Chromatic Bounds for Some Subclasses of $(P_3 \cup P_2)$-free graphs

Athmakoori Prashant[1(✉)], P. Francis[2], and S. Francis Raj[1]

[1] Department of Mathematics, Pondicherry University, Puducherry 605014, India
11994prashant@gmail.com, francisraj_s@pondiuni.ac.in
[2] Department of Computer Science, Indian Institute of Technology, Palakkad 678557, India
pfrancis@iitpkd.ac.in

Abstract. The class of $2K_2$-free graphs has been well studied in various contexts in the past. It is known that the class of $\{2K_2, 2K_1 + K_p\}$-free graphs and $\{2K_2, (K_1 \cup K_2) + K_p\}$-free graphs admits a linear χ-binding function. In this paper, we study the classes of $(P_3 \cup P_2)$-free graphs which is a superclass of $2K_2$-free graphs. We show that $\{P_3 \cup P_2, 2K_1 + K_p\}$-free graphs and $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graphs also admits a linear χ-binding function. In addition, we give tight chromatic bounds for $\{P_3 \cup P_2, HVN\}$-free graphs and $\{P_3 \cup P_2, diamond\}$-free graphs, and it can be seen that the latter is an improvement of the existing bound given by A. P. Bharathi and S. A. Choudum [1].

Keywords: Chromatic number · χ-binding function · $(P_3 \cup P_2)$-free graphs · Perfect graphs

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1 Introduction

All graphs considered in this paper are simple, finite and undirected. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. For any positive integer k, a *proper k-coloring* of a graph G is a mapping $c : V(G) \to \{1, 2, \ldots, k\}$ such that adjacent vertices receive distinct colors. If a graph G admits a proper k-coloring, then G is said to be *k-colorable*. The *chromatic number*, $\chi(G)$, of a graph G is the smallest k such that G is k-colorable. Let P_n, C_n and K_n respectively denote the path, the cycle and the complete graph on n vertices. For $S, T \subseteq V(G)$, let $N_T(S) = N(S) \cap T$ (where $N(S)$ denotes the set of all neighbors of S in G), let $\langle S \rangle$ denote the subgraph induced by S in G and let $[S, T]$ denote the set of all edges with one end in S and the other end in T. If every vertex in S is adjacent with every vertex in T, then $[S, T]$ is said to be complete. For any graph G, let $\overline{G}$ denote the complement of G.

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Let $\mathcal{F}$ be a family of graphs. We say that G is *$\mathcal{F}$-free* if it does not contain any induced subgraph which is isomorphic to a graph in $\mathcal{F}$. For a fixed graph H, let us denote the family of H-free graphs by $\mathcal{G}(H)$. For any two disjoint graphs G_1 and G_2 let $G_1 \cup G_2$ and $G_1 + G_2$ denote the *union* and the *join* of G_1 and G_2 respectively. Let $\omega(G)$ and $\alpha(G)$ denote the *clique number* and *independence number* of a graph G respectively. When there is no ambiguity, $\omega(G)$ will be denoted by ω. A graph G is said to be *perfect* if $\chi(H) = \omega(H)$, for every induced subgraph H of G.

In order to determine an upper bound for the chromatic number of a graph in terms of their clique number, the concept of χ-binding functions was introduced by A. Gyárfás in [4]. A class $\mathcal{G}$ of graphs is said to be *χ-bounded* [4] if there is a function f (called a χ-binding function) such that $\chi(G) \leq f(\omega(G))$, for every $G \in \mathcal{G}$. We say that the χ-binding function f is *special linear* if $f(x) = x + c$, where c is a constant.

The family of $2K_2$-free graphs has been well studied. A. Gyárfás in [4] posed a problem which asks for the order of magnitude of the smallest χ-binding function for $\mathcal{G}(2K_2)$. In this direction, T. Karthick et al. in [5] proved that the families of $\{2K_2, H\}$-free graphs, where $H \in \{HVN, diamond, K_1 + P_4, K_1 + C_4, \overline{P_5}, \overline{P_2 \cup P_3}, K_5 - e\}$ admit a special linear χ-binding functions. The bounds for $\{2K_2, K_5 - e\}$-free graphs and $\{2K_2, K_1 + C_4\}$-free graphs were later improved by Athmakoori Prashant et al., in [7,8]. In [2], C. Brause et al., improved the χ-binding function for $\{2K_2, K_1 + P_4\}$-free graphs to $\max\{3, \omega(G)\}$. Also they proved that for $s \neq 1$ or $\omega(G) \neq 2$, the class of $\{2K_2, (K_1 \cup K_2) + K_s\}$-free graphs with $\omega(G) \geq 2s$ is perfect and for $r \geq 1$, the class of $\{2K_2, 2K_1 + K_r\}$-free graphs with $\omega(G) \geq 2r$ is perfect. Clearly when $s = 2$ and $r = 3$, $(K_1 \cup K_2) + K_s \cong HVN$ and $2K_1 + K_r \cong K_5 - e$ which implies that the class of $\{2K_2, HVN\}$-free graphs and $\{2K_2, K_5 - e\}$-free graphs are perfect for $\omega(G) \geq 4$ and $\omega(G) \geq 6$ respectively which improved the bounds given in [5].

Motivated by C. Brause et al., and their work on $2K_2$-free graphs in [2], we started looking at $(P_3 \cup P_2)$-free graphs which is a superclass of $2K_2$-free graphs. In [1], A. P. Bharathi et al., obtained a $O(\omega^3)$ upper bound for the chromatic number of $(P_3 \cup P_2)$-free graphs and obtained sharper bounds for $\{P_3 \cup P_2, diamond\}$)-free graphs. In this paper, we obtain linear χ-binding functions for the class of $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graphs and $\{P_3 \cup P_2, 2K_1 + K_p\}$-free graphs. In addition, for $\omega(G) \geq 3p - 1$, we show that the class of $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graphs admits a special linear χ-binding function $f(x) = \omega(G) + p - 1$ and the class of $\{P_3 \cup P_2, 2K_1 + K_p\}$-free graphs are perfect. In addition, we give a tight χ-binding function for $\{P_3 \cup P_2, HVN\}$-free graphs and $\{P_3 \cup P_2, diamond\}$-free graphs. This bound for $\{P_3 \cup P_2, diamond\}$-free graphs turns out to be an improvement of the existing bound obtained by A. P. Bharathi et al., in [1].

Some graphs that are considered as forbidden induced subgraphs in this paper are given in Fig. 1. Notations and terminologies not mentioned here are as in [10].

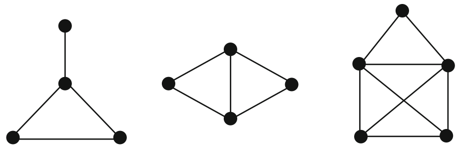

Fig. 1. Some special graphs

2 Preliminaries

Throughout this paper, we use a particular partition of the vertex set of a graph G as defined initially by S. Wagon in [9] and later improved by A. P. Bharathi et al., in [1] as follows. Let $A = \{v_1, v_2, \ldots, v_\omega\}$ be a maximum clique of G. Let us define the *lexicographic ordering* on the set $L = \{(i,j) : 1 \leq i < j \leq \omega\}$ in the following way. For two distinct elements $(i_1, j_1), (i_2, j_2) \in L$, we say that (i_1, j_1) precedes (i_2, j_2), denoted by $(i_1, j_1) <_L (i_2, j_2)$ if either $i_1 < i_2$ or $i_1 = i_2$ and $j_1 < j_2$. For every $(i,j) \in L$, let $C_{i,j} = \{v \in V(G)\backslash A : v \notin N(v_i) \cup N(v_j)\} \setminus \left\{ \underset{(i',j') <_L (i,j)}{\cup} C_{i',j'} \right\}$. Note that, for any $k \in \{1, 2, \ldots, j\}\backslash\{i,j\}$, $[v_k, C_{i,j}]$ is complete. Hence $\omega(\langle C_{i,j}\rangle) \leq \omega(G) - j + 2$.

For $1 \leq k \leq \omega$, let us define $I_k = \{v \in V(G)\backslash A : v \in N(v_i)$, for every $i \in \{1, 2, \ldots, \omega\}\backslash\{k\}\}$. Since A is a maximum clique, for $1 \leq k \leq \omega$, I_k is an independent set and for any $x \in I_k$, $xv_k \notin E(G)$. Clearly, each vertex in $V(G)\backslash A$ is non-adjacent to at least one vertex in A. Hence those vertices will be contained either in I_k for some $k \in \{1, 2, \ldots, \omega\}$, or in $C_{i,j}$ for some $(i,j) \in L$. Thus $V(G) = A \cup \left(\overset{\omega}{\underset{k=1}{\cup}} I_k \right) \cup \left(\underset{(i,j) \in L}{\cup} C_{i,j} \right)$. Sometimes, we use the partition $V(G) = V_1 \cup V_2$, where $V_1 = \underset{1 \leq k \leq \omega}{\cup} (\{v_k\} \cup I_k) = \underset{1 \leq k \leq \omega}{\cup} U_k$ and $V_2 = \underset{(i,j) \in L}{\cup} C_{i,j}$.

Let us recall a result on $(P_3 \cup P_2)$-free graphs given by A. P. Bharathi et al., in [1].

Theorem 1 [1]**.** *If a graph G is $(P_3 \cup P_2)$-free, then $\chi(G) \leq \frac{\omega(G)(\omega(G)+1)(\omega(G)+2)}{6}$.*

Without much difficulty one can make the following observations on $(P_3 \cup P_2)$-free graphs.

Fact 2. *Let G be a $(P_3 \cup P_2)$-free graph. For $(i,j) \in L$, the following holds.*

(i) Each $C_{i,j}$ is a disjoint union of cliques, that is, $\langle C_{i,j}\rangle$ is P_3-free.

(ii) For every integer $s \in \{1, 2, \ldots, j\}\backslash\{i,j\}$, $N(v_s) \supseteq \{C_{i,j} \cup A \cup (\overset{\omega}{\underset{k=1}{\cup}} I_k)\}\backslash\{v_s \cup I_s\}$.

3 $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graphs

Let us start Sect. 3 with some observations on $((K_1 \cup K_2) + K_p)$-free graphs.

Proposition 1. *Let G be a $((K_1 \cup K_2) + K_p)$-free graph with $\omega(G) \geq p + 2$, $p \geq 1$. Then G satisfies the following.*

(i) For $k, \ell \in \{1, 2, \ldots, \omega(G)\}$, $[I_k, I_\ell]$ is complete. Thus, $\langle V_1 \rangle$ is a complete multipartite graph with $U_k = \{v_k\} \cup I_k$, $1 \leq k \leq \omega(G)$ as its partitions.
(ii) For $j \geq p + 2$ and $1 \leq i < j$, $C_{i,j} = \emptyset$.
(iii) For $x \in V_2$, x has neighbors in at most $(p - 1)$ U_ℓ's where $\ell \in \{1, 2, \ldots, \omega(G)\}$.

Proposition 2. *Let G be a $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graph with $\omega(G) \geq p + 2$, $p \geq 1$. Then G satisfies the following.*

(i) For $(i, j) \in L$ such that $j \leq p+1$, if $\omega(\langle C_{i,j} \rangle) \geq p - j + 4$, then $\omega(\langle C_{k,j} \rangle) \leq 1$ for $k \neq i$ and $1 \leq k \leq j - 1$.
(ii) If $\omega(G) = (p + 2 + k)$, $k \geq 0$, then $\left\langle \left(\bigcup_{j=\max\{2, p+1-\lfloor \frac{k}{2} \rfloor\}}^{p+1} \left(\bigcup_{i=1}^{j-1} C_{i,j} \right) \right) \right\rangle$ is P_3-free.

As a consequence of Proposition 1, we obtain Corollary 1 which is a result due to S. Olariu in [6].

Corollary 1 [6]**.** *Let G be a connected graph. Then G is paw-free graph if and only if G is either K_3-free or complete multipartite.*

Now, for $p \geq 1$ and $\omega \geq \max\{3, 3p - 1\}$, let us determine the structural characterization and the chromatic number of $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graphs.

Theorem 3. *Let p be a positive integer and G be a $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graph with $V(G) = V_1 \cup V_2$. If $\omega(G) \geq \max\{3, 3p - 1\}$, then (i) $\langle V_1 \rangle$ is a complete multipartite graph with partition $U_1, U_2, \ldots, U_\omega$, (ii) $\langle V_2 \rangle$ is P_3-free graph and (iii) $\chi(G) \leq \omega(G) + p - 1$.*

Proof. Let $p \geq 1$ and G be a $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graph with $\omega(G) \geq \max\{3, 3p-1\}$. By (i) of Proposition 1, we see that $\langle V_1 \rangle$ is a complete multipartite graph with the partition $U_k = \{v_k\} \cup I_k$, $1 \leq k \leq \omega$. By (i) of Fact 2, each $\langle C_{i,j} \rangle$ is P_3-free, for every $(i, j) \in L$. Also without much difficulty we can show that $\langle V_2 \rangle$ is P_3-free. Now, let us exhibit an $(\omega + p - 1)$-coloring for G using $\{1, 2, \ldots, \omega + p - 1\}$ colors. For $1 \leq k \leq \omega$, give the color k to the vertices of U_k. Let H be a component in $\langle V_2 \rangle$. Clearly, each vertex in H is adjacent to at most $p - 1$ colors given to the vertices of V_1 and is adjacent to $\omega(H) - 1$ vertices of H. Since $\omega(H) \leq \omega(G)$, each vertex in H is adjacent to at most $\omega(G) + p - 2$ colors. Hence, there is a color available for each vertex in H. Similarly, all the components of $\langle V_2 \rangle$ can be colored properly. Hence, $\chi(G) \leq \omega(G) + p - 1$.

Even though we are not able to show that the bound given in Theorem 3 is tight, we can observe that the upper bound cannot be made smaller than $\omega + \lceil \frac{p-1}{2} \rceil$ by providing the following example. For $p \geq 1$, consider the graph G^* with $V(G^*) = X \cup Y \cup Z$, where $X = \bigcup_{i=1}^{\omega} x_i$, $Y = \bigcup_{i=1}^{\omega} y_i$ and $Z = \bigcup_{i=1}^{p-1} z_i$ if $p \geq 2$ else $Z = \emptyset$ and edge set $E(G^*) = \{\{x_i x_j\} \cup \{y_i y_j\} \cup \{z_r z_s\} \cup \{x_m y_n\} \cup \{y_m z_n\} \cup \{x_n z_n\}$, where $1 \leq i, j, m \leq \omega$, $1 \leq r, s, n \leq p-1$, $i \neq j$, $r \neq s$ and $m \neq n\}$. Without much difficulty, one can observe that G^* is a $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graph, $\omega(G^*) = \omega$ and $\alpha(G^*) = 2$. Hence, $\chi(G^*) \geq \left\lceil \frac{|V(G^*)|}{\alpha(G^*)} \right\rceil = \omega(G^*) + \lceil \frac{p-1}{2} \rceil$.

Next, we obtain a linear χ-binding function for $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graphs with $\omega \geq 3$.

Theorem 4. *Let p be an integer greater than 1. If G is a $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graph, then*

$$\chi(G) \leq \begin{cases} \omega(G) + \sum\limits_{j=2}^{p+1} (j-1)(p-j+3) & \text{for } 3 \leq \omega(G) \leq p+1 \\ \omega(G) + 7(p-1) + \sum\limits_{j=4}^{p-\lfloor \frac{k}{2} \rfloor} (j-1)(p-j+3) & \text{for } \omega(G) = (p+2+k), 0 \leq k \leq 2p-5 \\ \omega(G) + 4p - 3 & \text{for } \omega(G) = 3p-2 \\ \omega(G) + p - 1 & \text{for } \omega(G) \geq 3p-1. \end{cases}$$

Proof. Let G be a $\{P_3 \cup P_2, (K_1 \cup K_2) + K_p\}$-free graph with $p \geq 2$. For $\omega(G) \geq 3p-1$, the bound follows from Theorem 3. By (ii) of Proposition 1, we see that $C_{i,j} = \emptyset$ for all $j \geq p+2$. We know that $V(G) = V_1 \cup V_2$, where $V_1 = \bigcup_{1 \leq \ell \leq \omega} (\{v_\ell\} \cup I_\ell)$ and $V_2 = \bigcup_{(i,j) \in L} C_{i,j}$. Clearly, the vertices of V_1 can be colored with $\omega(G)$ colors. Let us find an upper bound for $\chi(\langle V_2 \rangle)$. First, let us consider the case when $3 \leq \omega(G) \leq p+1$. For $1 \leq i < j \leq p+1$, one can observe that $\omega(\langle C_{i,j} \rangle) \leq \omega(G) - j + 2 \leq p - j + 3$. Thus one can properly color the vertices of $\left(\bigcup_{i=1}^{j-1} C_{i,j} \right)$ with at most $(j-1)(p-j+3)$ colors and hence $\chi(G) \leq \chi(\langle V_1 \rangle) + \chi(\langle V_2 \rangle) \leq \omega(G) + \sum\limits_{j=2}^{p+1} (j-1)(p-j+3)$.

Next, let us consider $\omega(G) = (p+2+k)$, where $0 \leq k \leq 2p-4$. By using (ii) of Proposition 2, $\left\langle \left(\bigcup_{j=p-\lfloor \frac{k}{2} \rfloor+1}^{p+1} \left(\bigcup_{i=1}^{j-1} C_{i,j} \right) \right) \right\rangle$ is a P_3-free graph. As in Theorem 3, one can color the vertices of $V(G) \backslash \left\{ \bigcup_{j=2}^{p-\lfloor \frac{k}{2} \rfloor} \left(\bigcup_{i=1}^{j-1} C_{i,j} \right) \right\}$ with at most $\omega(G)+p-1$ colors. For $k = 2p-4$, $\omega(G) = 3p-2$ and we see that $\langle V_2 \backslash C_{1,2} \rangle$ is P_3-free and $\chi(\langle V(G) \backslash C_{1,2} \rangle) \leq \omega(G)+p-1$. Therefore, $\chi(G) \leq \chi(\langle V(G) \backslash C_{1,2} \rangle) + \chi(\langle C_{1,2} \rangle) = \omega(G) + 4p - 3$. Finally, for $0 \leq k \leq 2p-5$, by using (i) of Proposition 2 and by using similar strategies but with a little more involvement, we can show that $\left(\bigcup_{i=1}^{j-1} C_{i,j} \right)$, where $2 \leq j \leq p - \lfloor \frac{k}{2} \rfloor$ can be properly colored using $\omega(G)$ colors when $\omega(\langle C_{i,j} \rangle) \geq p - j + 4$, for some $i \in \{1, 2, \ldots, j-1\}$ or by using

$(j-1)(p-j+3)$ colors when $\omega(\langle C_{i,j}\rangle) \leq p-j+3$, for every $i \in \{1,2,\ldots,j-1\}$. For $4 \leq j \leq p-\lfloor \frac{k}{2} \rfloor$, one can observe that $(j-1)(p-j+3) \geq \omega(G)$ and hence the vertices of $\left(\bigcup_{j=4}^{p-\lfloor \frac{k}{2} \rfloor}\left(\bigcup_{i=1}^{j-1} C_{i,j}\right)\right)$ can be properly colored with $\sum_{j=4}^{p-\lfloor \frac{k}{2} \rfloor}(j-1)(p-j+3)$ colors. When $j=3$, $(j-1)(p-j+3)=2p$. Since $2p-5 \geq 0$, $p \geq 3$. Also, since $\omega(G) \geq 4$, the vertices of $C_{1,2}$ and $(C_{1,3} \cup C_{2,3})$ can be properly colored with at most $(3p-3)$ colors each. Hence, $\chi(G) \leq (\omega(G)+p-1)+2(3p-3)+\sum_{j=4}^{p-\lfloor \frac{k}{2} \rfloor}(j-1)(p-j+3) = \omega(G)+7(p-1)+\sum_{j=4}^{p-\lfloor \frac{k}{2} \rfloor}(j-1)(p-j+3)$.

The bound obtained in Theorem 4 is not optimal. This can be seen in Theorem 5. Note that when $p=2$, $(K_1 \cup K_2)+K_p \cong HVN$.

Theorem 5. *If G is a $\{P_3 \cup P_2, HVN\}$-free graph with $\omega(G) \geq 4$, then $\chi(G) \leq \omega(G)+1$.*

The graph G^* (defined next to Theorem 3) shows that the bound given in Theorem 5 is tight.

4 $\{P_3 \cup P_2, 2K_1 + K_p\}$-free graphs

Let us start Sect. 4 by observing that any $\{P_3 \cup P_2, 2K_1+K_p\}$-free graph is also a $\{P_3 \cup P_2, (K_1 \cup K_2)+K_p\}$-free graph. Hence the properties established for $\{P_3 \cup P_2, (K_1 \cup K_2)+K_p\}$-free graphs is also true for $\{P_3 \cup P_2, 2K_1+K_p\}$-free graphs.

One can observe that by using techniques similar to the one's used in Theorem 3 and by Strong Perfect Graph Theorem [3], any $\{P_3 \cup P_2, 2K_1+K_p\}$-free graph is perfect, when $\omega \geq 3p-1$.

Theorem 6. *Let p be a positive integer and G be a $\{P_3 \cup P_2, 2K_1+K_p\}$-free graph with $V(G)=V_1 \cup V_2$. If $\omega(G) \geq 3p-1$, then $\langle V_1\rangle$ is complete, $\langle V_2\rangle$ is P_3-free and G is perfect.*

As a consequence of Theorem 4 and Theorem 6, without much difficulty one can observe Proposition 3 and Corollary 2.

Proposition 3. *Let G be a $\{P_3 \cup P_2, 2K_1+K_p\}$-free graph. If $\omega(\langle C_{1,2}\rangle) \geq 2p$, then $\chi(G)=\omega(G)$.*

Corollary 2. *Let p be an integer greater than 1. If G is a $\{P_3 \cup P_2, 2K_1+K_p\}$-free graph, then*

$$\chi(G) \leq \begin{cases} \omega(G)+\sum_{j=2}^{p+1}(j-1)(p-j+3) & \text{for } 3 \leq \omega(G) \leq p+1 \\ \omega(G)+2p-1+\sum_{j=3}^{p-\lfloor \frac{k}{2} \rfloor}(j-1)(p-j+3) & \text{for } \omega(G)=(p+2+k), 0 \leq k \leq 2p-5 \\ \omega(G)+2p-1 & \text{for } \omega(G)=3p-2 \\ \omega(G) & \text{for } \omega(G) \geq 3p-1. \end{cases}$$

When $p = 2$, $2K_1 + K_p \cong diamond$ and hence by Theorem 6, $\{P_3 \cup P_2, diamond\}$-free graphs are perfect for $\omega(G) \geq 5$ which was shown by A. P. Bharathi et al., in [1].

Theorem 7 [1]. *If G is a $\{P_3 \cup P_2, diamond\}$-free graph then* $\chi(G) \leq \begin{cases} 4 \text{ for } \omega(G) = 2 \\ 6 \text{ for } \omega(G) = 3 \\ 5 \text{ for } \omega(G) = 4 \end{cases}$ *and G is perfect if $\omega(G) \geq 5$.*

We can further improve the bound given in Theorem 7 by obtaining a $\omega(G)$-coloring when $\omega(G) = 4$.

Theorem 8. *If G is a $\{P_3 \cup P_2, diamond\}$-free graph then* $\chi(G) \leq \begin{cases} 4 \text{ for } \omega(G) = 2 \\ 6 \text{ for } \omega(G) = 3 \\ 4 \text{ for } \omega(G) = 4. \end{cases}$ *and G is perfect if $\omega(G) \geq 5$.*

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