functions of analysis, management. These functions are required to be provided in the architecture of a functioning integrated platform of a cyber-physical system. When implementing these functions, significant difficulties arise: limited resources, possible lack of feedback in the subsystem: the governing body is a controlled socio-economic system.

A cyber-physical system includes a set of interacting components to achieve a single goal in the control process [17]. Mathematical models of such complex complexes generally correspond to the main provisions of synergetic [18] and are created on the basis of methods developed in physics, chemistry, and biology [19].

The difficulty of using mathematical models and methods is associated with the lack of measurement scales that are used in natural sciences (measurement of speed in m / s, mass in kg, etc.). To overcome it, it is advisable to use linguistic variables to describe, for example, psychophysiological indicators in ergatic systems based on the theory of fuzzy sets [20], as well as to interpret data obtained in the course of field experiments using the so-called status functions (SF).

The SF-based algorithm allows you to enter linguistic assessments and represent them with complex-valued functions, called status functions. Mathematical models

Table 1 Classification of approaches to the digitalization of SES

Method of digitalization	Characteristics of the digitalization method
Process Automation (RBA)	Implementation of robotization of operational activities, which allows to reduce the duration of these operations and increase efficiency and / or efficiency by up to 80% based on a decrease in the payroll, as well as reducing the risks of operational activities
Big Data and related methods of analysis and interpretation	Improving the performance of the company based on increasing the speed and volume of processed information. Using predictive analytics to build forecasting procedures when processing large amounts of data. At the same time, statistical methods for processing multidimensional data, analysis of historical data, and procedures for planning expected results are used
Reference and information retrieval and reference complexes (systems)	Implementation of the functions of forming the required information, obtaining the required competencies for the systematization and analysis of data
Chat robots	Software tools that run in applications. At the same time, they can imitate speech and text for organizing interaction to find the required data
Artificial Intelligence (AI)	Used to perform complex tasks by programs and optimize the use of human resources
Virtual and Augmented Reality	Technologies of projection or augmentation of reality by technical means to reduce the cost of the implemented processes
Internet of things (IoT)	A network of components that interact, for example, a set of sensors for collecting data, remote information exchange to improve the efficiency of solutions
Pattern recognition OCR/ICR	The technology is capable of replacing a person in the process of receiving, analyzing, and verifying documents; it is designed to digitize document flow, security systems, and customer service
Blockchain	Technologies make it possible to form various training situations in which the learning process is carried out in the form of a game

based on SF are devoid of many of the shortcomings of classical models of socio-economic processes, such as static nature and the impossibility of taking into account the intersection and mutual influence of the studied components of the processes.

The main trends in the synthesis of CSD of socio-economic systems (SES) are as follows:

1. an increase in the number of states that are evaluated, as well as the components of the data structure that are subject to analysis;

2. increasing the productivity and complexity of information platforms for the implementation of problem-oriented software tools;
3. the complication of requirements for the structure and quality of the estimated values of the SES parameters, the multicomponent nature of the nested structures of the estimates, the overlap and complexity of the processes that are implemented in socio-economic systems;
4. activation of subjects and processes of interaction in organizing management to increase the degree of its automation.

This raises the problem of creating mathematical models that allow increasing the number of controlled variables and parameters to automate the used decision support process (DSS). This initiates the use of modern technologies, techniques and corresponding mathematical methods [19, 20]. So, when developing new methods of managing the educational process, various models are actively used, fuzzy, boolean, etc.

Thus, the development of new methods for assessing the state of socio-economic systems, taking into account the complexity, subjectivity and multidimensionality is a very urgent problem. It is advisable to apply the proposed mathematical method in the construction of an intelligent monitoring system, as well as PM in the analysis of indicators of the functioning of the SES, such as, for example, individual enterprises or the structure of indicators of the activities of the regions of the Russian Federation to increase the effectiveness and efficiency of management according to the characteristics of the SES and the successful solution of problems caused by the goals of functioning. management systems for individual enterprises or regions.

3 The Concept of Development and Integration of New Mathematical Models

The concept of mathematical modelling based on effective mathematical methods and building a model that describes ergatic interactions in the form of corresponding communications of digital twins based on an integrated platform for various cyber-physical systems, it is advisable to display as a sequence of five stages (see Fig. 1).

Stage 1. Interpretation of data obtained as a result of field experiments. At this stage, an analysis of the data of the controlled variables of the SES is required. So, for example, the analysis of indicators of socio-economic activity of the regions of the Russian Federation and models was carried out in works [22, 23].

Stage 2. Mathematical modelling of socio-economic processes and objects. The best basis that is advisable to use for the synthesis of mathematical models of digital twins is SF. From their combination, it is possible to form the final mathematical models, which are proposed and presented in the article [14]. The new mathematical model allows building an integrated platform for cyber-physical systems. On this platform, it is advisable to implement training for the synthesis of hybrid ergatic systems.

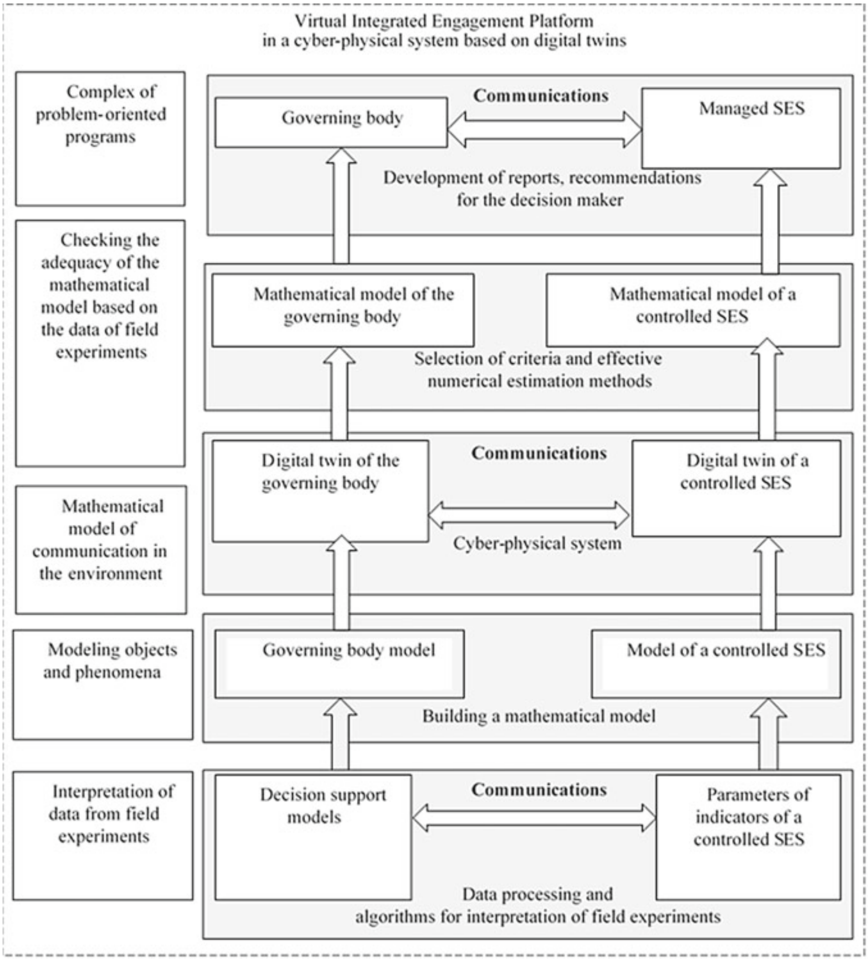


Fig. 1 Architecture of an integrated cyber-physical system platform using mathematical modelling to apply effective mathematical methods based on status functions

Stage 3. Mathematical model of communications in the environment. We will focus on meta-communication in a cyber-physical system. Moreover, the direct communication and interaction of the governing body—the managed socio-economic object are replaced by meta-communication of digital twins: the governing body—the central office—the managed socio-economic object, the managed socio-economic object—the central office—the governing body.

Stage 4. Checking the adequacy of the proposed mathematical models using data from field experiments. It is advisable to interpret the information obtained as a result of experiments by the SF method, as well as on the basis of the Kolmogorov-Chapman equations [25, 26] based on the data obtained in the FIRA PRO system.

Stage 5. The complex of problem-oriented software. Below is a description of the architecture for assessing the parameters for assessing the activities of SES on the example of assessing the competitiveness of the regions of the Russian Federation.

4 Example of Implementation of the Concept

Let us describe the 5th stage of building a complex of problem-oriented programs. First, based on the collection of data, a linguistic description of the assessment results is formed. To assess the state of the variables "Degree of depreciation of fixed assets", "The number of populations with incomes below the subsistence level", "The number of official unemployed", "Dilapidated and dilapidated housing", "Debt on taxes and fees", Table 2 with the corresponding linguistic description is used. For the generated assessment tables, SFs are put into correspondence, which are presented in Table 3.

Basic SF values consist of two parts: amplitude and phase. The amplitude parts are real and are shown in Table 4. They are the orthonormal basis of the system and represent a set of basis functions that can describe all possible states of the system. The phase part of the SF is complex and indicates the direction of the trend—or in which direction the shift of the mathematical value of the SF is expected.

As a result, the corresponding complex-valued status functions (SF) are formed.

Secondly, the formation of hierarchical interconnected structures is carried out. The result is a causal graph (see Fig. 2).

In the diagram shown in Fig. 2 structure of the following indicators: the cost of fixed assets (E29); depreciation rate of fixed assets (E30); density of railway tracks

Table 2 Linguistic description of variables for data from the FIRA PRO system

	Interpretation of parameter values based on FIRA PRO data		
Trend assessment	High values of the indicator, it has been growing for the last 3 years, more than 10% higher than the same indicator in neighboring regions	The maximum permissible threshold value, reached as a result of a decrease in the indicator	Low values of the indicator, it has been growing for the last 3 years, more than 10% lower than the same indicator in neighboring regions
	High values, changes over 3 years within the margin of error	Maximum permissible threshold value, changes within 3 years within the margin of error	Low values, changes over 3 years within the margin of error
	High values of the indicator, it has been growing for the last 3 years	The maximum permissible threshold value, reached as a result of an increase in the indicator	High values of the indicator, decreasing for the last 3 years

Table 3 Expressions of basic SFs for substitution in accordance with the table of linguistic description of variable values for data from the FIRA PRO system

	Mathematical expressions of estimates in accordance with the parameter interpretation table		
Evaluation of the trend yes	Low = 3.37619 (e-50. r+0.142-0.580252e-22.2222 r2 e-i2πr	Middle = 1.9394e-22.2222r2 e-i2πr	High = 1.37668 e-(14. -50.r)r +1.87105e(14. -50.r)r --5.02117e-22.2222r2 e-i2πr
	Low = 3.37619 (e-50. r+0.142-0.580252e-22.2222 r2	Middle = 1.9394 e-22.2222r2	High = 1.37668 e-(14. -50.r)r +1.87105e(14. -50.r)r --5.02117e-22.2222r2
	Low = 3.37619 (e-50. r+0.142-0.580252e-22.2222 r2 ei2πr	Middle = 1.9394 e-22.2222r2 ei2πr	High = 1.37668 e-(14. -50.r)r +1.87105e(14. -50.r)r --5.02117e-22.2222r2 ei2πr

(E31); road density (E32); per capita money income (E33); population with incomes below the subsistence level (E34); the number of officially unemployed people (E35); life expectancy (E36); population growth rate (E37); migration growth rate (E38); dilapidated and dilapidated housing (E39); the number of doctors per 10,000 population (E40); the number of reported crimes (E41); emissions of polluting products (E42); discharge into water bodies of polluted wastewater (E43) was obtained as a result of data collection and analysis [28, 29]. The rest of the indicators uniting leaf vertices into a causal graph were introduced as a result of logical operations presented in [16].

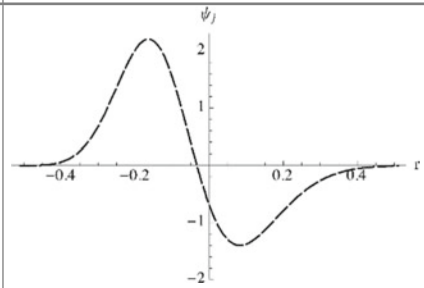
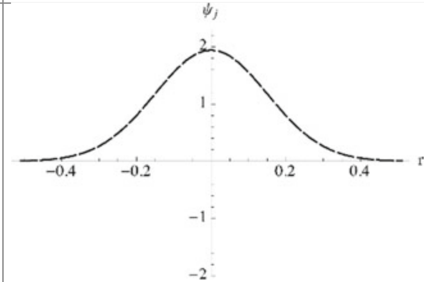
Third, a system of differential operators is formed for the structure of groups of higher indicators. As a result, the system of differential equations based on SF becomes an analogue of the described structure. The system is obtained similarly to Forrester's equations of world dynamics.

Note that recently, in general, interest in the use of modern mathematical models, digital technologies for mathematical modelling of socio-economic systems using the ideas of digital twins have increased significantly [29, 30].

5 Conclusion

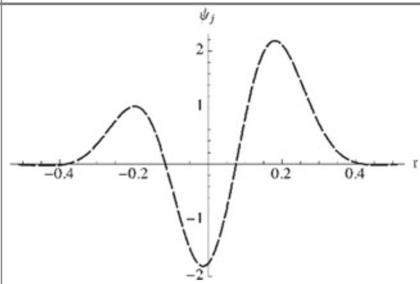
Thus, based on the estimates presented by the SF method, it is possible to construct dynamic models of digital twins for use in cyber-physical systems. Structures that can be built on the basis of status functions can be simply formed from models for evaluating indicators based on linguistic terms. Status functions are a "bridge" between a person's simple ideas about the world around him and phenomena to the

Table 4 Examples of linguistic estimates, mathematical expressions and the form of the amplitude part of the basic SF

Example of linguistic assessment	Status function	Graphics
High values of the indicator, it has been growing for the last 3 years, more than 10% higher than the same indicator in neighboring regions	Low = 3.37619 (e-50. r + 0.142–0.580252e-22.2222 r2	
High values, changes over 3 years within the margin of error	middle = 1.9394 e-22.2222r2	

(continued)

Table 4 (continued)

Example of linguistic assessment	Status function	Graphics
High values of the indicator, it has been growing for the last 3 years	high = 1.37668 e-(14. -50.r)r + 1.87105e(14. -50.r)r –5.02117e-22.2222r2	

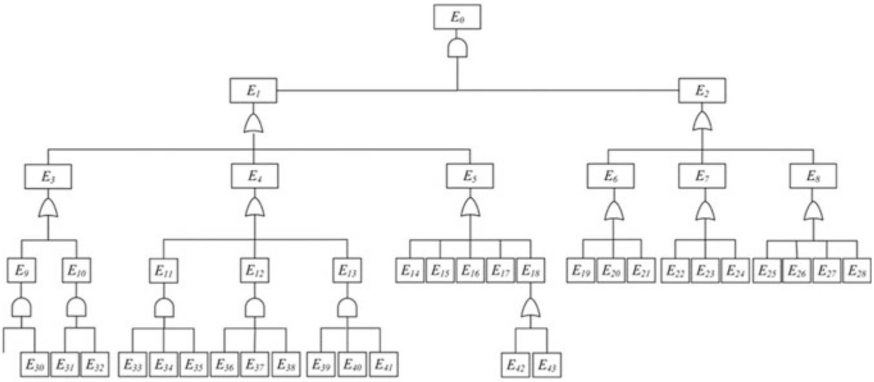


Fig. 2 A causal graph for assessing the risks of regional competitiveness

strict uncertainty of quantum mechanics. The advantage of the model is that it takes into account the complexity, high degree of uncertainty, and the intersection of socio-economic processes. Models allow building-level structures of possible states. The main advantage is the predictivity of the model and its suitability both for predictions and for describing meta-interactions in an integrated environment of cyber-physical systems.

The proposed generalized scheme of mathematical modeling based on the use of effective numerical methods and a complex of problem-oriented programs is advisable to use to assess communications in an integrated environment of cyber-physical systems based on status functions.

Acknowledgements This research was partially supported by the Russian Fund of Basic Research (grant No. 20-010-00465).

References

1. Porter, M.: The economic performance of regions. Reg. Stud. **37**(6–7), 549–578 (2008). https://doi.org/10.1080/0034340032000108688
2. Kadir, B.A., Broberg, O.: Human-centered design of work systems in the transition to industry 4.0. Appl. Ergon., 92. https://doi.org/10.1016/j.apergo.2020.103334 (2021)
3. .Beliatis, M.J., Jensen, K., Ellegaard, L., Aagaard, A., Presser, M.: Next generation industrial IoT digitalization for traceability in metal manufacturing industry: a case study of industry 4.0 Electronics (Switzerland), **10** (5), 1–14. https://doi.org/10.3390/electronics10050628 (2021)
4. Hoque, M.A., Azad, M., Ashik-Uz-Zaman, M.: IoT and machine learning based smart garbage management and segregation approach for Bangladesh. ICIET 2019—2nd International Conference on Innovation in Engineering and Technology. https://doi.org/10.1109/ICIET48527.2019.9290551 (2019)

5. Dinesh, K., Gobinath, M., Subathra, M.: A survey on intelligent internet of things-technology and its application. Proceedings of the International Conference on Inventive Research in Computing Applications, ICIRCA 2018, 81–84. https://doi.org/10.1109/ICIRCA.2018.8597205 (2018)
6. Buldakova, T.I., Suyatinov, S.I.: Assessment of the state of production system components for digital twins technology. Springer Nature Switzerland AG 2020. In: Kravets, A.G. et al. (eds.) Cyber-Physical Systems: Advances, in Design & Modelling, Studies in Systems, Decision and Control, vol. 259, pp. 253–262. https://doi.org/10.1007/978-3-030-32579-4_20 (2020)
7. Gorsse, S., Hutchinson, C., Gouné, M., Banerjee, R.: Additive manufacturing of metals: a brief review of the characteristic microstructures and properties of steels, Ti-6Al-4V and high-entropy alloys. Sci. Technol. Adv. Mater. **18**(1), 584–610 (2017). https://doi.org/10.1080/14686996.2017.1361305
8. Maroni, A., Melocchi, A., Parietti, F., Foppoli, A., Zema, L., Gazzaniga, A.: 3D printed multi-compartment capsular devices for two-pulse oral drug delivery. J. Controlled Release **268**, 10–18. https://doi.org/10.1016/j.jconrel.2017.10.008 (2017)
9. Golafshan, N., Vorndran, E., Zaharievski, S., Brommer, H., Kadumudi, F.B., Dolatshahi-Pirouz, A., Gbureck, U., van Weeren, R., Castilho, M., Malda, J.: Tough magnesium phosphate-based 3D-printed implants induce bone regeneration in an equine defect model. Biomaterials **261**, 120302. https://doi.org/10.1016/j.biomaterials.2020.120302 (2020)
10. Stergiou, C.L., Psannis, K.E., Gupta, B.B.: Iot-based big data secure management in the fog over a 6G wireless network. IEEE Internet Things J. **8**(7), 5164–5171 (2021). https://doi.org/10.1109/JIOT.2020.3033131
11. Dahiya, V., Dalal, S.: Fog computing: a review on integration of cloud computing and internet of things. 2018 IEEE International Students' Conference on Electrical, Electronics and Computer Science, SCEECS 2018. https://doi.org/10.1109/SCEECS.2018.8546860 (2018)
12. Kuhn, M.: Big Data, AI, and the pleasures of engineering. article and big data, AI und die Freude am Ingenieurwesen]. Chemie-Ingenieur-Technik **93**(3), 364–372. https://doi.org/10.1002/cite.202000221 (2021)
13. Asghar, M.R., Hu, Q., Zeadally, S.: Cybersecurity in industrial control systems: Issues, technologies, and challenges. Comput. Netw. **165**. https://doi.org/10.1016/j.comnet.2019.106946 (2019)
14. Chandan, A.R., Khairnar, V.D.: Security testing methodology of IoT. Proceedings of the International Conference on Inventive Research in Computing Applications, ICIRCA 2018, 1431–1435. https://doi.org/10.1109/ICIRCA.2018.8597192 (2018)
15. Barykin, S.Y., Kapustina, I.V., Kirillova, T.V., Yadykin, V.K., Konnikov, Y.A.: Economics of digital ecosystems. J. Open Innov.: Technol. Market Complexity **6**(4), 1–16. https://doi.org/10.3390/joitmc6040124 (2020)
16. Veshneva, I.V., Chistyakova, T.B., Bolshakov, A.A.: The status functions method for processing and interpretation of the measurement data of interactions in the educational environment. SPIIRAS Proc **6**(49), 144–166. https://doi.org/10.15622/sp.49.8.ISSN:20789181 (2016)
17. Bolshakov, A., Veshneva, I., Lushin, D.: Mathematical model of integration of cyber-physical systems for solving problems of increasing the competitiveness of the regions of the Russian federation studies in systems. Society 5.0: Cyberspace for Advanced Human-Centered Society. Studies in Systems, Decision and Control, vol. 333, pp. 129–139. https://doi.org/10.1007/978-3-030-63563-3_11 (2021)
18. Yakovenko, V., Rosser, J.B., Colloquium, Jr.: Statistical mechanics of money, wealth, and income. Rev. Mod. Phys. **81**, 1703 (2009)
19. Shvartsev, S.L.: Evolution in nonliving matter: nature, mechanisms, complication, and self-organization. Her. Russ. Acad. Sci. **87**(6), 518–526 (2017). https://doi.org/10.1134/S1019331617050069
20. Xie, J.S., Korniss, S.G., Zhang, W., Lim, C., Szymanski1, B.K.: Social consensus through the influence of committed minorities. Phys. Rev. E. http://arxiv.org/PS_cache/arxiv/pdf/1102/1102.3931v2.pdf Last accessed (2021)

21. Bolshakov, A.A., Veshneva, I.V.: Assessment of the effectiveness of decision support in the application of the information system for monitoring the process of forming competences based on status functions. International Conference on Actual Problems of Electron Devices Engineering, APEDE, Art. No. 8542462, 75–82 (2018).https://doi.org/10.1109/APEDE.2018.8542462
22. Bender, C.M., Brody, H.F.: Extension of PT—symmetric quantum mechanics to quantum field theory with cubic interaction. Phys. Rev. D. **70**, 025001 (2004) Erratum. Phys. Rev. D., **71**, 049901 (2005)
23. Castellano, C., Fortunato, S., Loreto, V.: Statistical physics of social dynamics. Rev. Modern Phys **81**(2), 591–659 (2009). https://arxiv.org/pdf/0710.3256.pdf. Accessed 14 March 2021
24. Sokolovski, D.: Are the 'weak measurements' really measurements? Quanta **2**, 50–57. https://arxiv.org/pdf/1305.4809.pdf. Accessed 2 March 2021
25. Veshneva, I., Chernyshova, G., Bolshakov, A.: Regional competitiveness research based on digital models using Kolmogorov-chapman equations studies in systems. Society 5.0: Cyberspace for Advanced Human-Centered Society. Editors Alla G. Kravets. Alexander A. Bolshakov. Maxim Shcherbakov. Series Title Studies in Systems, Decision and Control. Series Volume 333. Publisher Springer International Publishing. Vol. 333, pp. 141–154 (2021). https://doi.org/10.1007/978-3-030-63563-3_12 (2021)
26. Matsena Zingoni, Z., Chirwa, T.F., Todd, J., Musenge, E.: A review of multistate modelling approaches in monitoring disease progression: Bayesian estimation using the Kolmogorov-Chapman forward equations. Stat. Methods Med. Res. https://doi.org/10.1177/0962280221997507 (2021)
27. Rahman, P.A.: A calculation method for estimation of the mean time to first failure of the technical systems on basis of the topological conversion of the markov reliability model
28. ARPN Journal of Engineering and Applied Sciences, **13** (5), 1819–1826 (2018)
29. EU Regional Competitiveness Index, RCI. https://ec.europa.eu/regional_policy/sources/docgener/work/201701_regional_competitiveness2016.pdf. Accessed 12 March 2021
30. Regional Innovation Scoreboard, RIS. https://ec.europa.eu/growth/industry/innovation/facts-figures/regional_en. Accessed 4 March 2021
31. Davtian, A., Shabalina, O., Sadovnikova, N, Parygin D.: Cyber-social system as a model of narrative management. Springer Nature Switzerland AG 2020. In: Kravets et al. A. G. (eds.) Society 5.0: Cyberspace for Advanced Human-Centered Society. Decision and Control, vol. 333, pp. 4–14 (2021). https://doi.org/10.1007/978-3-030-63563-3_1
32. Dorrer M., Dorrer, A , Zyryanov, A.: Numerical modeling of business processes using the apparatus of GERT networks. Springer Nature Switzerland AG 2020. In: Kravets et al. A. G. (eds.) Society 5.0: Cyberspace for Advanced Human-Centered Society. Decision and Control, vol. 333, pp. 47–55. https://doi.org/10.1007/978-3-030-63563-3_5 (2021)