Chapter 6
Loan Pricing

Look beneath the surface; let not the several quality of a thing nor its worth escape thee.

(Marcus Aurelius)

Loan origination is the lifeblood of any lending institution. This activity involves an enormous range of complicated and difficult qualitative elements that go far beyond any financial modelling discussion.[1] It should be evident, by this point in the proceedings, that the ideas discussed in this book are ill-equipped to tackle the non-technical aspects of loan origination. Skipping over these important—and not-to-be-ignored details—we will focus on the aspect where quantitative tools can add value: the lending decision point. When faced with a potential loan proposal, a lending institution ultimately has to determine to extend credit or not. From a quantitative perspective, it is immensely useful to conceptualize the possible addition of each individual loan to one's portfolio as a distinct investment decision or project. This allows us to frame this choice within a wider framework of very useful ideas in corporate finance and risk management.

Informing investment decisions is a fundamental branch of the corporate-finance literature.[2] Sometimes more broadly referred to as capital budgeting, it addresses the central question of how a firm should prioritize the allocation of its assets to risky, but potentially profit-generating activities. Brealey et al. [11] describe it as the answer to the following question:

> [H]ow much should the firm invest, and what specific assets should the firm invest in?

This is necessarily very broad. Taking a medium to long-term perspective, typical examples include whether or not to build a new manufacturing facility, trial a new drug, or develop a new product brand. This idea applies equally well to potential

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[1] The reader is directed to EBA [16] for an interesting (and timely) dip into these conceptual waters.

[2] There are literally too many references to effectively cite. There is a deep and extensive discussion ranging from a 50-year chestnut in Mao [26] to the standard textbook treatment found in Peterson and Fabozzi [31].

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mergers and acquisitions. While the possible variations are endless, it is not terribly difficult to see how the loan decision fits into this broad mould. Each loan spans a reasonable amount of time, consumes a firm's scarce resources and any decision can, quite possibly, crowd out other lending prospects. A clear and consistent approach is thus required to rank various lending options and ultimately ensure that good lending decisions are taken.

The lending institution, as is often the case in capital budgeting, has limited power over the circumstances surrounding lending decisions.[3] When granting a loan, the Bank has no appreciable control over the firm's current creditworthiness, its competitors, its operating environment nor over the future development of economic conditions. It does, however, control some aspects. A lending institution can, within reason, exert some influence on the term, the pricing, and the convenants of the loan. These are the basic parameters of a lending institution's capital-budgeting activity.

We will, in this chapter, operationalize the lender's control, or choice, variable as the magnitude of its lending margins.[4] In short, every financial institution requires a holistic approach towards setting sensible lending margins (i.e., prices)—for all of its stakeholders—for each loan that it extends to a credit obligor. This is a particularly challenging task for, at least, *three* reasons. A loan price:

1. has to cover the institution's costs including a required return on capital;
2. must be broadly consistent with market prices; and
3. has to be consistent with the institution's mandate or strategy.

Every financial institution thus needs to walk a fine line between cost recovery, market consistency, and strategic direction. Figure 6.1 provides a schematic description of this challenge. Costs, to be clear, need to be defined in a larger sense. They naturally need to include funding costs, administration expenses, and expected credit losses. Loans also need to be priced to recover the cost of capital allocated to them. If this is not the case, then eventually loan activity will need to be curtailed. This links the pricing of a given loan closely to its consumption of capital as well as the balance-sheet composition of the institution. Indeed, as will soon become evident, the simple question of accepting or rejecting a loan proposal encompasses almost every aspect of the lending institution's structure.

The principal objective of this chapter is to describe the conceptual framework underlying NIB's loan-pricing methodology in significant detail; this development will, nonetheless, apply more generally to a broad range of financial institutions. We will begin with some fundamental notions associated with asset pricing; this will provide us with the necessary machinery for the construction of loan-pricing

[3] This may be hard to believe for a reader having struggled to get a mortgage loan, but this point is quite true in commercial lending.

[4] It is, as we'll see, broader than just the lending margin. Tenor, for example, is also quite important. Lending margin, however, is a useful one-dimensional description of the principal lever in the lending decision.

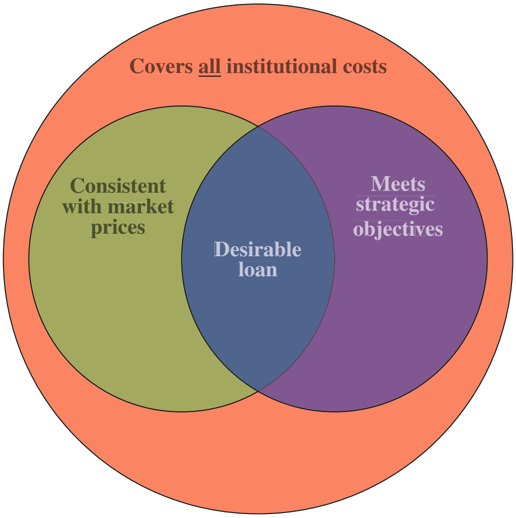

Fig. 6.1 *Key lending criteria*: This picture displays, in a simplified and schematic manner, the three key criteria involved in identifying a desirable loan: cost recovery, market consistency, and strategic relevance. Finding the intersection of all three elements is often challenging.

expressions. The next step involves a close look at the lender's balance-sheet. This provides insight into the interlinkages between assets, liabilities, and equity. Ultimately, the lessons learned in this section will permit us to confidently estimate the financing costs associated with an arbitrary loan transaction. The final sections handle the fine print of marrying notions of risk and return within the context of the firm's capital structure.

The end product is a risk-adjusted return measure that seeks to (fairly) inform the appropriate choice of lending margin (i.e., price). One can think of this as a common yardstick, or benchmark, permitting impartial comparison between disparate loan proposals, thereby supporting the capital-budgeting process. To do this sensibly requires both the incorporation of various specialized pricing features and explicit links back to the institution's strategic objectives and economic-capital framework. While helpful and centrally important, it bears repeating that this analysis captures only the quantitative dimension of the lending decision. Numerous other qualitative factors enter into these decisions that, although not addressed in this chapter, play a central role in this process. Returning to Fig. 6.1, this chapter will have much to say about the outer circle, but rather little to add regarding the important inner circles touching on market conditions and strategic objectives.

6.1 Some Fundamentals

People have been valuing assets as long as assets have existed. Barter and trade would hardly be possible absent such practice; our focus, however, is on financial assets. Consider some arbitrary contingent claim; an asset whose value depends (i.e., is contingent) upon the realization of some future (unknown) financial-market event (or events). This could be a bond, an equity, a deposit or some form of derivative product. Values need to be assigned to these assets. Said valuation might be required for financial reporting or, more fundamentally, because we wish to sell or purchase it. In some cases, there are active markets that can be used to determine this value. Other times, we must take one or more logical steps to infer a valuation. This is referred to as the asset-pricing problem; it is an important sub-field of mathematical finance.

Our focus is financial assets in general and loan contracts in particular. The loan-pricing problem falls into this long tradition and the finance literature is literally teeming with useful tools to address this question.[5] Treynor [39], Sharpe [33, 34] and Lintner [25] introduced the capital-asset pricing model (CAPM) in the early 1960's. Designed mainly for equity pricing, it clearly illustrates the fundamental importance of systemic risk. Although, it has been twisted and revised over the years, the central notions of CAPM still lie at the heart of many modern financial tools. Black and Scholes [7], Merton [28], and Vasicek [40] ushered in the era of modern derivative-contract asset-pricing in the early to mid 1970's. Exploiting the notions of arbitrage and market completeness, and using continuous time mathematics, this work set the stage for most current valuation models. In the economics literature, conversely, general equilibrium approaches are used (mostly in theoretical settings) for price determination.[6]

Although our application is less complex than many pricing problems, it still fits into the general asset-valuation framework. As a consequence, this framework sets the stage for the subsequent discussion. Let us introduce these ideas in the context of pricing a risk-free bond. Ultimately, once you peel away all the details, pricing involves the discounting of cash-flows. This case is no exception. Managing the book-keeping of these cash-flows, however, requires organization and clarity. Pinching some common, but useful, notation from Brigo and Mercurio [12], we can establish some important basic quantities.

Many commercial loans have amortized notional repayment schedules, which implies a need to be quite cautious about the outstanding amounts on payment dates. To sort this out, let β denote the total number of payment dates and define the notional payment schedule as

$$\left\{-N_0, N_1, N_2, \cdots, N_\beta\right\}, \tag{6.1}$$

for payment dates $\left\{t \equiv T_0, T_1, T_2, \cdots, T_\beta \equiv T\right\}$ where $-N_0$ is the initial disbursement. We define

$$X_i = \sum_{j=i}^{\beta} N_j, \tag{6.2}$$

[5] It's hard to be entirely definitive, but modern asset pricing probably started with Williams [41].

[6] See Mas-Colell et al. [27, Part IV] for more on this important, but complex question.

as the notional between the $(i-1)$th and ith payment dates.[7] X_i thus denotes the outstanding loan amount associated with the ith payment.

Let us further write c_i as the coupon rate for the ith payment date.[8] The (ex-disbursement) loan cash-flow stream would look something like:

$$\left\{\underbrace{c_1X_1+N_1}_{\substack{\text{Coupon and}\\ \text{Principal}}}, c_2X_2+N_2, \cdots, c_\beta X_\beta + N_\beta\right\}, \tag{6.3}$$

where

$$c_i = \underbrace{\Delta(T_{i-1}, T_i)}_{\Delta_i} K_i. \tag{6.4}$$

K_i is the ith coupon rate and $\Delta_i \equiv \Delta(T_{i-1}, T_i)$ is the day-count convention.[9] Understanding the cash-flow stream, three challenges immediately arise. First, the coupon element must be represented in a less vague manner to provide insight into the lending margins. Second, we need to determine how to actually discount these future values back to the current time. Finally, it will be necessary to incorporate the potential for credit default. Let's examine each in turn.

The first challenge is easy to handle. Our loans—as is the case with many international financial institutions—typically pay a floating-rate coupon based on a reference index.[10] When $K_i \neq K$—that is, the coupon is not a fixed rate—then we have that

$$K_i = \underbrace{L(T_{i-1}, T_i)}_{L_i} + m, \tag{6.5}$$

where L_i is a—typically six-month—LIBOR tenor and m is a lending spread or margin. With the advent of reference-rate reform—see, for example, Hou and Skeie [20], Duffie and Stein [15], or Bailey [5]—the structure of the LIBOR is changing.[11] Conceptually, however, the notion of some common underlying reference rate will not. We can, therefore, think of L_i as a generic floating reference rate; currently, it is

[7] During the grace period, where the loan outstanding does not change, the associated N values can naturally be zero.

[8] Strictly speaking, the term *coupon* should be used exclusively with bonds, not loans. Old habits, and expressions, nevertheless die hard. The reader should feel free to replace the term "coupon" with *interest-rate payment* as desired.

[9] This is the concrete representation of the number of days between payment dates used to determine the magnitude of the cash-flow.

[10] There is the possibility of providing fixed-rate loans to lending clients, but these would be swapped back to floating anyway.

[11] Moreover, by the publication of this book, it will certainly have been replaced.

traditional LIBOR, in the future it will take an alternative (but conceptually similar) form.

The second challenge involves, in principle, a huge amount of finance theory. Pricing a loan-commitment might look like the simple discounted sum of cash-flows, but a startling amount of complexity is lurking just under the surface. We will not grapple with all of it, but some of it will be necessary for our purposes. Formally, the price of any contingent claim can, on the probability space, $(\Omega, \mathcal{F}, \mathbb{P})$, be written as its expectation taken with respect to the equivalent martingale measure, $\mathbb{Q}$.[12] This is quite a mouthful, but it is the result of a significant amount of practical and theoretical research.[13]. The consequence is that the price of our loan may be written as,

$$
\begin{aligned}
\mathbb{E}^{\mathbb{Q}}\left(P \middle| \mathcal{F}_t\right) &= \mathbb{E}^{\mathbb{Q}}\left(\sum_{i=1}^{\beta}\left(\underbrace{\Delta_i(L_i+m)X_i}_{\text{Coupon}}+N_i\right)e^{-\int_t^{T_i} r_u du}\middle|\mathcal{F}_t\right), \qquad (6.6)\\
&= \sum_{i=1}^{\beta}\mathbb{E}^{\mathbb{Q}}\left(\left(\Delta_i(L_i+m)X_i+N_i\right)e^{-\int_t^{T_i} r_u du}\middle|\mathcal{F}_t\right),\\
&= \sum_{i=1}^{\beta}\underbrace{\mathbb{E}^{\mathbb{Q}^{T_i}}\left(\left(\Delta_i(L_i+m)X_i+N_i\right)\middle|\mathcal{F}_t\right)P(t,T_i)}_{\text{Björk [6, Proposition 19.12]}},\\
&= \sum_{i=1}^{\beta}\left(\Delta_i\left(F(t,T_{i-1},T_i)+m\right)X_i+N_i\right)\underbrace{P(t,T_i)}_{\delta_i},\\
P_t &= \underbrace{\sum_{i=1}^{\beta}\Delta_i\left(F(t,T_{i-1},T_i)+m\right)X_i\delta_i}_{\text{Coupons}}+\underbrace{\sum_{i=1}^{\beta}N_i\delta_i}_{\text{Principal}},
\end{aligned}
$$

where r_t denotes the instantaneous, risk-free, short-term interest rate at time t. This requires some unpacking.[14] Much of Eq. 6.6—such as notional amounts, spreads,

[12] We further assume that this (transformed) measure is induced with the money-market account as the choice of numeraire asset. This is typically generically referred to as the risk-neutral measure.

[13] See Harrison and Kreps [18], Harrison and Pliska [19], and Duffie [13] for foundational discussion on asset-pricing theory.

[14] The measure $\mathbb{Q}^{T_i}$ is induced with the zero-coupon bond with maturity T_i as the choice of numeraire asset. This is typically referred to as the forward measure; see, for example, Brigo and Mercurio [12, Section 2.5] or Björk [6, Chapter 19]. Implicitly, there is a change-of-measure from $\mathbb{Q}$ to $\mathbb{Q}^{T_i}$; this is accomplished with none other than the Radon-Nikodym derivative, $\frac{d\mathbb{Q}^{T_i}}{d\mathbb{Q}}$. This clever choice of numeraire allows us to simplify dramatically our integrands by separating out the dependent instantaneous short rate, r, and the LIBOR rate, L_i.

and day-count fractions—is deterministic. These values are constant under the expectation operator. The only random variables are the future LIBOR rates and the path of the instantaneous risk-free, short rate. The (forward-measure) expectation of future LIBOR is written as,

$$\mathbb{E}^{\mathbb{Q}^{T_i}}\left(L_i \middle| \mathcal{F}_t\right) \equiv \mathbb{E}^{\mathbb{Q}^{T_i}}\left(L(T_{i-1}, T_i)\middle| \mathcal{F}_t\right) = F(t, T_{i-1}, T_i), \tag{6.7}$$

where $F(t, \tau, T)$ denotes the forward interest rate prevailing at time t for a contract starting at time $\tau \geq t$ with a tenor $T - \tau$.[15] In plain English, therefore, this confirms the common practice of using implied forward rates to represent (unknown) future LIBOR rates when generating loan cash-flows. The following quantity is another well-known character in asset-pricing circles,

$$\delta_i \equiv P(t, T_i) \equiv \delta(t, T_i) = \mathbb{E}_t^{\mathbb{Q}}\left(e^{-\int_t^{T_i} r_u du}\right). \tag{6.8}$$

Depending on your perspective and preferences, this might be referred to as the pure-discount bond price, an Arrow-Debreu security, or the pricing kernel. For the purposes of loan pricing, we will unceremoniously refer to it as the discount factor.[16]

The important takeaway from this unprovoked application of financial theory is a pricing relationship that is on firm ground. We may replace unknown future LIBOR outcomes with the implied forward rates extracted from the swap curve and discount future cash-flows using the risk-free overnight-index-swap (OIS) interest rate.[17]

Colour and Commentary 64 (RISK-FREE PRICING): *Every student who has taken a base course in finance is aware that the price of a risk-free, fixed-income security is simply the discounted sum of its cash-flows. The theoretical foundations of this general approach are often less well understood. There is a random element to both the magnitude and form of future cash-flows and their associated discount factors. More formally, the price of a fixed-income security is characterized as its* expected *discounted cash-flows under the appropriate pricing measure discounted at the risk-free interest rate. Both quantities must be inferred from the financial market. In the loan-pricing setting, this reduces to a replacement of (unknown) future LIBOR rates with*

(continued)

[15] Brigo and Mercurio [12] provide much more background and detail on this point.

[16] An Arrow-Debreu security is, of course, rather more general; this would be a special case. See Arrow and Debreu [3] for the gory details.

[17] OIS discounting remains a relatively recent practice and is (probably) poised to change somewhat with the advent of reference-rate reform. See Hull and White [21] for a discussion of its origins.

Colour and Commentary 64 (continued)
implied forward rates and discounting with risk-free rates.[a] *This may feel like financial semantics, but appreciating this aspect is central to placing our upcoming risk-adjusted return calculation on a sound theoretical footing.*

[a] Overnight-interest-rate swaps rates, while not entirely risk-free, are generally employed as an acceptable proxy.

If there was no possibility of default from the loan obligor, we would be done. Default is possible, of course, so it cannot be ignored. Following Jeanblanc [23], we can explicitly introduce the default time τ as a positive-valued random-variable defined on the probability space, $(\Omega, \mathcal{F}, \mathbb{Q})$. You can think of it as a random future time. Defining t as the current time and T as the loan's terminal date, then default occurs if $\tau \in (t, T]$. Practically, therefore, this random default time shows up in pricing formulae as the following indicator variable,

$$\mathbb{I}_{\tau > T} = \begin{cases} 1 : \tau > T \text{ (or survival to } T) \\ 0 : \tau \leq T \text{ (or default prior to } T) \end{cases}. \tag{6.9}$$

This a useful object taking the value of one if default occurs over $(t, T]$ and zero otherwise. It is the main tool for the incorporation of default into pricing logic.

Making generous use of Eq. 6.6—and leaning on the theoretical background from Duffie and Singleton [14] and Bluhm et al. [8]—we can re-write our pricing expression—without expectation—as,

$$P = \underbrace{\sum_{i=1}^{\beta} \mathbb{I}_{\tau > T_i} \left(\Delta_i \left(L_i + m \right) X_i + N_i \right) e^{-\int_t^{T_i} r(u)du}}_{\text{Conditional interest and principal payments}} + \underbrace{R(\tau) X(\tau) e^{-\int_t^{\tau} r(u)du}}_{\text{Conditional default recovery}}, \tag{6.10}$$

where $R \equiv R(\tau)$ is the amount of the defaulted loan one expects to recover. Default conditionality has now been introduced.

The remaining effort involves computation, as done in Eq. 6.6, of the $\mathbb{Q}$-expectation of Eq. 6.10. This requires a few assumptions. The first, common choice is to assume that τ and $r(t)$ are independent. That is, that the default event does not depend upon risk-free interest rates.[18] This means that we can readily evaluate

[18] The link between short-term interest rates, the business cycle, and default probabilities probably argues against this decision. For simplicity of pricing, however, it is probably best to avoid this potential rabbit hole of complex economic relationships.

Eq. 6.10 as

$$
\begin{aligned}
\mathbb{E}_t^{\mathbb{Q}}(P) &= \mathbb{E}_t^{\mathbb{Q}}\left(\sum_{i=1}^{\beta}\mathbb{I}_{\tau>T_i}\left(\Delta_i\left(L_i+m\right)X_i+N_i\right)e^{-\int_t^{T_i}r(u)du}\right.\\
&\quad\left.+R(\tau)X(\tau)e^{-\int_t^{\tau}r(u)du}\right), \qquad (6.11)\\
&= \sum_{i=1}^{\beta}\mathbb{E}_t^{\mathbb{Q}}\left(\mathbb{I}_{\tau>T_i}\left(\Delta_i\left(L_i+m\right)X_i+N_i\right)e^{-\int_t^{T_i}r(u)du}\right)\\
&\quad+\mathbb{E}_t^{\mathbb{Q}}\left(R(\tau)X(\tau)e^{-\int_t^{\tau}r(u)du}\right),\\
&= \sum_{i=1}^{\beta}\underbrace{\mathbb{E}_t^{\mathbb{Q}}\left(\mathbb{I}_{\tau>T_i}\right)\cdot\mathbb{E}_t^{\mathbb{Q}}\left(\left(\Delta_i\left(L_i+m\right)X_i+N_i\right)e^{-\int_t^{T_i}r(u)du}\right)}_{\text{By independence}}\\
&\quad+\mathbb{E}_t^{\mathbb{Q}}\left(R(\tau)X(\tau)e^{-\int_t^{\tau}r(u)du}\right),\\
&= \sum_{i=1}^{\beta}\mathbb{Q}(\tau>T_i)\underbrace{\left(\Delta_i\left(F(t,T_{i-1},T_i)+m\right)X_i+N_i\right)\underbrace{P(t,T_i)}_{\delta_i}}_{\text{Equation 6.6}}\\
&\quad+\mathbb{E}_t^{\mathbb{Q}}\left(R(\tau)X(\tau)e^{-\int_t^{\tau}r(u)du}\right),\\
&= \underbrace{\sum_{i=1}^{\beta}\left(\Delta_i\left(F(t,T_{i-1},T_i)+m\right)X_i+N_i\right)\overbrace{\delta_i\mathbb{Q}(\tau>T_i)}^{\tilde{\delta}_i}}_{\text{Regular cash-flows}}\\
&\quad+\underbrace{\mathbb{E}_t^{\mathbb{Q}}\left(R(\tau)X(\tau)e^{-\int_t^{\tau}r(u)du}\right)}_{\text{Recovery}},
\end{aligned}
$$

where we use $\mathbb{E}_t^{\mathbb{Q}}(\cdot) \equiv \mathbb{E}^{\mathbb{Q}}(\cdot|\mathcal{F}_t)$ to (slightly) ease the notation. Equation 6.11 looks qualitatively quite similar to the risk-free setting summarized in Eq. 6.6. There are *two* main differences: the discount factors have an alternative form and there is an ugly recovery expression hanging around at the end.

The discount factors are the easiest to explain. From first principles, we have that

$$\mathbb{Q}(\mathbb{I}_{\tau>T_i}) = \underbrace{\mathbb{Q}(\tau > T_i)}_{S(T_i)}, \tag{6.12}$$
$$= 1 - \underbrace{\mathbb{Q}(\tau \leq T_i)}_{F_\tau(T_i)},$$

where risk-neutral $S(T_i)$ and $\mathbb{Q}(\tau \leq T_i)$ represent the survival and default probabilities, respectively.[19] The survival probability is, of course, a number between zero and 1. If $S(T) = 0.9$, then we would conclude that there is a 90% probability of survival between t and T. It works much like a discount factor. If we take the product of a survival probability and risk-free discount factors yield, we get the following, very useful, object

$$\tilde{\delta}_i \equiv \tilde{\delta}(t, T_i) = \underbrace{\delta(t, T_i)}_{\delta_i} S(T_i). \tag{6.13}$$

This is referred to as the credit-risk-adjusted discount factor. It is quite intuitive. If the survival probabilities are always and everywhere equal to one, then we recover the risk-free discount rates. If they all take the value of zero, conversely, the security has no value. Under normal circumstances, their values are not so extreme. Instead, in the context of Eq. 6.13, they act as a modifier to the risk-free discount rate. They push down the discount function in a manner directly proportionate to the credit risk of the obligor.[20]

The final, less obvious term, in Eq. 6.11 requires a bit of algebraic wrestling. Making a few assumptions and recalling the basic principles of statistical distributions, it is accurately approximated as

$$\mathbb{E}_t^{\mathbb{Q}}\left(R(\tau)X(\tau)e^{-\int_t^\tau r(u)du}\right) = \int_t^T R(u)X(u)\overbrace{e^{-\int_t^u r(v)dv}}^{\delta(t,u)} f_\tau(u)du, \tag{6.14}$$
$$= \underbrace{\sum_{i=1}^{\beta}\int_{T_{i-1}}^{T_i} R(u)X(u)\delta(t,u)f_\tau(u)du}_{\text{Partition the integral range}},$$

[19] $F_\tau(\cdot)$ denotes the cumulative distribution function of τ. The default probability is, by its very form, equivalent to this (as-yet-unspecified) function of τ.

[20] Indeed, one can think of the difference between the risk-free and survival rates as a kind of representation of the individual firm's credit spread.

$$\approx \underbrace{\sum_{i=1}^{\beta} R \int_{T_{i-1}}^{T_i} X(u)\delta(t,u) f_\tau(u)du,}_{\text{Assume constant recovery}}$$

$$\approx \underbrace{\sum_{i=1}^{\beta} R X_i \delta_i \int_{T_{i-1}}^{T_i} F'_\tau(u)du,}_{\substack{\text{Constant exposure and discount} \\ \text{factor on each sub-interval}}}$$

$$\approx \sum_{i=1}^{\beta} R X_i \delta_i \bigg(F_\tau(T_i) - F_\tau(T_{i-1}\bigg),$$

$$\approx \sum_{i=1}^{\beta} R X_i \delta_i \underbrace{\bigg(\mathbb{Q}(\tau \leq T_i) - \mathbb{Q}(\tau \leq T_{i-1}) \bigg)}_{\text{Equation 6.12}},$$

$$\approx \sum_{i=1}^{\beta} R X_i \delta_i \mathbb{Q}\bigg(\tau \in [T_{i-1}, T_i] \bigg).$$

This approximation has a very practical form; it permits us to write the recovery aspect as a simple product of recovery rates, exposures, risk-free discount factors, and risk-neutral default probabilities.[21]

Gathering all of the details together, we arrive at the final product. The (risk-neutral) expectation of a floating-rate loan obligation can be approximated as,

$$\mathbb{E}_t^{\mathbb{Q}}(P) \equiv P_t \approx \sum_{i=1}^{\beta} \underbrace{\bigg(\Delta_i \bigg(F(t, T_{i-1}, T_i) + m \bigg) X_i + N_i \bigg) \tilde{\delta}_i}_{\text{Discounted (risky) expected cash-flows}} + \underbrace{R X_i \mathbb{Q}\bigg(\tau \in [T_{i-1}, T_i] \bigg) \delta_i}_{\text{Expected recovery}}. \tag{6.15}$$

The logic is that the individual cash-flows are discounted at a rate that reflects the creditworthiness of the underlying obligor. It also includes a conditional amount for the recovery of any funds in the event of default.

[21] It is common, at this point in most default-risk pricing discussions, to place some additional structure onto the survival probabilities. This involves the (very useful) notion of the hazard function. For our purposes, however, this quantity will not be required. The interested reader is referred to Taylor and Karlin [38, Chapter 1] or Stuart and Ord [37, Section 5.34] for more background on hazard functions.

This is not quite the end of the story. Equation 6.15 can also be (usefully) written in an alternative, but equivalent form. We could discount the cash-flows, without any notion of default, as in Eq. 6.6. If we did that, however, we would have to subtract off the expected credit loss and then, as before, add back the expected recovery. Let's try this with Eq. 6.15,

$$
\begin{aligned}
P_t &\approx \sum_{i=1}^{\beta} \underbrace{\left(\Delta_i \left(F(t, T_{i-1}, T_i) + m \right) X_i + N_i \right) \delta_i}_{\text{Equation 6.6}} - \underbrace{X_i \mathbb{Q}\left(\tau \in [T_{i-1}, T_i] \right) \delta_i}_{\text{Expected loss}} \\
&\quad + \underbrace{R X_i \mathbb{Q}\left(\tau \in [T_{i-1}, T_i] \right) \delta_i}_{\text{Expected recovery}}, \qquad (6.16) \\
&\approx \sum_{i=1}^{\beta} \underbrace{\left(\Delta_i \left(F(t, T_{i-1}, T_i) + m \right) X_i + N_i \right) \delta_i}_{\text{Discounted (riskless) expected cash-flows}} \\
&\quad - \underbrace{\overbrace{(1 - R)}^{\text{LGD}} X_i \mathbb{Q}\left(\tau \in [T_{i-1}, T_i] \right) \delta_i}_{\text{Net expected loss}},
\end{aligned}
$$

where the well-known quantity, loss-given-default or LGD, makes a surprise appearance. While Eqs. 6.15 and 6.16 are equivalent, the latter expression offers the (not-to-be-underestimated) benefit of being able to adjust all cash-flows directly using risk-free discount factors.

Colour and Commentary 65 (DEFAULT-RISK PRICING)*: A second financial pricing adage is that cash-flows should be discounted with a rate that is commensurate to their riskiness. How such discount rates are constructed requires theoretical structure. It turns out that these rates are based on the interaction between risk-free rates and survival probabilities. The risk-free dimension captures the time value of money, while the survival aspect explicitly addresses the payment-default dimension. This clever decomposition has useful consequences. It permits us—when making the appropriate adjustments for default and recovery—to continue to discount expected future cash-flows at risk-free rates. In other words, as long as the key actors are present in the correct form, there are a number of possible configurations for their representation. We will make generous use of this flexibility in the construction of risk-adjusted loan returns.*

Actually evaluating Eq. 6.16 requires some effort. One needs to identify cash-flows dates, outstanding amounts, margins, and day-count fractions. These will need to be sourced from internal systems. Risk-free discount, implied LIBOR forward-rate, and (risk-neutral) default probability curves will also be required. This likely involves additional systems and input data. Discount and projection curves need to be extracted from market instruments such as overnight-index and interest-rate swap securities.[22] Risk-neutral default probabilities are, most typically, extracted from credit-default-swap markets. This requires some additional theoretical machinery and statistical assumptions.[23]

The bigger question relates the conceptual notion of so-called risk-neutral default probabilities. The risk-neutral, or pricing, or equivalent martingale measure—which we have denoted as $\mathbb{Q}$—is a central object in asset pricing. It is a subtle object arising from assumptions about arbitrage and market completeness. Risk-neutral probabilities are the correct quantity for pricing, but they unfortunately do not correspond to one's typical idea of a default probability. We generally like to think of a probability in relative frequency terms. That is, if a random event was repeated 100 times, then a specific outcome might be sensibly assigned a probability of 0.1 if it occurs on ten occasions. In a financial setting, this more natural definition of probability is—in a loose sense—referred to as the natural or physical probability measure. We write it as $\mathbb{P}$ to differentiate it from the equivalent martingale measure, $\mathbb{Q}$.

These probability measures, quite simply, serve different purposes. The risk-neutral measure arises naturally in pricing applications, whereas the physical probability measure stars in strategic and risk-management settings.[24] Often both make an appearance in a given analysis, but when this occurs, they stick to their area of relevance. While this is a well-understood area of finance theory, it bears repeating and underscoring in this analysis. The computation of risk-adjusted rates of return—the objective of this chapter—involves both pricing and risk-management dimensions. Particular care will need to be taken to use the correct quantity in the correct place.

Colour and Commentary 66 *($\mathbb{P}$'s and $\mathbb{Q}$'s): To* mind your p's and q's *is an old-fashioned English expression counselling caution and attentiveness in one's behaviour. It presumably comes from the fact that these two letters*

(continued)

[22] This can get quite involved. See, for example, Ametrano and Bianchetti [2] for a discussion of some of the finer points involved in this exercise.

[23] More details can be found in Bolder [10, Chapter 9].

[24] Meucci [29] provides an insightful description of these two different *tracks* in quantitative financial analysis.

Colour and Commentary 66 (continued)
look strikingly similar, but mean rather different things.[a] *A similar degree of vigilance is recommended when working with risk-neutral and physical probabilities—denoted as* $\mathbb{P}$ *and* $\mathbb{Q}$*, respectively—in financial problems. Analogous to our English proverb, these measures serve rather different purposes despite some structural parallels. The risk-neutral measure applies when pricing securities; quantities computed under this measure cannot be interpreted in the typical way. The physical measure applies to real-world empirical problems such as risk measurement or strategic analysis. Using the wrong measure in the wrong context can lead to potentially disastrous results. Since computation of risk-adjusted returns touch on both these areas, we are well advised to tread carefully.*

[a] The actual origins of this saying, as one might expect, are a matter of some debate. See Knight [24] for some (rather old) thoughts on the matter.

6.2 A Holistic Perspective

The determination of appropriate lending margins needs to capture both risk and return dimensions. As a result, we do *not* face a pure loan-pricing problem. Equation 6.16 is certainly useful and (if) applied correctly, were we so inclined, would represent an excellent starting point for the fair-value liquidation of a lending asset. Most financial institutions are not in the business, however, of liquidating their lending assets.[25] Loan-price valuations have many applications, but they are not directly informative to lending decisions. Why, therefore, have we invested several pages of effort in rather dense asset-pricing theory? The answer is that the forthcoming analysis will incorporate an important pricing dimension. We will need this knowledge. There is, nevertheless, an important dimension *not* directly present in our pricing formulae: worst-case risk.

Asset pricing incorporates average, or expected, risk. It does not include any explicit notion of extreme, downside (or worst-case) risks. This is the job of economic capital. Risk-adjusted pricing might incorporate this aspect, conceptually at least, as follows:

$$\text{Risk-adjusted price} = \underbrace{\text{Risk-free price} - \text{Expected loss}}_{\text{Equation 6.16}} - \text{Worst-case loss.} \tag{6.17}$$

[25] Nor are these loans typically fair-valued, beyond informational purposes, in financial reporting. The clear exception is loan securitization, of course, but that is not the current topic of discussion.

This might be a useful quantity, but it is denominated in currency terms making comparison between individual loans somewhat difficult. A risk-adjusted return might solve this issue with the following (very generic form):

$$\text{Risk-adjusted return} = \frac{\overbrace{\text{Risk-free price} - \text{Expected loss}}^{\text{Equation 6.16}} - \text{Other relevant costs}}{\text{Worst-case loss}}. \tag{6.18}$$

Risk-adjusted returns have a long history in finance. The Sharpe ratio, which is a rather famous case of the more general information ratio, is still used extensively in asset-management circles and dates back to the late 1960s.[26] The usefulness of these ratios is that they normalize some notion of return by some measure of risk. The return is thus deflated, or discounted, by the amount of risk it generates. High-return, but high risk investments are thus made comparable to their low-return, low-risk equivalents in a consistent manner. Such a quantity is thus of significant usefulness in informing investment, or lending, decisions. We thus find ourselves back to the firm's capital budgeting problem, thus explaining our interest.

The big question, of course, is what precisely to put into the numerator and denominator of Eq. 6.18 to make the precise structure of risk-adjusted return the most meaningful and representative of one's business. The key to answering these questions is found in the financial statements. We need to understand how lending activities are financed. How much is financed, for example, by liabilities and how much by equity? This is a capital structure question. We also need to investigate how an individual loan contributes to the firm's overall equity position. What, therefore, are the relevant revenues and expenses associated with an individual loan? This is a profit-and-loss question. Finally, we must identify the specific risks that need to be compensated. This goes somewhat beyond the firm's financial statements and enters into the realm of economic capital.

6.2.1 The Balance-Sheet Perspective

The most natural starting place is the firm's balance sheet. Among other things, a balance sheet illustrates asset composition and capital structure. These two dimensions provide useful insight into investment and financing decisions, which will help us organize the actors appearing in the numerator of Eq. 6.18.

Figure 6.2 begins this process with the graphical representation of two stylized balance sheets. The left-hand graphic includes the useful high-level view of assets, liabilities, and shareholder's equity. The right-hand side specializes to the type of

[26] See Sharpe [35, 36] for the origins of the Sharpe ratio and Bacon [4] for a good discussion on the information ratio. Bolder [9, Chapter 14] also touches on these ratios and some of their cousins.

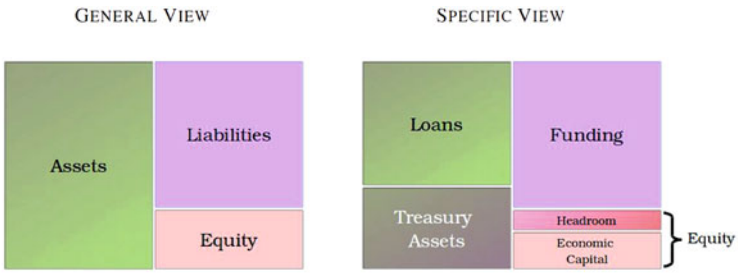

Fig. 6.2 *Asset composition and capital structure*: The preceding graphics illustrate—from a few complementary perspectives—the typical (albeit somewhat stylized) balance sheet of a lending institution.

balance sheet one would expect to see from a garden-variety lending institution.[27] The asset side is broken into two main categories: lending and treasury assets. Loans, appearing in amortized-cost form, are probably best treated as a monolithic category. It is useful, conversely, to consider the various fair-valued elements of treasury assets. Many of these instruments will serve short-term liquidity needs—such as facilitating loan disbursements and financing repayments—but some portion will also relate to medium-term investments. A fraction of the treasury assets will also include derivative securities—such as interest-rate or cross-currency swaps—employed to hedge various aspects of the lending operations. The market values of such instruments are generally only a small proportion of their notional values and, depending on market conditions, they may appear as either assets or liabilities.[28]

As we move to the specific balance-sheet view in Fig. 6.2, it is also helpful to broadly characterize firm liabilities as funding. While some other liabilities naturally occur, the vast majority relates to financial-market funding. NIB, like most international financial institutions, funds itself in global capital markets at various tenors across a range of currencies.[29] The equity allocation is also further subdivided into two categories: economic-capital and headroom. The magnitude of the economic-capital value is, of course, directly proportional to the riskiness of the assets on the left-hand side of the balance sheet. Recall further that the headroom is defined as the residual between a firm's actual equity position and its economic-capital consumption. A positive headroom indicates capacity for further risk taking, while a small or negative figure often spells trouble.

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[27] This viewpoint would, with a few small label changes and brushing over some firm-specific elements, equally apply to virtually any financial institution.

[28] Typically, derivative valuations on both sides of the balance sheet roughly cancel one another out. While centrally important for the operations of almost any financial institution, they do not figure importantly in this discussion. Chapter 10 touches on how derivative contracts enter into the economic-capital picture.

[29] For many financial institutions, funding is typically augmented by (or even predominately comprised of) client deposits.

While it is obvious that the right-hand side of the balance sheet finances the left-hand assets, it is less evident what specifically finances what. It is not uncommon for financial institutions to, conceptually at least, imagine that lending activity is financed by funding, whereas treasury investments are funded with equity. A firm may, through their asset-and-liability management function, build explicit (albeit partial) links between the two sides of the balance sheet. Conceptually, however, this is something of a fiction. A firm should be viewed, and is typically managed, from a holistic or macro perspective. Each asset held by a firm is being financed by both funding and equity. This is, in many ways, the central message of any balance sheet and the reason for the primacy of capital-structure questions in the corporate-finance literature.

This seemingly innocuous conclusion—that assets are financed by the entire right-hand side of the balance sheet—nonetheless has some important ramifications. Most particularly, the revenues generated by the firm's assets need to cover one's costs. Funding costs, while potentially multifaceted and complex, are readily tabulated and determined. The cost of equity, however, is rather less clear cut. In the simplest sense, assets need to generate sufficient revenues to pay an annual dividend. This, however, is only part of the equation. If the firm wishes to grow its balance sheet—as most do—the assets also need to permit corresponding growth in the equity position.[30] Increasing levels of assets will—even at unchanged levels of riskiness—consume more economic capital. Absent a commensurate increase in equity, the headroom will shrink and, ultimately, business activity will likely need to be curtailed.

The actual required level of equity growth is something of an open question. Some firms, and industries, have a broad range of growth prospects. Others do not. These prospects are generally somehow linked to their products' life cycles, the competitive structure of the industry, and the state of technology. It is probably a reasonable assumption that, over the medium to long-term, all firms should seek to grow, at least, at the same pace as their general macroeconomy.[31] Over shorter time periods, of course, firms might target higher (or lower) levels of growth. Again, if assets do not generate sufficient revenues to fund the necessary degree of equity growth then—without an external injection of capital—challenges may arise. For a financial institution, the shrinkage in headroom might ultimately lead to foregoing future lending opportunities, issues with regulators, or even downgrade by credit agencies.

What precisely is the mechanism for equity growth? It follows a rather well-known channel through the firm's profit-and-loss statement. The generation of

[30] An old paper—which still nicely frames the key issues—about the interplay between growth, dividends, and stocks prices is found in Gordon [17].

[31] A randomly selected Swedish firm's long to medium-term growth target, for example, should probably not be determined independently of the expected evolution of Swedish output.

asset revenues involves incurrence of costs. For a financial institution, the revenue element will take the form of loan margins, associated lending fees, and financial security coupon payments.[32] The expense side will include financing costs, administrative expenses, fair-value adjustments, and loan impairments.[33] On a periodic basis, the asset revenues and expenses are summed and netted, the dividend payment is subtracted and the remainder is added to the equity position. The consequence is an adjustment—made on a quarterly or annual basis—to the equity position; this is often referred to as retained earnings. More simply, these retained earnings are an internal asset-driven source of financing. In principle, a firm can only grow in two ways: via this internal, organically driven, route or from additional external injections of capital.

Colour and Commentary 67 (GROWING A FIRM)*: There are, when you sift through all of the details, only* two *possible ways to grow a firm. The first involves organically increasing one's equity position through retained earnings. The second requires an external injection of capital. Both, as is usual, offer advantages and disadvantages. External injections can lead to issues surrounding information asymmetries between management and shareholders.*[a] *For international financial institutions, with government shareholders, capital replenishment can also become a political question. These issues notwithstanding, new external capital can be extremely effective in helping an entity expand its activities. Internal growth has fewer implications for the institution's status quo, but it can limit growth opportunities and, in certain situations, prove rather slow. In day-to-day loan pricing discussions—given the infrequent and unpredictable nature of external equity inflows—it is nonetheless customary to abstract from possible future capital injections and focus on internal growth. Organic capital expansion places a burden on firm assets to contribute sufficient return to make one's desired growth possible; one's loan-pricing framework must reflect this central fact.*

[a] This relates to Akerlof [1]'s famous lemons problem.

[32] If a firm owns equity assets, this would also include dividend income. Marketable securities' revenue is also a bit more complex than simply coupons, but here we seek to keep it conceptually simple.

[33] Because loans are typically held at amortized cost, changes in their fair value do not flow through profit-and-loss. To account for expected lending losses, a separate loan-impairment computation is performed. While structurally slightly different, it is conceptually analogous to the final *net expected loss* term in Eq. 6.16. This element, in all its glory, will be considered more formally in Chap. 9.

6.2.2 Building the Foundation

The discussion in the preceding section will, almost certainly for most readers, contain few (if any) new ideas. These details nevertheless bear explicit repetition, because they provide part of the foundation for the computation of risk-adjusted returns. The fundamental assumption, at this point, is that a firm's assets need to internally create sufficient return to finance current and future growth prospects. Let us try to work out, from first principles, what precisely these assets need to return. We begin with the definition of a firm's return on equity:

$$\text{Return on equity (ex dividend)} = \frac{\overbrace{\text{Revenues} - \text{Expenses}}^{\text{Firm P\&L}} - \text{Dividends}}{\text{Equity}},$$

$$\underbrace{\text{Return on equity (ex dividend)} + \frac{\text{Dividends}}{\text{Equity}}}_{\text{Return on equity}} = \frac{\overbrace{\text{Revenues} - \text{Expenses}}^{\text{Firm Net income}}}{\text{Equity}}. \tag{6.19}$$

This is a familiar ratio; it is simply the quotient of net income and equity. Equation 6.19 attempts to include dividend policy and raises the point that it may be—according to one's tastes and objectives—placed on either side of the identity. Moreover, Eq. 6.19 also explicitly incorporates the central role of assets in the generation of net income. This is important, because asset composition is our main focus of attention and principal decision variable.

The return on equity identity, from Eq. 6.19, is a good jumping off point. One can establish a sensible growth target for this quantity. That said, it has a few critical shortcomings; these are the same issues that precipitated the construction of the economic-capital measure. In particular, it is silent on worst-case risk. One (perhaps slightly dodgy) strategy to meet a return-on-equity target would be to add very risky assets to one's balance sheet. In the short term, at least, this would—despite increased loan impairments and fair-value volatility—typically enlarge net income. It would, however, expose the firm to greater risk and potentially disastrous future net-income outcomes. This argues against the use of the return-on-equity measure for decisions on asset allocation.[34]

Wholesale addition of riskier assets to one's balance sheet is *not* necessarily a sub-optimal strategy. The important element, of course, is that the asset return is commensurate to the risk taken; moreover, the portfolio-level implications of these risks need to be understood.[35] This argues, therefore, for an adjustment—or perhaps restatement—of Eq. 6.19 introducing the risk dimension. Abstracting from dividend

[34] It is naturally part of the analysis, but it should not be the sole decision criterion.

[35] It also must be consistent with the firm's overall risk appetite and capital position.

policy, let us consider:

$$\text{Return on economic capital} = \frac{\overbrace{\text{Asset revenues} - \text{Asset (and other) expenses}}^{\text{Net asset income}}}{\text{Economic capital}}, \tag{6.20}$$

with a slight specialization relative to Eq. 6.19 towards the asset dimension. The differences between Eqs. 6.19 and 6.20 are not dramatic, but they are nonetheless definitive. The revised denominator captures the riskiness of the assets. The high asset-risk strategy, mentioned above, can only work if those assets also generate a correspondingly high degree of return. Conversely, low-risk assets—that in the previous analysis might look uninteresting—can also usefully contribute to this ratio. Equation 6.20 thus appears to be a sensible candidate for a risk-adjusted return ratio.

While a portfolio level risk-adjusted return ratio is fantastic, actually decisions occur by individual asset. Somehow, therefore, we need to break Eq. 6.20 out by asset. With N individual assets, the following decomposition is always possible,

$$\underbrace{\text{Return on economic capital}}_{\text{RAROC}} = \sum_{n=1}^{N} \frac{\overbrace{\text{Asset revenues}_n - \text{Asset (and other) expenses}_n}^{\text{Net asset income}_n}}{\text{Economic capital}}. \tag{6.21}$$

The numerator in Eq. 6.21 represents the marginal contribution to net income (and eventually equity) from each asset, whereas the denominator is unchanged. This is progress. Equation 6.21 tells us how each asset contributes, on a risk-adjusted basis, to the total return. This is referred to as the risk-adjusted return on capital or RAROC. It has a long and storied history in the banking community; it was initially developed roughly 50 years ago by Banker's Trust.[36] Depending on the exact construction of the numerator and denominator of Eq. 6.21, this measure can provide rather different guidance. Thus, while we will use the generic term RAROC throughout the following discussion, be aware that there is no unique definition.[37]

Equation 6.21, while respecting the rules of algebra, does *not* quite go far enough. Our ultimate interest would involve the marginal contribution to *both* return and risk

[36] James [22] is an interesting look into the genesis and rationale behind this measure.

[37] As usual, it is always a good idea to understand the specific recipe used to cook the dish.

at the asset level. We thus propose something like the following:

$$\underbrace{\text{Return on economic capital}_n}_{\text{RAROC}_n} = \frac{\overbrace{\text{Asset revenues}_n - \text{Asset (and other) expenses}_n}^{\text{Net asset income}_n}}{\text{Economic capital}_n}, \\ = \frac{\text{Marginal asset-income contribution}_n}{\text{Marginal asset-risk consumption}_n}, \tag{6.22}$$

for $n = 1, \ldots, N$ individual assets. This is precisely what we seek. The numerator of Eq. 6.22 captures an asset's marginal contribution to income, while the denominator normalizes by its risk consumption.

Sadly, although Eq. 6.22 is conceptually ideal, it an arithmetic mess. It is clear that

$$\underbrace{\text{Return on economic capital}}_{\text{Portfolio RAROC}} \neq \sum_{n=1}^{N} \frac{\text{Marginal asset-income contribution}_n}{\text{Marginal asset-risk consumption}_n}. \tag{6.23}$$

In plain English, our preferred asset-level, risk-adjusted return measure cannot be summed, in an intuitive way, to yield the overall portfolio metric. Instead, a bit more flexibility is required. Equation 6.22 is aggregated to the portfolio level by summing over both the numerator and denominator simultaneously. More specifically, this looks like

$$\underbrace{\text{Return on Economic Capital}}_{\text{Portfolio RAROC}} = \frac{\sum_{n=1}^{N} \text{Marginal asset-income contribution}_n}{\sum_{n=1}^{N} \text{Marginal asset-risk consumption}_n}, \\ = \frac{\sum_{n=1}^{N} \text{Asset revenues}_n - \text{Asset expenses}_n}{\sum_{n=1}^{N} \text{Economic capital}_n}, \\ = \frac{\text{Net income}}{\underbrace{\text{Economic capital}}_{\text{Equation 6.20}}}. \tag{6.24}$$

This minor inconvenience would seem to be the price to pay for a meaningful working definition of risk-adjusted return (or RAROC).[38]

[38] In practice, Eq. 6.24 allows us to compute the RAROC of any possible sub-portfolio. It is only necessary to keep track of the marginal contribution and consumption to economic capital for each individual asset.

Colour and Commentary 68 (A MARGINAL PERSPECTIVE)*: In our search for a sensible risk-adjusted return ratio, a natural point of departure is the return on equity. Sole examination of return on equity is nonetheless sub-optimal, because it ignores important (worst-case) aspects of the underlying asset risks. This argues for updating the return-on-equity ratio's denominator with economic capital. This small change permits explicit consideration of asset riskiness. A further adjustment reduces this to the individual asset level. Our proposed risk-adjusted return—or risk-adjusted return on capital (RAROC)—measure is thus the ratio of an asset's marginal contribution to asset income divided by its marginal consumption of economic capital. This RAROC measure provides a sensible, risk-adjusted metric for asset-selection decisions. A bit of caution and extra arithmetical gymnastics are nonetheless required when aggregating this measure over individual assets or subsets of one's portfolio.*

6.3 Estimating Marginal Asset Income

Equipped with a workable and concrete risk-adjusted ratio, we may proceed beyond word equations and build a specific prescription for its calculation. This process will bring us back to the previously introduced fundamentals of loan pricing. A few new bells and whistles will, however, be added. The typical loan-valuation problem, for example, does not concern itself with how the loan is actually financed. Equation 6.22 involves the explicit inclusion of relevant asset expenses. This will be our starting point.

Figure 6.3 provides a high-level schematic of the typical inflows and outflows of a lending transaction. As highlighted in the initial section of this chapter, loan obligors pay (usually six-month) LIBOR plus a lending margin denoted m. These

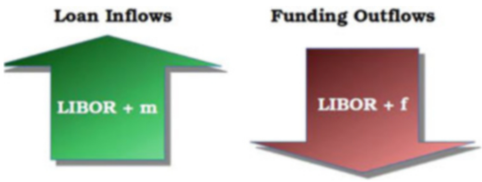

Fig. 6.3 *Loan mechanics*: This graphic provides a simplified schematic of the principal elements of loan profit-and-loss in a LIBOR-based—or, if you prefer, reference-rate-based—financial institution.

loans are funded with a (typically three-month) LIBOR-based funding transaction plus a funding spread, which we'll call f. A few points arise. First of all, there may be some form of basis risk between the six-month LIBOR lending rate and the three-month LIBOR funding values. While important, let us collect this aspect and push it into the funding spread, f. The second point relates to an institution's attitude towards interest-rate risk. Many financial institutions immunize themselves from interest rate movements by hedging all lending and funding activity back into some fixed (typically LIBOR) reference rate. This also helps manage tenor mismatches between assets and liabilities.[39] Occasionally, however, some aspect is not completely hedged. This amounts to a perfectly legitimate view on interest rates, but it does complicate our analysis.

Let's take the perspective, for the moment at least, of a fully LIBOR-based bank.[40] Any fixed-rate loans, funding, and (most) investment operations are dully swapped—individually or on a macro basis—back into either three- or six-month LIBOR.[41] For the purposes of determining the numerator of our risk-adjusted ratio in Eq. 6.22, even when this hedging does not occur, we may always conceptually determine the equivalent m and f values associated with any position. This relatively straightforward computation involves the instrument's cash-flows flows and the appropriate swap curve. On this basis, we may (cautiously) conclude that Fig. 6.3 provides a reasonable characterization of lending inflows and outflows.[42]

To arrive at a sensible expression for the numerator of our risk-adjusted return ratio, we will begin slowly. We attend first to the loan revenues less funding expenses. This has the following form,

$$\begin{aligned}\begin{array}{c}\text{Loan}\\ \text{Revenues}\end{array} - \begin{array}{c}\text{Funding}\\ \text{Expenses}\end{array} &= \sum_{i=1}^{\beta} \underbrace{\left(\Delta_i\left(L_i+m\right)X_i\right)e^{\int_t^{T_i} r(u)du}}_{\text{Discounted (riskless) inflows}} \\ &\quad - \underbrace{\left(\Delta_i\left(L_i+f\right)X_i\right)e^{\int_t^{T_i} r(u)du}}_{\text{Discounted (riskless) outflows}}, \end{aligned} \tag{6.25}$$

[39] Most retail commercial banks, for example, fund themselves with short-term deposits, but make long-term mortgages loans. Swapping both sides back into a common reference rate enormously helps manage net margins.

[40] This is defensible, because the hedging decision can be thought of separately and independently from the lending choice.

[41] Exchange-rate risks are also hedged, but even if they were not, this element would show up in the economic-capital calculation.

[42] Following this reasoning, an m value is also readily calculable for fixed-rate treasury asset investments. This would permit extension of these ideas across all firm assets.

$$\mathbb{E}_t^{\mathbb{Q}}\left(\begin{matrix}\text{Loan}\\ \text{Revenues}\end{matrix} - \begin{matrix}\text{Funding}\\ \text{Expenses}\end{matrix}\right) = \mathbb{E}_t^{\mathbb{Q}}\left(\sum_{i=1}^{\beta}\left(\Delta_i\left(L_i + m\right)X_i\right)e^{\int_t^{T_i} r(u)du}\right.$$
$$\left. -\left(\Delta_i\left(L_i + f\right)X_i\right)e^{\int_t^{T_i} r(u)du}\right),$$
$$\begin{matrix}\text{Expected}\\ \text{Marginal}\\ \text{Loan Income}\end{matrix} = \sum_{i=1}^{\beta}\Delta_i \underbrace{\left(m - f\right)}_{\substack{\text{Internal}\\ \text{margin}}} X_i\delta_i.$$

This is a lovely result. Abstracting from other expenses and impairments, the economic lending contribution to net income is simply the exposure-weighted, discounted sum of net margins over the loan's lifetime. This expression does, however, require some additional explanation. The notional repayments—from both loan and funding perspectives—are not considered because they do not have a profit-and-loss impact. We apply the (risk-neutral) expectation operator, because we are trying to compute market-consistent economic values for these cash-flows. The structural relationships are, of course, motivated by financial-statement accounting principles, but this remains (at least, partially) a pricing problem. The pleasant aspect of Eq. 6.25 is the cancellation of the LIBOR component.[43] This is a practical and conceptual benefit of running a LIBOR-based financial institution.[44] The other important dimension is that, unlike the accounting problem, we are not focusing on a single reporting period. We seek to consider the financial impact of the loan instrument over its entire tenor; this further supports the application of (risk-neutral) expectations.

Unfortunately, it is necessary to complicate the marvellously parsimonious expression from Eq. 6.25. Embedded in its construction is an important, and debatable, assumption. It assumes that the loan is entirely funded with market-based liabilities. As explicitly highlighted in Fig. 6.3, however, loan financing stems from both funding and equity. This would imply that both these elements need to be incorporated. We can take another crack at the form of the expected marginal loan income through a slight restatement of Eq. 6.25. This simply requires the

[43] We need not even make use of the change-of-measure trick to evaluate our expectation taken with respect to the equivalent martingale measure (induced by a collection of forward measures).

[44] Recall that the (expected) basis risk associated with the different LIBOR tenors is embedded in f. The worst-case aspect of this risk should also find its way into the market-risk component of economic capital.

re-weighting of a loan's financing sources as,

$$\begin{aligned}\begin{pmatrix}\text{Marginal}\\\text{Loan Income}\end{pmatrix} &= \sum_{i=1}^{\beta}\left[\underbrace{\Delta_i\Big(L_i+m\Big)X_i}_{\text{Inflows}} - \underbrace{\Delta_i\Bigg(\overbrace{(1-\xi_i)\Big(L_i+f\Big)}^{\text{Funding}} + \overbrace{\xi_i\mathcal{H}}^{\text{Equity}}\Bigg)X_i}_{\text{Financing outflows}}\right]e^{\int_t^{T_i} r(u)du},\end{aligned} \tag{6.26}$$

$$\begin{aligned}\mathbb{E}_t^{\mathbb{Q}}\begin{pmatrix}\text{Marginal}\\\text{Loan Income}\end{pmatrix} &= \mathbb{E}_t^{\mathbb{Q}}\left(\sum_{i=1}^{\beta}\left[\Delta_i\Big(L_i+m\Big)X_i - \Delta_i\Big((1-\xi_i)\Big(L_i+f\Big)+\xi_i\mathcal{H}\Big)X_i\right]e^{\int_t^{T_i} r(u)du}\right),\\
&= \sum_{i=1}^{\beta}\mathbb{E}_t^{\mathbb{Q}}\left(\left[\Delta_i\Big(L_i+m\Big)X_i - \Delta_i\Big((1-\xi_i)\Big(L_i+f\Big)+\xi_i\mathcal{H}\Big)X_i\right]e^{\int_t^{T_i} r(u)du}\right),\\
&= \underbrace{\sum_{i=1}^{\beta}\mathbb{E}_t^{\mathbb{Q}^{T_i}}\left(\left[\Delta_i\Big(L_i+m\Big)X_i - \Delta_i\Big((1-\xi_i)\Big(L_i+f\Big)+\xi_i\mathcal{H}\Big)X_i\right]\right)\underbrace{P(t,T_i)}_{\delta_i},}_{\text{Using the logic in Eq. 6.6}}\end{aligned}$$

$$\begin{pmatrix}\text{Expected}\\\text{Marginal}\\\text{Loan Income}\end{pmatrix} = \sum_{i=1}^{\beta}\Delta_i\left(m - \underbrace{(1-\xi_i)f}_{\substack{\text{Funding}\\\text{financed}}} - \underbrace{\xi_i\Big(\mathcal{H}-F(t,T_{i-1},T_i)\Big)}_{\substack{\text{Equity}\\\text{financed}}}\right)X_i\delta_i,$$

where ξ_i denotes the proportion of equity financing associated with the ith loan payment and $\mathcal{H}$ represents the target rate of return on equity. This restatement is not as immediately intuitive as Eq. 6.25. As we can see, the LIBOR based element does not completely cancel out.[45] We are also left to interpret the new equity financing term.

[45] This consequently forces us to make use of the forward-measure numeraires introduced in our initial loan-pricing development.

If we set $\xi_i = 0$ then Eq. 6.26 collapses to Eq. 6.25. This is reassuring. Conversely, setting ξ_i to unity (i.e., fully financing the loan with equity) yields,

$$\begin{array}{c}\text{Expected}\\ \text{(Equity-Financed)}\\ \text{Marginal}\\ \text{Loan Income}\end{array} = \sum_{i=1}^{\beta} \Delta_i \left(m - \left(\mathcal{H} - F(t, T_{i-1}, T_i) \right) \right) X_i \delta_i, \tag{6.27}$$

$$= \sum_{i=1}^{\beta} \Delta_i \left(m + \underbrace{F(t, T_{i-1}, T_i)}_{\text{Expected LIBOR}} - \mathcal{H} \right) X_i \delta_i.$$

How should we interpret this result? If the loan was entirely financed with equity, it would (as usual) earn the margin *plus* expected LIBOR. These inflows would need to be compared to the expected return on equity, $\mathcal{H}$. In the more general case, however, the funding costs—with a weight of $(1 - \xi)$—need to be offset. The difference between expected LIBOR and the required equity return also arises. The larger point is that, in a LIBOR bank, the equity financed proportion of a loan—or an investment for that matter—earns LIBOR.[46]

Colour and Commentary 69 (LIABILITY VS. EQUITY FINANCING)*: If we assume that a loan obligation is solely financed with market funding, we may derive a streamlined representation of expected marginal loan income. It is essentially the sum of the discounted differences between lending and funding margins over the instrument's lifetime. In reality, however, a proportion of every loan is also financed through equity. Inclusion of this dimension creates two practical problems: determination of the required return for equity and assignment of appropriate weights to the funding and equity components. Leaving these practical details for future sections, a revised loan-income expression may be derived. While less parsimonious, the result is nonetheless quite interesting. While the reference rate (i.e., LIBOR) component cancels out from the funding-only perspective, it remains for the equity. This implies that, in addition to the margin, the equity financed aspect of any loan earns the full amount of LIBOR. This naturally leads to a comparison of this LIBOR amount to the expected return associated with equity. Both the funding and equity sources of financing, of course, earn the lending margin.*

[46] Funding financed lending activity also, of course, earns LIBOR. Since it simultaneously costs LIBOR, this effect cancels out.

6.3.1 Weighting Financing Sources

Equation 6.26, despite its useful structure, cannot be implemented without a clearer view on the value of each ξ_i. This quantity tells us the proportion of loan financing stemming from equity. In any introductory corporate finance textbook—Brealey et al. [11] is an excellent choice—one can find a treatment of the so-called *weighted-average cost of capital*. As the name strongly suggests, this involves weighting the relative costs of market funding and equity. If we denote D_t and E_t as the time t balance-sheet values of the firm's debt and equity, respectively, then a possible candidate for the equity weight is

$$\xi_t = \frac{E_t}{D_t + E_t}. \tag{6.28}$$

This particular choice offers a number of benefits. It is easy to compute and intuitive. Moreover, it directly reflects the balance-sheet structure of the financial institution. This makes it an excellent choice for computation of a firm's cost of capital.

For our purposes, however, there are also a few shortcomings. Logistically, we need to consider future cash-flows over potentially long-term horizons. The precise capital structure, at these future points, will be difficult or impossible to know. A possible solution would be to fix our weight, ξ, to the current time. Perhaps more worrying, however, is the relatively homogeneous treatment of all loans implied by Eq. 6.28. Irrespective of the loan's riskiness, it will be financed in the same proportions with funding and equity. This does not follow the spirit of the calculation. Some loan transactions will consume—through their inherent riskiness—more of the firm's equity. As a consequence, they need not cover only their funding costs, but also contribute more to equity than less risky substitute loans.

An alternative weighting scheme that explicitly incorporates the (worst-case) risk dimension might look like

$$\xi_t = \frac{\mathcal{E}_t}{X_t}, \tag{6.29}$$

where $\mathcal{E}_t$ and X_t represent the loan's economic capital and outstanding notional values at time t, respectively. The intuition, and logic, behind this choice is that a loan's amount of equity financing is directly proportional to its consumption of equity.[47] Equity consumption is proxied with its economic-capital allocation. A low-risk loan might only have an equity financing weight of a few percentage points, whereas this could rise to 20 or even 30% for highly risky lending ventures. The corollary—which will be addressed in later sections—is that the required return on

[47] These asset-specific values are also, from a logistical perspective, more readily available over time for inclusion in the calculation.

equity for such loans may be higher than that necessitated by market-based funding. It also means that riskier assets face a higher burden.

Using Eq. 6.29, we may return to our expression for expected marginal loan income and make it a bit more concrete. In particular,

$$\begin{aligned}\begin{array}{c}\text{Expected}\\\text{Marginal}\\\text{Loan Income}\end{array} &= \underbrace{\sum_{i=1}^{\beta} \Delta_i \left(m - \left(1 - \underbrace{\frac{\mathcal{E}_i}{X_i}}_{\xi_i} \right) f - \underbrace{\frac{\mathcal{E}_i}{X_i}}_{\xi_i} \Big(\mathcal{H} - F(t, T_{i-1}, T_i) \Big) \right) X_i \delta_i}_{\text{Equation 6.26}}, \\ &= \sum_{i=1}^{\beta} \Delta_i \left(\underbrace{\left(m - \left(1 - \frac{\mathcal{E}_i}{X_i} \right) f \right) X_i}_{\text{Funding component}} - \underbrace{\Big(\mathcal{H} - F(t, T_{i-1}, T_i) \Big) \mathcal{E}_i}_{\text{Equity component}} \right) \delta_i .\end{aligned} \tag{6.30}$$

The result is quite sensible. The funding component involves the lending and (weighted) funding margins relative to the outstanding loan amount. The equity piece, as before, describes the difference between the required equity return and expected forward LIBOR rates. The magnitude of this component, however, depends on the loan's economic-capital consumption. If we set $\mathcal{E}_i$ to zero—the loan is considered to be riskless—this is equivalent to a zero weight on equity financing. In this case, the equity piece vanishes and Eq. 6.30 collapses to Eq. 6.25.

Colour and Commentary 70 (FINANCING WEIGHTS)*: The firm's current capital structure is a natural touchstone for the determination of the relative weights on the cost of funding and equity. While simple and intuitive, this approach fails to capture the specific risks associated with a given credit obligor. In other words, the static, homogeneous nature of the capital structure doesn't allow us to differentiate loan riskiness. We resolve this issue by specifying the weight on equity financing as the ratio of the loan's economic capital to its outstanding amount. The consequence of this choice is that the cost of equity financing is directly proportional to a loan's economic-capital allocation. Alternative choices are naturally possible, but this appears to provide an intuitive link to worst-case risk and questions of capital adequacy. In other words, it establishes a logical correspondence between determination of one's lending margin and the riskiness of the firm's balance sheet.*

6.3.2 Other Income and Expenses

Before moving to actually construct our risk-adjusted return ratio, some additional elements must be incorporated into loan income. These have been ignored, up to this point, to avoid notational clutter. At this point, however, it is necessary to ensure that all possible cost and revenue criteria are included.

Let us begin with an important source of expense: administration. There are various ways to describe this, but our approach is to denote administrative expenses in percentage terms. We will denote it as a. This is multiplied—as with lending and funding margins—by the current loan outstanding amount. The actual magnitude of a specific loan's administrative expenses can, and does, depend upon a number of things: creditworthiness, tenor, or geographical location to name a few. These would, of course, have some link to, or influence on, the composition of administrative expenses. Since we are focused on recurring (discounted) administrative expenses over the loan's lifetime, we can embed it into the existing machinery as,

$$\begin{matrix}\text{Administrative} \\ \text{Expenses}\end{matrix} = \sum_{i=1}^{\beta} \Delta_i a X_i \delta_i. \tag{6.31}$$

The next element relates to a feature of certain loans, which certainly also applies in other lending settings. This is referred to as an up-front fee; we will denote it as u. Again, it is a percentage quantity that is conceptually similar to lending or funding margins or administrative expenses. The twist in this case, as the name implies, is that it is a one-time event.[48] An upfront-free occurs only at inception. We can readily handle this with an appropriately structured indicator function. $\mathbb{I}_{i=1}$ dutifully takes a value of unity at the first cash-flow date and zero everywhere else. This permits us to write the lifetime loan income associated with up-front fees as,

$$\begin{matrix}\text{Up-front} \\ \text{Fee Income}\end{matrix} = \sum_{i=1}^{\beta} \Delta_i \left(\mathbb{I}_{i=1} u \right) X_i \delta_i. \tag{6.32}$$

This might seem like overkill, but the intention is to inject this aspect organically into our risk-adjusted return ratio formula.

The notion of (expected) default risk—introduced in the first section—has not yet been included into our net loan-income construction.[49] If we let γ represent our estimated loss-given-default value, then we could simply borrow the formally derived expected credit loss expression from Eq. 6.16 on page 352. For convenience,

[48] Indeed, in many cases, we may have $u \equiv 0$.

[49] This idea is intimately related to loan impairments, covered in great detail in Chaps. 7 to 9. In loan pricing, we use a stylized statistical measure of expected default loss and assume away the complex accounting details.

we restate its approximate form

$$\begin{matrix}\text{Risk-Neutral}\\\text{Expected}\\\text{Loss}\end{matrix} \approx \sum_{i=1}^{\beta} \Delta_i \gamma \mathbb{Q}\Big(\tau \in [T_{i-1}, T_i]\Big) X_i \delta_i. \tag{6.33}$$

This structure, while technically correct for pricing, is not ideal for our purposes. We will need to make some concessions for the financial-statement perspective adopted in the construction of our ratio. In brief, we seek to maintain the spirit of Eq. 6.33, but simultaneously bring it closer to the loan-impairment computation flowing through the profit-and-loss statement.

Practically, this transformation is accomplished by simply replacing the risk-neutral default probabilities with their real-world equivalents. The result is,

$$\begin{matrix}\text{Loan}\\\text{Impairment}\end{matrix} \approx \sum_{i=1}^{\beta} \Delta_i \gamma \underbrace{\mathbb{P}\Big(\tau \in [T_{i-1}, T_i]\Big)}_{p_i} X_i \delta_i. \tag{6.34}$$

The loan-impairment calculation is not a price; it is a risk measure. As a consequence, we need to change our underlying probability measure. Instead of extracting our probabilities from financial markets, they now need to be estimated from historical data. This implies a slightly lower degree of flexibility in their form. These physical-measure default probabilities are typically estimated at an annual frequency. To ease the notational burden somewhat, we will denote p_i as the appropriate annual default rate associated with the ith cash-flow date.[50]

Pulling all of these disparate pieces together, we finally arrive at a comprehensive—if somewhat stylized—description of the marginal income associated with a specific loan. It takes the following, admittedly noisy, form

$$\begin{aligned}\begin{matrix}\text{Expected}\\\text{Marginal}\\\text{Loan}\\\text{Income}\end{matrix} &= \begin{matrix}\text{Upfront}\\\text{Fees}\end{matrix} + \begin{matrix}\text{Lending}\\\text{Margin}\end{matrix} - \begin{matrix}\text{(Weighted)}\\\text{Funding}\\\text{Margin}\end{matrix} - \begin{matrix}\text{Admin}\\\text{Expenses}\end{matrix}\\ &\quad - \begin{matrix}\text{Loan}\\\text{Impairments}\end{matrix} - \begin{matrix}\text{(Weighted)}\\\text{Equity}\\\text{Cost}\end{matrix}, \\ &= \sum_{i=1}^{\beta} \Delta_i \Bigg[\Bigg(\mathbb{I}_{i=1} u + m - \left(1 - \frac{\mathcal{E}_i}{X_i}\right) f - a - \gamma p_i\Bigg) X_i \\ &\quad - \Big(\mathcal{H} - F(t, T_{i-1}, T_i)\Big)\mathcal{E}_i\Bigg] \delta_i.\end{aligned} \tag{6.35}$$

[50] This necessitates the scaling by the appropriate day count.

Equation 6.35 represents the numerator of our risk-adjusted return ratio. All that remains is to identify the appropriate denominator.

6.4 Risk-Adjusted Returns

Equation 6.22 indicates that the RAROC computation requires the marginal asset-risk contribution in the denominator; in short, it is the economic capital. That much is obvious. The practical challenge, however, is to find the most reasonable possible form for our purposes. The key questions surround timing and discounting.

Economic capital is computed at a given point in time, with a fixed portfolio composition, and with a predefined set of model parameters. Equation 6.35, which is the comparable term for the numerator of our risk adjusted ratio, involves the discounted sums of cash-flows over a single loan's lifetime. This puts us in something of an apples-and-pears situation.

The most natural solution to this mismatch is to organize the denominator in a similar manner. We seek the total marginal economic-capital consumption of a given loan over its lifetime. A direct way to construct such a quantity would look like:

$$\begin{matrix}\text{Discounted}\\ \text{Marginal}\\ \text{Economic-Capital}\\ \text{Consumption}\end{matrix} = \sum_{i=1}^{\beta} \mathcal{E}_i \delta_i . \tag{6.36}$$

This is basically the sum of discounted future economic-capital consumptions associated with each individual payment date. This would create a logical correspondence between the numerator and denominator.

There is just one problem: we do not know the future economic-capital consumptions associated with a given loan at each of its future cash-flow dates. There are, at least, *two* annoying reasons. First, we simply do not know the portfolio composition at these future times. This implies that we cannot really accurately determine future concentration effects. Second, even if we did have this information, we do not have the computational capacity to perform so many complex stochastic simulations. Economic capital, after all, is *not* a fast computation.

The first annoyance can be solved by assuming that future portfolio will have a similar composition to the current portfolio. This is something of a heroic assumption and is subject to criticism, but there are really no other practical alternatives. The second issue requires some mathematical machinery. It is solved with the use of a complex, non-linear approximation of the economic-capital consumption.[51] This approximation depends on a set of regression parameters and explanatory variables

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[51] The mathematical structure of this approximation—treated here in a very abstract manner—is discussed in detail in Chap. 5.

as well as the specific details of the loan.[52] These parameters are updated every day to reflect the current structure of the portfolio; forecasts of *future* economic capital, to repeat, necessarily assume a similar portfolio composition. Denoting all of this approximation-level information as θ, we may write the economic-capital approximation at the ith payment date as $\hat{\mathcal{E}}_i(\theta)$.

Using our approximation, we can rewrite Eq. 6.36 as,

$$\begin{matrix}\text{Estimated}\\ \text{Discounted}\\ \text{Marginal}\\ \text{Economic-Capital}\\ \text{Consumption}\end{matrix} = \sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i. \tag{6.37}$$

This is an object that can be sensibly computed.[53] Equation 6.37 will play the role of denominator for our risk-adjusted return ratio. We will also use this approximation, where appropriate, to replace the funding-and-equity cost weights introduced in Eqs. 6.29 and 6.30.

We now have all the necessary components to actually explicitly write out a possible RAROC computation for an arbitrary loan obligation. Combining our previous development, we have

$$\begin{aligned}
\text{RAROC} &= \frac{\overbrace{\text{Asset revenues} - \text{Asset expenses}}^{\text{Marginal net asset income: Eq. 6.35}}}{\underbrace{\text{Economic capital}}_{\text{Equation 6.37}}}, \qquad (6.38)\\
&\approx \frac{\sum_{i=1}^{\beta} \Delta_i \left(\left(\mathbb{I}_{i=1} u + m - \left(1 - \frac{\hat{\mathcal{E}}_i(\theta)}{X_i}\right) f - a - \gamma p_i \right) X_i - \Big(\mathcal{H} - F(t, T_{i-1}, T_i)\Big) \hat{\mathcal{E}}_i(\theta) \right) \delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i},\\
&\approx \frac{\sum_{i=1}^{\beta} \Delta_i \overbrace{\left(\mathbb{I}_{i=1} u + m - \left(1 - \frac{\hat{\mathcal{E}}_i(\theta)}{X_i}\right) f - a - \gamma p_i \right) X_i \delta_i}^{\eta_i}}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i}\\
&\quad - \frac{\sum_{i=1}^{\beta} \Delta_i \Big(\mathcal{H} - F(t, T_{i-1}, T_i)\Big) \hat{\mathcal{E}}_i(\theta)\delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i},
\end{aligned}$$

[52] A non-exhaustive list would include the regional and sector identity of the obligor, the amount outstanding, the credit class, the loss-given-default, the and size of firm.

[53] Albeit subject to the reasonableness of the variety of assumptions made in Chap. 5.

$$\approx \underbrace{\frac{\sum_{i=1}^{\beta} \Delta_i \eta_i X_i \delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i}}_{\text{Lending RAROC}}$$

$$- \underbrace{\left(\frac{\sum_{i=1}^{\beta} \Delta_i \mathcal{H} \hat{\mathcal{E}}_i(\theta)\delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i} - \frac{\sum_{i=1}^{\beta} \Delta_i F(t, T_{i-1}, T_i) \hat{\mathcal{E}}_i(\theta)\delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i} \right)}_{\text{Equity-return target}},$$

where we have tried to reduce the notational clutter with the following definition,

$$\eta_i \stackrel{\triangle}{=} \underbrace{\eta_i \Big(u, m, f, a, \gamma, p_i, X_i, \theta \Big)}_{i\text{th payment loan profit-and-loss}}, \tag{6.39}$$

$$= \mathbb{I}_{i=1} u + m - \left(1 - \frac{\hat{\mathcal{E}}_i(\theta)}{X_i} \right) f - a - \gamma p_i.$$

The result is a prescriptive—if not necessarily parsimonious—formula for the construction of a risk-adjusted return on (economic) capital.

The lending RAROC piece of Eq. 6.38 is relatively intuitive. It is a risk-deflated (marginal) net loan income. The equity return aspects on the right-hand-side, however, are independent of the size of the position. They are also, approximately at least, independent of the loan's riskiness. This requires a bit of effort to see. Inspection reveals that both elements of the equity-return target are essentially weighted averages. Let's begin with the required equity return piece, where

$$\mathcal{H}\bar{\Delta} \approx \mathcal{H} \underbrace{\left(\frac{\sum_{i=1}^{\beta} \Delta_i \hat{\mathcal{E}}_i(\theta)\delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i} \right)}_{\approx \bar{\Delta}}, \tag{6.40}$$

which is a kind average payment-period return; $\bar{\Delta}$ is a kind of scaling for the payment frequency.[54]

The second term is a bit more unpleasant since it relates to the economic-capital weighted implied forward rate component. This relationship between the spot rate and the future path of forward rates—and the assumption of their equality—is referred to as the expectations hypothesis. A huge literature—starting with Modigliani and Shiller [30]—deals with this issue. In a nutshell, it does not really hold, but the difference is typically small and relates to term premia. For our conceptual purposes, we can probably safely abstract from this element. LIBOR-based forward rates could thus usefully be replaced with averaged expected interest rates over the loan's lifetime (i.e., $T - t$). We might denote this rate as $\overline{L(t, T)}$ to reflect its origins in the LIBOR market.[55] We thus opt to approximate this LIBOR-related element as,

$$\overline{L(t, T)}\bar{\Delta} \approx \frac{\sum_{i=1}^{\beta} \Delta_i F(t, T_{i-1}, T_i)\hat{\mathcal{E}}_i(\theta)\delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i}. \tag{6.41}$$

Using these simplifications, the implication for our RAROC computation is a significantly simplified more compact form,

$$\text{RAROC} \approx \frac{\sum_{i=1}^{\beta} \Delta_i \eta_i X_i \delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i} - \left(\mathcal{H} - \overline{L(t, T)}\right)\bar{\Delta}. \tag{6.42}$$

Once again, the main contribution stems from the funding-financed net-income contribution; this is unchanged. Subtracted from this component is the difference between the required rate of return on equity and the loan's LIBOR-related earnings. The second aspect represents, in a simplified and approximate way, the required return on equity (approximately) decoupled from both the size and riskiness of the loan. As we will see in the following section, it may be useful to think of these two aspects as being conceptually distinct.

[54] Equation 6.40 would hold with equality if $\Delta_i \equiv 1$ for all $i = 1, \ldots, \beta$ or the payments are made annually. When the payment frequency is *not* annual, we will almost invariably annualize the RAROC estimate anyway.

[55] It is admittedly a kind of risk-weighted average, but this effect is relatively small and we'll ignore it for our purposes.

Colour and Commentary 71 (THE ROLE OF LIBOR)*: Working from a combination of asset-pricing and corporate-finance first principles, a risk-adjusted return ratio for loan obligations can be constructed. Interestingly—when explicitly allocating the financing costs into funding and equity proportions—the ratio can be split into* two *distinct parts. The first piece describes risk-adjusted net-income return; this is basically the ratio of marginal loan contribution to the marginal consumption of capital. The second aspect, however, is approximately equal to the difference between the expected cost of equity and the reference rate (i.e., LIBOR).*[a] *This has a pair of important implications. The expected rate of return on equity can be examined either together or separately from the main RAROC computation. Indeed, it can be considered as a kind of RAROC target. The second point has already been touched upon. Each loan contributes, for its equity component, an amount roughly equal to the full LIBOR rate. This quantity correspondingly represents the relevant point of comparison for the determination of an expected equity return.*

[a] Subject to simplifying assumptions surrounding payment frequencies and intertemporal risk weighting.

6.5 The Hurdle Rate

When firms are examining investment projects, a natural outcome is the associated expected rate of return. Indeed, this is precisely what we have been doing in the context of a specific loan project. Having computed such a return leads to the next inevitable question: is it sufficient? There is certainly some floor on the return below which the firm would simply not accept the project. Such a floor is often colloquially referred to as a *hurdle rate*.[56] Hence the use of the suggestive symbol, $\mathcal{H}$, to represent this quantity.

The most obvious choice for a hurdle rate is the firm's cost of capital. In practice, however, it is generally set to a level somewhat north of the cost-of-capital figure. The reasons relate to a focus on profit generation and a desire to incorporate the risk dimension. In our setting, these aspects have a different relevance. The risk component, to begin with, is already explicitly incorporated into our RAROC calculation; it need not be considered twice. The pure profit motive, as previously

[56] Presumably, although it is not easily verified, the origin of the term stems from the need for a project's expected return to *jump* over this rate to be accepted.

alluded to, does not entirely fit at the NIB. As an international financial institution, the mandate is not—unlike most commercial organizations—to maximize profit. Instead, it seeks to provide additionality in its area of operations—with a focus on productivity and the environment—in a (financially) sustainable manner.[57] Financial sustainability might also be interpreted as not needing to return to one's shareholders—outside of exceptional circumstances—for injections of capital. With this in mind, each firm needs (on a risk-adjusted basis) to generate a sufficient equity return to permit its targeted level of growth and relevance. Nevertheless, some lower-return loans with very high degrees of additionality might be considered. These, to meet the overall objectives and constraints, would need to be offset by other activities generating higher levels of return. This balance is central for attaining sustained organic growth.

There is a number of possible ways to derive such a hurdle rate. We will avoid the weighed average cost of capital approach, because we seek an ex-funding perspective.[58] Instead, our focus is rather on a required equity return. A default approach, as previously discussed, would be to focus on the (predicted or historical average) rate of economic growth, above the risk-free time value of money, in the firm's area of operation. If we denote this as π, then we might write our hurdle rate as

$$\mathcal{H}(t,T) = r(t,T) + \pi(t,T), \tag{6.43}$$

where $T - t$ is the horizon under consideration and r represents a risk-free interest rate.[59] Practically, therefore, if one expected the Nordic and Baltic regions to have medium to long-term economic growth in the neighbourhood of 3% with 2% risk-free interest rates, then the hurdle rate would take a value of about 5%.

The role of the risk-free rate is interesting. Equation 6.42 indicates that equity financed lending or treasury activity contributes, at least, LIBOR. For analytic purposes, we can consider this to be roughly equivalent to a risk-free rate. If we assume that our hurdle rate is equal to this risk-free rate, then the lending-financed RAROC calculation could be used on a standalone basis since—from Eq. 6.42—we have that $\mathcal{H} - L(t,T) \approx \mathcal{H} - r(t,T) = 0$. Since an entity's shareholders can earn the risk-free rate on their own, of course, this is a relatively poor choice of hurdle rate. The consequence, therefore, is that it makes logical sense to judge a hurdle rate relative to the risk-free rate.

[57] Other entities can, and certainly do, have analogous strategic objectives to be attained.

[58] This piece is, in fact, already incorporated into our net lending income.

[59] Whatever the time horizon, we have an annualized rate in mind.

To hammer down this point, let's revisit our RAROC calculation using the concrete definition of the hurdle rate from Eq. 6.43. Restating Eq. 6.42, we have

$$\text{RAROC} \approx \frac{\sum_{i=1}^{\beta} \Delta_i \eta_i X_i \delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i} - \Bigg(\underbrace{r(t,T) + \pi(t,T)}_{\text{Equation 6.43}} - \overbrace{L(t,T)}^{\approx r(t,T)} \Bigg) \bar{\Delta}, \tag{6.44}$$

$$\approx \frac{\sum_{i=1}^{\beta} \Delta_i \eta_i X_i \delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i} - \left(\cancel{r(t,T)} + \pi(t,T) - \cancel{r(t,T)} \right) \bar{\Delta},$$

$$\approx \frac{\sum_{i=1}^{\beta} \Delta_i \eta_i X_i \delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i} - \pi(t,T)\bar{\Delta}.$$

Using this form, in this case, the decision point is a RAROC of zero. That is, if the funding financed component covers $\pi(t, T)$, then the proposed lending project should—at least, on purely financial-return grounds—be accepted. This is because, once having covered all loan-related costs, its overall contribution meets the corporate equity return target. This result naturally leans on a few mathematical choices. One may naturally question the assumption that the risk-free and LIBOR rates roughly coincide.[60] Mechanically, however, this would lead to a slightly more conservative form for Eq. 6.44. So, while the assumption is clearly incorrect, it should not bias our decision-making in the wrong direction.

A challenge with the hurdle-rate definition in Eq. 6.43 relates to the level of risk-free rates. We have established that equity funded assets, in a LIBOR-based lending institution, are (at least) earning something roughly equivalent to the risk-free rate. For this reason, the risk-free rate falls out of our direct comparison. The issue is that this property makes LIBOR-based firms' overall income levels rather sensitive to the level of interest rates. If interest rates are zero—or, in some cases, negative as is the case in the current environment—focusing on long-term economic growth for one's hurdle rate definition may be insufficient. A firm will also require a general level of income to meet dividend obligations and basic absolute levels of balance-sheet growth.

[60] In reality, due to term premia and credit risk, the LIBOR rate should exceed the risk-free rate.

Another alternative, that might slightly assuage this aspect, would relate the hurdle rate to the firm's medium-term growth targets. Imagine, for example, that the institution wishes to attain an annual growth rate of $g(t, T)$ over the risk-free rate over the next $T - t$ years. This figure could come from internal analysis, mandate-driven objectives, or shareholder guidance. The logical process for selecting $g(t, T)$ could, and should, also incorporate assumptions and expectations regarding the path of risk-free interest rates. In this case, we could write our hurdle rate in a manner similar to Eq. 6.43 as,

$$\mathcal{H}(t, T) = r(t, T) + g(t, T). \tag{6.45}$$

Over the short-term, it is certainly possible for $g(t, T)$ to differ from $\pi(t, T)$. Over the medium to long-term, however, it will be difficult for any firm to attain growth levels that deviate importantly from broad-based economic growth.

Colour and Commentary 72 (SPECIFYING A HURDLE RATE)*: The choice of hurdle rate, defined as $\mathcal{H}$ in the previous discussion, is not specific to the individual loan under investigation. It is more broadly related to the general level of return required on a firm's equity. It must, therefore, be determined by reflection of firm-wide objectives. Two key elements enter into this choice. First, the equity financed component of asset returns—in a LIBOR-matched lending institution—will earn approximately the risk-free rate.*[a] *This argues for the specification of the hurdle rate relative to the general level of (risk-free) interest rates. The second point is that while it is difficult to target overall returns in excess of economic growth in the long term, any functional hurdle rate certainly needs to be calibrated to a firm's current and targeted growth objectives. Selection of one's hurdle rate is thus a central part of a lending institution's annual planning process; the corollary is that, depending on circumstances and future plans, it may vary over time.*

[a] There is a credit premium embedded in the LIBOR setting. During the reference-rate reform process as predominately overnight-funding, transaction-based replacements assume the role of reference rates, we should expect this risk-free assumption to move closer to the truth.

6.6 Allocating Economic Capital

Having covered the majority of the structural elements in the risk-adjusted capital return calculation, a bit more practical specificity regarding the economic-capital is required. Economic capital is, after all, a key component in the calculation. The question is: how precisely do we determine which elements of economic capital to include? Do we include everything—including regulatory or internal management buffers—for every type of asset investment? This would imply allocating all market, credit, and operational risk as well as the full set of buffers to every individual loan and treasury investment.

Some guidance would be welcome. If this was an accounting exercise, then full allocation might be a sensible objective. There is, however, also a behavioural aspect to a firm's RAROC calculation. It is, in fact, trying to help support sensible decision making. Aspects outside the decision-maker's control might not be useful inclusions into the calculation.[61] To reflect this fact, the basic premise is thus proposed:

> Economic capital should be allocated to those areas responsible for its management.

How this principle manifests itself depends, of course, on a firm's organizational structure. Asset-investment decisions are, after all, made in differing ways across institutions. At most lending institutions, for example, a lending origination department (or function) makes (most) loan proposals. They are not, however, responsible for determining how the associated financing is sourced and hedged. Loans also give rise to liquidity needs; again, the lending originator does not decide on the specific investment profile of these liquidity securities. The counterparty, default-credit, and market risk associated with borrowing, hedging, and liquidity investments are correspondingly not sensibly allocated to lending decisions.

A similar argument, operating in the opposite direction, applies to treasury operations. Treasury staff have no influence on the idiosyncratic credit-risk characteristics of a given loan investment. As a consequence, they should not be allocated lending related credit-risk economic capital. This makes sense despite the fact that treasury activity is largely driven, indeed caused, by lending business.

Operational risk and the capital buffers, conversely, are shared by both business lines.[62] It thus makes sense to allocate them accordingly. Moreover, both are readily determined through either an estimate of net interest income or risk-weighted assets. The (management) stress-testing buffer, however, is unallocated.[63] Figure 6.4 summarizes how the various elements of economic capital might sensibly be allocated to the lending and treasury business lines. This is merely one example that fits relatively well into our setting; compelling arguments can be made for alternative forms of risk allocation.

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[61] Such questions, often referred to as risk budgeting, can become quite involved. See Scherer [32] for a useful entry point into this world.

[62] Countercyclical buffers are (typically) excluded to avoid introduction of economic cyclicality into the pricing decision. This point, depending on one's perspective and objectives, is open to debate.

[63] Refer to Chap. 1 for a review of these economic-capital buffers.

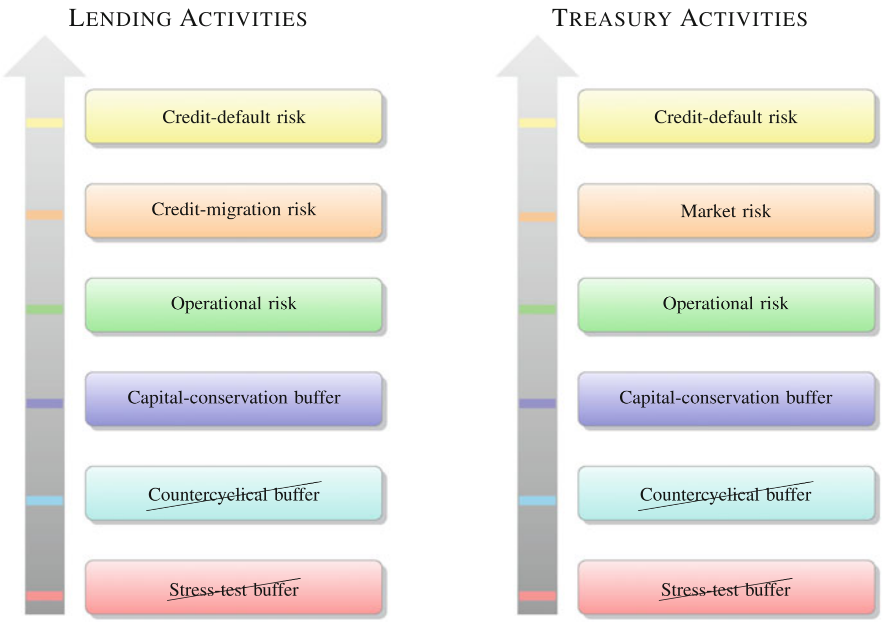

Fig. 6.4 *Business-line allocation*: The preceding schematic summarizes the allocation of economic capital by the abovementioned business lines. The amounts allocated, by risk and buffer type, are only those specific to assets within that business area.

6.7 Getting More Practical

To this point, our discussion has been heavy on the theoretical dimension, but rather light on practicalities. To really get a feel for the RAROC computation, it is thus useful to examine a detailed, practical example. Not only does an example make things more concrete, it also allows us to examine a few twists and turns in the computation presented by real-life situations. Due to privacy concerns, however, it is impossible to examine a real loan. To that end, Fig. 6.5 outlines the details of an entirely fictitious loan.

Our fabricated case takes the form of a Norwegian industrial corporate considering a five-year maturity loan with a total exposure of EUR 60 million. It furthermore resides in internal credit-rating class PD06, and has a loss-given-default value of 0.45. The range of fees and costs are also summarized in Fig. 6.5. This collection of data represents the information available to the loan originator. The key question is: how can we think about the return associated with this constellation of loan details on a risk-adjusted basis?

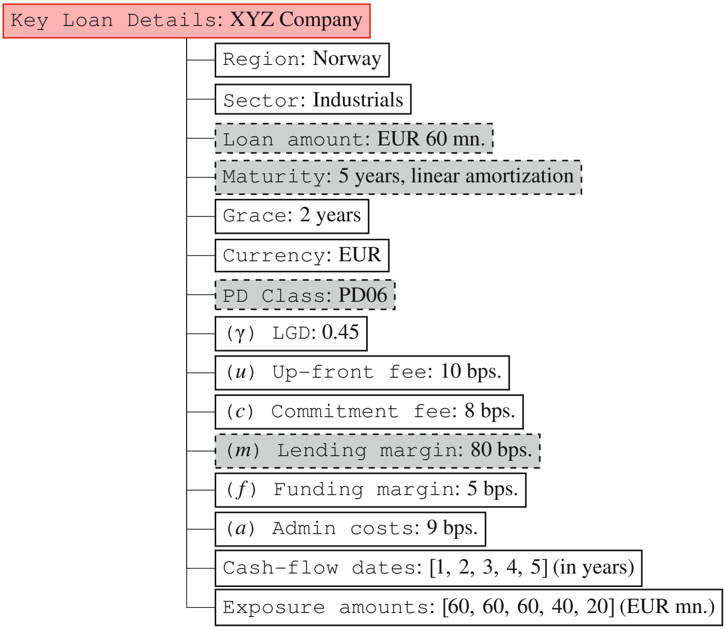

Fig. 6.5 *A concrete example*: This schematic outlines the qualitative and quantitative details associated with a fictitious, but illustrative and concrete, loan example.

6.7.1 Immediate Disbursement

The first step in addressing this question involves understanding the intertemporal structure of implied loan cash-flows. Figure 6.6 illustrates the expected cash-flows associated with our five-year loan, its two-year grace period, and a linear amortization structure. A key underlying assumption is that the full amount of the loan is immediately disbursed. Practically, this is a not only unlikely, but probably even impossible. For loans that are expected to disburse in the iminent future, however, this is a sensible simplification.[64]

Figure 6.6 clearly displays the equal capital repayment of EUR 20 million during the final three years of the loan's life. This creates a step-down effect on the loan outstanding, which needs to be reflected in the cash-flow and economic-capital values.

Table 6.1 jumps right to the details of the RAROC computation. In particular, it focuses attention on the various elements of the ratio's numerator. These aspects of expected marginal loan income were introduced in Eq. 6.35. The first point is that we assume each cash-flow occurs on an annual basis; this simplifies the

[64] We will relax this assumption in a moment and explore the implications for the RAROC calculation.

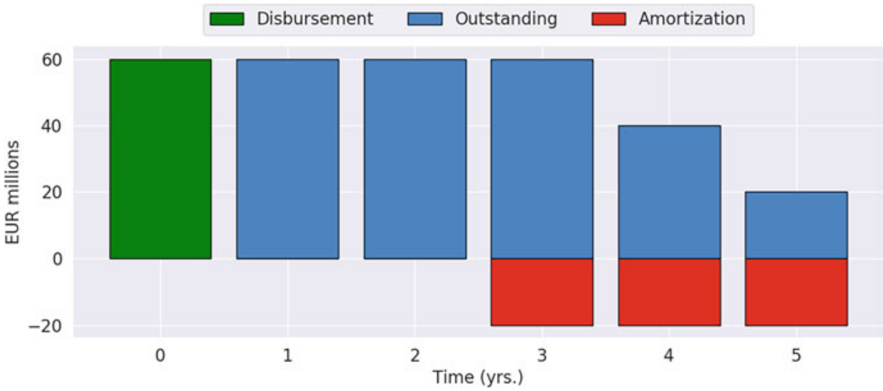

Fig. 6.6 *Outstanding profile*: This figure displays the initial disbursement, outstanding amount, and amortization payment schedule associated with the example loan introduced in Fig. 6.5. In this case, we observe immediate loan disbursement; while this is not always the case, it simplifies the computation.

illustration of results by keeping the total number of cash-flows low. It also implies that $\Delta_i = 1$ and can thus be ignored. The second key point is that the loan outstanding amounts—from Fig. 6.6—are summarized in the second column. These $\{X_i; i = 1, \ldots, 5\}$ quantities play a central role in determining each subsequent column.

The remaining values in Table 6.1 should be relatively self-explanatory. A 10 basis-point up-front fee, on a EUR 60 million loan, amounts to a single payment of EUR 60,000 on the first cash-flow date. The funding margin is slightly more complex since it involves weighting its values by the associated economic-capital consumption at period i.[65] Overall, on a discounted basis, the total (lifetime) marginal contribution to capital is roughly EUR 1.8 million. This forms the numerator of our RAROC ratio.

Table 6.2 turns our attention to the various components comprising the denominator of our RAROC calculation. The actual economic capital estimate at time i is written as,

$$\underbrace{\text{Economic Capital}}_{\mathcal{E}_i(\theta)} = \left(\text{Default Risk}\right) + \left(\text{Migration Risk}\right) + \left(\text{Buffers \& Operational Risk}\right), \tag{6.46}$$

$$\mathcal{E}_i(\theta) \approx \hat{\mathcal{E}}_i(\theta) = \hat{\mathcal{D}}_i(\theta) + \hat{\mathcal{M}}_i(\theta) + \hat{\mathcal{B}}_i,$$

[65] These values will be addressed in the subsequent table.

Table 6.1 *The RAROC numerator*: The underlying table illustrates, in the context of our practical example outlined in Fig. 6.5, the various elements of the RAROC numerator organized by individual cash-flow date.

i	X_i	$\mathbb{I}_{i=1} \cdot u \cdot X_i$	$m \cdot X_i$	$\left(1 - \frac{\hat{\mathcal{E}}_i(\theta)}{X_i}\right) \cdot f \cdot X_i$	$-a \cdot X_i$	$-\gamma \cdot p_i \cdot X_i$	$\eta_i \cdot X_i$	$\eta_i \cdot X_i \cdot \delta_i$
1	60,000,000	60,000	480,000	26,375	−54,000	−18,225	494,150	495,036
2	60,000,000	0	480,000	26,199	−54,000	−21,776	430,422	431,622
3	60,000,000	0	480,000	26,130	−54,000	−23,500	428,630	428,334
4	40,000,000	0	320,000	17,419	−36,000	−16,893	284,526	282,359
5	20,000,000	0	160,000	8727	−18,000	−9099	141,628	139,174
Total	240,000,000	60,000	1,920,000	104,849	−216,000	−89,493	1,779,356	**1,776,525**

Table 6.2 *The RAROC denominator*: This table outlines, following from the example in Fig. 6.5, the different aspects of the RAROC denominator organized by individual cash-flow date.

i	X_i	$\hat{\mathcal{D}}_i(\theta)$	$\hat{\mathcal{M}}_i(\theta)$	$\hat{\mathcal{B}}_i$	$\hat{\mathcal{E}}_i(\theta)$	$\hat{\mathcal{D}}_i(\theta) \cdot \delta_i$
1	60,000,000	7,136,138	113,521	1,296,378	8,546,037	8,561,362
2	60,000,000	7,515,214	87,218	1,296,378	8,898,810	8,923,618
3	60,000,000	7,680,664	60,154	1,296,378	9,037,197	9,030,964
4	40,000,000	5,132,042	30,390	864,252	6,026,684	5,980,788
5	20,000,000	2,536,025	10,236	432,126	2,978,388	2,926,779
Total	240,000,000	30,000,083	301,519	5,185,513	35,487,115	**35,423,511**

for $i = 1, \ldots, \beta$. In words, there are three main components to the approximated *lending* economic-capital consumption for each time period.[66] The first two relate to the default and migration credit-risk allocations; these are both based on separate approximation algorithms and depend on a parameter vector.[67] The final component combines the non-stress-test capital buffers and operational risk. It does not depend on the parameter vector, θ, since it does not involve any complicated mathematical approximations. The capital buffers, based on risk-weighted assets, are known with precision.[68] The overall quantity is still designated as a percentage, because determination of the operational risk is not exactly computed. It depends on a firm's historical accounting-based net-interest income, which can be readily determined in aggregate, but is less exact at the loan level.

Examination of Table 6.2 (unsurprisingly) reveals that the majority of economic capital, for this loan, stems from credit-default risk. Credit-migration risk is relatively modest for this loan; capital buffers and operational risk are also fairly small. The total (discounted) lifetime economic-capital consumption amounts to about EUR 35 million.

The RAROC estimate, quite mechanically, is simply the ratio of the total discounted capital contribution and consumption. This amounts to about $\frac{1.78}{35.4} \approx 5.0\%$. This result is displayed, more precisely, in Table 6.3, which further illustrates a separate RAROC figure for each individual cash-flow period. These annual figures do not have a direct application, but they are nonetheless interesting. They provide some insight into the evolution of annual capital contribution and consumption over the loan's lifetime. The various components of the numerators and denominators are also displayed. The elements of the numerator will change along with key inputs such as lending margin, administrative expenses, funding margin, and other fees. The denominator, which scales the income contribution by risk, varies by

[66] If we wish to compute a RAROC for a treasury investment, the denominator would include slightly different ingredients. See Fig. 6.4 for more detail.

[67] To be really precise, we should write the set of approximation parameters as $\theta_0 \equiv \theta$ to reflect their link to the starting time point—that is, $i = 0$—associated with the calculation.

[68] This depends on the level of the loan's associated risk-weighted assets; this quantity is extensively discussed in Chap. 11.

Table 6.3 *The RAROC calculation*: The underlying table combines the results from Tables 6.1 and 6.2 to illustrate the overall RAROC estimate along with associated values for each individual year.

i	X_i	$\eta_i \cdot X_i \cdot \delta_i$	$\hat{\mathcal{E}}_i(\theta) \cdot \delta_i$	$\text{RAROC}_i = \frac{\eta_i \cdot X_i \cdot \delta_i}{\hat{\mathcal{E}}_i(\theta) \cdot \delta_i}$
1	60,000,000	495,036	8,561,362	5.78%
2	60,000,000	431,622	8,923,618	4.84%
3	60,000,000	428,334	9,030,964	4.74%
4	40,000,000	282,359	5,980,788	4.72%
5	20,000,000	139,174	2,926,779	4.76%
Total	240,000,000	1,776,525	35,423,511	**5.02%**

what specific risks are allocated, the riskiness of the loan, and the current portfolio composition. The actual economic-capital computation also importantly takes into account any existing exposure to the same credit obligor. Adding EUR 60 million of new loan exposure is rather different—in terms of incremental economic capital—from combining this exposure to existing loans of, let's say, EUR 300 million. In the former case, this is a relatively average new exposure. In the latter, it appends to an already fairly important concentration. The consequence is a higher level of risk and thus a larger economic-capital allocation.

6.7.2 Payment Frequency

Questions naturally arise regarding the frequency of payments. The preceding illustrative calculations operate on the assumption that cash-flows occur only on an annual basis. Most real-life loans, however, actually pay on a semi-annual basis. In some institutions, payments might even occur at a quarterly, or even monthly, frequency. How would this impact the actual RAROC computation? We would hope that the impact would be minimal. The situation is analogous to the role of payment frequency on bond-yield calculations; there is an effect, but the result is modest. In the RAROC setting, the magnitude of this effect needs to be determined.

Figure 6.7 addresses this potential concern by illustrating the path of RAROC from the displayed annual frequency incrementing towards a daily payment. While daily payments are obviously somewhat extreme, we see a clear limiting behaviour in the RAROC evolution. The biggest jump occurs from annual to semi-annual payment frequency, although the total change is only a matter of a handful of basis points. Beyond monthly, the change is almost imperceptible as it tends towards continuous compounding. We can conclude, therefore, that there is no material difference between the use of annual or semi-annual cash-flows in the RAROC computation.

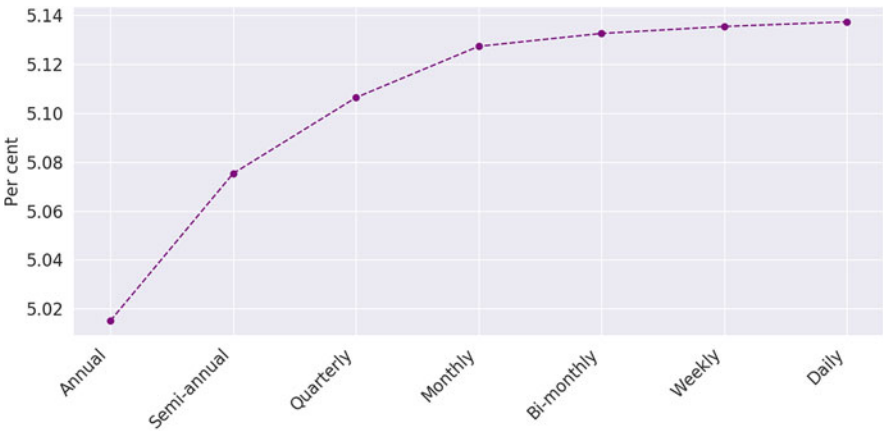

Fig. 6.7 *The role of payment frequency*: This graphic repeats the computation from Table 6.3 for an array of possible payment frequencies ranging from annual to daily. The impact on the final RAROC outcome is modest amounting to only a handful of basis points.

6.7.3 The Lending Margin

The risk-adjusted return on capital calculation is, at its core, a tool to determine an appropriate level of lending margin. Administrative expenses, discount factors, funding margins, and loan impairments are beyond the loan originator's control. Economic capital, determined principally by the obligor-level attributes, is also a given. The size and tenor of the loan are, of course, subject to some degree of negotiation. These are, however, largely determined by the obligor's preferences and the requirements of the underlying project. The only remaining degree of freedom for the loan originator relates to the income component: upfront and commitment fees as well as lending-related margins. Among these quantities, the lending margin has the greatest impact and, as a consequence, plays the most important role. Indeed, loan pricing is principally about finding an appropriate lending margin. As indicated in the introduction to this chapter, the lending margin is basically our *choice* variable.

Mechanically, loan pricing could be quite easy. One need only find the lending margin that equates the RAROC calculation with the firm's internal hurdle rate. Any level of lending margin above that point would meet the institution's internal growth target. The seemingly simple procedure masks an underlying problem: the lending institution does not operate in a vacuum. For many loan transactions, more specifically, there is an external market price. In some cases, this price relates to a deep and liquid set of external sources of financing. In others, it might represent a few indicative quotes from other potential lenders. Thus, although the depth of

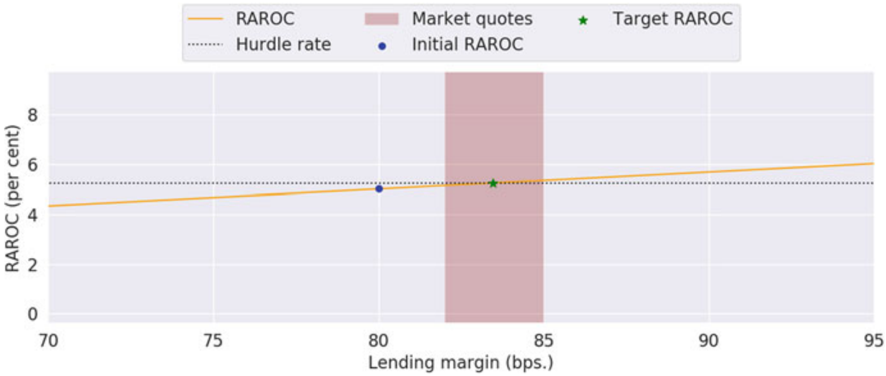

Fig. 6.8 *Finding the lending margin*: Finding the lending margin is a tricky affair. The lending originator must negotiate between meeting the firm hurdle rate and setting a margin consistent with market pricing. The lack, in many cases, of a single clear signal from the market confuses the task. The example above attempts, in an illustrative manner, to describe this process.

the market is variable, there is almost invariably an external comparator for loan pricing.[69] Lending margins need to be determined within this context.

Figure 6.8 highlights—within our simple loan example—the relationship between the lending margin and the RAROC value. Assuming an invented hurdle rate of $5\frac{1}{4}\%$, the lending margin would need to be increased from 80 to about 84 basis points. This is the simple intersection of the RAROC curve—as a function of lending margin—and the hurdle rate. For illustrative purposes, a range of possible market quotes are presented in Fig. 6.8 in the neighbourhood of 82 to 85 basis points. In a real-life situation, these could possibly deviate importantly from this hurdle-rate break-even point. The loan originator needs to find a balance between the internal-growth requirements of the lending institution and the competitive forces summarized by external market prices.

Colour and Commentary 73 (THE KEY CHOICE VARIABLE)*: Many elements in the loan origination process are beyond the lender's control. Administrative costs and funding margins are, in principle, controllable, but they involve larger processes outside the lending area. Loan impairments and economic-capital consumption relate principally to the riskiness of the credit obligor. The principal area of choice relates to the lending margin. Even in this case, however, flexibility is limited. Selection of the lending margin involves walking a fine line between the firm's hurdle rate and market prices.*

(continued)

[69] For small to medium firms, the process of price discovery is complicated by a relatively small number of potential lenders. Moreover, the flow of information regarding these lender's pricing intentions is not entirely transparent.

Colour and Commentary 73 (continued)
An overly aggressive lending margin might undercut the market and gain the business, but at the cost of internal profitability and future growth prospects. Too large a lending margin, however, may make the offer uncompetitive and undermine the firm's ability to make the loan. Despite the inherent challenges, determination of the lending margin is the central task of the loan-pricing problem. In short, it is our principal choice *variable.*

6.7.4 Existing Loan Exposure

In our straightforward example, the EUR 60 million deal was assumed to represent *new* lending for the institution. In other words, the loan portfolio does not include any other exposure to the same credit obligor. This is not always the case. Indeed, it is not uncommon for a firm to engage in multiple loans of various sizes and tenors with a single counterpart. Leveraging existing client relationships is, after all, simply good business. From a risk perspective, this is not immediately problematic, but it does require some adjustments. In particular, a large concentration with a single client involves more risk than numerous smaller exposures with a range of obligors. The RAROC calculation, and more particularly the economic-capital computation, needs to take this dimension into account.

Figure 6.9 illustrates the impact of varying levels of existing exposure—holding all other variables constant—associated with our sample loan.[70] The first point, which involves no existing client exposure, corresponds to the roughly 5% RAROC estimate from Table 6.3. As we increase existing obligor loans gradually to EUR 500 million, the RAROC falls non-linearly to approximately 4.3%. While this general trend will apply for other loans, the specific shape of the curve will depend on the obligor's risk attributes and the general composition of the portfolio.

The initial economic-capital consumption—as a percentage of the loan exposure—is also presented for the various existing loan values. We see that it starts around 14% (i.e., EUR 8.5 million) and increases (also non-linearly) by about 2.5% points to 16.5% (i.e., almost EUR 10 million). The concentration effect is thus sufficiently large that it cannot be ignored in the RAROC computation.[71]

[70] In this example, the existing loans are assumed to have an average maturity of four years.

[71] Practically, the calculation of this effect is conceptually straightforward. If N and O denote the *new* and *existing* (i.e., old) loan exposures, respectively, then

$$\hat{\mathcal{E}}_\theta(N) = \hat{\mathcal{E}}_\theta(O+N) - \hat{\mathcal{E}}_\theta(O), \tag{6.47}$$

where, $\hat{\mathcal{E}}_\theta(\cdot)$ denotes the economic-capital approximation. This computation thus requires *three* separate evaluations of the approximation approach from Chap. 5.

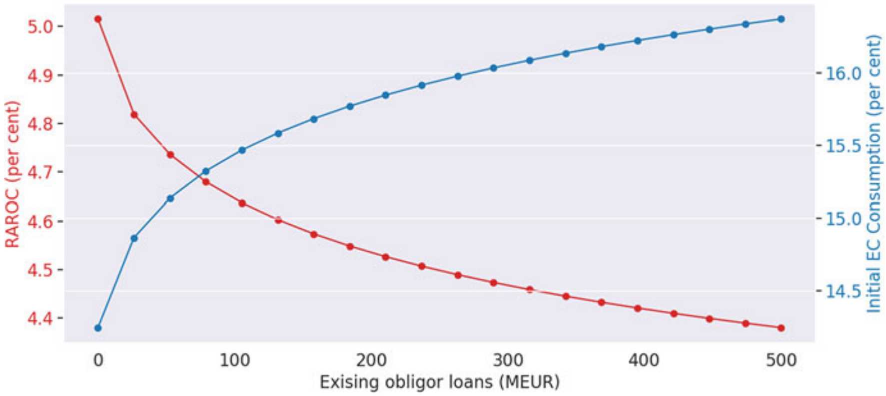

Fig. 6.9 *Concentration effects*: Economic-capital consumption is a (complicated) non-linear function of the overall exposure to an obligor. New loans, therefore, must be handled differently from loans adding to existing exposures. The preceding graphic outlines the RAROC impact, for our example loan, associated with varying levels of existing exposure (all else equal).

Colour and Commentary 74 (CONCENTRATION EFFECTS)*: The preceding chapters describe, in great detail, the technical aspects of the computation of economic capital. Much of this discussion surrounded the role of single-name and portfolio concentrations. The loan-pricing problem—and more particularly, the impact of existing obligor exposure for new loans—is a concrete, practical application of this modelling dimension. Incorporation of existing and outstanding loans into the economic-capital estimates within the RAROC computation is* not *optional. A simple example illustrates clearly that the economic-capital consumption increases significantly (and non-linearly), with a corresponding decrease in the associated RAROC value. This information is critical in the determination of efficient lending margins reflecting existing portfolio concentrations.*[a]

[a] This simple fact also explains why multiple lenders, when asked to provide quotes on a given deal, often provide such a surprisingly wide range of values. A given loan can, and will, impact their own portfolios in different ways.

6.7.5 Forward-Starting Disbursements

Not all loans disburse quickly after performance of one's RAROC analysis. Indeed, not all loans disburse in their entirety at one point in time. In some cases, multiple disbursements might occur over a relatively lengthy period of time. This presents a few new challenges. In particular,

1. committed, but-not-yet disbursed, loans are included in a firm's economic-capital calculation;
2. a fee is customarily charged for these committed loans; and
3. decisions need to be taken regarding the incurrence of administrative and expected-loss costs associated with undisbursed, committed loans.

Each of these issues will, of course, impact the final RAROC computation. Let's examine them all in the context of our running example.

Figure 6.10 displays a potential twist on our loan example introduced in Fig. 6.5. One half of the loan disburses in one year, with the remainder going out the door in two years' time. At the time of final disbursement, the five-year loan tenor clock begins. The final maturity occurs after seven years, but the basic amortization profile remains unchanged; it is simply shifted forward two years.

To accommodate this dimension, it is necessary to introduce two new concepts: the disbursement and the commitment. These have, of course, always been lurking around in the background. We now simply need to make them more visible. On the same time grid—$i = 0, 1, \ldots, \beta$—we can denote the loan disbursement at time i as D_i. If we let X represent the total loan notional amount, then the commitment amount at time i—referred to as C_i—can be inferred as,

$$\text{Commitment}_i = \text{Notional amount} - \text{Cumulative disbursements up until time } i,$$

$$C_i = X - \sum_{k=0}^{i-1} D_i, \tag{6.48}$$

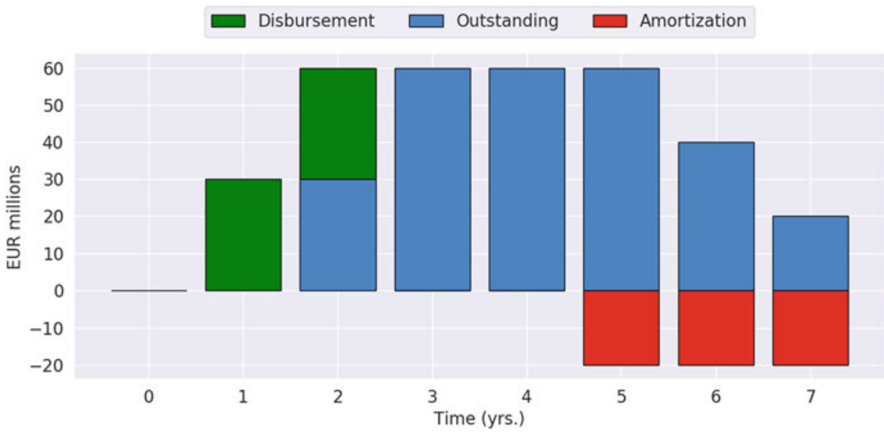

Fig. 6.10 *A forward starting loan*: This graph illustrates the same basic loan example, introduced in Figure 6.5, but changes the starting conditions. The disbursement occurs in two steps: one- and two-years in the future, respectively.

for $i = 1, \ldots, \beta$. The loan commitments, as we would expect, decrease as the amount of loan disbursements approach the agreed total loan level.[72] In our previous five-year, immediate-disbursement loan example, $D_0 = X$. The consequence was the rather uninteresting case of C_i being identically zero for all i. Naturally, in such cases, there is little incremental value associated with including them in the RAROC calculation. When disbursements are spread out over future periods, conversely, the commitments play a rather more important role.

To properly accommodate the fee income associated with these commitments, it is necessary to update Eq. 6.35 to include the associated fees. The expected marginal loan income thus becomes

$$\begin{aligned}\begin{array}{c}\text{Expected}\\\text{Marginal}\\\text{Loan}\\\text{Income}\end{array} &= \begin{array}{c}\text{Commit-}\\\text{ment}\\\text{Fees}\end{array} + \begin{array}{c}\text{Upfront}\\\text{Fees}\end{array} + \begin{array}{c}\text{Lending}\\\text{Margin}\end{array} - \begin{array}{c}\text{(Weighted)}\\\text{Funding}\\\text{Margin}\end{array} - \begin{array}{c}\text{Admin}\\\text{Expenses}\end{array}\\ &\quad - \begin{array}{c}\text{Loan}\\\text{Impair-}\\\text{ments}\end{array} - \begin{array}{c}\text{(Weighted)}\\\text{Equity}\\\text{Cost}\end{array}, \qquad (6.49)\\ &= \sum_{i=1}^{\beta} \Delta_i \left(cC_i + \left(\mathbb{I}_{i=1} u + m - \left(1 - \frac{\mathcal{E}_i}{X_i}\right) f - a - \gamma p_i \right) X_i \right.\\ &\quad \left. - \Big(\mathcal{H} - F(t, T_{i-1}, T_i) \Big) \mathcal{E}_i \right) \delta_i,\end{aligned}$$

where the RAROC computation itself has the following revised form:

$$\text{RAROC} \approx \frac{\sum_{i=1}^{\beta} \Delta_i \Big(cC_i + \eta_i X_i \Big) \delta_i}{\sum_{i=1}^{\beta} \hat{\mathcal{E}}_i(\theta)\delta_i} - \Big(\mathcal{H} - L(t, T) \Big), \tag{6.50}$$

where, to be clear, c represents the percentage commitment fee.

Table 6.4 provides a revised version of Table 6.1 for this two-year, gradual forward-start version of our loan. Beyond two additional years of cash-flows, there are new columns for commitments and commitment fees. After one year, the loan commitment remains at EUR 60 million. Half of the loan was disbursed on this date, but the commitment fee is earned on the undisbursed balance over the entire first year. In the second year, by contrast, the commitment fees are only earned

[72] A lag is necessary, since these values are used for cash-flows and economic-capital calculations. At time i, we need to know the commitment as of time $i - 1$ to properly compute these quantities.

Table 6.4 *The forward RAROC numerator*: The underlying table illustrates, in the context of our practical example outlined in Fig. 6.5, the various elements of the RAROC numerator organized by individual cash-flow date. It compares to Table 6.1, but has been modified to incorporate a gradual two-year forward start.

i	C_i	X_i	$\mathbb{I}_{i=1} u \cdot X_i$	$c \cdot C_i$	$m \cdot X_i$	$\left(1 - \frac{\hat{\mathcal{E}}_i(\theta)}{X_i}\right) \cdot f \cdot X_i$	$-a \cdot X_i$	$-\gamma \cdot p_i \cdot X_i$	$c \cdot C_i + \eta_i \cdot X_i$	$\eta_i \cdot X_i \cdot \delta_i$
1	60,000,000	0	60,000	48,000	0	0	-0	-0	108,000	108,194
2	30,000,000	30,000,000	0	24,000	240,000	11,644	-27,000	-10,888	237,755	238,418
3	0	60,000,000	0	0	480,000	26,116	-54,000	-23,500	428,616	428,320
4	0	60,000,000	0	0	480,000	26,047	-54,000	-25,340	426,707	423,458
5	0	60,000,000	0	0	480,000	25,978	-54,000	-27,297	424,681	417,323
6	0	40,000,000	0	0	320,000	17,310	-36,000	-19,586	281,725	273,500
7	0	20,000,000	0	0	160,000	8675	-18,000	-10,528	140,146	134,136
Total	90,000,000	270,000,000	60,000	72,000	2,160,000	115,770	-243,000	-117,139	2,047,631	**2,023,349**

Table 6.5 *The forward RAROC denominator*: This table outlines, following from the example in Fig. 6.5, the different aspects of the RAROC denominator organized by individual cash-flow date.

i	$\mu \cdot C_i + X_i$	$\hat{\mathcal{D}}_i(\theta)$	$\hat{\mathcal{M}}_i(\theta)$	$\hat{\mathcal{B}}_i$	$\hat{\mathcal{E}}_i(\theta)$	$\hat{\mathcal{D}}_i(\theta) \cdot \delta_i$
1	46,200,000	5,427,769	106,294	998,211	6,532,274	6,543,987
2	53,100,000	6,612,889	100,051	1,147,295	7,860,234	7,882,147
3	60,000,000	7,680,664	87,218	1,296,378	9,064,260	9,058,009
4	60,000,000	7,846,209	60,154	1,296,378	9,202,742	9,132,658
5	60,000,000	8,011,438	31,875	1,296,378	9,339,692	9,177,858
6	40,000,000	5,348,804	30,390	864,252	6,243,446	6,061,186
7	20,000,000	2,640,509	10,236	432,126	3,082,871	2,950,648
Total	339,300,000	43,568,281	426,217	7,331,019	51,325,518	**50,806,493**

on the EUR 30 million committed balance. The total marginal economic-capital contribution associated with the revised example increases to about EUR 2 million.

The remaining non-commitment column calculations in Table 6.4 operate in the same manner as before, but with one important exception. The assumption is that the administrative expenses and expected loss are only allocated to the disbursed loan amounts. This choice can be disputed. One might legitimately argue that administrative costs are also incurred for undisbursed loans. Moreover, there is typically an allowance for committed loans in the loan-impairment calculation. Arguments can be made in either direction and ultimately it comes down to a trade-off between accuracy and desired organizational incentives. Overloading forward-start loans with excessive costs might discourage such activity; this might create structural disadvantages for certain types of loans and obligors. Excluding these costs could, however, encourage them and lead to lower profitability and reduced capacity for capital growth.

Table 6.5—which is directly comparable to Table 6.2—turns to the denominator of our forward-start example. The same characters are present, albeit with two additional years of calculations. The second column has a slightly different form. A central input to the economic-capital calculation—often referred to as the exposure-at-default—is redefined as,

$$\text{Economic Capital Exposure}_i = \mu C_i + X_i, \tag{6.51}$$

where $\mu \in [0, 1]$. The remaining approximation inputs are unchanged. Loan commitments are an economic obligation on the part of the lending institution. To the extent that they earn commitment fees and represent potential future earnings, they are also a firm asset. As an (off-) balance sheet asset, it is logically important to also capture their risks. Ignoring them completely would be incorrect. The question is: to what extent should we incorporate these inherently uncertain commitments into our risk calculations? The constant, μ, is a multiplier that attempts to address this question. It modifies the commitments by a value in the unit interval. A value of zero ignores them, while a value of unity implies certain disbursement. In regulatory

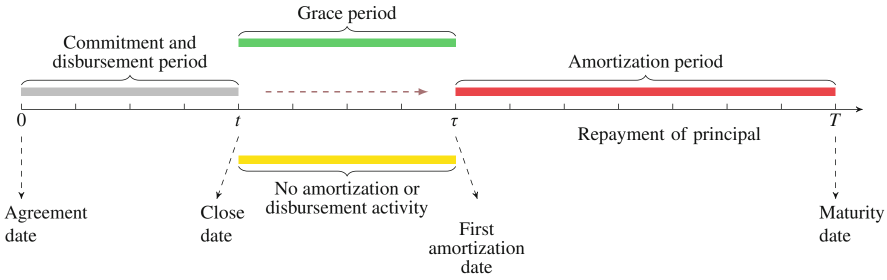

Fig. 6.11 *Loan life-cycle schematic*: This schematic illustrates, in a stylized manner, the life cycle of a loan. It helps us to visualize the commitment, grace, and amortization periods.

circles, this constant is referred to as a credit-conversion factor or CCF.[73] This example sets this value to 0.75. The consequence of this setting, and Eq. 6.51, is that despite the forward start, economic-capital is consumed from the agreement date of this loan. The total, discounted, marginal lifetime economic-capital consumption from the forward-start version of the loan is about EUR 50 million.

Figure 6.11 helps to visualize all of these moving parts by providing a schematic of the loan life cycle. It begins with the agreement date, followed by the commitment period. As we saw earlier, the loan is disbursed in one or more instalments during this time interval. Once the loan closes—or, alternatively, is fully disbursed—the grace period follows. During this time span, interest payments proceed as normal, but no principal is repaid.[74] After the expiry of the grace period, each payment is a combination of principal and interest until final maturity. Often the agreement and close dates are very close together—as in our original version of the loan example—so the commitment period is not particularly exciting. When this is not the case, however, this loan schematic needs to be kept in mind to correctly manage the RAROC calculation.

Table 6.6 closes out our loan example—in a manner similar to Table 6.3—by summarizing all of the numerator and denominator elements and combining them to illustrate the annual, and overall, RAROC figures. The total RAROC is mechanically computed as $\frac{2.0}{50.8} \approx 4.0\%$. This is significantly lower than in the immediate disbursement case. Examining the annual RAROC values explains the difference: the first few years—when we earn only the commitment fees—have a significantly lower level of return. This should be no surprise. The commitment fee is a relatively small value compared to the $\mu = 75\%$ consumption of economic capital. If

[73] As one would expect, regulators also put limits and provide guidance on the appropriate choice of μ. Chapter 4 discusses this point in more detail.

[74] For a bullet-style loan, the grace period extends to the final maturity, when the full notional amount is repaid.

Table 6.6 *The forward RAROC calculation*: The underlying table combines the results from Tables 6.4 and 6.5 to illustrate the overall RAROC estimate along with associated values for each individual year.

i	C_i	X_i	$\mu \cdot C_i + X_i$	$c \cdot C_i + \eta_i \cdot X_i \cdot \delta_i$	$\hat{\mathcal{E}}_i(\theta) \cdot \delta_i$	$\text{RAROC}_i = \frac{(c \cdot C_i + \eta_i \cdot X_i) \cdot \delta_i}{\hat{\mathcal{E}}_i(\theta) \cdot \delta_i}$
1	60,000,000	0	46,200,000	108,194	6,543,987	1.65%
2	30,000,000	30,000,000	53,100,000	238,418	7,882,147	3.02%
3	0	60,000,000	60,000,000	428,320	9,058,009	4.73%
4	0	60,000,000	60,000,000	423,458	9,132,658	4.64%
5	0	60,000,000	60,000,000	417,323	9,177,858	4.55%
6	0	40,000,000	40,000,000	273,500	6,061,186	4.51%
7	0	20,000,000	20,000,000	134,136	2,950,648	4.55%
Total	90,000,000	270,000,000	339,300,000	2,023,349	50,806,493	**3.98%**

administrative and impairment costs were included, this would act to further reduce the risk-adjusted return associated with this forward-start loan example.

6.7.6 Selecting Commitment Fees

In their basic form, the RAROC estimates of the immediate-disbursement and forward-start versions of our loan example differ by about 100 basis points. The reason stems from the inability of the 8 basis-point fee to compensate for the loan-commitment economic capital and the reduced lending-margin receipts during the commitment period. This raises an interesting question: what level of commitment fees would make the firm indifferent between the immediate and forward-start versions of our sample loan?

Figure 6.12 attempts to answer this question by illustrating the RAROC—under *ceteris paribus* conditions—of our forward-start loan for commitment fees ranging from zero to 80 basis points. The initial RAROC of 4% relates to a commitment fee, as seen previously, of eight basis points. The break-even commitment fee amounts to roughly 65 basis points. This makes logical sense; it is roughly equal to the product of the credit-conversion factor ($\mu = 0.75$) and the original lending margin of 75 basis points.

This analysis suggests that a likely unreasonably high level of commitment fee would be necessary to compensate for the forward-start nature of the loan. In practice, however, the commitment fee is not the only tool available to the loan originator. A combination of a small increase in lending margin and upfront fee—in addition to an augmentation of the commitment fee—might also accomplish the

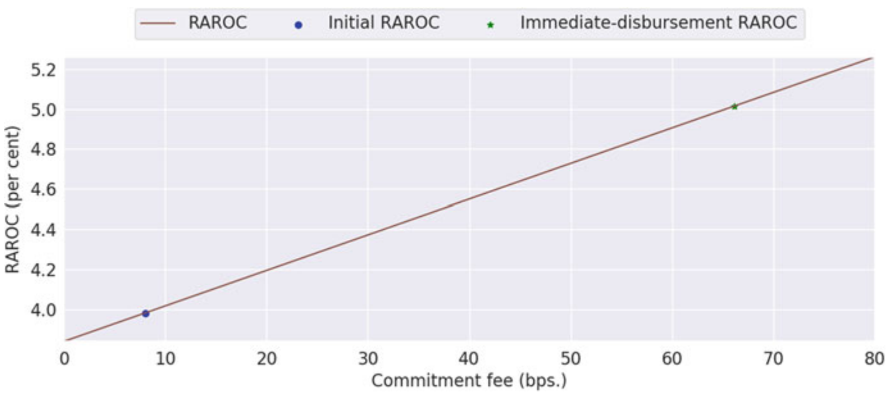

Fig. 6.12 *The commitment fee*: The difference between the commitment fee and the lending margin tends to lead, for forward-start loans, to a drop-off in risk-adjusted returns on capital. The preceding graphic illustrates the relationship between the RAROC and the commitment fee. All else equal, a value of approximately 65 basis points is required to break-even with the immediate disbursement loan.

same result. Naturally, the institution (and the client) may not view these various fee elements as equivalent, but the option still remains. The larger picture is that, without some adjustment, a forward-start loan appears to structurally lead to lower RAROC outcomes.

Colour and Commentary 75 (NON-IMMEDIATE LOAN DISBURSEMENT)*: Not all loans are immediately, or rather quickly, disbursed. Some involve a rolling forward start with multiple disbursement instalments ranging over a relatively lengthy time period. These loans, once agreed, find themselves in a kind of limbo. They are not fully loans, but they do represent an economic obligation to the lending institution. We refer to the associated undisbursed loan balances as* commitments. *Computation of risk-adjusted return on capital for such forward-starting loans requires making practical (and defensible assumptions) regarding these loan commitments. In particular, one needs to decide upon the amount of capital they consume, how much fee income they earn, and the treatment of administrative and loan-impairment expenses during the commitment period. These choices have an important influence on the RAROC computations and, ultimately, the desirability of such forward-starting loan activity.*

6.8 Wrapping Up

This chapter addresses the thorny, but central, question of loan pricing. Determining an appropriate pricing structure for a given loan includes both qualitative and quantitative elements. Although extremely important, the preceding discussion does not spend much time on the qualitative dimension. Exclusion of these strategic elements from our focus—principally due to difficulties in their quantitative model-based assessment—should *not* be interpreted as a statement on their importance. On the contrary, we must not lose sight of their central role in the loan-origination process.

This chapter is essentially about building a supporting quantitative infrastructure for the complicated business of originating loans. Some heavy lifting was required. The core idea is to treat each loan decision as an investment project or, as it is referred in the literature, as a capital-budgeting decision. Asset-pricing theory and key corporate-finance concepts were combined to construct a structural description of the risk-adjusted return on capital. Further embedded in this calculation is the full machinery of the economic-capital simulation engine and associated approximations developed in Chaps. 2 to 5. Such technical detail is inevitable for building a coherent approach to loan pricing; the consequence of this effort is the useful and flexible RAROC calculation.

The embedded theoretical elements of the RAROC calculation provide insight into a range of practical questions. Where should one set the lending margin to meet internal growth targets? What is the impact associated with delayed disbursement, or forward-start, loans? How might the magnitude of the commitment fee influence the forward-start decision? Does the payment frequency make an important difference? How should existing loans with a given obligor influence our decision to extend additional credit? These, and many more, legitimate queries need to be addressed in the loan-origination process. While they need to be married with a range of important qualitative factors, these critical practical questions build the foundation for supporting decisions on lending proposals.

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