Chapter 20
The Origins of Modern Logic

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Abstract These are unpublished lecture notes from Popper's Nachlass.

Editorial notes: The source typescript is from KPS Box 366, Folder 19. The text refers to Popper in the third person, which suggests that these are lecture notes that were prepared by one of Popper's students. There is a reference to Popper's lecture notes on logic of 1939–1941 (cf. this volume, Chapter 19), which indicates that he gave these lectures during the same period or shortly afterwards. The page numbers skip number 9, but no page seems to be missing. The typescript uses the notation $\overline{p}$ for the negation of p.

1 Modern logic has had a recent and rapid growth and represents a complete break away from the traditional or Aristotelian logic. We do not mean by this that no relationships can be established between the two logics, on the contrary the Aristotelian logic of syllogism is to be regarded as a small, and not very well formulated portion of the whole field of modern logic, but that the new logic owes only little of its development to the older logic.

The first steps towards a new logic came from the work of mathematicians such as Frege and Peano who used deductive logical methods in working out the foundations of Arithmetic, and Cantor who in his work on infinity introduced the notion of classes or aggregates.

From another angle Boole and Peirce contributed to the rise of the new logic by using mathematical symbolism and methods to state and work out the principles of logic.

Russell's work in Principia Mathematica[a] consists largely in bringing together the work of all these men.

These sources of modern logic emphasize its main differences from traditional logic:

1. The traditional logic had no contact with mathematics. Now whatever view one takes of the nature of mathematics, it is apparent that it is the field of study in which really deductive logical methods are used with most effect. Mathematics makes more use of deductive logical methods than any other science. Exponents of

[a] Whitehead and Russell (1925–1927).

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the traditional logic however confined themselves to the examination of the logic of everyday discourse and were thus divorced from logical practice and unable to understand the character of deductive systems by which a vast superstructure is built upon a few initial postulates.

2. The traditional logic showed little capacity for development and expansion while the new logic derived from the rich source of mathematics, has developed and
2 expanded with great rapidity. |

3. Modern logic realizes the significance and importance of the paradoxes, which the traditional logic failed to do. Epimenides' paradox for example is not even formulated within Aristotelian formalism. It was soon realized that the concept of classes, which was employed in the interpretation of the Boolean Algebra, led to the same paradoxes as those which involved the concept of predicate.

Modern logic unlike the old logic has set itself the task of eliminating the paradoxes, realizing that until this is done there can be no adequate system of deductive logic. If in a system there is a contradiction then in that system one can prove any sentence whatsoever (and therefore its negation), and from contradictory premises follows any statement whatever and its negative. A deductive system which did not *both* permit something and exclude something else would be useless.

To show this: –[b]

The following rules of inference are valid

$$\text{(a)}\quad \begin{array}{l} p \\ \text{not } p \text{ or } q \text{ or both} \\ \hline \therefore\ q \end{array} \qquad \Bigg] \qquad \begin{array}{l} p \\ \overline{p} \vee q \\ \hline \therefore\ q \end{array}$$

$$\text{(b)}\quad \begin{array}{l} \overline{p} \\ \hline \text{not } p \text{ or } q \text{ or both} \end{array} \qquad \Bigg] \qquad \begin{array}{l} \overline{p} \\ \hline \therefore\ \overline{p} \vee q \end{array}$$

To prove that from the two premises p, $\overline{p}$ follows any proposition whatever:

$$\begin{array}{ll} p & \\ \overline{p} & \\ \hline \therefore\ \overline{p} \vee q & \text{rule (b) (where } q \text{ may be any proposition)} \\ \hline \therefore\ q & \text{rule (a) (using premise } p\text{)} \end{array}$$

3 | Thus modern logic is distinguished from the traditional logic by its contact with mathematics, its richness of material, its capacity to develop, and by the serious consideration which it gives to the paradoxes. These rather than the superficial characteristic of using a symbolism resembling that of mathematics are the fundamental features of the new logic.

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[b] In the following paragraph the principle of *ex contradictione quodlibet sequitur* (that from p and its negation $\overline{p}$ follows any proposition q) is demonstrated using a form of disjunctive syllogism (rule (a)) and disjunction introduction (rule (b)). The rules are given in semi-symbolic and symbolic notation. We have deleted some duplications, corrected typographical errors, and have rendered the proof (with some simplifications) in symbolic instead of semi-symbolic notation to improve readability.

Note: Paradox of the classes of all classes which do not contain themselves as an element

1. A class is *normal* if it does not contain itself as an *element*
2. The class of all normal classes
3. Is this class (2) itself normal or not?

(a) *Assume* it is *normal*
(then it does not contain itself as an element)
but it is the class of *all* normal classes therefore if it is normal it must itself be an element of that class of *all* normal classes and therefore of itself
therefore it is *abnormal*

(b) *Assume* it is *abnormal*
then it must contain itself as an element
But it contains only such classes as elements which are normal
therefore it must be *normal*

1. *Modus Tollens*

$$\begin{array}{l} p > q \\ \underline{\overline{q}\quad\;\;} \\ \therefore\ \overline{p} \end{array}$$

2. *Reductio ad absurdum*

from p follows q
now q is impossible or absurd (for some reason)
———
$\therefore$ p is impossible too
$\therefore$ $\overline{p}$

3. Special case of the reductive *ad absurdum – the indirect proof*

from p follows $\overline{p}$
therefore
from p follows $p\ .\ \overline{p}$ which is absurd
———
$\therefore$ $\overline{p}$

4. *Paradox*
Z is a paradoxical statement if from Z follows $\overline{Z}$ *and* from $\overline{Z}$ follows Z

4 | Logic like all studies investigates certain types of objects. These objects are linguistic, consisting of statements which are often premises or conclusions. Logic is mainly concerned with the analysis of the relationships existing between premises and conclusions so that its task may be stated broadly as the investigation of the conditions of *inference*.

In order to refer to objects we commonly use *names*. Now in other subjects of investigation people are not likely to use the thing itself in place of its name. Thus the botanist though he may use actual plants as illustrations is never likely to insert a specimen in place of the name of a plant when he is writing about his subject. The logician however who has groups of words for his object of study is often tempted to put down the words themselves instead of the name for them. The easiest way to make the distinction between words as ordinarily used and words as names in a statement about linguistic entities is by the use of inverted commas. Here are two sentences for logical comparison:

"Tom is dark. Mary is fair." Now when we write *about these sentences* we must show that we are referring to these sentences and not to the situations represented by the sentences. Thus we should write: "Tom is dark" is compatible with "Mary is fair." If the inverted commas were omitted then it would not be apparent that we were referring to the sentences themselves.

This is a mistake that is frequently made in logical textbooks and one that is liable to lead to serious confusion.

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Our consideration of logic as concerned with the kind of relationships which hold between linguistic expressions is valid whether we prefer to talk about thoughts, judgements, propositions, statements or sentences because for logical analysis these must be linguistically expressed.

We cannot pass *logical* judgements on a man's thoughts or actions until he or someone else expresses them in sentences or some such linguistic form. Indeed logical relationships *as such* only occur between linguistic forms and not between the objects represented by those forms. For example "*A* follows from *B*" is a statement
5 about | statements made with the help of the name *A* and the name *B*.

A may stand for the statement "Auckland is to the north of Wellington and Wellington is to the north of Christchurch" and *B* may stand for the statement, that, "Auckland is to the north of Christchurch". We cannot say that the *fact* that Auckland is to the north of Christchurch *follows from* the *fact* that Auckland is to the north of Wellington and Wellington is to the north of Christchurch. It is only of the statements that we can say that one follows from the other. The relationship "follows from" *may be* based on some relationship between the facts referred to by the statements, but if it says something about these facts it is only in a secondhand way.

That logical relationships differ from factual relationships is clearly seen from the logical relationship of contradiction. "Jones is six feet tall" contradicts "Jones is less than six feet tall" but it is only the statements which contradict each other; there ⟨do⟩ not exist two lots of fact about Jones in contradiction with each other. Although the contradiction of statements may correspond to the exclusiveness of facts we have to recognize that logic is concerned with one and not directly with the other.

In this connection it is important for us to make clear the distinction between *compound* statements and *meta* statements. Both contain statements within statements but in a different way. Thus "Humpty Dumpty sat on a wall and Humpty Dumpty had a great fall" is a compound statement about fact containing two distinct factual statements. On the other hand the sentence "'Humpty Dumpty sat on a wall' is compatible with the statement 'Humpty Dumpty had a great fall'" while it contains the same sentences (but in inverted commas) treats them in an entirely different way. In the meta statement it is the sentences as such which are referred to and not the facts they express. In the first case the words in the two sentences are used as statements of fact while in the second case, when put into inverted commas, they are the *names of these statements.*

On this basis we may distinguish between an object and a meta language. The object language is (usually) the language of everyday discourse or the language of the sciences, and it is the object of our interest – that which we are going to analyse.
6 But to refer to it we must use a language and this we call the meta language. | About the meta language we *need* not talk at all, we just talk in it. The two may coincide (e.g. they may be English) but they may not. In one context only one language should be used; there should be no mixture. This does not mean that we cannot refer to the other language but when we do so we must use some such device as inverted commas or preface it by the phrase "the statement which says that," thus in effect translating it into the meta language, just as we do for example when referring to a French word in the English language, e.g. the French word "garçon" has the same meaning as the English word "boy".

In "Principia Mathematica"[c] Russell has not kept strictly to the meta language. His formula $p \supset q$ does not express the logical relationship of deducibility i.e. $p \supset q$ is not equivalent to q follows from p, instead its equivalent is $\overline{p} \vee q$.

The following are important meta-linguistic terms referring to relationships which hold between statements:

"*A* contradicts *B*"
"*A* and *B* are compatible"
"*A* follows from *B*"
"*A* is analytic"
"*A* is synthetic"
"*A* is self-contradictory"
"*A* is consistent"
"*A* is true"
"*A* is a generalization of *B*"
"*A* is a negation of *B*"

7 Note on the Concept of TRUTH

(a) If truth is a property of certain statements and implies a relationship to facts then the meta language must not be confined to speaking only or exclusively about linguistic objects otherwise the concept of truth could not occur within the meta language.
(b) The definition truth means correspondence between the sentence and the fact does not cover analytical true sentences. However, the definition can be extended in such a way that this difficulty disappears.

(A propositional function is a sentence which contains variables. Names which can be substituted for these variables are said to *satisfy* the propositional function

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[c] Whitehead and Russell (1925–1927).

while the fact or object represented by the name is said to *fulfil* the propositional function. A proposition may be regarded as a propositional function with no variables in which case it may be regarded as being fulfilled by the null class of facts).

We can extend our definition by saying that a true proposition or statement is one which is fulfilled by certain facts. Analytic statements would then be propositions that were fulfilled by any facts whatever.

Remarks about Truth

A technical question: property and class.

Truth is a property or a class. In saying a sentence is true we are saying that it has a certain property or belongs to a class of sentences. We are thus talking *about* a *sentence* and therefore statements about truth belong to the *meta language*.

If “*A*”, “*B*”, “*C*” are names of sentences then I can write symbolically “$A \in$ Tr” (“*A* is true”) where “$\in$” means “is an element of”, and “Tr” is the name of the class of true sentences.

Note: A sentence of the form “Socrates $\in$ man” must always be constructed so that on the left of the symbol “$\in$” there is a *name* of an *element* and on the right there is a *name* of a *class*.

8 | Therefore, if I am stating that a certain proposition belongs to a certain class (or has a certain property) then I can express this only in a statement where the name of a proposition occurs (e.g. on the left) the⟨n⟩ some symbol which symbolizes the individual class relationship and then (e.g. on the right) the name of a class.

For the proposition itself is the element of the class not what is named by it.

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If we have the concept of truth at our disposal in the meta language then we can define with the help of this concept most of the *logical constants* of our language – words like these: “and”, “are”, “is”, “if”, “or”, “some”.

Logical constants of the Aristotelian syllogism e.g. such words as “all”, “some” and the copula. We distinguish between *logical constants* and *descriptive constants* like “table”, “chair”, “Socrates”, and between *variables* (which themselves can be either logical or descriptive; usually variables representing descriptive constants are employed.)

Definition of “and” (as used between *sentences*) with the help of the concept of truth.

A sentence of the form “*p* and *q*” or symbolically “$p \, . \, q$”, where “*p*” and “*q*” may be replaced by any sentence of the language under consideration, *is true if* and *only if* both of the constituent sentences (represented by “*p*” and “*q*” are true).

Definition of “or” “*p* or *q*” (symbolically “$p \vee q$”) is true if and only if at least one of its constituents is true.

To define Conjunction and Disjunction

A Conjunction is a sentence composed of two sentences in such a way that the whole sentence is true if and only if both of its constituents are true.

A Disjunction is a sentence composed of two sentences in such a way that the whole sentence is true if at least one of its constituents is true.

The words “conjunction” and “disjunction” obviously are terms of the meta language on the same level as “true”.

The words “and” and “or” or “.” and “$\lor$” are words of | the object language, they do not themselves describe anything, they only express a certain way of composition of sentences. Which way can be expressed in meta language with the help of constants.

Truth Tables

“p”	“$\sim p$”
T	F
F	T

Constituent sentences		Whole sentences		
		Conjunction	Disjunction	Implication
“p”	“q”	“$p \,.\, q$”	“$p \lor q$”	“$p > q$”
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Thus they show us in what way we have to make use of the meta language in constructing or analysing an object language.

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The class of formally true or analytical sentences will be named by the symbol “An”. Thus “The sentence *A* is analytic” can be symbolized by “$A \in$ An”. Now we have in the meta language a theorem An $\subset$ Tr ($\subset$ subclass relation). If a sentence “*A*” follows from a sentence “*B*” then the sentence which is the implication with the antecedent *B* and the consequent *A* must be analytic and vice versa.

Thus if “p” and “q” represent sentences of the object language, the sentence of the meta language “‘$p > q$’$\in$ An” expresses that the second of these sentences of the object language i.e. a sentence “q”, follows from the first “p”.

Examples showing the relationships set out in the truth table:
Conjunction “$p \,.\, q$” e.g. Tom is dark and Mary is fair. Unless it is true that both, “Tom is dark” and “Mary is fair” the statement as a whole is a false one.
Disjunction “$p \lor q$” e.g. Jones has a horse or Jones has a car. This sentence may be true as a whole if ⟨one or both sentences are true.⟩[d]

[d] The sentence ends abruptly in the typescript.

11 Criticism of W. E. Johnson

Johnson[e] distinguishes between primary and secondary propositions. He says that there may even be tertiary propositions. He defines “a secondary proposition is one which predicates some character of a primary proposition”[f].

Examining this definition we see that if a secondary proposition predicates some character of a primary proposition then it must refer to the primary proposition with the help of some sort of description or name. For, if a certain statement, whether it is secondary or primary, some character of some object, then it must refer to the object with the help of a name; e.g. ⟨if⟩ I want to predicate of my fountain pen some characteristic – for instance that it is black – then I have to use a sentence or proposition like “My fountain pen is black”.

If we look at this proposition then we see that neither my fountain pen nor the characteristic in question occurs in it, but *words* or *names* referring to my fountain pen and to the characteristic. Namely, the names “fountain pen” and “black”.

In the same way if a secondary proposition predicates something of a primary proposition then the primary proposition cannot occur itself in it but a name of it must occur in the secondary proposition.

Johnson writes “taking p to stand for any proposition we may construct such secondary propositions as: p is true, p is false, p is certainly true, ...”

Let us criticise this sentence of Johnson’s. In saying “taking p to stand for any proposition” he is indicating that he is using the letter “p” not as a name (variable name) referring to, but as a symbol standing for, or representing some proposition itself, but in that case we cannot say that “p is true” is a correct example of a secondary proposition. It is not correct to say “It is now raining is true” but rather “‘It is now raining’ is true” or even clearer “The proposition ‘It is now raining’ is true”.

12 The Consequence Relation and Analytic Implication

In what follows we will make use of the following symbols: “p”, “q” and “r” *represent* certain statements like “It is raining” (they represent not they denote or they name i.e. “p” is not used as a name for e.g. “It is raining” but the letter “p” is used instead of some series of letters like “It is raining”). We say “p” is a variable representing or standing for sentences.

We use symbols like “A”, “B”, “C” as *names* of sentences and we use symbols like “P”, “Q”, “R” as *variables* representing or standing for not sentences but *names of sentences*.

We want to express that a certain sentence B follows from another sentence, say A. Every statement like “B follows from A” or “from A follows B” must be a sentence

[e] William Ernest Johnson (1858–1931), British philosopher and logician who wrote the three-volume *Logic* (1921–24).

[f] The quote is from Johnson (1921, p. 50)

of the meta language because this sentence states that a relationship between two propositions holds (similar to a sentence which states a property, say truth, of a proposition or say that a proposition is analytic.)

Now we want to show that if the proposition named by "*A*" is, for instance, "It is raining now" and the proposition named by "*B*" is, for instance, "It is sometimes raining" then instead of saying "From *A* follows *B*" we can say "'It is raining now implies it is sometimes raining' is analytic". Now, instead of writing "It is raining" we will write "*a*" and instead of writing "It sometimes rains" we will write "*b*" (i.e. the small letters "*a*", "*b*" and "*c*" shall represent constant propositions – certain givens i.e. they shall be just shortenings of these propositions).
Now what we want to show is: the sentences

$$\text{“From } A \text{ follows } B\text{”} \quad \text{and} \quad \text{“‘}a > b\text{’} \in \text{An”}$$

express the same thing.

It has to be noted that both these sentences are sentences of the meta language the one making use of the relationship "follows" the other of the predicate "analytic" which are both in terms of the meta language.

We can use a third way to express it: if we introduce the name "$A \rightarrow B$" as a *name* for the proposition "$a > b$" then we can write "$A \rightarrow B \in$ An". In other words, we maintain that if implication of the object language is analytic i.e. is formally true
13 then the | two components of this implication stand in such a relation that the second follows from the first.

If an implication of the object language is not analytic i.e. not formally true but for instance true (not formally) or false (either formally or not formally) then the second of the components does not follow from the first. In other words, the symbol of implication (">" or "If . . . then") does not express the relationship of deducibility or consequence, it cannot because it is the symbol of the object language.

To show that the two sentences

$$\text{“From } A \text{ follows } B\text{”} \quad \text{and} \quad \text{“}A \rightarrow B \in \text{An”}$$

express the same thing. We have seen in the lecture notes[g] that there are two characteristics of the consequence relation

(1) Transmission of truth
(2) This transmission must be based on the *formal* structure of the sentence involved.

We may apply this to the sentence "From *A* follows *B*". This is a sentence in the meta language expressing the fact that sentence *B* is deducible from sentence *A* (i.e. the truth of *A* guarantees the truth of *B*). This inference is formal because even if *A* and *B* were both false the inference would not be rendered invalid, and also the inference is made solely from an examination of the structure of the sentences *A* and *B*.

We may express these two requirements of a valid inference of the form "*A* follows from *B*" by using "$A \rightarrow B$" to indicate the relationship of sentence *A* guaranteeing the truth of sentence *B*. And we may indicate the formal nature of this inference

[g] Cf. this volume, Chapter 19, § IV.

relation by stating that this statement belongs to the class of analytical statements. "From A follows B" would then be fully expressed by the statement "$A \rightarrow B \in \mathrm{An}$".

14 Remarks on the Theory of Deducibility

What we have already said serves only to make clear in what way to speak about deducibility.

Now we will proceed to questions like – under what conditions is a certain sentence of the object language, say Q, deducible from another sentence of the object language, say P. The rules which answer this question can be called Rules of Inference:

1. (The Principle of Inference) Q is deducible from P, if P is a conjunction, the one component of which is some proposition, say R, whilst the other component of the conjunction consists of an implication with the implicant R and the implicate Q.

In symbols:

Q is deducible from P if P has the form "$R \,\&\, (R \rightarrow Q)$" or "$(R \rightarrow Q) \,\&\, R$".

Where we use the symbol & in order to form the *name* of a conjunctino as we have done with the symbol "→" in order to form the name of an implication. Therefore we can say

$$\begin{array}{l} R \\ \underline{R \rightarrow Q} \\ \therefore\ Q \end{array} \qquad \text{or} \qquad \begin{array}{l} \underline{R \,\&\, (R \rightarrow Q)} \\ \therefore\ Q \end{array}$$

That shall be used as some symbolism belonging to the meta language and expressing the principle of inference.

Other rules of inference

Rule 2 From any statement whatsoever can be deduced the same statement

$$\begin{array}{l} \underline{P} \\ \therefore\ P \end{array}$$

Rule 3 From any given statement whatsoever can be deduced any disjunction of which the given statement is one of the components

$$\begin{array}{l} \underline{P} \\ \therefore\ P \vee Q \end{array}$$

Rule 4 From any conjunction whatsoever can be deduced either of the components

$$\begin{array}{l} \underline{P \,\&\, Q} \\ \therefore\ Q \end{array}$$

Rule 5 From any sentence whatsoever can be deduced any implication of which
15 the given sentence is the implicate |

$$\begin{array}{l} \underline{P} \\ \therefore\ Q \rightarrow P \end{array}$$

Rule 6 From any given sentence can be deduced an implication of which the negation of the given sentence is the implicant

$$\frac{P}{\therefore\ \overline{P} \to Q}$$

Rule 7 From an implication of the form $P \to \overline{P}$ can be deduced

$$\frac{P \to \overline{P}}{\therefore\ \overline{P}}$$

Rule 8 From any implication with an analytic implicant can be deduced a sentence identical with the implicate

$$\frac{\begin{array}{l} P \to Q \\ P \in \text{An} \end{array}}{\therefore\ Q}$$

Rule 9 From any implication with a contradictory implicate can be deduced the negation of the implicans.

$$\frac{\begin{array}{l} P \to Q \\ Q \in \text{Con} \end{array}}{\therefore\ \overline{P}}$$

(1)[h] $\vdash :: p : p . \supset . q : \supset : .q$

(2) $\vdash : p . \supset . p$

(3) $\vdash :. p \supset : p \vee q$

(4) $\vdash :. p . q . \supset : q$

(5) $\vdash :. p . \supset : q . \supset . p$

(6) $\vdash :. p . \supset : \overline{p} . \supset . q$

(7) $\vdash :. p . \supset . \overline{p} \supset : \overline{p}$

(8) $$\frac{\begin{array}{l} \vdash p \supset q \\ \vdash p \text{ (analytic)} \end{array}}{\therefore \vdash q}$$

(9) $$\frac{\begin{array}{l} \vdash p \supset q \\ \vdash q \text{ (contradictory)} \end{array}}{\therefore \vdash \overline{p}}$$

[h] The typescript uses "+", which we have replaced by "⊢". In (8) we have added "(analytic)", corresponding to the restriction given in Rule 8. We have added the rule in (9), which is missing in the typescript.

16 The Concept of Truth

Ramsey and Johnson maintain that the Concept of Truth is redundant. We do not agree with this, but fully agree with the evidence they offer for their opinion. They have misinterpreted their own evidence. What is this evidence? Both emphasize that the statement "It is true that it rains" conveys exactly as much, not more or less than the statement "It rains".

From our standpoint truth is a property of a statement and Johnson and Ramsay's evidence can be termed a criterion for the right and correct definition of this property – namely:

We can say that the word "truth" or rather the predicate "Tr" is properly defined if it will always be that "$X \in \mathrm{Tr}$" is analytically equivalent with the proposition *named* by X (and expressed in the meta language). For instance if English is the meta language and German the object language, and if the English translation of the German sentence "Es regnet" is "It rains" then the following sentence (of the meta language)

"Es regnet $\in$ Tr" or

"The German sentence which consists of the two words 'es' and 'regnet' is true" must be analytically equivalent with the following sentence of the meta language, "It rains". This can be shortly expressed in the following way: $X \in \mathrm{Tr} \equiv S$
Where X is the metalinguistic name of some sentence of the object language, and where S is the meta linguistic translation of that sentence.

Now, in case the object language and meta language coincide or rather where the object language is a certain part of the meta language (for instance where the object language is the part of English which is not covered by linguistic or logical entities) "X" would be a name of a sentence of the object language and S would be simply that sentence itself (which would in this connection i.e. in a metalinguistic connection belong to the meta language) e.g. "'It rains' is true" must be equivalent with "It rains".

Tarski said that the criterion for a correct definition of truth was that from a correct definition of truth all statements of the described form "$X \in \mathrm{Tr} \equiv S$" must be deducible.

17 | We must distinguish between the criterion for a definition and the definition itself.

When we are required to give a definition we must know what precisely is the *task* which has been set for us. The demand for a definition of say, time or truth remains utterly vague if there is not given

(a) a clear indication as to the terms admissible as definitions
(b) a criterion with the help of which we can decide whether or not the task to define "truth" or "time" is successfully carried out.

Tarski succeeded in giving such a criterion, having done so it was comparatively easy to do the task set. These considerations lead to a more general statement of procedure, namely, do not try to solve a problem before you have stated it. The difficulty presented by many philosophical problems is very often due to the fact that the problem has not been stated in a clear and precise fashion. Often we become aware of problems in a vague way without being able to give a precise account of what is involved. Until the problem is correctly formulated there can be no hope of solving it.

The Logic of Thought and the Logic of Linguistic Form

We may contrast two opposing attitudes towards the study of logic. W. E. Johnson in his "Logic"[i] defines his subject as follows: "Logic is most comprehensively and least controversially defined as the analysis and criticism of thought", and again he says, "Adopting as we do the general view that no logical treatment is finally sound which does not take account of the mental attitude in thought, it follows that the fundamental terms 'true' and 'false' can only derive their meaning from the point of view of criticising a certain possible mental attitude."

The opposing point of view is represented by R. Carnap in his "Logical Syntax of Language" where he says, "The development of logic during the past ten years has shown clearly that it can only be studied with any degree of accuracy when it is based, not on judgments (thoughts, or the contents of thoughts) but rather on linguistic expressions, of which sentences are the most important because only for them is it possible to lay down sharply defined rules"[j].

18 | Of these two methods the latter has been by far the more useful and fruitful for the study of logic, but it may be that it is not so far from achieving the aims of the former method as it may appear.

The study of the purely linguistic form of sentences may reveal some very important facts about "thoughts" and "Judgments" which would not have been discovered by the direct method of treating logic as concerned with mental attitudes. To use a metaphor. The economist who stays at home and makes a comprehensive study of the statistics concerning the economic condition of a certain remote country may have a far better conception of the conditions prevailing in that country than a person who has been there and relied merely on his own observations. The indirect method may be the more fruitful one.

19 | Logic has been regarded from various points of view as being the study of judgments, thoughts, propositions or sentences.

The most extreme views are held by those who on the one side regard logic as the study of thoughts, and by those on the other side who regard logic as the study of sentences.

On the one hand thought need not be formulated at all and on the other hand the sentence may be regarded as just a series of symbols, like black marks on white paper, or else a collection of sounds.

Those who emphasise the fact that they are only interested in a logic of *meaningful* sentences and who identify this with a logic of thoughts we may call psychologists (logical psychologism).

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[i] Johnson (1921).

[j] Carnap (1937, § 1).

The opposing wing who emphasise their interest in black marks on white paper we may call the logical *formalists*.

Now between the extremes of logic as a study of unformulated thought and logic as a study of sentences there are many intermediate stages, like the one expressed in the definition, “A judgement is a linguistically formulated thought.”

Before we proceed to a logical analysis of the matter we will first give some terminological analysis which although it is rather problematic, will help us to see how the different times can be differentiated. This analysis goes back to that used by the Stoics.

1. We can distinguish between three principle entitites involved in an ordinary judgment as when someone says “It is raining.” There is an entity – a certain mental act which we can call a mental attitude of assertion taking place in the head of this person (to express it rather crudely).

The next entity is a certain part of the world or a certain spatial-temporal aggregate in which there are falling drops of water i.e. in which it is raining. (We assume that the judgement is true).

The third entity is a series of symbols which may be either spoken or written or
20 perhaps even only thought of namely, the | symbols “It”, “is”, and “raining” which go to make up the sentence “It is raining.”

Let us name these three entitites as follows:

(i) active assertion
(ii) objective basis
(iii) the sentence

In addition to these three there is a further entity namely the *feature* of the objective basis which is *designated* by the sentence. This is not the whole objective basis itself. We will call it the *fact* designated by the sentence. However, if we do not look at it from the side of the objective basis, but from the side of the sentence, we could say that it is what is *meant* by the sentence, or that it is the objective content of the sentence.

21 | The logicians who maintain that they are only interested in the meaning, or what is meant by sentences, or in the content of the sentence have never succeeded either in:

(a) saying what they mean by this, i.e. in saying *how* to distinguish this content from

    (i) the asserting act
    (ii) the sentence
    (iii) the objective basis

    i.e. some of them emphasized that the content is not the psychological attitude but rather its object, and all of them emphasized that it is not the sentence.

(b) or in showing the significance of their emphasis i.e. in showing what difference it does make to logical theory or (especially) in showing its advantage for logical theory. It is clear that only in contrasting their results with the results of a purely formal analysis, could the significance of their emphasis be shown. This has never been done because of the universal agreement of the insignificance of the verbal formal investigations.

Dr. Popper's view is that hardly any progress has been made since the Stoics to analyse the meaning of meaning (c.f. Ogden and Richards quotation of Gomperz[k]) and especially (b) (see above) was entirely overlooked. Dr. Popper maintains further that the purely formal analysis has opened the way for deeper penetration into all logical problems – especially is this the case with the theory of meta logic.

The development of this theory which is based upon the formal analysis of sentences has exposed many simple though serious confusions which may arise in treating logic as a study of thoughts. Of these we have already noted the confusion of the proposition with its name, and the mistaken conclusion that the concept of truth is redundant. Thus, the formal treatment of logic has cleared up issues which the treatment of logic as thought clouded over.

22 | 2. Most logicians (until recently) have assumed that a sentence consists of black marks on white paper or something of that sort – and that is all. Its meaning or content or anything of that kind must be found in something outside – something mental (or factual) for instance.

They did not see that apart from the factors mentioned, a language is more than a mass of symbols – that it involves a certain system of rules which declare how to use such symbols. These rules may in part be non formal, like for instance the rules for using the different descriptive terms of a language or the rules of the use of words like "rain" or "apple", and so on. Anyone learning a language discovers such rules by finding out in what kind of situation he has to use the words "rain" or "apple".

These rules refer to certain situations of a practical kind, that is, they refer to something outside of the language and correspond therefore to the old idea of meaning as referring to something outside the sentence.

But the formal analysis has shown that the problem of the meaning of the descriptive constants is of comparatively small significance compared with the problem of the meaning of such words (constants) like "is" ("is an element of", "has the property", "is a part of", "is a sub-class of"), "and", "if . . . then".

The meaning of such words is of the greatest importance – their analysis shows that loosely speaking they make a language *to be* a language, and their analysis shows that the meaning of such words can be found by analysing their *rules of use* which turn out to be entirely *formal* rules i.e. rules which have only to do with the handling of symbols and which do not refer in any way to something outside of the language. These rules are of the following type: ". . . ∈ . . ." forms a sentence if on the left of "∈" and on the right of "∈" there are two different kinds of symbols; thus: If "Socrates ∈ man" is a sentence, "man ∈ Socrates" *cannot* be a sentence.

23 | The problem of meaning can be approached by two radically different methods.

The traditional approach is by way of psychology, that is by relating the word or sentence to someone's field of experience, placing it in its psychological background.

The opposing method is to treat it in a purely formal linguistic manner; the meaning of a word or sentence being the place it occupies in a linguistic context and the purely formal rules which govern its use in that context.

Carnap asserts that the whole problem of meaning can be solved by the second

[k] Cf. Ogden and Richards (1923).

method. Dr. Popper thinks that this is going further than the evidence warrants but that nevertheless the purely formal analysis does go a long way towards solving the problem. He thinks that it has been justified by its results and that it has certainly been a more successful approach than the psychological one.

Tarski, who also developed the formal side upon which Carnap places such emphasis was particularly concerned with showing the relations between the purely formal structure and the context of a sentence. While Carnap developed a language which deals only with matters of syntax, Tarski developed a language capable of referring to the *object* as well as to meta linguistic entities. Thus his language is able to show the connection between the two. Tarski divides the subject of meta-logic into two parts:

1. The part dealing with the purely formal structure of language, and
2. The portion which treats of the relation between this formal structure and the context of the sentence. This he calls semantic.

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24 | One may distinguish between a more radical and a less radical attitude towards the role of logical analysis and the nature of philosophical problems.

Wittgenstein first took the standpoint that philosophical problems were nothing but linguistic confusions and that when these were analysed the problems were not solved but just disappeared. Wittgenstein thought that philosophical problems were of the following type. Instead of saying that one's watch was going and has now stopped one might say that the "go" of the watch had gone and might then proceed to ask, "where has the 'go' gone?" This example would be analogous for instance, with the philosophical problem of mind and body.

This radical attitude was also the one adopted by Carnap in his "Logical Syntax of Language"[1]. He said that all the problems of philosophy could be reduced to those of syntax, that is to questions concerning the rules of the use of language – problems which could not be solved in these terms were mere linguistic confusions. Thus all that remained of philosophy was the study of the formal structure of sentences.

Dr. Popper adopts a less radical attitude. He thinks that while the purely formal study of the structure of language is undoubtedly of great importance and clears up many philosphical difficulties yet residual problems may remain which are not purely a matter of linguistic structure. This attitude was confirmed by the work of Tarski who showed that the problem of truth could only be dealt with satisfactorily by getting beyond the formal structure of the sentence to its relations with the object.

One of the main tasks of analysis is to reformulate the problems of philosophy. This may be done partly by the use of more accurate terminology. For example, for the term "knowledge" should be substituted "scientific statements", for "sources of knowledge" should be substituted "method of testing scientific statements", for Kant's "limits of knowledge" should be substituted "characterisation of the method of science." Such
25 a change of terminology is necessary when discussing the philosophy | of science for the term knowledge implies "truth". We cannot, strictly speaking, use the phrase

[1] Carnap (1937).

"scientific knowledge" for if you find that what you professed to know was incorrect then you cannot say that you had knowledge at all. Since no scientific statements can be conclusively proved it is misleading to talk about scientific knowledge.

The Viennese Circle has been concerned with the following main problems:

1. The Nature of Philosophy
2. The Nature of Logic and Mathematics
3. The Nature of Science – especially its empirical basis which involves the problem of induction
4. The Nature of Language
5. The special problem, traditionally known as the body-mind problem, formulated in the typical form of "Is psychology dealing with something different from physics".

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26 | In 1931–32 Carnap, influenced by Neurath, developed a standpoint which he called "Physicalism" (from Neurath's side the intention was to develop something like a modernised materialism, that is connected with the fact that Neurath was interested in Marxism though not an orthodox Marxist).

The philosophical idea was the following: Behaviourism is *methodologically* correct i.e. a psychological statement can only be tested with the help of *observation of the behaviour* in the widest sense of the word. Even the so-called introspective method can be said to be based on observation of the behaviour, because its results must be formulated with the help of sentences, but these are communications i.e. bound up with behaviour such as mouth movements or movements of the hand in writing and so on. The only empirical methods available in psychology are observing what a man does or what a man says and writes.

Carnap expressed this in the following way: All terms of psychology are physical terms or rather must be capable of being reformulated within the language of physics. If I say "Mr. A is excited" then I refer to certain typical reactions.

A statement of that kind is fundamentally not different from a statement like, "Mr. A is ill" or "Mr. A has a broken leg" (the last is obviously a sentence of a physical kind). The thesis of Physicalism is therefore the following: Every scientific theory, if scientific must be able to be formulated in the language of physics. This last formulation indicates already a certain connection with what Dr. Popper calls the Kantian problem – namely the characterisation of science and its limits towards metaphysics; i.e. an attempt to characterise the empirical feature of science. This is here identified with being able of formulation in physical terms. Because all our empirical observations are of temporal and spatial happenings.

27 | Wittgenstein attacks metaphysics from the point of view of the verification of sentences. A sentence is only verifiable when it is possible to analyse it into its constituent atomic propositions. By atomic proposition he means *given sense data* as expressed by such propositions as "The grass is green", "Water is wet" and so on. These atomic propositions or given sense data may be verified immediately by empirical observation. Any sentence therefore which cannot be reduced to these

immediately verifiable atomic propositions cannot itself be verified and hence is meaningless. Such sentences are typically those of metaphysics.

Dr. Popper criticises this standpoint from certain aspects. It is psychologically wrong – there are no *givens*, what we have is always an interpretation – we can either take it as a basis for further interpretation or else analyse it into its components – but however far we analyse we never get beyond an interpretation. Wittgenstein has just reproduced in the dialect of the Viennese Circle the older positivist view namely, that the mind contains ultimate sense data to which science must reduce all knowledge.

Wittgenstein's view has affinities with the Kantian view of the empirically given upon which, according to Kant, the mind worked to produce scientific theories – the empirically given was not itself sufficient for science, it had to be combined with the a priori forms of the mind. Wittgenstein's theory has the further consequence that the generalizations of science must also be classed with the sentences of metaphysics, as meaningless. For they cannot be completely reduced to their constituent atomic propositions and therefore they are not verifiable. This question of verification is bound up with the problem of induction.

We may formulate this problem by indicating the apparent contradiction involved between the following statements:

1. Hume's analysis proved that induction cannot be based on a pure observational basis.
2. The fundamental thesis of every empiricist is that only observation and experiment are *decisive authorities* about all kinds of scientific sentences.
3. The fact that science consists mainly of strictly universal sentences or theories.

28 | 1 and 3 apparently contradict 2, and this represents Kant's view. He held that there must be a non-empirical basis for scientific theories. 2 and 3 contradict ⟨1⟩ and represent the view of Bacon and Mill (who ignored Hume's criticism of induction).

1 and 2 contradict 3, and this is the consequence of Wittgenstein's view.

These contradictions however are only apparent. There would be a real contradiction if in 2 instead of saying that observation and experiment are *decisive authorities*, we said that they were a *complete verification* of scientific sentences.

All three standpoints, however, are correct, for although only experience *decides about* scientific theories there is no *conclusive positive decision*.

By experiment and observation we test theories i.e. we attempt to show that they are false, but we can never succeed in conclusively verifying them.

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29 | In any deductive inference there is no transmission of truth from the conclusion to the premises. But does not this depend upon the precision and completeness with which the conclusion is stated? In the following argument for example,

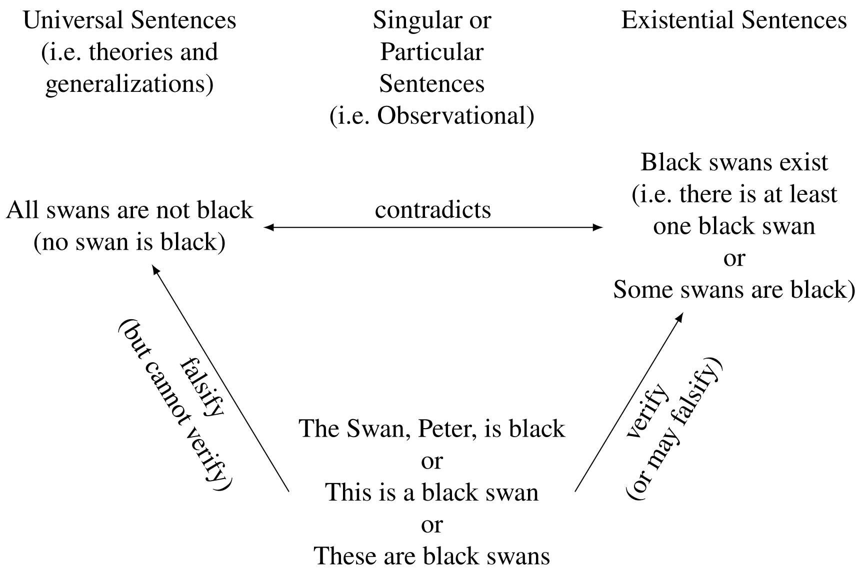
Prussic acid is a deadly poison
A took a dose of Prussic acid
therefore *A* is dead

we cannot argue from the statement "*A* is dead" to the statement "Prussic acid is a deadly poison," or to the statement "*A* took a dose of Prussic acid", for it might be the case that he died a natural death, or was shot or stabbed. We have not, however, stated the conclusion precisely enough. If we said

> Prussic acid is a deadly poison which acts in such
> and such a way
> *A* took a dose of Prussic acid
> therefore *A* died such and such a death,

we could argue from the conclusion, "*A* died such and such a death" to the premise "*A* took a dose of Prussic acid", with the help of the generalization "Prussic acid is a deadly poison which acts in such and such a way".

Our first conclusion above was "weak", we could not use it in arguing back to the truth of one of the premises, but our second more complete conclusion was "strong" since with the help of a generalization we could do this. Sometimes the conclusion is more than strong, in which case it is in itself sufficient to demonstrate the truth of its premise. In such a case we have two sentences which are mutually deducible – they are said to be equipollent i.e. when $A > B$, and $B > A$ then $A \equiv B$. When it is the case, however, that from a generalization and certain initial conditions we deduce certain consequences which can be expressed as observational sentences or basic propositions, then if the conclusion is true we are unable to deduce from it the truth of the generalization: if it is false, however, we may conclude that the generalization is false. In this connection we may note that there is a symmetry between the pure
30 universal and the pure existential sentence and there is a | symmetry between each of them and a singular or observational sentence. The universal and existential sentences may contradict each other, while the existential sentence may be verified by the observational sentence and the universal may be falsified by it.



2. Can a statement be conclusively falsified any more then it can be conclusively verified? To falsify a theory we must show that it leads to a conclusion which can be

contradicted by observational statements or basic propositions. But no such empirical statements are beyond the possibility of error, therefore it cannot be shown that any theory is certainly false any more than it can be shown that it is certainly true.

It is true that any statement of fact may be false and that therefore in the last analysis we cannot be certain that we have falsified a theory any more than we can be certain that we have verified it. But this is a ground for the uncertainty of falsification and verification alike. We can see, however, that the verification of theories is impossible even before we cast doubts upon the truth of our basic propositions (i.e. observational sentences) and for an entirely different reason.

In any investigation or process of reasoning we must be in by taking something as given – the empiricist regards it as safer and offering less possibility of error to
31 take basic propositions, or observational sentences, as given, in contrast | with the rationalist who assumes certain general laws.

On the empiricist basis we may examine the claims of verifiability and falsifiability. As we have seen above a conclusion (or prediction) which consists solely of basic propositions cannot transmit truth back to its premises. Thus verifiability fails to fulfil the logical requirements necessary for making a deduction.

It might be argued that every basic proposition itself implies a theory, for if doubt is expressed concerning it we proceed to test it on the grounds of some generalization or theory. But this occurs on a different level of argument, we are no longer taking our basic propositions as given but are treating them as objects for investigation. The point is that before an argument can be developed we must take something as given, although on another occasion, or at a later stage, we may cast doubt on, or inquire into the truth and falsity of what we previously took as given. Thus, falsification while sharing with verifiability the uncertainty of basic propositions does not, like the latter, lack logical justification.

3. The problem of Induction[m]

The problem of induction was raised by Hume when he showed that it was impossible to make logically justifiable *positive* decisions concerning theories and hypotheses like those formulated by science.

If we ask, why does science formulate theories and hypotheses? – then the answer is that it does so in order to make predictions.

Then the main question arises – On what grounds does it prefer one theory or hypothesis to another? While, as Hume showed, it cannot make conclusive positive assertions about the truth of its hypotheses it can *test* them by trying to falsify them.

32 | While it cannot arrive at positive conclusions about the truth of a theory – it can come to negative conclusions to the effect that a theory is false since the predictions based on it were not fulfilled.

[m] The number 3 probably refers to the corresponding list item on p. 25 of the typescript.

Thus a solution is given to the problem of induction by characterising the aims and methods of scientific procedure.

The aim of science is to make predictions for which purpose it frames hypotheses – it tests these hypotheses in the course of using them i.e. the very making of predictions constitutes the testing of the hypotheses – if the predictions are untrue the hypotheses are falsified.

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33 | Basic propositions are statements that some characteristic can be observed at a particular time and place. We may call the particular spatial area indicated the "surrounding". Hence basic propositions predicate a characteristic of a particular "surrounding".

Universal propositions, on the other hand, do not assert that anything exists e.g. a statement about "all X are Y" is not falsified if in fact there are no X's, but it is falsified if in fact there exists one or more X's that are not Y's. The universal proposition "All X's are Y's" does not assert X but it does exclude from existence any X that is not Y.

We may bring out the relationship between the various types of propositions by using the following symbolism. ( ) is the universal operator, or generalisator so that $(X)(X \in \text{swan} \supset X \in \text{white})$ is equivalent to "All swans are white" (X is a variable satisfied by "swan")

$(\exists)$ is the existential operator, or particularisator, so that $(\exists X)(X \in \text{swan} \;.\; X \in \text{black})$ is equivalent to "There is at least one swan which is black". Substituting the propositional function φX for $(X \in \text{swan} \supset X \in \text{white})$

$$\overline{(X)(\varphi X)} \equiv (\exists X)\overline{(\varphi X)}$$

i.e. the contradictory of the universal proposition is equivalent to an existential proposition, and also

$$\overline{(\exists X)(\varphi X)} \equiv (X)\overline{(\varphi X)}$$

i.e. the contradictory of the existential proposition is equivalent to a universal proposition. Substituting ψ for φ such that $\psi X \equiv \overline{\varphi X}$

$$(X)(\psi X) \equiv \overline{(\exists X)(\varphi X)}$$

then if "a" is a particular surrounding[n]

$$\varphi a \rightarrow (\exists X)(\varphi X) \longleftrightarrow \overline{(X)(\psi X)}$$

i.e. a basic proposition implies an existential proposition which is equivalent to the contradictory of a universal proposition.

34 | A basic proposition is concerned only with what can be observed at a certain time and place, hence its contradictory *may* not be a basic proposition itself. While the contradictory of the basic proposition "Here is an elephant," namely the proposition,

[n] What is meant in the following is: $\varphi a \rightarrow (\exists X)(\varphi X)$, and $(\exists X)(\varphi X) \equiv \overline{(X)(\psi X)}$.

"There is no elephant here" may be regarded as a basic proposition also, there are occasions when inability to detect the presence of something is no guarantee of its absence. Thus, while "There is a needle in this haystack" is a basic proposition, its contradictory "There is no needle in this haystack" is not.

However, there is another and perhaps more important reason for stressing this fact and that is that basic propositions should not be deducible from universal propositions.

We can arrive at basic propositions only by observation, or by deduction, from a universal (general law) *and* another basic proposition (initial condition).

But the contradictories of some basic propositions *follow from* universals and therefore these contradictions cannot be themselves basic propositions.

G = general law I = initial condition F = forecast

$$\begin{array}{ll} G & \text{universal proposition} \\ \underline{I} & \text{basic proposition} \\ \therefore\ F & \text{basic proposition} \end{array}$$

$$\begin{array}{ll} \underline{\overline{G}} & \\ \therefore\ I + \overline{F} & \text{basic proposition} \end{array}$$

$$\begin{array}{ll} \underline{G} & \\ \therefore\ \overline{I + F} & \text{not a basic proposition} \end{array}$$

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35 The Development of the Vienna Circle

The movement originated with the appointment to a Chair of Philosophy at Vienna of men such as Mach and Boltzmann whose training had been in the physical sciences and who were themselves eminent physicists. Their interests were naturally in the methods of science rather than in metaphysical speculation. Schlick who was appointed to this chair in 1922, also had training in the physical sciences and around him developed the Vienna Circle.

Russell's work in logic and the foundations of mathematics was introduced to the circle by mathematicians at Vienna, and interest was also aroused by Russell's attempts to explain the concept of physics as logical constructions, and by his work "Our Knowledge of the External World"°.

Up to this point the Viennese Circle had not developed positivist views, being more inclined towards realism. They were not, however, very interested in such questions

° Russell (1914).

of[p] They had already adopted an anti-metaphysical attitude and were inclined to believe that many philosophical problems were due to confusion of words.

A new stage was reached with the appearance of Wittgenstein's ⟨Tractatus⟩ which greatly influenced the circle. The conception of atomic propositions as statements concerning given sense data, and the notion that any statement not reducible to these elements was meaningless, introduced a positivist attitude, gave a definite ground for the attack upon metaphysics and strengthened the influence of Russell's logical work.

Then came Carnap who had been very much influenced by Russell. He wished to give an account of the whole field of experience in terms of logical constructions. This was termed logical solipsism by which he meant that each person's conception of the world was a logical construction of his own observations.

Carnap thought that he could show how such concepts were built up and give all definitions in these terms with the help of only one non-logical relationship, namely
36 the relationship | expressed by the sentence "This reminds me of that"[q]. From such a relationship Carnap hoped to build up all his definitions. Thus the notion of before and after could be defined by saying, that when *A reminds me of B*, then *B* is before *A* and *A* is after *B*.

Dr. Popper pointed out the fundamental defect of this system namely that if everything that we were acquainted with is a logical construction out of our immediate observations, then we could not make predictions or talk about the future in any way since we had not observed and could not as yet have made a logical construction out of it.

Since logical constructions could not be made prior to the event and could have no reference to the future, it followed that there was no place for scientific laws within this system and the activity of scientific investigation was left unaccounted for.

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37 Criticism of Physicalism and the Unity of Science

Carnap and Neurath developed from Wittgenstein's conception of atomic propositions, the theses of "Physicalism" and the "Unity of Science".

They said that statements with an empirical content such as those of science could only be verified and thus rendered significant by the protocol statements which they implied.

A protocol statement was the statement of the intermediate sense experience of some particular person and as such it could only be verified by that person. Thus while protocols are (according to this view) the ultimate source of verification yet they are not intersubjective, for each person's protocols are private to himself.

Therefore, said Carnap, the protocols must be translated into a universal and intersubjective language, i.e. a language in terms of which all states of affairs can be

[p] Sentence ends abruptly in the typescript.

[q] This theory is developed in Carnap (1928).

expressed. “Such a language”, said Carnap[r], “is the physical language which expresses a quantitatively determined property of a definite position at a definite time.” This introduces the notion of the Unity of Science, for although each science may have its own terminology it must refer to certain physical determinations expressable in the physical language, and the definition of all such terminology must be in relation to these physical determinations.

There are two problems involved in the conception of Physicalism. One could say that sentences like, “I have a toothache” i.e. “at this time, and at a particular spot in my jaw there is a pain”, or for that matter one could say “I feel sad”, i.e. “somewhere in the region defined by my body there is a certain feeling which occurs when I am aware of certain situations.” There is no reason why we should not call the states indicated by these sentences, physical determinations, and we might regard it as something in common to all empirical observations, and all empirical sciences that they deal with qualities or characteristics of a certain spatial-temporal region. Carnap, however, |
38 although he may have this in mind means something further. An observer cannot feel *my* toothache or *my* sorrow, all that he observes about me when I tell him that “I have a toothache” or “I am sad” are certain facial expressions and actions. These Carnap regards as the physical determinations, and the statement of them as the sentences of the physical language. Thus my statement “I have a toothache” and “I am sad” in the physical language can only be translated as “I have such and such an expression on my face”.

Now Carnap regards these latter sentences as *translation* of the former ones – that is they are exactly equivalent. The sentence, “I have a toothache” ≡ the sentence “I have such and such an expression on my face.”

This view, however, is not correct. It may be that when I truthfully assert the psychological sentence A, that what is asserted by the physical sentence B is also always the case, and it may be in some cases (perhaps all) that what is asserted by the physical statement is an essential part of the state of affairs referred to by the psychological statement, but it is certainly not *all* that is meant, or referred to, by the psychological statement, and it is not all that the observer understands by it; e.g. he sympathises not about the aspect of my face but about what is going on in my tooth. Furthermore, I may make psychological statements without being aware of what my overt behaviour looks like, while the overt behaviour corresponding to two different statements like “I have a toothache” and “I am sad” may be exactly the same.

Therefore, it seems false to speak of *translation* into the physical language, or to imply that the physical language expresses precisely the same thing as is expressed, say, in the language of psychology. It may express part of it, or it may express an accompaniment, and it may be possible to infer from a statement of the physical language a certain statement of the psychological language (usually with the aid of some generalisation).

It is very important to note this criticism for Carnap makes use of this equivalence of statements in the following way. Let us imagine that one statement in the material
39 | mode (i.e. non-physical) of speech contradicts some other statement in the same

[r] Carnap (1934b, p. 52f.).

mode. Then let us *translate* them into the physical language. In the physical language they do not contradict each other, hence the equivalent statements in the material mode of speech do not really contradict each other. But if the relationship between the two languages is not one of equivalence then it will not be possible to solve problems in this way.

The whole theory outlined and criticised above is typically an idealist approach to such problems – the attempt is made to guarantee the truth of certain propositions by equating the mind with its objects, in this case protocol sentences with sentences about physical determinations, the former always containing a reference to the observer and the latter being the common object of knowledge.

In criticism of this view Dr. Popper pointed out that the so-called protocol sentences are exceedingly difficult to verify, and that science does not test its theories by reference to statements like "I *perceive* so and so", but by statements on predictions such as "At a certain time and place there is so and so", such sentences unlike the former are not bound up with the experience of a particular person who is inaccessible to other people. On the contrary *anyone* can proceed to test the statements of science by proceeding to see if "so and so" is "there" at "a certain time". Such simple objective statements with which scientific theories are tested Dr. Popper calls "basic propositions".

Carnap acknowledged the validity of this criticism and having failed to provide definitions by means of translations into the physical language and attempted to provide definitions in another way e.g. Green could be defined as the property of the class composed of grass, trees, and so on, naming all the particular green objects with which one is acquainted. Dr. Popper pointed out that here the old problem of induction appeared in that one could never be sure of having exhausted the total enumeration of green objects – hence if one came across an object with which one
40 was not previously acquainted, say a piece of jade, then either it would not | *have* the property green since it did not belong to the class of enumerated objects or if one granted that it was green then the previous definition of green must have been false.

Dr. Popper considers that definitions should be considered as an operation with terms analogous to the deduction of sentences. The following table shows the points
41 of comparison between these two operations. |

	Deduction of Sentences	*Definition of Terms*
1.	All scientific theories are deductive systems	
2.	They start with certain sentences which are just assumed.	They make use of certain terms, or concepts, or ideas which are just assumed.
3.	That is not deduced within the theory	That is not defined within the theory
4.	We can call them *axioms* or postulates or fundamental *hypotheses*	We can call them primitive *ideas* or primitive *concepts* of the theory, or fundamental terms or *universals*

5. From the axioms or postulates we can deduce a certain body of theory	With the help of the primitive ideas we can define a certain body of *derived* terms
6. The deduction – every deduction starts with one ore more axioms	Every definition starts with one or more primitive terms
7. and it presents in such a way that a series of sentences is constructed such that every sentence of the series is either an axiom or immediately deduced from one of the foregoing sentences.	and it proceeds in such a way that a series of terms is constructed such that every term of the series is either a primitive idea or immediately derived from one of the foregoing terms.
Thus the procedure is based upon some rules which describe certain transformations of sentences as permissible thus defining what is meant by immediately deducible from	Thus the procedure is based upon some rules which describe certain substitutions of terms as permissable, thus defining what is meant by immediately derivable from
8. Rules of deduction	Rules of definition
9. Positivism with regard to sentences: – Criterion of verifiability	Positivism with regard to terms: – the terms must be such that they can be constituted.
Verification means deduction from recognized basic atomic propositions with the help of some logical rules and analytical sentences	Constitution means definition with the help of some terms, based on observation (basic or atomic terms) with the help of certain logical rules and purely logical terms
BUT	
hypotheses cannot be verified only particulars can be	Universals cannot be constituted only particular terms of proper names can be.

Thus we can see that the problem is one of universal sentences and universals.

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42 The Nature of Logic and Mathematics

In his works on Logic and the foundations of Mathematics Russell speaks of mathematics as being a part of logic. Strictly, however, in addition to the rules of logic, mathematics requires certain postulates which are not themselves logical. Russell, in fact, uses three such postulates although perhaps only one, the axiom of infinity is necessary.

We can apply all mathematical systems to nature although some are more convenient than others in this respect. Must we in the final analysis decide between various mathematical and logical systems on empirical rather than formal grounds? The evidence as yet is not conclusive but it appears as if we can make our preference on purely formal considerations.

Axiom Systems

Descriptive terms can be replaced by, or interpreted as variables. A set of things which fulfils this system is called a model of the system (cf. Carmichael, The Logic of Discovery, Chapter concerning deductive systems)[s]. A system is called categorical if any two models of it are *isomorphic*, that is if the relations between their terms have the same structure (structural characteristics are symmetry, reflexiveness and so on).

An Axiom system can be said to define or determine its primitive terms in an implicit manner so far as it determines the models in a certain way. It can never characterize them entirely, and only in terms of a structure. Thus it cannot do more in that direction than to determine the structure of the model completely, i.e. to be categorical.

We have to distinguish from categoricalness another sort of perfectness, or completeness.

A system can be called complete if every sentence formulated in terms of the system can be either deduced or refuted with the mere help of the axioms of the system. Arithmetic is categorical but not complete in the second sense, as Gödel has
43 shown. (But it is complete in a third sense, namely it is | possible to show in the meta theory that every sentence of Arithmetic, even a "Gödel" sentence, is either true or false and whether it is true or false – if the axioms of arithmetic are accepted as being true.)

The possibility of characterising systems, not only as consistent or inconsistent but also from the standpoint of their different degrees of completeness (in at least three different senses or dimension) shows that there are certain possibilities formally to distinguish between the merits of different concurring deductive systems.

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[s] Carmichael (1930, Ch. III, cf. also Ch. II).

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