Single-Valued Neutrosophic Set: An Overview

26

Surapati Pramanik

As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.

Albert Einstein

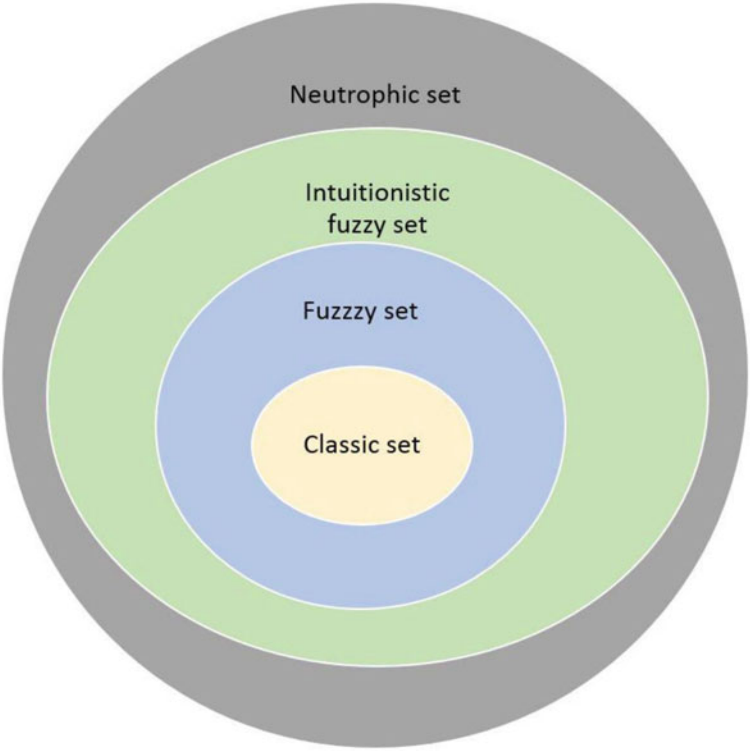
Summary

The purpose of this chapter is to present an overview of neutrosophic sets. Professor Florentin Smarandache defined the neutrosophic set and helped popularize the concept to deal with uncertainty, inconsistency, and indeterminacy common in human existence. The chapter presents the basic definitions of neutrosophic sets, single-valued neutrosophic sets, single-valued neutrosophic numbers, score functions, accuracy and certainty functions, the ranking of neutrosophic numbers, and some extensions of neutrosophic sets. It describes different types of neutrosophic sets and a few examples of their applications in social sciences. The chapter also presents a critical discussion and the future scope of research.

S. Pramanik (✉)
Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, Narayanpur, West Bengal 743126, India

Integrated Science Association (ISA), Universal Scientific Education and Research Network (USERN), Kolkata, India

N. Rezaei (ed.), *Transdisciplinarity*, Integrated Science 5,
https://doi.org/10.1007/978-3-030-94651-7_26



The relation between classic set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set

The code of this chapter is 01001110 01100101 01110101 01110100 01110010 01101111 01110011 01101111 01110000 01101000 01111001.

1 Introduction

Since Zadeh [1] grounded the Fuzzy Set (FS) in 1965, a plethora of new theories dealing with imprecision and uncertainty has been reported in the fuzzy literature. Neutrosophic Sets (NSs) grounded by Smarandache [2–6] constitute a generalization of FS and Intuitionistic FS (IFS) [7]. While the FS offers the degree of Truth Membership (TM) of an element in a prescribed set, IFS offers both a degree of TM and a degree of False Membership (FM), whereas NS offers a degree of TM, a degree of Indeterminacy Membership (IM), and a degree of FM. In IFS, TM function and FM function are independent, but in FS, FM function is dependent on TM function. In NS, TM function, IM function, and FM function are independent. Smarandache [8] presented the differences between NSs [2] and various extensions of FSs. Single-Valued NS (SVNS) [9–11] is an instance of NS that has a root in the work of Smarandache [2]. Haibin Wang, the first author, has presented the SVNS [9] in the international seminar in Salt Lake City, USA. NSs [2–6] and SVNSs [9–11] have been presented in different seminars and published in different proceedings and journals to draw much attentions from the researchers.

The popularity of NSs [2] gains momentum after the publication of SVNS [11] and the international journal "Neutrosophic Sets and Systems". NSs and SVNSs have been widely used in Multi-Attribute Decision-Making (MADM) [12–21] and Multi-Attribute Group Decision-Making (MAGDM) [22–24]. Interval Neutrosophic Set (INS) [25, 26] has been proposed as an instance of NS. INS is a subclass of NS, and it considers only subunitary intervals of [0, 1].

NSs have drawn much attention from the researchers. Various applications such as NS-based models have been introduced for options market [27], financial market [28], image denoising [29–32], cluster analysis [33], information retrieval [34, 35], love dynamics [36], video tracking [37], fault diagnosis [38], air surveillance [39, 40], and so on.

After 2010, various extensions of NS have been rapidly proposed in the literature. Ye [41] defined the Simplified NS (SNS) in terms of three numbers in [0, 1]. SNS is a subclass of NS. SNS includes of an INS and an SVNS. SNSs have been used in decision-making [42–46], medical diagnosis [47], and so on.

In 2003, Kandasamy and Smarandache [48] defined the Neutrosophic Number (NN) by combining real numbers and indeterminate parts of the form $u + Iv$, where I is an indeterminate component, and u, v are real numbers. Several strategies for MAGDM have been proposed in NN environment [49–54]. Du et al. [55] combined SNS and NN to define the Simplified Neutrosophic Indeterminate Set (SNIS) and presented a new decision-making strategy. Köseoğlu et al. [56] extended SNS and defined the Simplified Neutrosophic Multiplicative Set (SNMS) and Simplified Neutrosophic Multiplicative Preference Relation (SNMPR).

Ye [57] defined the Single-Valued Neutrosophic Hesitant Fuzzy Set (SVNHFS) that encompasses the FS [1], IFS [7], hesitant FS [58], dual hesitant FS [59], and SVNS [10]. Theoretical developments and applications of SVNHFSs have been presented in several studies [60–67].

Neutrosophic Cubic Set (NCS) [68] was proposed by extending the cubic set [69]. Jun et al. [70] also proposed the NCS. Some theoretical developments and applications of NCSs have been made in MADM [71–74] and MAGDM [75, 76].

Bipolar NS (BNS) [77] was developed by extending bipolar FS [78] and NS [2]. BNSs have been utilized in dealing with MADM [79–83] and MAGDM [84].

Maji [85] grounded the Neutrosophic Soft Set (NSS) by combining Soft Set (SS) [86] and NS [2]. The NSSs have been utilized in MADM [87–92] and MAGDM [93–96]. Ali et al. [97] defined the bipolar neutrosophic SS by combining SS [86] and BNS [77].

Many investigators paid wide attention to the research of hybridizing Rough Set (RS) [98] and NS [2]. Broumi et al. [99, 100] defined the Rough Neutrosophic Set (RNS). Broumi and Smarandache [101] grounded the Interval RNS (IRNS). Yang et al. [102] defined the Single-Valued Neutrosophic RS (SVNRS) by exploring constructive and axiomatic strategies. Single-Valued Neutrosophic Multi-Granulation

RS (SVNMGRS) [103] was proposed by combining multi-granulation rough sets [104] with SVNS [10]. Jiao et al. [105] developed the three-way decision models utilizing Decision-Theoretic RS (DTRS) [106] with SVNS [10]. New theoretical developments and applications of the RNSs have been presented in the current neutrosophic literature [107–119]. Pramanik and Mondal [120] proposed the Rough BNS (RBNS). Recently, Pramanik [121] documented an overview of RNSs.

Yager [122] introduced the fuzzy bag or the Fuzzy Multi-Set (FMS) by extending the bag or multi-set [123, 124]. An element of a FMS can assume the same or different membership values more than once. In case of Intuitionistic Fuzzy Multi-Set (IFMS) [125], an element can assume membership and falsity values more than once. To overcome the shortcomings of FMS and IFMS, Smarandache [126] presented the n-valued refined neutrosophic logic. Smarandache [126] paved the way to define the Neutrosophic Refined (NR) set. Several investigators [127–136] studied the application of NR sets. Bao and Yang [137, 138] proposed the Single-Valued Neutrosophic Refined Rough Set (SVNRRS) by hybridizing NR sets with RSs.

Ye [139] extended the intuitionistic linguistic set [140] to Single-Valued Neutrosophic Linguistic (SVNL) set and SVNL number. Li et al. [141] defined the Neutrosophic Linguistic Set (NLS) and presented a comparison strategy for Neutrosophic Linguistic Numbers (NLNs). Ye [142] defined the Interval NLN (INLN) and employed it for MADM. Tian et al. [143] defined the Simplified NLS (SNLS) that combines the SNS and linguistic term set [144]. SNLS is capable of describing linguistic information to some extent. Tian et al. [143] employed the Simplified NLNs (SNLNs) for MADM.

Biswas et al. [145] and Ye [146] presented the Trapezoidal Neutrosophic Fuzzy Number (TrNFN) by extending the trapezoidal fuzzy numbers. TrNFNs have been utilized in MADM and MAGDM [147–153]. Biswas et al. [154] presented the ranking strategy for Single-Valued Neutrosophic Trapezoidal Number (SVNTrN) and employed it for MADM. Liang et al. [155] presented the Single-Valued Trapezoidal Neutrosophic Preference Relation (SVTrNPR) and the completely consistent SVTrNPR to solve MADM problems. Biswas et al. [156] extended the SVNTrN to interval trapezoidal neutrosophic number and employed it for MADM. Biswas et al. [157] defined the triangular fuzzy NSs and employed them for MADM. Abdel- Basset et al. [158] extended neutrosophic triangular number to propose the type-2 neutrosophic number. Chakraborty et al. [159] defined the neutrosophic pentagonal number and studied some of its properties. Karaaslan [160] presented the Gaussian Single-Valued Neutrosophic Number (SVNN) and employed it for MADM.

Researchers extended NSs to different sets such as bipolar neutrosophic refined sets [161], tri-complex rough neutrosophic set [162], rough neutrosophic hyper-complex set [163], quadripartitioned SVNS [164], plithogenic set [165, 166]. Smarandache [167, 168] further proposed the neutrosophic off/under/over sets by extending NSs.

NSs, SVNSs, and their hybrid extensions and applications can be found in the studies [121, 169–177].

Rest of the chapter is designed as follows: Sect. 2 presents the basics of the NSs. Section 3 presents the triangular fuzzy number NS. Section 4 presents the trapezoidal fuzzy NS. Section 5 describes single-valued pentagonal neutrosophic numbers. Section 6 describes the cylindrical neutrosophic single-valued sets. Section 7 describes the neutrosophic numbers. Section 8 describes some applications of NSs. Section 9 describes the extensions of the NSs. Section 10 presents the direction of new research. Section 11 presents conclusions.

2 Basics of Neutrosophic Sets

2.1 Neutrosophic Set

Let Q be a space of points with a generic element ω in Q. An NS [2] ϕ in Q is characterized by a truth. Membership Function (MF) ξ_ϕ, an indeterminacy MF ψ_ϕ, a falsity MF ζ_ϕ and is presented as:

$$\phi = \langle \omega, \xi_\phi(\omega), \psi_\phi(\omega), \zeta_\phi(\omega) > /, \omega \in \Omega \rangle.$$

Here, $\xi_\phi(\omega), \psi_\phi(\omega), \zeta_\phi(\omega)$ in Ω denote the subsets of $]^-0, 1^+[$ such that $\xi_\phi(\omega)$: $Q \rightarrow]^-0, 1^+[$, $\psi_\phi(\omega)$: $Q \rightarrow]^-0, 1^+[$, and $\zeta_\phi(\omega)$: $\Omega \rightarrow]^-0, 1^+[$.

Then,

$^-0 \leq \sup \xi_\phi(\omega) + \sup \psi_\phi(\omega) + \sup \zeta_\phi(\omega) \leq 3^+$.

2.2 Single-Valued Neutrosophic Set

An SVNS [10] χ in a universal set Ω is presented by a truth MF $\xi_\chi(\omega)$, an indeterminacy MF $\psi_\chi(\omega)$, and a falsity MF $\psi_\chi(\omega)$ such that.

$\xi_\chi(\omega), \psi_\chi(\omega), \zeta_\chi(\omega)$ are in [0, 1] for all $\omega \in \Omega$.

If Ω is continuous, χ is presented as

$$\chi = \int_\omega \langle \xi_\chi(\omega), \psi_\chi(\omega), \zeta_\chi(\omega) \rangle / \omega, \ \forall \omega \in \Omega.$$

If Ω is discrete, χ is presented as

$$\chi = \sum \langle \xi_\chi(\omega), \ \psi_\chi(\omega), \zeta_\chi(\omega) \rangle / \omega, \ \forall \omega \in \Omega$$

with $0 \leq \sup \xi_\chi(\omega) + \sup \psi_\chi(\omega) + \zeta_\chi(\omega) \leq 3, \omega\forall \in \Omega$.

An SVNS χ is also presented as.

$\chi = \langle \omega, \xi_\chi(\omega), \psi_\chi(\omega), \zeta_\chi(\omega) > /, \omega \in \Omega \rangle$, where $\xi_\chi(\omega), \psi_\xi(\omega), \zeta_\xi(\omega) \in [0, 1]$, for each ω in Ω. Therefore,

$$0 \leq \sup \xi_\chi(\omega) + \sup \psi_\chi(\omega) + \sup \zeta_\chi(\omega) \leq 3.$$

Note: Since SVNS is a subclass of NS, we use NS and SVNS equivalently throughout chapter.

For convenience, the triplet $(\xi_\chi(\omega), \psi_\chi(\omega), \zeta_\chi(\omega))$ is called as the SVNN and simply presented as $(\xi_\chi, \psi_\chi, \zeta_\chi)$.

2.2.1 Some Operations of SVNNs

Let $\eta_1 = (\alpha_1, \beta_1, \gamma_1)$ and $\eta_2 = (\alpha_2, \beta_2, \gamma_2)$ be any two SVNNs with $\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2 \in [0, 1]$, $(\alpha_1 + \beta_1 + \gamma_1) \in [0, 3]$ and $(\alpha_2 + \beta_2 + \gamma_2) \in [0, 3]$.

The following operations for SVNNs [171] hold

i. $\eta_1 \oplus \eta_2 = (\alpha_1 + \alpha_2 - \alpha_1\alpha_2, \beta_1, \beta_2, \gamma_1\gamma_2)$ [Summation]
ii. $\eta_1 \otimes \eta_2 = (\alpha_1\alpha_2, \beta_1 + \beta_2 - \beta_1\beta_2, \gamma_1 + \gamma_2 - \gamma_1\gamma_2)$ [Multiplication]
iii. $\lambda\eta_1 = (1 - (1 - \alpha_1)^\lambda, \beta_1^\lambda, \gamma_1^\lambda)$ [Scalar multiplication]
iv. $\eta_1^\lambda = (\alpha_1^\lambda, 1 - (1 - \beta_1)^\lambda, 1 - (1 - \gamma_1)^\lambda)), \lambda > 0$.

2.2.2 Score Function and Accuracy Function of SVNNs

Assume that $\eta_1 = (\alpha_1, \alpha_2, \alpha_3)$ is an SVNN. Score function denoted by $\Gamma(n_1)$, accuracy function denoted by $H(n_1)$ [178] of η_1 are, respectively, represented as.

i. $\Gamma(n_1) = \frac{(1 + \alpha_1 - 2\alpha_2 - \alpha_3)}{2}$, where $\Gamma(n_1) \in [-1, 1]$
ii. $H(n_1) = \alpha_1 - \alpha_2(1 - \alpha_1) - \alpha_3(1 - \alpha_2)$, where $H(n_1) \in [-1, 1]$

Nancy and Garg [179] presented the improved score function as follows:

iii. $S(n_1) = \frac{(1 + (\alpha_1 - 2\alpha_2 - \alpha_3)(2 - \alpha_1 - \alpha_3))}{2}$,

Clearly, if $\alpha_1 + \alpha_3 = 1$, $S(n_1)$ reduces to $\Gamma(n_1)$.

2.2.3 Comparison of SVNNs

Assume that $\chi_1 = (\alpha_1, \alpha_2, \alpha_3)$ and $\chi_2 = (\beta_1, \beta_2, \beta_3)$ be any two SVNNs. Comparison strategy [179] between χ_1 and χ_2 is presented as.

i. if $\Gamma(\chi_1) < \Gamma(\chi_2))$, then $\chi_1 \prec \chi_2$
ii. if $\Gamma(\chi_1) = \Gamma(\chi_2)$, then

- İf $S(\chi_1) < S(\chi_2)$, then $\chi_1 \prec \chi_2$
- $S(\chi_1) > S(\chi_2)$, then $\chi_1 \succ \chi_2$
- $S(\chi_1) = S(\chi_2)$, then $\chi_1 \approx \chi_2$.

2.3 Interval Neutrosophic Set

Let P be a space of points having generic element p in P.

An INS [25] φ in P is presented as $\varphi = \langle p, M_\varphi(p), N_\varphi(p), O_\varphi(p) > /, p \in P \rangle$.

Here, $M_\varphi(p) = [M_\varphi^L(p), M_\varphi^U(p)]$, $N_\varphi(p) = [N_\varphi^L(p), N_\varphi^U(p)]$, $O_\varphi(p) = [O_\varphi^L(p), O_\varphi^U(p)]$ and for each $p \in P$, $M_\varphi(p), N_\varphi(p), O_\varphi(p) \subseteq [0, 1]$.

For convenience, an Interval Neutrosophic Number (INN) η_1 is presented in the form: $\eta_1 = \langle [M_1^L, M_1^U], [N_1^L, N_1^U], [O_1^L, O_1^U] \rangle$

2.3.1 Operations on INNs

Let $\eta_1 = \langle [M_1^L, M_1^U], [N_1^L, N_1^U], [O_1^L, O_1^U] \rangle$ and $\eta_2 = \langle [M_2^L, M_2^U], [N_2^L, N_2^U], [O_2^L, O_2^U] \rangle$ be any two INNs. The operations for INNs [180] are presented as:

i. $\eta_1 \oplus \eta_2 = \langle [M_1^L + M_2^L - M_1^L M_2^L, M_1^U + M_2^U - M_1^U M_2^U], [N_1^L N_2^L, N_1^U N_2^U], [O_1^L O_2^L, O_1^U O_2^U] \rangle$

ii. $\eta_1 \otimes \eta_2 = \langle [M_1^L M_2^L, M_1^U M_2^U], [N_1^L + N_2^L - N_1^L N_2^L, N_1^U + N_2^U - N_1^U N_2^U], [O_1^L + O_2^L - O_1^L O_2^L, O_1^U + O_2^U - O_1^U O_2^U] \rangle$

iii. $\gamma\eta_1 = \langle [1 - (1 - M_1^L)^\gamma, 1 - (1 - M_1^U)^\gamma)], [(N_1^L)^\gamma, (N_1^U)^\gamma], [(O_1^L)^\gamma, (O_1^U)^\gamma] \rangle$

iv. $\eta_1^\gamma = \langle [(M_1^L)^\gamma, (M_1^U)^\gamma)], [1 - (1 - N_1^L)^\gamma, 1 - (1 - N_1^U)^\gamma)], [1 - (1 - O_1^L)^\gamma, 1 - (1 - O_1^U)^\gamma)] \rangle$ where $\gamma > 0$.

2.3.2 Score Function and Accuracy Functions of INNs

Assume that $\eta_1 = \langle [M_1^L, M_1^U], [N_1^L, N_1^U], [O_1^L, O_1^U] \rangle$ be an INN. The score function $\Gamma(\eta_1)$, accuracy function $H(\eta_1)$ [178] of η_1 are, respectively, presented as:

i. $\Gamma(\eta_1) = \left(\frac{1}{4}\right) \times [2 + M_1^L + M_1^U - 2N_1^L - 2N_1^U - O_1^L - O_1^U]$, $\Gamma(\eta_1) \in [-1, 1]$

ii. $H(\eta_1) = \frac{M_1^L + M_1^U - N_1^U(1 - M_1^U) - N_1^L(1 - M_1^L) - O_1^U(1 - N_1^U) - \zeta_1^L(1 - N_1^L)}{2}$, $H(\eta_1) \in [-1, 1]$

2.3.3 Comparison INNs

The convenient strategy for comparing INNs [178] is described as follows:

Let $\eta_1 = \langle [M_1^L, M_1^U], [N_1^L, N_1^U], [O_1^L, O_1^U] \rangle$ and $\eta_2 = \langle [M_2^L, M_2^U], [N_2^L, N_2^U], [O_2^L, O_2^U] \rangle$ be any two INNs. Then,

i. If $\Gamma(\eta_1) \succ \Gamma(\eta_2)$, then $\eta_1 \succ \eta_2$
If $\Gamma(\eta_1) = \Gamma(\eta_2)$,and $H(\eta_1) \succ H(\eta_2)$, then $\eta_1 \succ \eta_2$.

2.4 Spherical Neutrosophic Set

Smarandache presented spherical NS [8], which is a generalization of spherical FS [181].

2.4.1 Single-Valued Spherical NS

A Single-Valued Spherical NS [8] of the universe of discourse Θ is presented as follows:

$$\chi = \langle \omega, \xi_\chi(\omega), \ \psi_\chi(\omega), \zeta_\chi(\omega) > /, \ \omega \in \Theta \rangle.$$

Here, $\forall w \in \Theta$, the functions $\xi_\chi(\omega), \ \psi_\chi(\omega), \zeta_\chi(\omega) : \Omega \to [0, \sqrt{3}]$ indicate the degrees of truth, indeterminacy, and falsity MF, respectively, and $0 \leq \xi_\chi^2(\omega) + \psi_\chi^2(\omega) + \ \zeta_\chi^2(\omega) \leq 3$.

Single-Valued Spherical Neutrosophic Number (SVSpNN)

Smarandache [182] presented the SVSpNN having the form: (q, r, s) where $q, r, s \in [0, \sqrt{3})$ and $q^2 + r^2 + s^2 \leq 3$.

SVSrNN is the generalization of Single-Valued Pythagorean Fuzzy Number (SVPFN) having the form: (q, r) with $q, \ r \in [0, \ 2]$ and $q^2 + r^2 \leq 2$.

Interval-Valued Spherical Neutrosophic Number (ISpNN)

Smarandache [182] presented the ISpNN that has the form:$(q, \ r, s)$ where the real intervals $q, \ r, s \subseteq [0, \ \sqrt{3}]$ and $q^2 + r^2 + s^2 \subseteq [0, \ 3]$.

2.4.2 n-Hyper Spherical Neutrosophic Set (n-HSpNS)

Single-valued n-HSpNS [8] is a generalization of the spherical NS in the universe of discourse Ω, for $n \geq 1$. It is defined as $\chi = \langle \omega, \xi_\chi(\omega), \ \psi_\chi(\omega), \zeta_\chi(\omega) > /, \omega \in \Omega \rangle$, where, $\forall \omega \in \Omega$, the functions $\xi_\chi(\omega), \ \psi_A(\omega), \zeta_A(\omega) : \Omega \to [0, \ \sqrt[n]{3}]$ represent the degrees of truth, indeterminacy, and falsity MF, respectively, and $0 \leq \xi_\chi^n(\omega) + \ \psi_\chi^n(\omega) + \zeta_\chi^n(\omega) \leq 3$.

3 Triangular Fuzzy Number Neutrosophic Set (TFNNS)

Biswas et al. [157] hybridized the Triangular Fuzzy Number (TFN) with SVNSs to define the TFNNS.

3.1 TFNNS

Let Ω be the finite universe of discourse and $\theta[0, 1]$ be the set of all TFNs on [0, 1]. A TFNNS φ in Ω is presented by

$$\varphi = \{\langle \omega, \xi_\varphi(\omega), \psi_\varphi(\omega), \zeta_\varphi(\omega) \rangle | \omega \in \Omega\},$$

where $\xi_\varphi(\omega) : \Omega \to \theta[0, 1]$,$\psi_\varphi(\omega) : \Omega \to \theta[0, 1]$, and $\zeta_\varphi(\omega) : \Omega \to \theta[0, 1]$.

The TFNs $\xi_\varphi(\omega) = \left(\xi^1_\varphi(\omega), \xi^2_\varphi(\omega), \xi^3_\varphi(\omega)\right)$, $\psi_\varphi(\omega) = \left(\psi^1_\varphi(\omega), \psi^2_\varphi(\omega), \psi^3_\varphi(\omega)\right)$, and $\zeta_\varphi(\omega) = \left(\zeta^1_\varphi(\omega), \zeta^2_\varphi(\omega), \zeta^3_\varphi(\omega)\right)$ present the degree of truth, indeterminacy, and falsity MF, respectively, of $\omega \in \varphi$, $\forall \omega \in \Omega$ and $0 \le \xi_\varphi(\omega) + \psi_\varphi(\omega) + \zeta_\varphi(\omega) \le 3$.

3.2 Triangular Fuzzy Neutrosophic Number (TFNN)

For notational convenience, we consider $\phi = \langle(\alpha, \beta, \gamma), (\rho, \sigma, \tau), (l, m, n)\rangle$ as a TFNN [157] where $\left(\xi^1_\varphi(\omega), \xi^2_\varphi(\omega), \xi^3_\varphi(\omega)\right) = (\alpha, \beta, \gamma)$, $\left(\psi^1_\varphi(\omega), \psi^2_\varphi(\omega), \psi^3_\varphi(\omega)\right) = (\rho, \sigma, \tau)$, and $\left(\zeta^1_\varphi(\omega), \zeta^2_\varphi(\omega), \zeta^3_\varphi(\omega)\right) = (l, m, n)$.

3.3 Operations on TFNNs

Let $\chi_1 = \langle(\alpha_1, \beta_1, \gamma_1), (\rho_1, \sigma_1, \tau_1), (l_1, m_1, n_1)\rangle$ and $\chi_2 = \langle(\alpha_2, \beta_2, \gamma_2), (\rho_2, \sigma_2, \tau_2), (l_2, m_2, n_2)\rangle$ be any two TFNNs in $\Re$. The basic operations for TFNNs [157] hold good:

i. $$\chi_1 \oplus \chi_2 = \langle(\alpha_1 + \alpha_2 - \alpha_1\alpha_2, \beta_1 + \beta_2 - \beta_1\beta_2, \gamma_1 + \gamma_2 - \gamma_1\gamma_2), (\rho_1\rho_2, \sigma_1\sigma_2, \tau_1\tau_2), (l_1 l_2, m_1 m_2, n_1 n_2)\rangle;$$

ii. $$\chi_1 \otimes \chi_2 = \left\langle \begin{array}{r} (\alpha_1\alpha_2, \beta_1\beta_2, \gamma_1\gamma_2), (\rho_1 + \rho_2 - \rho_1\rho_2, \sigma_1 + \sigma_2 - \sigma_1\sigma_2, \tau_1 + \tau_2 - \tau_1\tau_2), \\ (l_1 + l_2 - l_1 l_2, m_1 + m_2 - m_1 m_2, n_1 + n_2 - n_1 n_2) \end{array} \right\rangle;$$

iii. $$\lambda\chi_1 = \left\langle \left(1 - (1 - \varepsilon_1)^\lambda, 1 - (1 - \beta_1)^\lambda, 1 - (1 - \gamma_1)^\lambda\right), \left(\rho_1^\lambda, \sigma_1^\lambda, \tau_1^\lambda\right), \left(l_1^\lambda, m_1^\lambda, n_1^\lambda\right) \right\rangle$$ for $\lambda > 0$ and

iv. $$\chi_1^\lambda = \left\langle \begin{array}{r} \left(\alpha_1^\lambda, \beta_1^\lambda, \gamma_1^\lambda\right), \left(1 - (1 - \rho_1)^\lambda, 1 - (1 - \sigma_1)^\lambda, 1 - (1 - \tau_1)^\lambda\right), \\ \left(1 - (1 - l_1)^\lambda, 1 - (1 - m_1)^\lambda, 1 - (1 - n_1)^\lambda\right) \end{array} \right\rangle$$ for $\lambda > 0$.

The operations presented in Sect. 26.3 satisfy the following properties:

i. (Commutativity) : $\chi_1 \oplus \chi_2 = \chi_2 \oplus \chi_1$; $\chi_1 \otimes \chi_2 = \chi_2 \otimes \chi_1$;
ii. (Distributivity) : $\lambda(\chi_1 \oplus \chi_2) = \lambda\chi_1 \oplus \lambda\chi_2$; $(\chi_1 \otimes \chi_2)^\lambda = \chi_1^\lambda \otimes \chi_2^\lambda$, $\lambda > 0$, and
iii. (Associativity) : $\lambda_1\chi_1 \oplus \lambda_2\chi_1 = (\lambda_1 + \lambda_2)\chi_1$; $\chi_1^{\lambda_1} \oplus \chi_1^{\lambda_2} = \chi_1^{(\lambda_1 + \lambda_2)}$, $\lambda_1, \lambda_2 > 0$.

3.4 Score and Accuracy Function of TFNN

Let $\chi_1 = \langle(\alpha_1, \beta_1, \gamma_1), (\rho_1, \sigma_1, \tau_1), (l_1, m_1, n_1)\rangle$ be a TFNN in $\Re$. The score function [157] $\Gamma(\chi_1)$, the accuracy function [157] $H(\chi_1)$ of χ_1 are, respectively, presented as:

$$\Gamma(\chi_1) = \frac{1}{12}[8 + (\alpha_1 + 2\beta_1 + \gamma_1) - (\rho_1 + 2\sigma_1 + \tau_1) - (l_1 + 2m_1 + n_1)],$$
$$\Gamma(\chi_1) \in [1, 0]$$

$$H(\chi_1) = \frac{1}{4}[(\alpha_1 + 2\beta_1 + \gamma_1) - (l_1 + 2m_1 + n_1)],\ H(\chi_1) \in [-1, 1]$$

For $\chi^+ = \langle(1,1,1),(0,0,0),(0,0,0)\rangle$ and $\chi^- = \langle(0,0,0),(1,1,1),(1,1,1)\rangle$, $\Gamma(\chi^+)$ = 1 and $\Gamma(\chi^-) = 0$.

For $\chi^+ = \langle(1,1,1),(0,0,0),(0,0,0)\rangle$ and $\chi^- = \langle(0,0,0),(1,1,1),(1,1,1)\rangle$, $H(\chi^+)$ = 1 and $H(\chi^-) = 0$.

3.5 Comparison of TFNNs

Assume that $\chi_1 = \langle(\alpha_1, \beta_1, \gamma_1), (\rho_1, \sigma_1, \tau_1), (l_1, m_1, n_1)\rangle$ and $\chi_2 = \langle(\alpha_2, \beta_2, \gamma_2), (\rho_2, \sigma_2, \tau_2), (l_2, m_2, n_2)\rangle$ are any two TFNNs in $\Re$. If $\Gamma(\chi_i)$ and $H(\chi_i)$ denote, respectively, the score and accuracy function of $\chi_i (i = 1, 2)$, then the ranking of TFNNs [157] is presented as:

i. If $\Gamma(\chi_1) > \Gamma(\chi_2)$, then χ_1 is greater than χ_2 that is $\chi_1 \succ \chi_2$;
ii. If $\Gamma(\chi_1) = \Gamma(\chi_2)$ and $H(\chi_1) > H(\chi_2)$, then χ_1 is greater than χ_2, that is,$\chi_1 \succ \chi_2$;
iii. If $\Gamma(\chi_1) = \Gamma(\chi_2)$,$\Gamma(\chi_1) = \Gamma(\chi_2)$, then χ_1 is indifferent to χ_2, that is, $\chi_1 \approx \chi_2$.

3.6 Triangular Fuzzy Neutrosophic Number Arithmetic Averaging (TFNNAA) Operator

3.6.1 Triangular Fuzzy Neutrosophic Number Weighted Arithmetic Averaging (TFNNWAA) Operator

Let $\theta_\imath = \langle(c_\imath, d_\imath, e_\imath), (f_\imath, g_\imath, h_\imath), (o_\imath, p_\imath, q_\imath)\rangle (\imath = 1, 2, ..., r)$ be a collection TFNNs in $\Re$ and let TFNNWAA : $\Sigma^r \to \Sigma$.

If $\text{TFNNWAA}(\theta_1, \theta_2, ..., \theta_r) = \omega_1\theta_1 \oplus \omega_2\theta_2 \oplus \cdots \oplus \omega_r\theta_r = \mathop{\oplus}\limits_{k=1}^{r} (\omega_k\theta_k)$,

then the function $\text{TFNNWAA}(\theta_1, \theta_2, ..., \theta_r)$ is called the TFNNWAA operator, where the weight of $\theta_\imath (\imath = 1, 2, ..., r)$ is denoted by $\omega_\imath \in [0, 1]$ and $\sum\limits_{\imath=1}^{r} \omega_\imath = 1$.

If $\omega = (1/r, 1/r, ..., 1/r)^T$, then the TFNNWAA$(\theta_1, \theta_2, ..., \theta_r)$ operator reduces to TFNNAA operator:

TFNNAA$(\theta_1, \theta_2, ..., \theta_r) = \frac{1}{r}(\theta_1 \oplus \theta_2 \oplus \cdots \oplus \theta_r)$

Theorem 3.6.1 *Let* $\theta_\imath = \langle (c_\imath, d_\imath, e_\imath), (f_\imath, g_\imath, h_\imath), (o_\imath, p_\imath, q_\imath) \rangle (\imath = 1, 2, ..., r)$ *be a collection of* TFNNs *in* $\Re$. *Then,* TFNNWAA$_\omega(\theta_1, \theta_2, ..., \theta_r) = \omega_1\theta_1 \oplus \omega_2\theta_2 \oplus \cdots \oplus \omega_r\theta_r) = \overset{r}{\underset{\imath=1}{\oplus}} (\omega_\imath \theta_\imath)$.

$$= \left\langle \begin{array}{l} \left(1 - \prod_{\imath=1}^{r} (1 - c_\imath)^{\omega_\imath}, 1 - \prod_{\imath=1}^{r} (1 - d_\imath)^{\omega_\imath}, 1 - \prod_{\imath=1}^{r} (1 - e_\imath)^{\omega_\imath}\right), \\ \left(\prod_{\imath=1}^{r} f_\imath^{\omega_\imath}, \prod_{\imath=1}^{r} g_\imath^{\omega_\imath}, \prod_{\imath=1}^{r} h_\imath^{\omega_\imath}\right), \left(\prod_{\imath=1}^{r} o_\imath^{\omega_\imath}, \prod_{\imath=1}^{r} p_\imath^{\omega_\imath}, \prod_{\imath=1}^{r} q_\imath^{\omega_\imath}\right) \end{array} \right\rangle,$$

where $\omega_\imath \in [0, 1]$ *denotes the weight of* $\theta_\imath (\imath = 1, 2, ..., r)$ and $\sum_{\imath=1}^{r} \omega_\imath = 1$.

Proof For proof, see Biswas et al. [157].

3.6.2 Triangular Fuzzy Number Neutrosophic Geometric Averaging (TFNNGA) Operator

Triangular Fuzzy Number Neutrosophic Weighted Geometric Averaging (TFNNWGA) Operator

Let $\theta_\imath = \langle (c_\imath, d_\imath, e_\imath), (f_\imath, g_\imath, h_\imath), (o_\imath, p_\imath, q_\imath) \rangle$ $(\imath = 1, 2, ..., r)$ be a collection of TFNNs in $\Re$ and TFNNWGA : $\Pi^r \to \Pi$.

If

TFNNWGA$_\omega(\theta_1, \theta_2, ..., \theta_r) = \theta_1^{\omega_1} \otimes \theta_2^{\omega_2} \otimes \cdots \otimes \theta_r^{\omega_r} = \overset{r}{\underset{\imath=1}{\otimes}} \left(\theta_\imath^{\omega_\imath}\right)$

then TFNNWGA$_\omega(\theta_1, \theta_2, ..., \theta_r)$ is called the TFNNWGA operator where $\omega_\imath \in [0, 1]$ is the exponential weight of $\theta_k (\imath = 1, 2, ..., r)$ such that $\sum_{i=1}^{r} w_k = 1$.

If $\omega = (1/r, r, ..., 1/r)^T$, then the TFNNWGA$(\theta_1, \theta_2, ..., \theta_r)$ operator reduces to TFNNGA operator:

TFNNGA$(\theta_1, \theta_2, ..., \theta_r) = (\theta_1 \otimes \theta_2 \otimes \cdots \otimes \theta_r)^{\frac{1}{r}}$.

Theorem 3.6.2 *Let* $\theta_\imath = \langle (c_\imath, d_\imath, e_\imath), (f_\imath, g_\imath, h_\imath), (o_\imath, p_\imath, q_\imath) \rangle (\imath = 1, 2, ..., r)$ *is a collection of TFNNs in* $\Re$. *Then,*TFNNWGA$_\omega(\theta_1, \theta_2, ..., \theta_r) = \theta_1^{\omega_1} \otimes \theta_2^{\omega_2} \otimes \cdots \otimes \theta_n^{\omega_\imath} = \overset{r}{\underset{\imath=1}{\otimes}} \left(\theta_\imath^{\omega_\imath}\right)$

$$\left(\prod_{\imath=1}^{r} c_{\imath}^{\omega_{\imath}}, \prod_{\imath=1}^{r} d_{\imath}^{\omega_{\imath}}, \prod_{\imath=1}^{r} e_{\imath}^{\omega_{\imath}}\right),$$

$$= \left\langle \begin{array}{c} \left(1-\prod_{\imath=1}^{r}(1-f_{\imath})^{\omega_{\imath}}, 1-\prod_{\imath=1}^{r}(1-g_{\imath})^{\omega_{\imath}}, 1-\prod_{\imath=1}^{r}(1-h_{\imath})^{\omega_{\imath}}\right), \\ \left(1-\prod_{\imath=1}^{r}(1-o_{\imath})^{\omega_{\imath}}, 1-\prod_{\imath=1}^{r}(1-p_{\imath})^{\omega_{\imath}}, 1-\prod_{\imath=1}^{r}(1-q_{\imath})^{\omega_{\imath}}\right) \end{array} \right\rangle \textit{ is a TFNN,}$$

where $\omega_k \in [0,1]$ *denotes the weight vector of* TFNN $\theta(\imath = 1, 2, \ldots, r)$ *such that* $\sum_{\imath=1}^{r} \omega_{\imath} = 1$.

Proof For proof, see Biswas et al. [157].

4 Trapezoidal Fuzzy Number NS

Ye [146] and Biswas et al. [145] combined Trapezoidal Fuzzy Number (TrFN) with SVNS to define Trapezoidal Fuzzy Number NS (TrFNNS).

4.1 TrFNNS

Assume that Θ is the finite universe of discourse. A TrFNNS θ in Θ is presented as:

$\theta = \{\langle x, o_\theta(x), p_\theta(x), q_\theta(x)\rangle | x \in \Theta\}$, where $O_\theta(x) \subset [0,1]$,$p_\theta(x) \subset [0,1]$ $q_\theta(x) \subset [0,1]$ are trpezoidal fuzzy numbers and $o_\theta(x) = \left(o_\theta^1(x), o_\theta^2(x), o_\theta^3(x), o_\theta^4(x)\right) : \Theta \to [0,1]$,$p_\theta(x) = \left(p_\theta^1(x), p_\theta^2(x), p_\theta^3(x), p_\theta^4(x)\right) : \Theta \to [0,1]$, and $q_\theta(x) = \left(q_\theta^1(x), q_\theta^2(x), q_\theta^3(x), q_\theta^4(x)\right) : \Theta \to [0,1]$ present, respectively, the degrees of truth, indeterminacy, and falsity MF of x in θ and for every $x \in \Theta$ and $0 \le o_\theta^4(x) + p_\theta^4(x) + q_\theta^4(x) \le 3$.

4.2 Trapezoidal Fuzzy Neutrosophic Number (TrFNN)

A TrFNN [145] χ_φ is presented as:

$\chi_\varphi = \langle(\alpha_1, \alpha_2, \alpha_3, \alpha_4), (\beta_1, \beta_2, \beta_3, \beta_4), (\gamma_1, \gamma_2, \gamma_3, \gamma_4)\rangle$ in a universe of discourse W with $\alpha_1 \le \alpha_2 \le \alpha_3 \le \alpha_4$, $\beta_1 \le \beta_2 \le \beta_3 \le \beta_4$ and $\gamma_1 \le \gamma_2 \le \gamma_3 \le \gamma_4$. Here,$\chi_\varphi$ is defined as follows:

Its truth MF is presented as:

$$\xi_{\chi_\varphi}(\omega) = \begin{cases} \frac{\omega-\alpha_1}{\alpha_2-\alpha_1}, & \alpha_1 \le \omega \le \alpha_2 \\ 1, & \alpha_2 \le \omega \le \alpha_3 \\ \frac{\alpha_4-\omega}{\alpha_4-\alpha_3}, & \alpha_3 \le \omega \le \alpha_4 \\ 0, & \text{otherwise}. \end{cases}$$

Its indeterminacy MF is presented as:

$$\psi_{\chi_\varphi}(\omega) = \begin{cases} \frac{\omega-\beta_1}{\beta_2-\beta_1}, & \beta_1 \le \omega \le \beta_2 \\ 1, & \beta_2 \le \omega \le \beta_3 \\ \frac{\beta_4-\omega}{\beta_4-\beta_3}, & \beta_3 \le \omega \le \beta_4 \\ 0, & \text{otherwise}. \end{cases}$$

and its falsity MF is presented as:

$$\zeta_{\chi_\varphi}(\omega) = \begin{cases} \dfrac{\omega-\gamma_1}{\gamma_2-\gamma_1}, & \gamma_1 \le \omega \le \gamma_2 \\ 1, & \gamma_2 \le \omega \le \gamma_3 \\ \dfrac{\gamma_4-\omega}{\gamma_4-\gamma_3}, & \gamma_3 \le \omega \le \gamma_4 \\ 0 & \text{otherwise}. \end{cases}$$

Let $\chi_1 = \langle(\alpha_1, \beta_1, \gamma_1, \delta_1), (\rho_1, \sigma_1, \tau_1, \upsilon_1), (l_1, m_1, n_1, o_1)\rangle$ and $\chi_2 = \langle(\alpha_2, \beta_2, \gamma_2, \delta_2), (\rho_2, \sigma_2, \tau_2, \upsilon_2), (l_2, m_2, n_2, 0_2)\rangle$. be any two TrFNNs in $\Re$. Then, the operational rules for χ_1 and χ_2 are presented as.

i. $\chi_1 \oplus \chi_2 = \left\langle \begin{array}{c} (\alpha_1+\alpha_2-\alpha_1\alpha_2, \beta_1+\beta_2-\beta_1\beta_2, \gamma_1+\gamma_2-\gamma_1\gamma_2, \delta_1+\delta_2-\delta_1\delta_2), \\ (\rho_1\rho_2+\sigma_1\sigma_2+\tau_1\tau_2+\upsilon_1\upsilon_2), (l_1l_2+m_1m_2+n_1n_2+o_1o_2) \end{array} \right\rangle$

ii. $\chi_1 \otimes \chi_2 = \left\langle \begin{array}{c} (\alpha_1\alpha_2, \beta_1\beta_2, \gamma_1\gamma_2, \delta_1\delta_2), \\ (\rho_1+\rho_2-\rho_1\rho_2, \sigma_1+\sigma_2-\sigma_1\sigma_2, \tau_1+\tau_2-\tau_1\tau_2, \upsilon_1+\upsilon_2-\upsilon_1\upsilon_2), \\ (l_1+l_2-l_1l_2, m_1+m_2-m_1m_2, n_1+n_2-n_1n_2, o_1+o_2-o_1o_2) \end{array} \right\rangle;$

iii. $\eta\chi_1 = \left\langle \begin{array}{c} \begin{pmatrix} 1-(1-\alpha_1)^\eta, 1-(1-\beta_1)^\eta, \\ 1-(1-\gamma_1)^\eta, 1-(1-\delta_1)^\eta \end{pmatrix}, \\ (\rho_1^\eta, \sigma_1^\eta, \tau_1^\eta, \upsilon_1^\eta), (l_1^\eta, m_1^\eta, n_1^\eta, o_1^\eta) \end{array} \right\rangle$ for $\eta > 0$

iv. $(\chi_1)^\eta = \left\langle \begin{array}{c} (\alpha_1^\eta, \beta_1^\eta, \gamma_1^\eta, \delta_1^\eta), \\ (1-(1-\rho_1)^\eta, 1-(1-\sigma_1)^\eta, 1-(1-\tau_1)^\eta, 1-(1-\upsilon_1)^\eta), \\ (1-(1-l_1)^\eta, 1-(1-m_1)^\eta, 1-(1-n_1)^\eta, 1-(1-o_1)^\eta), \end{array} \right\rangle$ for $\eta > 0$.

v. $\chi_1 = \chi_2$, if $(\alpha_1, \beta_1, \gamma_1, \delta_1) = (\alpha_2, \beta_2, \gamma_2, \delta_2)$; $(\rho_1, \sigma_1, \tau_1, \upsilon_1) = (\rho_2, \sigma_2, \tau_2, \upsilon_2)$; $(l_1, m_1, n_1, o_1) = (l_2, m_2, n_2, 0_2)$

4.3 Expected Value (EV) and Expected Interval (EI) of TrFNN

The EI and the EV [145] of the truth MF $\xi_{\chi_1}(\omega) = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \begin{cases} \frac{\omega-\alpha_1}{\alpha_2-\alpha_1}, & \alpha_1 \le \omega \le \alpha_2 \\ 1, & \alpha_2 \le \omega \le \alpha_3 \\ \frac{\alpha_4-\omega}{\alpha_4-\alpha_3}, & \alpha_3 \le \omega \le \alpha_4 \\ 0, & \text{otherwise}. \end{cases}$ of χ_1 in a universe of discourse W are defined as follows:

$$\mathrm{EI}\xi_{\chi_1}(\omega) = \left[\frac{(\alpha_1 + \alpha_2)}{2}, \frac{(\alpha_3 + \alpha_4)}{2}\right]$$

$$\mathrm{EV}\xi_{\chi_1}(\omega) = \frac{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)}{4}$$

In similar way, the EI and the EV of the indeterminacy MF $\psi_{\chi_1}(\omega) = \begin{cases} \frac{\omega-\beta_1}{\beta_2-\beta_1}, & \beta_1 \le \omega \le \beta_2 \\ 1, & \beta_2 \le \omega \le \beta_3 \\ \frac{\beta_4-\omega}{\beta_4-\beta_3}, & \beta_3 \le \omega \le \beta_4 \\ 0, & \text{otherwise}. \end{cases}$

of χ_1 are defined as:

$$\mathrm{EI}\psi_{\chi_1}(\omega) = \left[\frac{(\beta_1 + \beta_2)}{2}, \frac{(\beta_3 + \beta_4)}{2}\right]$$

$$\mathrm{EV}\psi_{\chi_1}(\omega) = \frac{(\beta_1 + \beta_2 + \beta_3 + \beta_4)}{4}$$

and the EI and the EV of the falsity MF

$$\zeta_{\chi_1}(\omega) = \begin{cases} \dfrac{\omega - \gamma_1}{\gamma_2 - \gamma_1}, & \gamma_1 \le \omega \le \gamma_2 \\ 1, & \gamma_2 \le \omega \le \gamma_3 \\ \dfrac{\gamma_4 - \omega}{\gamma_4 - \gamma_3}, & \gamma_3 \le \omega \le \gamma_4 \\ 0 & \text{otherwise}. \end{cases}$$

of χ_1 are defined as follows:

$$\mathrm{EI}(\zeta_{\chi_1}(\omega)) = \left[\frac{(\gamma_1 + \gamma_2)}{2}, \frac{(\gamma_3 + \gamma_4)}{2}\right]$$

$$\mathrm{EV}(\zeta_1(\omega)) = \frac{(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4)}{4}$$

4.4 Truth Favourite Relative Expected Value (TrFREV) of TrFNN

Let $a = \langle(\alpha_1, \beta_1, \gamma_1, \delta_1), (\rho_1, \sigma_1, \tau_1, \upsilon_1), (l_1, m_1, n_1, o_1)\rangle$ be a TrFNN in $\Re$. Then TrFREV [145] of a is defined as:

$$\mathrm{EV}^{\mathrm{truth}}(a) = \frac{3EV(\xi_a(\omega))}{\mathrm{EV}(\xi_a(\omega)) + \mathrm{EV}(\psi_a(\omega)) + \mathrm{EV}(\zeta_a(\omega))},$$

where $\mathrm{EV}(\xi_a(w))$, $\mathrm{EV}(\psi_a(\omega))$, and $\mathrm{EV}(\zeta_a(\omega))$ denote, respectively, the EVs of truth, indeterminacy, and falsity MF of a.

4.5 Expected Value Theorem

Assume that $\phi_1 = \langle(\alpha_1, \beta_1, \gamma_1, \delta_1), (\rho_1, \sigma_1, \tau_1, \upsilon_1), (l_1, m_1, n_1, o_1)\rangle$ is a TrFNN in $\Re$, then the TFREV of ϕ_1 is

$$\mathrm{EV}^{\mathrm{truth}}(a) = \frac{3\sum_{i=1}^{4} \alpha_i}{\left(\sum_{i=1}^{4} (\alpha_i + \beta_i + \gamma_i)\right)}$$

Proof For proof, see [145].

4.6 Single-Valued Trapezoidal Neutrosophic Number (SVTrNN)

An SVTrNN [146] (SVTNN) $\alpha = \langle(\alpha_1, \alpha_2, \alpha_3, \alpha_4); (\xi_\alpha, \psi_\alpha, \zeta_\alpha)\rangle$ is a special NS on the real number set R, whose truth MF, indeterminacy MF, and a falsity MF are presented as:

$$\xi_\alpha(\omega) = \begin{cases} (\omega - \alpha_1)\xi_\alpha / (\alpha_2 - \alpha_1), & (\alpha_1 \le \omega \le \alpha_2) \\ \xi_\alpha, & (\alpha_2 \le \omega \le \alpha_3) \\ (\alpha_4 - \omega)\xi_\alpha / (\alpha_4 - \alpha_3), & (\alpha_3 \le \omega \le \alpha_4) \\ 0, & \text{otherwise} \end{cases}$$

$$\psi_\alpha(\omega)=\begin{cases}(\alpha_2-\omega+\psi_a(\omega-\alpha_1))/_{(\alpha_2-\alpha_1)}, & (\alpha_1\le\omega\le\alpha_2)\\ \psi_\alpha, & (\alpha_2\le\omega\le\alpha_3)\\ (\omega-\alpha_3+\psi_\alpha(\alpha_4-\omega))/_{(\alpha_4-\alpha_3)}, & (\alpha_3\le x\le\alpha_4)\\ 1, & \text{otherwise}\end{cases}$$

$$\zeta_\alpha(\omega)=\begin{cases}(\alpha_2-\omega+\zeta_a(\omega-\alpha_1))/_{(\alpha_2-\alpha_1)}, & (\alpha_1\le\omega\le\alpha_2)\\ \zeta_\alpha, & (\alpha_2\le\omega\le\alpha_3)\\ (\omega-\alpha_3+\zeta_\alpha(\alpha_4-\omega))/_{(\alpha_4-\alpha_3)}, & (\alpha_3\le x\le\alpha_4)\\ 1, & \text{otherwise}\end{cases}$$

where $0\le\ \xi_\alpha\le\ 1,\ 0\le\ \psi_\alpha\le\ 1,\ 0\le\ \zeta_\alpha\le\ 1;$ and $0\le\xi_\alpha+\psi_\alpha+\zeta_\alpha\le 3;$ $\alpha_1,\alpha_2,\alpha_3,\alpha_4\in\Re$.

Here, $\alpha=\ <(\alpha_1,\alpha_2,\alpha_3,\alpha_4);(\xi_\alpha,\psi_\alpha,\zeta_\alpha)>$ is called a positive (+ve) SVTrNN, if $\alpha_1>0$.

Similarly, $\alpha=\ <(\alpha_1,\alpha_2,\alpha_3,\alpha_4);\ (\xi_\alpha,\psi_\alpha,\zeta_\alpha)>$ reduces to a negative (–ve) SVTrNN, if $\alpha_4\le 0$,

When $0\le\alpha_1\le\alpha_2\le\alpha_3\le\alpha_4\le 1)$, and $0\le\ \xi_\alpha\le\ 1,\ 0\le\ \psi_\alpha\le\ 1,\ 0\le\ \zeta_\alpha\le\ 1$, α is called a normalized SVTrNN.

4.6.1 Score Function of SVTrNNs

Let $\alpha=\langle(\alpha_1,\alpha_2,\alpha_3,\alpha_4);(\xi_\alpha,\psi_\alpha,\zeta_\alpha)\rangle$ be an SVTrNN. Then, the score and accuracy function [183] of α are denoted by $\Gamma(\alpha)$ and $H(\alpha)$, respectively, and are presented as:

i. $\Gamma(\alpha)=\left(\frac{1}{12}\right)(\alpha_1+\alpha_2+\alpha_3+\alpha_4)\times(2+\xi_\alpha-\psi_\alpha-\zeta_\alpha)$
ii. $H(\alpha)=\left(\frac{1}{12}\right)(\alpha_1+\alpha_2+\alpha_3+\alpha_4)\times(2+\xi_\alpha-\psi_\alpha+\zeta_\alpha)$

4.6.2 Ranking of SVTrNN

Let $\alpha=\langle(\alpha_1,\alpha_2,\alpha_3,\alpha_4);(\xi_\alpha,\psi_\alpha,\zeta_\alpha)\rangle$ and $\beta=\left\langle(\beta_1,\beta_2,\beta_3,\beta_4);(\xi_\beta,\psi_\beta,\zeta_\beta)\right\rangle$ be any two SVTrNNs. Ranking of SVTrNNs [183] is presented as follows:

(i) When $\Gamma(\alpha)<\Gamma(\beta)$, then $\alpha<\beta$
(ii) When $\Gamma(\alpha)=\Gamma(\beta)$ and if $H(\alpha)<H(\beta)$, then $\alpha<\beta$
(iii) When $\Gamma(\alpha)=\Gamma(\beta)$ and $H(\alpha)=H(\beta)$, then $\alpha=\beta$.

4.6.3 Centre of Gravity

Let $\alpha=[\alpha_1,\alpha_2,\alpha_3,\alpha_4]$ be a TrFN on $\Re$, and $\alpha_1\le\alpha_2\le\alpha_3\le\alpha_4$; then, the centre of gravity (COG) of α [155, 184, 185] defined by

$$\mathrm{COG}(\alpha)=\left\{\begin{array}{ll}\alpha_1, & \text{if}\quad \alpha_1=\alpha_2=\alpha_3=\alpha_4\\ \frac{1}{3}\left[\alpha_1+\alpha_2+\alpha_3+\alpha_4-\frac{\alpha_4-\alpha_3-\alpha_{21}-\alpha_1}{\alpha_4+\alpha_3-\alpha_2-\alpha_1}\right], & \text{otherwise}\end{array}\right\}$$

4.6.4 Score function SVTrNN

Assume that $\alpha = \langle(\alpha_1, \alpha_2, \alpha_3, \alpha_4); (\xi_\alpha, \psi_\alpha, \zeta_\alpha)\rangle$ is an SVTrNN. Then, the score, accuracy, and certainty function [155] of α are denoted by $\Gamma(\alpha)$, $H(\alpha)$, and $\kappa(\alpha)$), respectively, and presented as:

$$\Gamma(\alpha) = COG(\alpha) \times \frac{(2 + \xi_\alpha - \psi_\alpha - \zeta_\alpha)}{3}$$
$$H(\alpha) = COG(\alpha) \times (\xi_\alpha - \zeta_\alpha)$$
$$\kappa(\alpha) = COG(\alpha) \times \xi_\alpha$$

4.6.5 Comparison of SVTrNNs

Assume that $\alpha = \langle(\alpha_1, \alpha_2, \alpha_3, \alpha_4); (\xi_\alpha, \psi_\alpha, \zeta_\alpha)\rangle$, and $\beta = \langle(\beta_1, \beta_2, \beta_3, \beta_4);\ (\xi_\beta, \psi_\beta, \zeta_\beta)\rangle$ are any two SVTRNNs. Comparison between SVTrNNs [155] is presented as follows:

(i) When $\Gamma(\alpha) > \Gamma(\beta)$, then $\alpha > \beta$
(ii) When $\Gamma(\alpha) = \Gamma(\beta)$ and if $H(\alpha) > H(\beta)$, then $\alpha > \beta$
(iii) When $\Gamma(\alpha) = \Gamma(\beta)$ and $H(\alpha) < H(\beta)$, then $\alpha < \beta$
(iv) When $\Gamma(\alpha) = \Gamma(\beta)$, $H(\alpha) = H(\beta)$, and

- $\kappa(\alpha) > \kappa(\beta)$, then $\alpha > \beta$
- $\kappa(\alpha) < \kappa(\beta)$, then $\alpha < \beta$
- $\kappa(\alpha) = \kappa(\beta)$, then $\alpha = \beta$.

4.7 Interval TrNN (ITrNN)

Suppose that χ is an SVTrNN [156]. Its truth MF, indeterminacy MF, and falsity MF are defined as

$$\xi_\chi(\omega) = \begin{cases} \dfrac{(\omega - \alpha_1)a'_\chi}{(\alpha_2 - \alpha_1)}, & \alpha_1 \le \omega \le \alpha_2 \\ a'_\chi, & \alpha_2 \le \omega \le \alpha_3 \\ \dfrac{(\alpha_4 - \omega)a'_\chi}{(\alpha_4 - \alpha_3)}, & \alpha_3 \le \omega \le \alpha_4 \\ 0, & \text{otherwise.} \end{cases}$$

$$\psi_\chi(\omega) = \begin{cases} \dfrac{(\alpha_2 - \omega) + (\omega - \alpha_1)b'_\chi}{(\alpha_2 - \alpha_1)}, & \alpha_1 \le \omega \le \alpha_2 \\ b'_\chi, & \alpha_2 \le \omega \le \alpha_3 \\ \dfrac{\omega - \alpha_3 + (\alpha_4 - \omega)b'_\chi}{\alpha_4 - \alpha_3}, & \alpha_3 \le \omega \le \alpha_4 \\ 0, & \text{otherwise.} \end{cases}$$

$$\zeta_\chi(\omega) = \begin{cases} \dfrac{\alpha_2 - \omega + (\omega - \alpha_1)c'_\chi}{\alpha_2 - \alpha_1}, & \alpha_1 \le \omega \le \alpha_2 \\ c'_\chi, & \alpha_2 \le \omega \le \alpha_3 \\ \dfrac{\omega - \alpha_3 + (\alpha_4 - \omega)c'_\chi}{\alpha_4 - \alpha_3}, & \alpha_3 \le \omega \le \alpha_4 \\ 0, & \text{otherwise.} \end{cases}$$

where $0 \le \xi_\chi(\omega) \le 1,\ 0 \le \psi_\chi(\omega) \le 1, 0 \le \zeta_\chi(\omega) \le 1,\quad 0 \le \xi_\chi(\omega) + \psi_\chi(\omega) + \zeta_\chi(\omega) \le 3,\ \alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4 \in \Re$ and $a'_\chi, b'_\chi, c'_\chi \in [0,\ 1]$. Then, neutrosophic trapezoidal number χ is presented as $\chi = (\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4; a'_\chi, b'_\chi, c'_\chi)$.

4.7.1 Definition of ITrNN

Assume that $a'_\chi = [a'^{l}_\chi, a'^{u}_\chi], b'_\chi = [b'^{l}_\chi, b'^{u}_\chi], c'_\chi = [c'^{l}_\chi, c'^{u}_\chi,]$. Then, an ITrNN [156] χ denoted by $\chi = ([\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4]; a'_\chi, b'_\chi, c'_\chi)$ is defined as

$$\xi_\chi(\omega) = \begin{cases} \dfrac{(\omega - \alpha_1)a'_\chi}{(\alpha_2 - \alpha_1)}, & \alpha_1 \le \omega \le \alpha_2 \\ a'_\chi, & \alpha_2 \le \omega \le \alpha_3 \\ \dfrac{(\alpha_4 - \omega)a'_\chi}{(\alpha_4 - \alpha_3)}, & \alpha_3 \le \omega \le \alpha_4 \\ 0, & \text{otherwise.} \end{cases}$$

$$\psi_\chi(\omega) = \begin{cases} \dfrac{(\alpha_2 - \omega) + (\omega - \alpha_1)b'_\chi}{(\alpha_2 - \alpha_1)}, & \alpha_1 \le \omega \le \alpha_2 \\ b'_\chi, & \alpha_2 \le \omega \le \alpha_3 \\ \dfrac{\omega - \alpha_3 + (\alpha_4 - \omega)b'_\chi}{\alpha_4 - \alpha_3}, & \alpha_3 \le \omega \le \alpha_4 \\ 0, & \text{otherwise.} \end{cases}$$

$$\zeta_\chi(\omega) = \begin{cases} \dfrac{\alpha_2 - \omega + (\omega - \alpha_1)c'_\chi}{\alpha_2 - \alpha_1}, & \alpha_1 \le \omega \le \alpha_2 \\ c'_\chi, & \alpha_2 \le \omega \le \alpha_3 \\ \dfrac{\omega - \alpha_3 + (\alpha_4 - \omega)c'_\chi}{\alpha_4 - \alpha_3}, & \alpha_3 \le \omega \le \alpha_4 \\ 0, & \text{otherwise.} \end{cases}$$

where $a'_\chi, b'_\chi, c'_\chi \subset [0, 1]$ denote interval numbers, $0 \le \sup(a'_\chi) + \sup(b'_\chi) + \sup(c'_\chi) \le 3$ and $\chi = ([\alpha_1, \alpha_2, \alpha_3, \alpha_4]; [a'^l_\chi, a'^u_\chi], [b'^l_\chi, b'^u_\chi], [c'^l_\chi, c'^u_\chi,]$ is said to be positive ITrNN if $\chi > 0$ and one of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ is not equal to zero.

4.7.2 Operations on ITrNNs

Let $\chi_1 = \left([\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}]; [p^l_1, p^u_1], [q^l_1, q^u_1], [r^l_1, r^u_1]\right)$ and $\chi_2 = ([\alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}]; [p^l_2, p^u_2], [q^l_2, q^u_2], [r^l_2, r^u_2])$ be any two ITrNNs and $\lambda > 0$. Then, the following operations hold good [156].

i. $\chi_1 \oplus \chi_2 == ([\alpha_{11} + \alpha_{21}, \alpha_{12} + \alpha_{22}, \alpha_{13} + \alpha_{23}, \alpha_{14} + \alpha_{24}];$
$[p^l_1 + p^l_2 - p^l_1 p^l_2, p^u_1 + p^u_2 - p^u_1 p^u_2], [q^l_1 q^l_2, q^u_1 q^u_2], [r^l_1 r^l_2, r^u_1 r^u_2])$

$$\chi_1 \otimes \chi_2 = ([\alpha_{11}\,\alpha_{21}, \alpha_{12}\alpha_{22}, \alpha_{13}\,\alpha_{31}, \alpha_{14}\alpha_{24}]; [p^l_1 p^l_2,\ p^u_1 p^u_2],$$
$$\left[q^l_1 + q^l_2 - q^l_1 q^l_2, {}^u_1 + q^u_2 - q^u_1 q^u_2\right],$$
$$\left[r^l_1 + r^l_2 - r^l_1 r^l_2,\ r^u_1 + r^u_2 - r^u_1 r^u_2\right])$$

$$\lambda\chi_1 = ([\lambda\alpha_{11}, \lambda\,\alpha_{12}, \lambda\alpha_{13}, \lambda\alpha_{14}];$$
$$[1 - (1 - p^l_1)^\lambda, 1 - (1 - p^u_2)^\lambda], [(q^l_1)^\lambda, (q^u_2)^\lambda], [(r^l_1)^\lambda, (r^\lambda_2)^\lambda])$$

$$(\chi_1)^\lambda = (\left[(\alpha_{11})^\lambda, (\alpha_{12})^\lambda, (\alpha_{13})^\alpha, (\alpha_{14})^\alpha\right];$$
$$[(p^l_1)^\lambda, (p^u_1)^\lambda], [1 - (1 - q^l_1)^\lambda, 1 - (1 - q^u_1)^\lambda], [1 - (1 - r^l_1)^\lambda, 1 - (1 - r^u_1)^\lambda])$$

5 A Single-Valued Pentagonal Neutrosophic Number (SVPNN)

An SVPNN [159] χ is defined as

$$\chi = \left\langle \omega, \xi_\chi(\omega),\ \psi_A(\omega), \zeta_A(\omega) > /,\ \omega \in \Omega \right\rangle.$$

The truth MF $\xi_\chi(\omega) : \Re \to [0, \alpha]$, the indeterminacy MF $\psi_\chi(\omega) : \Re \to [\beta, 1]$, and the falsity MF $\zeta_\chi(\omega) : \Re \to [\gamma, 1]$ are presented as:

$$\xi_\chi(\omega) = \begin{cases} \xi_{\chi_{L1}}(\omega),\ p'_1 \le \omega \le q'_1 \\ \xi_{\chi_{L2}}(\omega), q'_1 \le \omega \le r'_1 \\ \tau, \quad \omega = r'_1 \\ \xi_{\chi_{U1}}(\omega),\ r'_1 \le \omega \le s'_1 \\ \xi_{\chi_{U2}}(\omega),\ s'_1 \le \omega \le t'_1 \\ 0, \quad \text{otherwise} \end{cases},$$

$$\psi_\chi(\omega) = \begin{cases} \psi_{\chi_{L1}}(\omega),\ p'_2 \le \omega \le q'_2 \\ \psi_{\chi_{L2}}(\omega), q'_2 \le \omega \le r'_2 \\ \iota, \quad \omega = r'_2 \\ \psi_{\chi_{U1}}(\omega),\ r'_2 \le \omega \le s'_2 \\ \psi_{\chi_{U2}}(\omega),\ s'_2 \le \omega \le t'_2 \\ 1, \quad \text{otherwise} \end{cases},$$

$$\zeta_\chi(\omega) = \begin{cases} \zeta_{\chi_{L1}}(\omega),\ p'_3 \le \omega \le q'_3 \\ \zeta_{\chi_{L2}}, q'_3 \le \omega \le r'_3 \\ \kappa, \quad \omega = r'_3 \\ \zeta_{\chi_{U1}}(\omega),\ r'_3 \le \omega \le s'_3 \\ \zeta_{\chi_{U2}}(\omega),\ s'_3 \le \omega \le t'_3 \\ 1, \quad \text{otherwise} \end{cases}$$

5.1 Score Function of SVPNN

Assume that $\iota = \langle(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5); (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5); (\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5))\rangle$ is an SVPNN.

Beneficiary degree of truth indicator $= (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)/5$.

Hesitation degree of indeterminacy indicator = $(\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5)/5$.

Non-beneficiary degree of falsity indicator = $(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5)/5$.

Chakraborty et al. [159] defined score function $\Gamma(\iota)$ and accuracy function $H(\iota)$ of ι as follows:

$$\Gamma(\iota) = \frac{1}{3}\left(2 + \frac{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)}{5} - \frac{(\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5)}{5} - \frac{(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5)}{5}\right)$$

Here, $\Gamma(\iota) \in [0,\ 1]$.

$H(\iota) = \left(\frac{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)}{5} - \frac{(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5)}{5}\right)$, $H(\iota) \in [-1, 1]$.

5.2 Comparison of SVPNNS

Assume that $\iota_1 = \langle(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5); (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5); (\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5))\rangle$ and $\iota_2 = \langle(c_1, c_2, c_3, c_4, c_5); (d_1, d_2, d_3, d_4, d_5); (e_1, e_2, e_3, e_4, e_5))\rangle$ be any two SVPNNs.

Comparison between any two SVPNNs [159] ι_1 and ι_2 is presented as

i. If $\Gamma(\iota_1) > \Gamma(\iota_2)$, then $\iota_1 > \iota_2$;
ii. If $\Gamma(\iota_1) < \Gamma(\iota_2)$, then $\iota_1 < \iota_2$;
iii. If $\Gamma(\iota_1) = \Gamma(\iota_2)$, and

  - if $H(\iota_1) > H(\iota_2)$, then $\iota_1 > \iota_2$;
  - $H(\iota_1) < H(\iota_2)$, then $\iota_1 < \iota_2$;
  - $H(\iota_1) = H(\iota_2)$, then $\iota_1 \approx \iota_2$.

6 Cylindrical Neutrosophic Single-Valued (CNSV) Set

Let W be a space of objects with generic element ω in W. A CNSV set [8] χ in W is presented as: $\chi = \left\langle \omega, \xi_\chi(\omega),\ \psi_\chi(\omega), \zeta_\chi(\omega) > /,\ \omega \in W \right\rangle$, where χ $\xi_\chi(\omega)$, $\psi_\chi(\omega), \zeta_\chi(\omega)$ denote the truth MF, the indeterminacy MF, and the falsity MF, respectively.

Here, $\left(\xi_\chi(\omega)\right)^2 + \left(\psi_\chi(\omega)\right)^2 \leq 1^2,\ \zeta_\chi(\omega) \leq 1$.

For convenience, $\chi = \left(\xi_\chi,\ \psi_\chi, \zeta_\chi\right)$ is simply defined as a CNSV Number (CNSVN).

6.1 Score Function of CNSVN

For any CNSVN $\chi = \left(\xi_\chi,\ \psi_\chi, \zeta_\chi\right)$.

- Beneficiary degree of truth MF = $2\left(\xi_\chi\right)^2$
- Indeterminacy degree of indeterminacy MF = $2(\psi)^2$
- Non-beneficiary degree of falsity MF = $2\left(\zeta_\chi\right)^2$.

The score function and accuracy function are, respectively, denoted by $\Gamma(\chi)$ and $A(\chi)$ [186] and presented as:

i. $\Gamma(\chi) = \left(2(\xi_\chi)^2 - (\psi_\chi)^2 - (\zeta_\chi)^2\right)$, with $\Gamma(\chi) = [-1, 1]$ and the accuracy function $A(\chi)$ is defined as:

ii. $A(\chi) = \frac{2(\xi_\chi)^2 + (\psi_\chi)^2 + (\zeta_\chi)^2}{2}$, $A(\chi) \in [0, 2]$

6.2 Comparıson of CNSVNs

Assume that $\imath_1 = \left(\xi_{\tau_1}, \psi_{\tau_1}, \zeta_{\tau_1}\right)$ and $\imath_2 = \left(\xi_{\tau_2}, \psi_{\tau_2}, \zeta_{\tau_2}\right)$ are any two CNSVNs. Then, comparison between $\imath_1$ and $\imath_2$ [186] is presented as:

i. If $\Gamma(\imath_1) > \Gamma(\imath_2)$, then $\imath_1 > \imath_2$;
ii. If $\Gamma(\imath_1) < \Gamma(\imath_2)$, then $\imath_1 < \imath_2$;
iii. If $\Gamma(\imath_1) = \Gamma(\imath)$, and

- if $A(\imath_1) > A(\imath_2)$, then $\imath_1 > \imath_2$;
- if $A(\imath_1) < A(\imath_2)$, then $\imath_1 < \imath_2$;
- if $A(\imath_1) = A(\imath_2)$, then $\imath_1 \approx \imath_2$.

7 Neutrosophic Number (NN)

Kandasamy and Smarandache [48, 54] introduced the NN of the structure $\eta = \alpha + \beta\imath$, where α, β denote real or complex numbers, and "$\imath$" denotes the indeterminacy component of η.

An NN η can be presented as $\eta = [\alpha + \beta\imath^l,\ \alpha + \beta\imath^u]$, $\eta \in N$, N denotes the set having $\forall$ NNs and $\imath \in [\imath^l, \imath^u]$. The interval $\imath \in [\imath^l, \imath^u]$ is called an indeterminate interval.

- When $\beta = 0$, η reduces to crisp number $\eta = \alpha$
- When $\alpha = 0$, then η reduces to the indeterminate number $\eta = \beta I$
- When $\imath^l = \imath^u$, then η reduces to a crisp number.

Assume that $\eta_1 = \alpha_1 + \beta_1\imath$ and $\eta_2 = \alpha_2 + \beta_2\imath$ for η_1, $\eta_2 \in$ N and $\imath \in [\imath^l, \imath^u]$ are two NNs. Some basic operational laws [187] for η_1 and η_2 are presented as:

(1) $\imath^2 = \imath$
(2) $\imath.0 = 0$
(3) $\frac{\imath}{\imath} =$ Undefined
(4) $\eta_1 + \eta_2 = \alpha_1 + \alpha_2 + (\beta_1 + \beta_2)\imath = [\alpha_1 + \alpha_2 + (\beta_1 + \beta_2)\imath^l,\ \alpha_1 + \alpha_2 + (\beta_1 + \beta_2)\imath^u]$
(5) $\eta_1 - \eta_2 = \alpha_1 - \alpha_2 + (\beta_1 - \beta_2)\imath = [\alpha_1 - \alpha_2 + (\beta_1 - \beta_2)\imath^l, \alpha_1 - \alpha_2 + (\beta_1 - \beta_2)\imath^u]$
(6) $\eta_1 \times \eta_2 = \alpha_1\alpha_2 - \alpha_2 + (\alpha_1\beta_2 + \alpha_2\beta_1)\imath + \beta_1\beta_2\imath^2 = \alpha_1\alpha_2 - \alpha_2 + (\alpha_1\beta_2 + \alpha_2\beta_1 + \beta_1\beta_2)\imath$

(7) $\frac{\alpha_1+\beta_1\iota}{\alpha_2+\beta_2\iota} = \frac{\alpha_1}{\alpha_2} + \frac{\alpha_2\beta_1-\alpha_1\beta_2}{\alpha_2(\alpha_2+\beta_2)}\iota$; $\alpha_2 \neq 0$, $\alpha_2 \neq -\beta_2$

(8) $\eta_1^2 = \alpha_1^2 + (2\alpha_1\beta_1 + \beta_1^2)\iota$

(9) $\lambda\eta_1 = \lambda\alpha_1 + \lambda\beta_1\iota$

Here, indeterminacy "ι" and the imaginary $i = \sqrt{-1}$ are different concepts. In general, $\iota^p = \iota$ if $p > 0$, and it is undefined if $p < 0$.

7.1 Score Function of NNs

Assume that an *NN is of the form:* $\tau = c + \iota d = [c + d\iota^l, c + d\iota^u]$, *where* c and d *are* not simultaneously zeroes. The score function $\Xi(\chi)$ [54] and the accuracy function $A(\chi)$ [54] are, respectively, presented as

$$\Xi(\tau) = \left|\frac{c + d(\iota^u - \iota^l)}{2\sqrt{c^2 + d^2}}\right|, \Xi(\tau) \in [0, 1]$$

$$A(\tau) = 1 - \exp(-\left|c + d(\iota^l - \iota^u)\right|),$$

7.2 Comparison of NNs

Assume that $\eta_1 = c_1 + \iota d_1$ and $\eta_2 = c_2 + \iota d_2$ be any two NNs. Comparison between η_1 and η_2 [54] is presented as follows:

i. If $\Xi(\eta_1) > \Xi(\eta_2)$, then $\eta_1 > \eta_2$;
ii. $\Xi(\eta_1) < \Xi(\eta_2)$, then $\eta_1 < \eta_2$;
iii. If $\Xi(\eta_1) = \Xi(\eta_2)$, and

- $A(\eta_1) > A(\eta_2)$, then $\eta_1 > \eta_2$;
- $A(\eta_1) < A(\eta_2)$, then $\eta_1 < \eta_2$;
- $A(\eta_1) = A(\eta_2)$, then $\eta_1 \approx \eta_2$.

7.3 Neutrosophic Refined Number (NRN)

NRN [188] was defined as:$[p + q_1\iota_1 + q_2\iota_2 + \ldots + q_m\iota_m]$, where $p, q_1, q_2, \ldots, q_m$ denote a real or complex number, and $\iota_1, \iota_2, \ldots, \iota_m$ denote sub-indeterminacies, for $m \geq 1$.

8 Some Applications of Neutrosophic Sets

NSs have been applied in different fields. Few applications are depicted in Table 1.

8.1 Sentiment Analysis

Smarandache et al. [189] studied the similarity measure by defining the words' sentiment scores. In their study, Smarandache et al. [189] dealt with the sentiment characteristics of the words only. They developed a novel word-level similarity measure and found promising results.

8.2 Cosmology

Christianto and Smarandache [190] argued that neutrosophic logic resolved the dispute dealing with the "*beginning and the eternity of the Universe*". In their study, Christianto and Smarandache [190] agreed that "the universe could have both a beginning and an eternal existence" leading to the paradox that "*there might have been a time before time or a beginning of time in time*".

8.3 Neutrosophic Cognitive Map (NCM) for Social Problems

Using the NCM, Devadoss et al. [191] studied to evaluate the impact of playing violent video games among the teenagers (13–18 years) in Chennai. In their study, Devadoss et al. [191] considered nine concepts and presented the outcome of the study by comparing the results derived from Fuzzy Cognitive Map (FCM) and NCM.

8.4 Neutrosophic Strategy to Combat COVID-19

Yasser et al. [192] developed a novel health-fog framework in assisting diagnosis and treatment for COVID-19 patients efficiently based on neutrosophic classifier. The study [192] integrated the information scattered among different medical centres and health organizations to combat with COVID-19.

8.5 Social Network Analysis e-Learning Systems via NSs

Using NSs, Salama et al. [193] integrated social activities in the environment of e-learning and developed a social learning management system. Radwan [194] presented the current trends and challenges in e-learning processes in NS environment.

Table 1 Some applications of NSs and neutrosophic logic

S. No.	Contributors	Contribution of the study
1	Yasser, Twakol, El-Khalek, Samrah, Salama [192]	The study developed a deep learning model to detect COVID-19 patient by employing neutrosophic classifier to extract visual features from volumetric exams.
2	Vasantha, Kandasamy, Smarandache, Devvrat, Ghildiyal [210]	It studied the imaginative play of children by utilizing single valued refined NSs
3	Kandasamy, Vasantha, Obbineni, Smarandache [211]	It analyzed ten political or social datasets of tweets for sentiment analysis using Python and necessary libraries for natural language processing in multi refined NS environment. It presented a more efficient strategy in capturing the opinion of the tweets with best accuracy
4	Vasantha, Kandasamy, Devvrat, Ghildiyal [212]	It studied the imaginative play of children (1–10 years) by employing the NCM
5	Devadoss and Rekha [213]	It analyzed the girls' problems faced by child marriage using the neutrosophic associative FCM
6	Mondal and Pramanik [214]	It analyzed the problems of Hijras in West Bengal using NCMs
7	Pramanik and Chackrabarti [215]	It presented the issues of construction workers in West Bengal using the NCMs
8	Jousselme and Maupin [216]	It described the role of neutrosophy in situation analysis
9	Thiruppathi, Saivaraju, Ravichandran [217]	It studied the suicide problem using combined overlap block NCMs
10	Zafar and Anas [218]	Using NCM, it analyzed the situation of crime in Nigeria
11	William, Devadoss, Sheeba [219]	It analyzed the risk factors of breast cancer
12	Kandasamy and Smarandache [220, 221]	It analyzed the social issues of migrant workers having HIV/AIDS
13	Bernajee [222]	It presented the decision support tool for knowledge based institution
14	Radwan [194]	It described the applications of NS in E-learning
15	Anitha and Gunavathi [195]	The study presented a classification employing musical features
16	Shadrach and Kandasamy [196]	The study provided an early leaf disease diagnosis
17	Pamucar et al. [197]	The study presented a potential energy storage options using fuzzy neutrosophic numbers
18	Ramalingam et al. [223]	The study presented the issues of traffic congestion problem in Indian context

8.6 Raga Classification

Anitha and Gunavathi [195] presented the study of Carnatic raga by classifying all 72 melakarta ragas by utilizing neutrosophic logic and NCM.

8.7 Early Diagnosis of Leaf Ailments

Shadrach and Kandasamy [196] presented a new feature selection strategy in detecting leaf disease using a computer-based method to classify the leaf diseases. In their study, Shadrach and Kandasamy [196] compared the eight existing selection strategies with their developed strategy to demonstrate the capability of their strategy. Their developed strategy [196] obtained 99.8% classification accuracy in selecting 11 characteristics for leaf disease diagnosis.

8.8 Potential Energy Storage Options.

Pamucar et al. [197] developed the neutrosophic Multi-Criteria Decision-Making (MCDM) strategy to evaluate potential energy storage options and conducted a case study in Romania by identifying four criteria and thirteen sub-criteria.

8.9 Neutrosophic Computational Model

Albeanu [198] reviewed the principles of computing and presented some new neutrosophic computational models to identify requirements for software implementation.

8.10 Finance and Economics

Bencze [199] clearly stated that Sukanto Bhattacharya made a significant contribution in neutrosophic research by reflecting the applications of NS-based models dealing with financial economic and social science problems [27, 200–204].

8.11 Conflict Resolution

Bhattacharya et al. [205] presented the arguments in applying the principles of the neutrosophic game theory in order to reflect Israel–Palestine conflict with regard to the goals and strategies of each side. Bhattacharya et al. [205] extended the game theoretic explanation of Plessner [206] and developed the neutrosophic game theoretic model.

Pramanik and Roy [207] presented the arguments in applying the principles of the game theory in order to understand the Indo-Pak conflict over Jammu and Kashmir (J&K) with regard to the goals and strategies of either country. In their study, Pramanik and Roy [207] presented the 2×2 zero-sum game theoretic crisp model of Indo-Pak conflict over J&K by identifying the goals, strategies, and options of either country. Pramanik and Roy [208] also extended the game theoretic crisp model of Pramanik and Roy [207] to neutrosophic game theoretic model to obtain the optimal solution of the ongoing Indo-Pak conflict over J&K.

Deli [209] initiated to study neutrosophic game theory and presented matrix games with simplified neutrosophic payoffs. New research is very important in this area.

8.12 Air Surveillance

Fan et al. [39] proposed the neutrosophic Hough transform (NHT) strategy to deal with the complex surveillance issues. NS is used to characterize the different targets, namely real, false, and uncertain (indeterminate) targets in surveillance environments. They proposed a new neutrosophic Hough transform-based track initiation (NHT-TI) strategy that performs better than modified HT-TI strategy and improved HT-TI strategy.

Air surveillance involves various uncertain factors. In sensor network, uncertain factors may result from unknown target dynamic models, unknown environmental disturbances, the imprecise data processing, and limited performance of sensors [39]. Fan et al. [40] reviewed the NS-based multiple target tracking (MTT) strategies and presented the NS-based MTT strategies and that help in improving the performance.

9 The Extensions of Neutrosophic Sets

NS generalizes the classic set, FS [1] and IFS [7] (see Graphical abstract). NSs have been widely studied, and many extensions have been proposed in the literature. Different applications of NSs are presented in Table 1. The hybrid and extensions of NSs and the contributing authors are shown in Table 2. Currently, there are 82 neutrosophic-related sets derived from neutrosophics in the literature.

10 New Directions

The management of neutrosophic information in human controlled real-world issue appears to be a complex and difficult task. NSs facilitate in handling inconsistency and indeterminacy caused by limited knowledge of the domain experts. This

Table 2 The various hybrid and extensions of NSs

S. No.	Name of the set	Acronym/ Abbreviation	Developed by
1	m-Generalized q-Neutrosophic Set	mGqNS	Saha et al. [224]
2	Generalized Neutrosophic b-Open Set	GNbOS	Das and Pramanik [225]
3	Linguistic neutrosophic set	LNS	Li et al. [141]
4	Plithogenic Set	PS	Smarandache [165]
5	Neutrosophic Crisp Set	NCrS	Salama and Smarandache [226]
6	Interval Neutrosophic Set	INS	Wang et al. [25, 26]
7	Dynamic Interval-Valued Neutrosophic Set	DIVNS	Thong et al. [227]
8	Interval Neutrosophic Linguistic Set	INLS	Ye [142]
9	Single-Valued Neutrosophic Set	SVNS	Wang et al. [9–11]
10	Single Valued Neutrosophic Linguistic Set	SVNLS	Ye [139]
11	Double-Valued Neutrosophic Set	DVNS	Kandasamy [228]
12	Type-2 Single-Valued Neutrosophic Set	T2SVNS	Karaaslan and Hunu [229]
13	Quadripartitioned Single Valued Neutrosophic Set	QSVNS	Chatterjee et al. [164]
14	Triangular Fuzzy Number Neutrosophic Set	TFNNS	Biswas et al. [157]
15	Trapezoidal Fuzzy Neutrosophic Set	TrFNS	Biswas et al. [145]
16	Trapezoidal Neutrosophic Set	TrNS	Ye [146]
17	Single Valued Neutrosophic Trapezoid Linguistic Set	SVNTrLS	Broumi and Smarandache [230]
18	Triangular Neutrosophic Set	TNS	Deli and Subas [231]
19	Simplified Neutrosophic Set	SNS	Ye [41]
20	Simplified Neutrosophic Multiplicative Set	SNMS	Köseoğlu et al. [56]
21	Possibility Simplified Neutrosophic Set	PSNS	Sahin and Liu [232]
22	Neutrosophic Soft Set	NSS	Maji [85]
23	Interval-Valued Neutrosophic Soft Set	IVNSS	Deli [233]
24	Generalized Neutrosophic Soft Set	GNSS	Broumi [234]
25	Generalized Interval Neutrosophic Soft Set	GINSS	Broumi et al. [235]
26	Time-Neutrosophic Soft Set	TNSS	Alkhazaleh [236]

(continued)

Table 2 (continued)

S. No.	Name of the set	Acronym/ Abbreviation	Developed by
27	Time-Neutrosophic Soft Expert Set	TNSES	Ulucay et al. [237]
28	ivnpiv-Neutrosophic Soft Set	ivnpiv-NSS	Deli et al. [92]
29	Interval Valued Neutrosophic Parameterized Soft Set	IVNPSS	Broumi et al. [238]
30	Interval-valued Possibility Quadripartitioned Single Valued Neutrosophic Soft Set	IPQSVNSS	Chatterjee et al. [239]
31	Neutrosophic Valued Linguistic Soft Set	NVLSS	Zhao and Guan [240]
32	Simplified Neutrosophic Uncertain Linguistic Set	SNULS	Tian et al. [143]
33	Bipolar Neutrosophic Set	BNS	Deli et al. [77]
34	Interval Valued Bipolar Neutrosophic Set	IVBNS	Deli et al. [241]
35	Interval Valued Bipolar Fuzzy Weighted Neutrosophic Set	IVBFWNS	Deli et al. [242]
36	Bipolar *Neutrosophic Refined Set*	BNRS	Deli and Subas [161]
37	Complex Neutrosophic Set	CNS	Ali and Smarandache [243]
38	Interval Complex Neutrosophic Set:	ICNS	Ali et al. [244]
39	Complex Neutrosophic Soft Expert Set	CNSES	Al-Quran and Hassan [245]
40	Single-Valued Linguistic Complex Neutrosophic Set	(SVLCNS)	Dat et al. [246]
41	Interval Linguistic Complex Neutrosophic Set	(ILCNS)	Dat et al. [246]
42	Bipolar Complex Neutrosophic Set	BCNS	Broumi et al. [247]
43	Neutrosophic Cubic Set	NCS	Ali et al. [68], Jun et al. [70]
44	Neutrosophic Soft Cubic Set	NSCS	Cruz and Irudayam [248]
45	Possibility Neutrosophic Cubic Set	PNCS	Xue et al. [249]
46	Rough Neutrosophic Set	RNS	Broumi et al. [99, 100]
47	Interval Rough Neutrosophic Set	IRNS	Broumi and Smarandache [101]
48	Single Valued Neutrosophic Rough Set	SVNRS	Yang et al. [102]
49	Single Valued Neutrosophic Multi-Granulation Rough Set	SVNMGRS	Zhang et al.[103]
50	Single Valued Neutrosophic Refined Rough Set	SVNRRS	Bao and Yang [137]

(continued)

Table 2 (continued)

S. No.	Name of the set	Acronym/ Abbreviation	Developed by
51	Interval-Valued Neutrosophic Soft Rough Set	IVNSRS	Broumi and Smarandache [250]
52	Rough Bipolar Neutrosophic Set	RBNS	Pramanik and Mondal [120]
53	Rough Neutrosophic Hyper-Complex Set	RNHCS	Mondal et al. [163]
54	Neutrosophic Hesitant Fuzzy Set	NHFS	Ye [57]
55	Interval Neutrosophic Hesitant Fuzzy Set	INHFS	Ye [251]
56	Triangular Neutrosophic Cubic Linguistic Hesitant Fuzzy	TNCLHF	Fahmi et al. [252]
57	Hesitant Bipolar-Valued Neutrosophic Set	HBVNS	Awang et al. [253]
58	Neutrosophic Refined Set	NRS	Smarandache [126]
59	Single Valued Neutrosophic Multiset	SVNM	Ye and Ye [254]
60	Triple Refined Indeterminate Neutrosophic Set	TRINS	Kandasamy and Smarandache [255]
61	Double Refined Indeterminacy Neutrosophic Set	DRINS	Kandasamy and Smarandache [256]
62	Single-Valued Spherical Neutrosophic Set	SVSp-NS	Smarandache [257]
63	Single-Valued n-Hyper Sphere Neutrosophic Set	SVn-HSNS	Smarandache [257]
64	Neutrosophic Fuzzy Set	NFS	Das et al. [258]
65	Neutrosophic Vague Set	NVS	Alkhazaleh [259]
66	Interval Neutrosophic Vague Set	INVS	Hashim et al. [260]
67	Neutrosophic Vague Soft Set	NVSS	Al-Quran and Hassan [261]
68	Neutrosophic Vague Soft Expert Set	NVSES	Al-Quran and Hassan [262]
69	Vague–valued Possibility Neutrosophic Vague Soft Expert Set	VPNVSPS	Mukherjee [263]
70	Neutrosophic Vague Soft Multiset	NVSM	Al-Quran and Hassan [264]
71	Possibility Neutrosophic Vague Soft Set	PNVSS	Hassan and Al-Quran [265]
72	Neutrosophic Bipolar Vague Set	NBVS	Hussain et al. [266]

(continued)

Table 2 (continued)

S. No.	Name of the set	Acronym/ Abbreviation	Developed by
73	Intuitionistic Neutrosophic Set	InNS	Bhowmik and Pal [267]
74	Intuitionistic Neutrosophic Soft Set	InNSS	Broumi and Smarandache [268]
75	Multi-Valued Neutrosophic Set	MVNS	Wang and Li [60]
76	N-Valued Interval Neutrosophic Set	NVINS	Broumi et al. [269]
77	Probability Multi-Valued Neutrosophic Set	PMVNS	Peng et al. [270]
78	Probability Multi-Valued Linguistic Neutrosophic Set	PMVLNS	Wang and Zhang [271]
79	Normal Neutrosophic Set	NNS	Liu and Teng [272]
80	Multi-Valued Neutrosophic Soft Set	MVNSS	Kamal and Abdullah [273]
81	Multi-Valued Interval Neutrosophic Soft Set	MVINSS	Kamal et al. [274]
82	n-Valued Refined Neutrosophic Soft Set	n-VRNSS	Alkhazaleh [275]

chapter has presented various neutrosophic concepts and tools to deal with neutrosophic information.

To manage neutrosophic information in complex problems, different theoretical strategies [169, 171] have been presented in the current neutrosophic literature. Some weaknesses of the neutrosophic studies are highlighted.

- Since the introduction of SVNS [11], many extensions and versions of NSs have been proposed. But some of the extensions of NSs are debatable with respect to their usefulness. These extensions should be able to solve real problems with uncertainty, inconsistency, and indeterminacy. Theoretical or practical dimensions of the extensions must be justified.
- Some papers with the same title and almost same content [2–6, 9–11, 99, 100] have been published in different journals leading to the confusion, self-plagiarism and the waste of time to find differences between them.
- Too many NSs and decision-making strategies based on these sets have been proposed in the literature without presenting a commanding justification of their applications in real-life problems. It appears that a commanding justification of their necessity and applicability are necessary.
- Researchers utilize different notations for presenting concepts, tools, and extensions to deal with neutrosophic information.
- The increase in volume, uncertainty, inconsistency, falsity, indeterminacy, and incompleteness in data reflects currently a major challenge. The current scenario demands the scientific analysis and new development of neutrosophic

frameworks that are capable of modelling, clustering, and data fusion problems in different fields of scientific study. Uncertainty, falsity and indeterminacy in prices are the key aspects in economic activities. So, new neutrosophic research should address the diverse topics such as stock trending analysis [276], performance of stock market [277], finance, economics and politics [27, 199–204], conflict resolution [208], air surveillance, and multiple target tracking strategy [37, 39].
- Neutrosophic game theoretic model [209] must be further studied.
- A new trend in research appears as the utilization of the neutrosophic theoretical models to realistic problems. Neutrosophic models should be a new and provide solutions to the problems, which cannot be solved by developed strategies in the literature.
- Since vague sets [278] are IFSs, and IFS is equivalent interval FS [279], the relations between neutrosophic FS [258], vague NS [259], and intuitionistic NS [267] should be deeply investigated.
- In upcoming 30 years, NS and its hybrid sets will highly be useful in artificial intelligence, automation, cybernetics, data analysis, engineering management science, mobile ad hoc network, neurosciences, operations research, interdisciplinary applications, multidisciplinary science, weather forecasting, etc.

11 Conclusion

Uncertainty, indeterminacy and inconsistency usually get involved in many human controlled complex real-world problems. NSs and their various extensions offer successful results in dealing with different neutrosophic decision-making problems. Much attentions have been given to some of them that manage neutrosophic situations, which often appear when indeterminacy must be dealt with. These new strategies have attracted the great attention of the investigators who deeply studied the diverse neutrosophic concepts, diverse hybrid extensions, similarity measures, and aggregation operators to deal with neutrosophic information.

The chapter has presented some directions and considerations of future researches that should be considered in the coming NS-based proposals. It has also recognizez that there exist many avenues of research in NSs and their extensions. It is to be noted that the results of this chapter offer a current overview of NSs and SVNSs. However, NSs will evolve and possibly develop in the future according to new ideas and topics that will dominate the current neutrosophic research arena.

Core Messages

- NS offers a natural foundation for dealing with the neutrosophic phenomena mathematically that exist widely in the real world.
- For realistic decision-making problems, the neutrosophic set is a very promising mathematical tool that can dominate the other mathematical tools for making pragmatic and rational decision-makıng in a complex and uncertain environment.
- NSs are capable of handling uncertainty, inconsistency, indeterminacy, and incompleteness and that is why they attract all branches of knowledge.
- NS is a promising mathematical tool for integrated science.

References

1. Zadeh LA (1965) Fuzzy sets. Inf Control 5:338–353
2. Smarandache F (1998) Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis. American Research Press, Rehoboth
3. Smarandache F (1999) Linguistic paradoxes and tautologies. Libertas Mathematica, University of Texas at Arlington XIX:143–154
4. Smarandache F (2005) Neutrosophic set–a generalization of the intuitionistic fuzzy set. Int J Pure Appl Math 24(3):287–297
5. Smarandache F (2006) Neutrosophic set-a generalization of the intuitionistic fuzzy set. In: 2006 IEEE international conference on granular computing. IEEE, New York, pp 38–42
6. Smarandache F (2010) Neutrosophic set–a generalization of the intuitionistic fuzzy set. J Defense Resources Manage 1(1):107–116
7. Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20(1):87–96
8. Smarandache F (2019) Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), pythagorean fuzzy set, spherical fuzzy set, and q-rung orthopair fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision (revisited). J New Theory 29:1–31
9. Wang H, Smarandache F, Zhang Y, Sunderraman R (2005) Single valued neutrosophic sets. In: Proceedings of 10th 476 International conference on fuzzy theory and technology
10. Wang H, Smarandache F, Zhang Y, Sunderraman R (2010) Single valued neutrosophic sets. Rev Air Force Acad 1:10–14
11. Wang H, Smarandache F, Zhang Y, Sunderraman R (2010) Single valued neutrosophic sets. Multispace Multistructure 4:410–413
12. Sodenkamp M (2013) Models, methods and applications of group multiple-criteria decision analysis in complex and uncertain systems. Dissertation, University of Paderborn, Germany
13. Ye J (2013) Multicriteria decision-making method using the correlation coefficient under single-value neutrosophic environment. Int J Gen Syst 42(4):386–394
14. Liu P, Wang Y (2014) Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Neural Comput Appl 25(7–8):2001–2010
15. Kharal A (2014) A neutrosophic multi-criteria decision making method. New Mathe Natural Comput 10(2):143–162

16. Biswas P, Pramanik S, Giri BC (2014) Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. Neutrosophic Sets Syst 2:102–110
17. Biswas P, Pramanik S, Giri BC (2014) A new methodology for neutrosophic multi-attribute decision-making with unknown weight information. Neutrosophic Sets Systs 3:42–50
18. Mondal K, Pramanik S (2015) Neutrosophic decision making model of school choice. Neutrosophic Sets Syst 7:62–68
19. Mondal K, Pramanik S (2015) Neutrosophic tangent similarity measure and its application to multiple attribute decision making. Neutrosophic Sets Syst 9:80–87
20. Şahin R, Liu P (2015) Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information. Neural Comput Appl 27(7):2017–2029
21. Pramanik S, Biswas P, Giri BC (2017) Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Neural Comput Appl 28(5):1163–1176
22. Biswas P, Pramanik S, Giri BC (2016) TOPSIS method for multi-attribute group decision making under single-valued neutrosophic environment. Neural Comput Appl 27(3):727–737
23. Biswas P, Pramanik S, Giri BC (2019) Non-linear programming approach for single-valued neutrosophic TOPSIS method. New Math Natural Comput 15(2):307–326
24. Biswas P, Pramanik S, Giri, BC (2019) Neutrosophic TOPSIS with group decision making. In: Fuzzy multi-criteria decision-making using neutrosophic sets. Springer, berlin, pp 543–585
25. Wang H, Madiraju P, Zhang YQ, Sunderraman R (2005) Interval neutrosophic sets. Int J Appl Math Stat 3:1–18
26. Wang H, Smarandache F, Zhang YQ, Sunderraman R (2005) Interval neutrosophic sets and logic: theory and applications in computing. Arizona, Hexis
27. Bhattacharya S (2005) Neutrosophic information fusion applied to the options market. Investment Manage Fin Innov 1:139–145
28. Khoshnevisan M, Bhattacharya S (2003) Neutrosophic information fusion applied to financial market. In: Sixth international conference of information fusion, 2003. Proceedings of the, vol 2. IEEE, New York, pp 1252–1257
29. Guo Y, Cheng HD, Zhang Y, Zhao W (2008) A new neutrosophic approach to image denoising. In: Proceedings of the 11th joint conference on information sciences. Atlantis Press, pp 1–6
30. Guo Y, Cheng HD (2009) New neutrosophic approach to image segmentation. Pattern Recogn 42(5):587–595
31. Ju W (2011) Novel application of neutrosophic logic in classifiers evaluated under region-based image categorization system. All Graduate Theses and Dissertations. 887. https://digitalcommons.usu.edu/etd/887
32. Koundal D, Gupta S, Singh S (2016) Applications of neutrosophic sets in medical image denoising and segmentation. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and application. Pons Editions, Brussels, pp 257–275
33. Ye J (2014) Single valued neutrosophic minimum spanning tree and its clustering method. J Intell Syst 23(3):311–324
34. Dutta AK, Biswas R, Al-Arifi NS (2011) A study of neutrosophic technology to retrieve queries in relational database. Int J Comput Sci Emerg Technol 2(1):133–138
35. Arora M, Pandey US (2011) Supporting queries with imprecise constraints in total neutrosophic data bases. Global J Sci Front Res 11(8):67–72
36. Patro SK (2016) On a model of love dynamics: a neutrosophic analysis. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and application. Pons Editions, Brussels, pp 279–287

37. Hu K, Ye J, Fan E, Shen S, Huang L, Pi J (2017) A novel object tracking algorithm by fusing color and depth information based on single valued neutrosophic cross-entropy. J Intell Fuzzy Syst 32:1775–1786
38. Ye J (2017) Single-valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine. Soft Comput 21(3):817–825
39. Fan E, Hu K, Li X (2019) Review of neutrosophic-set-theory-based multiple-target tracking methods in uncertain situations. In: 2019 IEEE international conference on artificial intelligence and computer applications (ICAICA). IEEE, New York, pp 19–27
40. Fan E, Xie W, Pei J, Hu K, Li X (2018) Neutrosophic Hough transform-based track initiation method for multiple target tracking. IEEE Access 6:16068–16080
41. Ye J (2014) Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. Int J Fuzzy Syst 16(2):204–211
42. Peng JJ, Wang JQ, Zhang HY, Chen XQ (2014) An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. Appl Soft Comput 25:336–346
43. Peng JJ, Wang JQ, Wang ZHY, Chen XH (2016) Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. Int J Syst Sci 47(10):2342–2358
44. Wu XH, Wang JQ, Peng JJ, Chen XH (2016) Cross-entropy and prioritized aggregation operator with simplified neutrosophic sets and their application in multi-criteria decision-making problems. Int J Fuzzy Syst 18:1104–1116
45. Şahin R, Küçük GD (2018) Group decision making with simplified neutrosophic ordered weighted distance operator. Math Methods Appl Sci 41(12):4795–4809
46. Du S, Yong R, Ye J (2020) Subtraction operational aggregation operators of simplified neutrosophic numbers and their multi-attribute decision making approach. Neutrosophic Sets Syst 33:157–167
47. Ye J (2015) Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. Artif Intell Med 63(3):171–179
48. Kandasamy WBV, Smarandache F (2003) Fuzzy cognitive maps and neutrosophic cognitive maps. ProQuest Information and Learning, Ann Arbor
49. Pramanik S, Roy R, Roy TK (2017) Teacher selection strategy based on bidirectional projection measure in neutrosophic number environment. In: Smarandache F, Basset MA, Chang V (eds) Neutrosophic operational research, vol II. Pons Edition, Brussels, pp 29–53
50. Ye J (2016) Multiple-attribute group decision-making method under a neutrosophic number environment. J Intell Syst 25(3):377–386
51. Ye J (20117). Bidirectional projection method for multiple attribute group decision making with neutrosophic number. Neural Comput Appl 28:1021–1029
52. Liu P, Liu X (2018) The neutrosophic number generalized weighted power averaging operator and its application in multiple attribute group decision making. Int J Mach Learn Cybern 9:347–358
53. Zheng E, Teng F, Liu P (2017) Multiple attribute group decision-making method based on neutrosophic number generalized hybrid weighted averaging operator. Neural Comput Appl 28:2063–2074
54. Mondal K, Pramanik S, Giri BC, Smarandache F (2018) NN-Harmonic mean aggregation operators-based MCGDM strategy in a neutrosophic number environment. Axioms 7(1):12
55. Du S, Ye J, Yong R, Zhang F (2020) Simplified neutrosophic indeterminate decision making method with decision makers' indeterminate ranges. J Civ Eng Manag 26(6):590–598
56. Köseoğlu A, Şahin R, Merdan M (2019) A simplified neutrosophic multiplicative set-based TODIM using water-filling algorithm for the determination of weights. Expert Syst. https://doi.org/10.1111/exsy.12515
57. Ye J (2015) Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment. J Intell Syst 24(1):23–36
58. Torra V (2010) Hesitant fuzzy sets. Int J Intell Syst 25(6):529–539

59. Zhu B, Xu Z, Xia M (2012) Dual hesitant fuzzy sets. J Appl Math. Article ID 879629. https://doi.org/10.1155/2012/879629
60. Wang JJ, Li XE (2015) TODIM method with multi-valued neutrosophic sets. Control Decis 30(6):1139–1142
61. Sahin R, Liu P (2016) Distance and similarity measure for multiple attribute with single – valued neutrosophic hesitant fuzzy information. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and applications. Pons Editions, Brussels, pp 35–54
62. Sahin R, Liu P (2017) Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making. Neural Comput Appl 28(6):1387–1395
63. Biswas P, Pramanik S, Giri BC (2016) GRA method of multiple attribute decision making with single valued neutrosophic hesitant fuzzy set information. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and applications. Pons Editions, Brussels, pp 55–63
64. Biswas P, Pramanik S, Giri BC (2016) Some distance measures of single valued neutrosophic hesitant fuzzy sets and their applications to multiple attribute decision making. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and applications. Pons Editions, Brussels, pp 55–63
65. Biswas P, Pramanik S, Giri, BC (2019) NH-MADM strategy in neutrosophic hesitant fuzzy set environment based on extended GRA. Informatica 30(2):1–30
66. Peng JJ, Wang J (2015) Multi-valued neutrosophic sets and its application in multi-criteria decision-making problems. Neutrosophic Sets Syst 10:3–17
67. Peng JJ, Wang J, Wu X, Wang J, Chen X (2014) Multi-valued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems. Int J Comput Intell Syst 8(2):345–363
68. Ali M, Deli I, Smarandache F (2016) The theory of neutrosophic cubic sets and their applications in pattern recognition. J Intell Fuzzy Syst 30(4):1957–1963
69. Jun YB, Kim CS, Yang KO (2012) Cubic sets. Ann Fuzzy Math Inform 4(3):83–98
70. Jun YB, Smarandache F, Kim CS (2017) Neutrosophic cubic sets. New Math Natural Comput 13(1):41–54
71. Banerjee D, Giri BC, Pramanik S, Smarandache F (2017) GRA for multi attribute decision making in neutrosophic cubic set environment. Neutrosophic Sets Syst 15:60–69
72. Pramanik S, Dey PP, Giri BC, Smarandache F (2017) An extended TOPSIS for multi-attribute decision making problems with neutrosophic cubic information. Neutrosophic Sets Syst 17:70–79
73. Lu Z, Ye J (2017) Cosine measures of neutrosophic cubic sets for multiple attribute decision-making. Symmetry 9(7):121
74. Zhan J, Khan M, Gulistan M, Ali A (2017) Applications of neutrosophic cubic sets in multi-criteria decision-making. Int J Uncertain Quantif 7(5):377–394
75. Pramanik S, Dalapati S, Alam S, Roy TK (2017) NC-TODIM-based MAGDM under a neutrosophic cubic set environment. Information 8(4):149
76. Pramanik S, Dalapati S, Alam S, Roy TK, Smarandache F (2017) Neutrosophic cubic MCGDM method based on similarity measure. Neutrosophic Sets Syst 16:44–56
77. Deli I, Ali M, Smarandache F (2015) Bipolar neutrosophic sets and their applications based on multicriteria decision making problems. In: International conference on advanced mechatronic systems (ICAMechs), pp 249–254
78. Lee KM (2000) Bipolar-valued fuzzy sets and their operations. In: Proceedings of the international conference on intelligent technologies, Bangkok, Thailand, pp 307–312
79. Uluçay V, Del I, Şahin M (2018) Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making. Neural Comput Appl 29:739–748
80. Dey PP, Pramanik S, Giri BC (2016) TOPSIS for solving multi-attribute decision making problems under bi-polar neutrosophic environment. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and applications. Pons Editions, Brussels, pp 65–77

81. Pramanik S, Dey PP, Giri BC, Smarandache F (2017) Bipolar neutrosophic projection based models for solving multi-attribute decision making problems. Neutrosophic Sets Syst 15:70–79
82. Wang L, Zhang HY, Wang JQ (2018) Frank Choquet Bonferroni mean operators of bipolar neutrosophic sets and their application to multi-criteria decision-making problems. Int J Fuzzy Syst 20(1):13–28
83. Fan C, Ye J, Feng S, Fan E, Hu K (2019) Multi-criteria decision-making method using Heronian mean operators under a bipolar neutrosophic environment. Mathematics 7(1):97
84. Jamil M, Abdullah S, Khan MY, Smarandache F, Ghani F (2019) Application of the bipolar neutrosophic Hamacher averaging aggregation operators to group decision making: an illustrative example. Symmetry 11(5):698
85. Maji PK (2013) Neutrosophic soft set. Ann Fuzzy Math Inform 5(1):157–168
86. Molodstov D (1999) Soft set theory-first results. Comput Math Appl 37:19–31
87. Deli I, Broumi S (2015) Neutrosophic soft matrices and NSM-decision making. J Intell Fuzzy Syst 28(5):2233–2241
88. Dey PP, Pramanik S, Giri BC (2016) Neutrosophic soft multi-attribute decision making based on grey relational projection method. Neutrosophic Sets Syst 11:98–106
89. Şahin R, Küçük A (2014) On similarity and entropy of neutrosophic soft sets. J Intell Fuzzy Syst 27(5):2417–2430
90. Deli I, Broumi S, Ali M (2014) Neutrosophic soft multi-set theory and its decision making. Neutrosophic Sets Syst 5:65–76
91. Jana C, Pal M (2019) A robust single-valued neutrosophic soft aggregation operators in multi-criteria decision making. Symmetry 11(1):110. https://doi.org/10.3390/sym11010110
92. Deli I, Erasla S, Çağman N (2018) ivnpiv-Neutrosophic soft sets and their decision making based on similarity measure. Neural Comput Appl 29:187–203
93. Dey PP, Pramanik S, Giri BC (2015) Generalized neutrosophic soft multi-attribute group decision making based on TOPSIS. Crit Rev 11:41–55
94. Dey PP, Pramanik S, Giri BC (2016) Neutrosophic soft multi-attribute group decision making based on grey relational analysis method. J New Res Sci 10:25–37
95. Pramanik S, Dalapati S (2016) GRA based multi criteria decision making in generalized neutrosophic soft set environment. Global J Eng Sci Res Manage 3(5):153–169
96. Das S, Kumar S, Kar S, Pal T (2017) Group decision making using neutrosophic soft matrix: an algorithmic approach. J King Saud Univ – Comput Inf Sci 31(4):459–546
97. Ali M, Son LH, Deli I, Tien ND (2017) Bipolar neutrosophic soft sets and applications in decision making. J Intell Fuzzy Syst 33(6):4077–4087
98. Pawlak Z (1982) Rough sets. Int J Comput Inf Sci 11:341–356
99. Broumi S, Smarandache F, Dhar M (2014) Rough neutrosophic sets. Italian J Pure Appl Math 32:493–502
100. Broumi S, Smarandache F, Dhar M (2014) Rough neutrosophic sets. Neutrosophic Sets Syst 3:60–66
101. Broumi S, Smarandache F (2015) Interval neutrosophic rough set. Neutrosophic Sets Syst 7:23–31
102. Yang HL, Zhang CL, Guo ZL, Liu YL, Liao X (2017) A hybrid model of single valued neutrosophic sets and rough sets: single valued neutrosophic rough set model. Soft Comput 21:6253–6267
103. Zhang C, Zhai Y, Li D, Mu Y (2016) Steam turbine fault diagnosis based on single-valued neutrosophic multigranulation rough sets over two universes. J Intell Fuzzy Syst 31(6):2829–2837
104. Qian YH, Liang JY (2006) Rough set method based on multi-granulations. In: 2006 5th IEEE international conference on cognitive informatics, vol 1. IEEE, New York, pp 297–304
105. Jiao L, Yang HL, Li SG (2020) Three-way decision based on decision-theoretic rough sets with single-valued neutrosophic information. Int J Mach Learn Cybern 11:657–665

106. Zhao XR, Hu BQ (2020) Three-way decisions with decision theoretic rough sets in multiset-valued information tables. Inf Sci 507:684–699
107. Mondal K, Pramanik S (2015) Rough neutrosophic multi-attribute decision-making based on grey relational analysis. Neutrosophic Sets Syst 7:8–17
108. Mondal K, Pramanik S (2015) Rough neutrosophic multi-attribute decision-making based on rough accuracy score function. Neutrosophic Sets Syst 8:14–21
109. Mondal K, Pramanik S, Giri BC (2019) Rough neutrosophic aggregation operators for multi-criteria decision-making. Fuzzy multi-criteria decision-making using neutrosophic set. Springer, Cham, pp 79–105
110. Mondal K, Pramanik S, Smarandache F (2016) Several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in decision making. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and application. Pons Editions, Brussels, pp 93–103
111. Mondal K, Pramanik S, Smarandache F (2016) Multi-attribute decision making based on rough neutrosophic variational coefficient similarity measure. Neutrosophic Sets Syst 13:3–17
112. Mondal K, Pramanik S (2015) Decision making based on some similarity measures under interval rough neutrosophic environment. Neutrosophic Sets Syst 10:46–57
113. Pramanik S, Mondal K (2015) Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Global J Adv Res 2(1):212–220
114. Pramanik S, Mondal K (2015) Some rough neutrosophic similarity measure and their application to multi attribute decision making. Global J Eng Sci Res Manage 2(7):61–74
115. Pramanik S, Mondal K (2015) Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. J New Theory 4:90–102
116. Mondal K, Pramanik S, Smarandache F (2016) Rough neutrosophic TOPSIS for multi-attribute group decision making. Neutrosophic Sets Syst 13:105–117
117. Pramanik S, Roy R, Roy TK, Smarandache F (2017) Multi criteria decision making using correlation coefficient under rough neutrosophic environment. Neutrosophic Sets Syst 17:29–36
118. Pramanik S, Roy R, Roy TK (2018) Multi criteria decision making based on projection and bidirectional projection measures of rough neutrosophic sets. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and applications. Pons Editions, Brussels, pp 175–187
119. Roy R, Pramanik S, Roy TK (2020) Interval rough neutrosophic TOPSIS strategy for multi-attribute decision making. In: Abdel-Basset M, Smarandache F (eds) Neutrosophic sets in decision analysis and operations research. IGI Global, pp 98–118
120. Pramanik S, Mondal K (2016) Rough bipolar neutrosophic set. Global J Eng Sci Res Manage 3(6):71–81
121. Pramanik S (2020) Rough neutrosophic set: an overview. In:Smarandache F, Broumi S (eds) Neutrosophic theories in communication, management and information technology. Nova Science Publishers, pp 275–311
122. Yager RR (1986) On the theory of bags. Int J General Syst 13(1):23–37
123. Knuth D (1981) The art of computer programming, vol 2: semi numerical Algorithms. Addison-Wesley, Reading, MA
124. Manna Z, Waldinger R (1985) The logical bases for computer programming, vol 1: deductive reasoning. Addison-Wesley, Reading, MA
125. Shinoj TK, John SJ (2012) Intuitionistic fuzzy multisets and its application in medical diagnosis. World Acad Sci Eng Technol 6(1):1418–1421
126. Smarandach F (2013) n-Valued refined neutrosophic logic and its applications in Physics. Prog Phys 4:143–146
127. Broumi S, Smarandache F (2014) Neutrosophic refined similarity measure based on cosine function. Neutrosophic Sets Syst 6:42–48

128. Broumi S, Deli I (2014) Correlation measure for neutrosophic refined sets and its application in medical diagnosis. Palestine J Math 3:11–19
129. Deli I, Broumi S, Smarandache F (2015) On neutrosophic refined sets and their applications in medical diagnosis. J New Theory 6:88–89
130. Mondal K, Pramanik S (2015) Neutrosophic refined similarity measure based on tangent function and its application to multi attribute decision making. J New Theory 8:41–50
131. Mondal K, Pramanik S (2015) Neutrosophic refined similarity measure based on cotangent function and its application to multi-attribute decision making. Global J Adv Res 2(2):486–494
132. Pramanik S, Banerjee D, Giri BC (2016) TOPSIS approach for multi attribute group decision making in refined neutrosophic environment. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and applications. Pons Editions, 79–91
133. Pramanik S, Banerjee D, Giri BC (2016) Multi-criteria group decision making model in neutrosophic refined set and its application. Global J Eng Sci Res Manage 3(6):12–18
134. Ye J, Smarandache F (2016) Similarity measure of refined single-valued neutrosophic sets and its multicriteria decision making method. Neutrosophic Sets Syst 12:41–44
135. Chen J, Ye J, Du S (2107) Vector similarity measures between refined simplified neutrosophic sets and their multiple attribute decision-making method. Symmetry 9(8):153
136. Pramanik S, Dey PP, Giri BC (2018) Hybrid vector similarity measure of single valued refined neutrosophic sets to multi-attribute decision making problems. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and applications, vol 2. Pons Editions, pp 156–174
137. Bao YL, Yang HL (2017) On single valued neutrosophic refined rough set model and its application. J Intell Fuzzy Syst 33:1235–1248
138. Bao YL, Yang HL (2019) On single valued neutrosophic refined rough set model and its application. In: Kahraman C, Otay I (eds) fuzzy multi-criteria decision-making using neutrosophic sets. Studies in fuzziness and soft computing, vol 369. Springer, Berlin, pp 107–143
139. Ye J (2015) An extended TOPSIS method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers. J Intell Fuzzy Syst 28:247–255
140. Wang JQ, Li HB (2010) Multi-criteria decision making based on aggregation operators for intuitionistic linguistic fuzzy numbers. Control Decis 25(10):1571–1574
141. Li YY, Zhang H, Wang JQ (2017) Linguistic neutrosophic sets and their application in multicriteria decision-making problems. Int J Uncertain Quantif 7(2):135–154
142. Ye J (2014) Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making. J Intell Fuzzy Syst 27:2231–2241
143. Tian ZP, Wang J, Zhang HY, Chen XH, Wang JQ (2016) Simplified neutrosophic linguistic normalized weighted Bonferroni mean operator and its application to multi-criteria decision-making problems. Filomat 30(12):3339–3360
144. Xu Z (2005) Deviation measures of linguistic preference relations in group decision making. Omega 33(3):249–254
145. Biswas P, Pramanik S, Giri BC (2014) Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Neutrosophic Sets Syst 8:46–56
146. Ye J (2015) Trapezoidal neutrosophic set and its application to multiple attribute decision-making. Neural Comput Appl 26:1157–1166
147. Deli I, Subas Y (2017) A ranking method of single valued neutrosophic numbers and its application to multi-attribute decision making problems. Int J Mach Learn Cybern 8 (4):1309–1322
148. Biswas P, Pramanik S, Giri BC (2018) Multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and applications, vol 2. Pons Editions, pp 103–124

149. Biswas P, Pramanik S, Giri BC (2018) TOPSIS strategy for multi-attribute decision making with trapezoidal neutrosophic numbers. Neutrosophic Sets Syst 19:29–39
150. Pramanik S, Mallick R (2018) VIKOR based MAGDM strategy with trapezoidal neutrosophic numbers. Neutrosophic Sets Syst 22:118–130
151. Pramanik S, Mallick R (2019) TODIM strategy for multi-attribute group decision making in trapezoidal neutrosophic number environment. Complex Intell Syst 5(4):379–389
152. Pramanik S, Mallick R (2020) Extended GRA-based MADM strategy with single-valued trapezoidal neutrosophic numbers. In: Abdel-Basset M, Smarandache F (eds) Neutrosophic sets in decision analysis and operations research. IGI Global, pp 150–179
153. Jana C, Pal M, Karaaslan F, Wang JQ (2020) Trapezoidal neutrosophic aggregation operators and their application to the multi-attribute decision-making process. Scientia Iranica. Transaction E, Ind Eng 27(3):1655–1673
154. Biswas P, Pramanik S, Giri BC (2016) Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. Neutrosophic Sets Syst 12:127–138
155. Liang RX, Wang JQ, Zhang HY (2018) A multi-criteria decision making method based on single valued trapezoidal neutrosophic preference relation with complete weight information. Neural Comput Appl 30:3383–3398
156. Biswas P, Pramanik S, Giri BC (2018) Distance measure based MADM strategy with interval trapezoidal neutrosophic numbers. Neutrosophic Sets Syst 19:40–46
157. Biswas P, Pramanik GBC (2016) Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. Neutrosophic Sets Syst 12:20–40
158. Abdel-Basset M, Saleh M, Gamal A, Smarandache F (2019) An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Appl Soft Comput 77:438–452
159. Chakraborty A, Broumi S, Singh PK (2019) Some properties of pentagonal neutrosophic numbers and its applications in transportation problem environment. Neutrosophic Sets Syst 28:200–215
160. Karaaslan F (2018) Gaussian single-valued neutrosophic numbers and its application in multi-attribute decision making. Neutrosophic Sets Syst 22:101–117
161. Deli I, Subas Y (2016) Bipolar neutrosophic refined sets and their applications in medical diagnosis. In: International conference on natural science and engineering (ICNASE'16), pp 1121–1132
162. Mondal K, Pramanik S (2015) Tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. Crit Rev 11:26–40
163. Mondal K, Pramanik S, Smarandache F (2016) Rough neutrosophic hyper-complex set and its application to multi-attribute decision making. Crit Rev 13:111–126
164. Chatterjee R, Majumdar P, Samanta SK (2016) On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets. J Intell Fuzzy Syst 30(4):2475–2485
165. Smarandache F (2017) Plithogeny, plithogenic set, logic, probability, and statistics. Pons asbl, Brussels
166. Smarandache F (2018) Plithogenic set: an extension of crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets–revisited. Neutrosophic Sets Syst 21:153–166
167. Smarandache F (2016) Operators on single valued neutrosophic oversets, neutrosophic undersets, and neutrosophic offsets. J Math Inf 5:63–67
168. Smarandache F (2016) Neutrosophic overset, neutrosophic underset, neutrosophic offset, similarly for neutrosophic over-/under-/off logic, probability, and statistic. Pons Editions
169. Smarandache F, Pramanik S (eds) (2016) New trends in neutrosophic theory and applications. Pons Editions
170. El-Hefenawy N, Metwally MA, Ahmed ZM, El-Henawy IM (2016) A review on the applications of neutrosophic sets. J Comput Theor Nanosci 13(1):936–944

171. Smarandache F, Pramanik S (eds) (2018) New trends in neutrosophic theory and applications, vol 2. Pons Editions
172. Khan M, Son LH, Ali M, Chau HT, Na NTN, Smarandache F (2018) Systematic review of decision making algorithms in extended neutrosophic sets. Symmetry 10(8):314
173. Pramanik S, Mallick R, Dasgupta A (2018) Contributions of selected Indian researchers to multi-attribute decision making in neutrosophic environment. Neutrosophic Sets Syst 20:108–131
174. Nguyen GN, Son LH, Ashour AS, Dey N (2019) A survey of the state-of-the-arts on neutrosophic sets in biomedical diagnoses. Int J Mach Learn Cybern 10:1–13. https://doi.org/10.1007/s13042-017-0691-7
175. Akram M, Shum, KP (2020) A survey on single-valued neutrosophic K−algebras. J Math Res Appl 40(3). https://doi.org/10.3770/j.issn:2095-2651.2020.03.000
176. Peng X, Dai J (2020) A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017. Artif Intell Rev 53(1):199–255
177. Muzaffar A, Nafis MT, Sohail SS (2020) Neutrosophy logic and its classification: an overview. Neutrosophic Sets Syst 35:239–251
178. Sahin R (2014) Multi-criteria neutrosophic decision making method based on score and accuracy functions under neutrosophic environment. http://arxiv.org/abs/1412.5202. Accessed on 7 July 2020
179. Nancy GH (2016) An improved score function for ranking neutrosophic sets and its application to decision-making process. Int J Uncertain Quantif 6(5):377–385
180. Chi P, Liu P (2013) An extended TOPSIS method for the multiple attribute decision making problems based on interval neutrosophic set. Neutrosophic Sets Syst 1:63–70
181. Kutlu Gündoğdu F, Kahraman C (2019) Spherical fuzzy sets and spherical fuzzy TOPSIS method. J Intell Fuzzy Syst 36(1):337–352
182. Smarandache F (2017) Neutrosophic perspectives: triplets, duplets, multisets, hybrid operators, modal logic, hedge algebras, and applications, 2nd edn. Pons Publishing House
183. Deli I, Subas Y (2014) Single valued neutrosophic numbers and their applications to multicriteria decision making problem. viXra preprint viXra 14120012
184. Wang JH, Hao JY (2006) A new version of 2-tuple fuzzy linguistic representation model for computing with words. IEEE Trans Fuzzy Syst 4(3):435–445
185. Wang YM (2009) Centroid defuzzification and the maximizing set and minimizing set ranking based on alpha level sets. Comput Ind Eng 57(1):228–236
186. Chakraborty A, Mondal SP, Alam S, Mahata A (2020) Cylindrical neutrosophic single-valued number and its application in networking problem, multi-criterion group decision-making problem and graph theory. CAAI Trans Intell Technol 5(2):68–77
187. Smarandache F (2014) Introduction to neutrosophic statistics. Sitech & Education Publishing, Columbus
188. Smarandache F (2105) Neutrosophic quadruple numbers, refined neutrosophic quadruple numbers, absorbance law, and the multiplication of neutrosophic quadruple numbers. Neutrosophic Sets Syst 10:96–98
189. Smarandache F, Colhon M, Vlăduţescu Ş, Negrea X (2019) Word-level neutrosophic sentiment similarity. Appl Soft Comput 80:167–176
190. Christianto V, Smarandache F (2019) A review of seven applications of neutrosophic logic: in cultural psychology, economics theorizing, conflict resolution, philosophy of science, etc. J — Multidiscip Sci J 2(2):128–137
191. Devadoss AV, Anand MCJ, Felix A (2012) A study on the impact of violent video-games playing among children in Chennai using neutrosophic cognitive maps (NCMs). Int J Sci Eng Res 3(8):1–4
192. Yasser I, Twakol A, El-Khalek AAA, Samrah A, Salama AA (2020) COVID-X: Novel health-fog framework based on neutrosophic classifier for confrontation Covid-19. Neutrosophic Sets Syst 35:1–21

193. Salama AA, El-Ghareeb HA, Mani AM, Lotfy MM (2014) Utilizing neutrosophic set in social network analysis e-learning systems. Int J Inf Sci Intell Syst 3(2):1–12
194. Radwan NM (2016) Neutrosophic applications in E-learning: outcomes, challenges and trends. In: Smarandache F, Pramanik S (eds) New trends in neutrosophic theory and application. Pons Editions, pp 177–184
195. Anitha R, Gunavathi K (2017) NCM-based raga classification using musical features. Int J Fuzzy Syst 19(5):1603–1616
196. Shadrach FD, Kandasamy G (2020) Neutrosophic cognitive maps (NCM) based feature selection approach for early leaf disease diagnosis. J Ambient Intell Humaniz Comput. https://doi.org/10.1007/s12652-020-02070-3
197. Pamucar D, Deveci M, Schitea D, Erişkin L, Iordache M, Iordache I (2020) Developing a novel fuzzy neutrosophic numbers based decision making analysis for prioritizing the energy storage technologies. Int J Hydrogen Energy. https://doi.org/10.1016/j.ijhydene.2020.06.016
198. Albeanu G (2013) Neutrosophic computational models–(I). Ann Spiru Haret Univ Math-Inf Ser 9(2):13–22
199. Bencze M (2006) Neutrosophic applications in finance, economics and politics a continuing bequest of knowledge by an exemplarily innovative mind. Scientia Magna 2(4):81–82
200. Bhattacharya S (2005) Utility, rationality and beyond-from behavioral finance to informational finance. American Research Press
201. Bhattacharya S, Smarandache F, Kumar K (2005) Conditional probability of actually detecting a financial fraud-a neutrosophic extension to Benford's law. Int J Appl Math 17 (1):7–14
202. Khoshnevisan M, Bhattacharya S (2002) A short note on financial data set detection using neutrosophic probability. In: Proceedings of the first international conference on Neutrosophy, neutrosophic logic, neutrosophic set, neutrosophic probability and statistics. Phoenix, Xiquan, pp 75–80
203. Khoshnevisan M, Bhattacharya S (2005) Neutrosophic information fusion applied to the options market. Investment Manage Fin Innov 1:139–145
204. Bhattacharya S, Khoshnevisan M, Smarandache F (2003) Artificial intelligence and responsive optimization. Xiquan, Phoenix
205. Bhattacharya S, Smarandache F, Khoshnevisan M (2006) The Israel-Palestine question—a case for application of neutrosophic game theory. Octogon Math Maga 14(1):165–173
206. Plessner Y (2001, February 15) The conflict between Israel and the Palestinians: a rational analysis. Jerusalem Letters/Viewpoints, No 448, 22 Shvat 5761
207. Pramanik S, Roy TK (2013) Game theoretic model to the Jammu-Kashmir conflict between India and Pakistan. Int J Math Archive 4(8):162–170
208. Pramanik S, Roy TK (2014) Neutrosophic game theoretic approach to Indo-Pak conflict over Jammu-Kashmir. Neutrosophic Sets Syst 2:82–101
209. Deli I (2019) Matrix games with simplified neutrosophic payoffs. In: Fuzzy multi-criteria decision-making using neutrosophic sets. Springer, Berlin, pp 233–246
210. Vasantha WB, Kandasamy I, Smarandache F, Devvrat V, Ghildiyal S (2020) Study of imaginative play in children using single-valued refined neutrosophic sets. Symmetry 12 (3):402
211. Kandasamy I, Vasantha WB, Obbineni JM, Smarandache F (2020) Sentiment analysis of tweets using refined neutrosophic sets. Comput Ind 115:103180. https://doi.org/10.1016/j.compind.2019.103180
212. Vasantha WB, Kandasamy I, Devvrat V, Ghildiyal S (2019) Study of imaginative play in children using neutrosophic cognitive maps model. Neutrosophic Sets Syst 30:241–252
213. Devadoss AV, Rekha M (2017) A study on child marriage using new neutrosophic associative fuzzy cognitive dynamical system. Global J Pure Appl Math 13(6):2255–2265
214. Mondal K, Pramanik S (2014) A study on problems of Hijras in West Bengal based on neutrosophic cognitive maps. Neutrosophic Sets Syst 5:21–26

215. Pramanik S, Chackrabarti SN (2013) A study on problems of construction workers in West Bengal based on neutrosophic cognitive maps. Int J Innov Res Sci Eng Technol 2(11):6387–6394
216. Jousselme AL, Maupin P (2004) Neutrosophy in situation analysis. In: Proceedings of fusion, pp 400–406
217. Thiruppathi P, Saivaraju N, Ravichandran KS (2010) A study on suicide problem using combined overlap block neutrosophic cognitive maps. Int J Algorithms Comput Math 3 (4):22–28
218. Zafar A, Anas M (2019) Neutrosophic cognitive maps for situation analysis. Neutrosophic Sets Syst 29:78–88
219. William MA, Devadoss AV, Sheeba JN (2013) study on neutrosophic cognitive maps (NCMs) by analyzing the risk factors of breast cancer. Int J Sci Eng Res 4(2):1–4
220. Kandasamy WBV, Smarandache F (2004) Analysis of social aspects of migrant laborers living with HIV/AIDS using fuzzy theory and neutrosophic cognitive maps. Xiquan, Phoenix
221. Kandasamy V, Smarandache, F (2005) Fuzzy neutrosophic analysis of women with HIV/AIDS. Hexis, Arizona
222. Bernajee G (2008) Adaptive fuzzy cognitive maps vs neutrosophic cognitive maps: decision support tool for knowledge based institution. J Sci Ind Res 67:665–673
223. Ramalingam S, Kandasamy WB, Broumi S (2019) An approach for study of traffic congestion problem using fuzzy cognitive maps and neutrosophic cognitive maps-the case of Indian traffic. Neutrosophic Sets Syst 30:273–283
224. Saha A, Smarandache F, Baidya J, Dutta D (2020) MADM using m-generalized q-neutrosophic sets. Neutrosophic Sets Syst 35:252–268
225. Das S, Pramanik S (2020) Generalized neutrosophic b-open sets in neutrosophic topological space. Neutrosophic Sets Syst 33:552–530
226. Salama AA, Smarandache F (2015) Neutrosophic crisp set theory. The Educational Publisher Inc., Columbus. http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf
227. Thong NT, Dat LQ, Hoa ND, Ali M, Smarandache F (2019) Dynamic interval valued neutrosophic set: modeling decision making in dynamic environments. Comput Ind 108:45–52
228. Kandasamy I (2018) Double-valued neutrosophic sets, their minimum spanning trees, and clustering algorithm. J Intell Syst 27(2):163–182
229. Karaaslan F, Hunu F (2020) Type-2 single-valued neutrosophic sets and their applications in multi-criteria group decision making based on TOPSIS method. J Ambient Intell Humaniz Comput. https://doi.org/10.1007/s12652-020-01686-9
230. Broumi S, Smarandache F (2014) Single valued neutrosophic trapezoid linguistic aggregation operators based multi-attribute decision making. Bull Pure Appl Sci- Math Stat 33(2):135–155
231. Deli I, Şubaş Y (2017) Some weighted geometric operators with SVTrN-numbers and their application to multi-criteria decision making problems. J Intell Fuzzy Syst 32(1):291–301
232. Şahin R, Liu P (2017) Possibility-induced simplified neutrosophic aggregation operators and their application to multi-criteria group decision-making. J Exp Theor Artif Intell 29(4):769–785
233. Deli I (2017) Interval-valued neutrosophic soft sets and its decision making. Int J Mach Learn Cybern 8(2):665–676
234. Broumi S (2013) Generalized neutrosophic soft set. Int J Comput Sci Eng Inf Technol 3 (2):17–30
235. Broumi S, Sahin R, Smarandache F (2014) Generalized interval neutrosophic soft set and its decision making problem. J New Res Sci 7:29–47
236. Alkhazaleh S (2016) Time-neutrosophic soft set and its applications. J Intell Fuzzy Syst 30 (2):1087–1098

237. Ulucay V, Şahin M, Olgun N (2018) Time-neutrosophic soft expert sets and its decision making problem. MATEMATIKA: Malaysian J Industrial Appl Math 34(2):246–260
238. Broumi S, Deli I, Smarandache F (2014) Interval valued neutrosophic parameterized soft set theory and its decision making. J New Res Sci 3(7):58–71
239. Chatterjee R, Majumdar P, Samanta SK (2016) Interval-valued possibility quadripartitioned single valued neutrosophic soft sets and some uncertainty based measures on them. Neutrosophic Sets Syst 14:35–43
240. Zhao AW, Guan HJ (2015) Neutrosophic valued linguistic soft sets and multi-attribute decision-making application. J Comput Theor Nanosci 12(12):6162–6171
241. Deli I; Şubaş Y, Smarandache F, Ali M (2016) Interval valued bipolar neutrosophic sets and their application in pattern recognition. https://arxiv.org/abs/1601.01266. Accessed on 9 July 2020
242. Deli I, Şubaş Y, Smarandache F, Ali M (2016) Interval valued bipolar fuzzy weighted neutrosophic sets and their application. In: 2016 IEEE international conference on fuzzy systems (FUZZ-IEEE). IEEE, New York, pp 2460–2467
243. Ali M, Smarandache F (2017) Complex neutrosophic set. Neural Comput Appl 28:1817–1834
244. Ali M, Dat LQ, Smarandache F (2018) Interval complex neutrosophic set: formulation and applications in decision-making. Int J Fuzzy Syst 20(3):986–999
245. Al-Quran A, Hassan N (2018) The complex neutrosophic soft expert set and its application in decision making. J Intell Fuzzy Syst 34(1):569–582
246. Dat LQ, Thong NT, Ali M, Smarandache F, Abdel-Basset M, Long HV (2019) Linguistic approaches to interval complex neutrosophic sets in decision making. IEEE Access 7:38902–38917
247. Broumi S, Bakali A, Talea M, Smarandache F, Singh PK, Uluçay V, Khan M (2019) Bipolar complex neutrosophic sets and its application in decision making problem. In: Fuzzy multi-criteria decision-making using neutrosophic sets. Springer, Berlin, pp 677–710
248. Cruz RA, Irudayam FN (2017) Neutrosophic soft cubic set. Int J Math Trends Technols 46 (2):88–94
249. Xue H, Yang X, Chen C (2020) Possibility neutrosophic cubic sets and their application to multiple attribute decision making. Symmetry 12(2):269
250. Broumi S, Smarandache F (2015) Interval-valued neutrosophic soft rough sets. Int J Comput Math Article Id:232919. https://doi.org/10.1155/2015/232919
251. Ye J (2016) Correlation coefficients of interval neutrosophic hesitant fuzzy sets and its application in a multiple attribute decision making method. Informatica 27(1):179–202
252. Fahmi A, Aslam M, Riaz M (2020) New approach of triangular neutrosophic cubic linguistic hesitant fuzzy aggregation operators. Granular Comput 5:527–543
253. Awang A, Ali M, Abdullah L (2019) Hesitant bipolar-valued neutrosophic set: formulation, theory and application. IEEE Access 7:176099–176114
254. Ye S, Ye J (2014) Dice similarity measure between single valued neutrosophic multisets and its application in medical diagnosis. Neutrosophic Sets Syst 6:49–54
255. Kandasamy I, Smarandache F (2016) Triple refined indeterminate neutrosophic sets for personality classification. In: 2016 IEEE symposium series on computational intelligence (SSCI). https://doi.org/10.1109/ssci.2016.7850153
256. Kandasamy I, Smarandache F (2017) Multicriteria decision making using double refined indeterminacy neutrosophic cross entropy and indeterminacy based cross entropy. Appl Mech Mater 859:129–143
257. Smarandache F (2019) Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), pythagorean fuzzy set, q-rung orthopair fuzzy set, spherical fuzzy set, and n-hyperspherical fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision (revisited). https://arxiv.org/abs/1911.07333

258. Das S, Roy BK, Kar MB, Kar S, Pamučar D (2020) Neutrosophic fuzzy set and its application in decision making. J Ambient Intell Humaniz Comput. https://doi.org/10.1007/s12652-020-01808-3
259. Alkhazaleh S (2015) Neutrosophic vague set theory. Crit Rev 10:29–39
260. Hashim H, Abdullah L, Al-Quran A (2019) Interval neutrosophic vague sets. Neutrosophic Sets Syst 25:66–75
261. Al-Quran A, Hassan N (2017) Neutrosophic vague soft set and its applications. Malaysian J Math Sci 11(2):141–163
262. Al-Quran A, Hassan N (2016) Neutrosophic vague soft expert set theory. J Intell Fuzzy Syst 30(6):3691–3702
263. Mukherjee A (2019) Vague–valued possibility neutrosophic vague soft expert set theory and its applications. Neutrosophic Sets Syst 29:142–157
264. Al-Quran A, Hassan N (2018) Neutrosophic vague soft multiset for decision under uncertainty. Songklanakarin J Sci Technol 40(2)
265. Hassan N, Al-Quran A (2017) Possibility neutrosophic vague soft expert set for decision under uncertainty. In: AIP conference proceedings, vol 1830, No 1. AIP Publishing LLC, p. 070007
266. Hussain SS, Hussai RJ, Jun YB, Smarandache F (2019) Neutrosophic bipolar vague set and its application to neutrosophic bipolar vague graphs. Neutrosophic Sets Syst 28:69–86
267. Bhowmik M, Pal M (2009) Intuitionistic neutrosophic set. J Inf Comput Sci 4(2):142–152
268. Broumi S, Smarandache F (2013) Intuitionistic neutrosophic soft set. J Inf Comput Sci 8 (2):130–140
269. Broumi S, Deli I, Smarandache F (2015) N-valued interval neutrosophic sets and their application in medical diagnosis. Crit Rev 10:45–69
270. Peng HG, Zhang HY, Wang JQ (2018) Probability multi-valued neutrosophic sets and its application in multi-criteria group decision-making problems. Neural Comput Appl 30 (2):563–583
271. Wang N, Zhang H (2017) Probability multivalued linguistic neutrosophic sets for multi-criteria group decision-making. Int J Uncertain Quantif 7(3):207–228
272. Liu P, Teng F (2018) Multiple attribute decision making method based on normal neutrosophic generalized weighted power averaging operator. Int J Mach Learn Cybern 9:281–293
273. Kamal NM, Abdullah L (2019) Multi-valued neutrosophic soft set. Malaysian J Math Sci 13:153–168
274. Kamal NLAM, Abdullah L, Abdullah I, Alkhazaleh S, Karaaslan F (2019) Multi-valued interval neutrosophic soft set: formulation and theory. Neutrosophic Sets Syst 30:149–170
275. Alkhazaleh S (2017) n-Valued refined neutrosophic soft set theory. J Intell Fuzzy Syst 32 (6):4311–4318
276. Jha S, Kumar R, Chatterjee JM, Khari M, Yadav N, Smarandache F (2019) Neutrosophic soft set decision making for stock trending analysis. Evol Syst 10(4):621–627
277. Tey DJY, Gan, YF, Selvachandran G, Quek SG, Smarandache F, Son LH, Long HV (2019) A novel neutrosophic data analytic hierarchy process for multi-criteria decision making method: a case study in Kuala Lumpur Stock exchange. IEEE Access 7:53687–53697
278. Bustince H, Burillo P (1996) Vague sets are intuitionistic fuzzy sets. Fuzzy Sets Syst 79:403–405
279. Deschrijver G, Kerre EE (2003) On the relationship between some extensions of fuzzy set theory. Fuzzy Sets Syst 133(2):227–235

Surapati Pramanik has been working as a teacher educator since 2006 in Nandalal Ghosh B.T. College, India. He received a Ph.D. in Mathematics from Indian Institute of Engineering Science and Technology (IIEST), Shibpur, Ph.D. in Education, an M.Sc., and an M.Ed. from University of Kalyani. He acts as a Ph.D. supervisor in Mathematics for Jadavpur University and IIEST, Shibpur. His research focuses on fuzzy multi-criteria decision-making (MCDM), mathematics education, soft computing, and neutrosophic fuzzy sets. His research earned outstanding paper awards several times in West Bengal State Science and Technology Congress in mathematics (2011) and Social Sciences (2010, 2013, 2019). He is an editorial board member of “Neutrosophic Sets and Systems”, and “Current Chinese Science: Artificial Intelligence & Robotics”. His publication includes 24 book chapters, 130 research papers, and two editorial books. He acts as a reviewer for more than 40 international journals with more than 100 reviews. He is a life member of thirteen scientific organizations including Calcutta Mathematical Society, and ISI, Kolkata. Currently, he is working on neutrosophic and fuzzy hybrid sets.