What Is the Relation Between Peirce's Logic and His Philosophy of Logic?

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Abstract C. S. Peirce was both a logician and a philosopher of logic. Strangely, these two approaches have not been much compared. This present paper intends to explicitly raise the question of the relation between logical formalism and the philosophical conceptions of logic that Peirce supported. The problem we consider is simply whether there is any connection between Peirce's advances in logic and how he conceived of logic, that is, between logical theory and logical practice. Is logic philosophically neutral or does it involve at least a certain conception of itself? We answer that Peirce's semiotic, iconic, relational, dialogical, inquiry-based formalism, that is, the pragmaticist conception of logic, stresses the insufficiency of deductive logic. Being mostly inductive and abductive, the inquiry cannot be expressed through formal logics. It shows that the contemporary meaning of logic cannot be separated from its Peircian use: Logic and philosophy of science come together, and it may paradoxically be this faith which motivated most of the formal advances of Peirce's logic.

Keywords C. S. Peirce · Philosophy of logic · Semiotics · Diagrams

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1 Introduction

Like B. Russell, C. S. Peirce was both a logician and the author of a considerable work in philosophy of logic. But unlike Russell, Peirce's two approaches of logic, technical and philosophical, have seldom been compared. It is not even sure that Peirce himself tried to unify his logical works and his conception of logic, at least

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until the development of existential graphs. This present paper intends to explicitly raise the question of the relation between logical "formalism" and the philosophical conceptions of logic that Peirce supported. It is a variation on the classical question in philosophy of logic: "Does the practice of logic rest on a theory or, conversely, does theory only make a practice explicit?" ([9], p. 162).

That there is a clear-cut distinction between logic and philosophy of logic is far from obvious. First, the terms were not used in the nineteenth century in the same way as we do. Peirce used "philosophy of notation" to refer to his symbolism, and "logic of science" was equivalent to our philosophy of science. In order to give a chance to this distinction, it will be requisite to understand the phrases "logic" and "philosophy of logic" in their contemporary sense. Second, philosophy of logic will be reduced in the main to the conceptions and definitions of logic and its role. As a consequence, thoughts on the connection between logic and the pragmatist maxim, for example, however important, will not be included in the present study.[1] Third, if the concept of philosophy of logic may seem fuzzy, that of logic is undoubtedly so. For instance, the logicists were reproached for calling logic what was actually set theory, but Putnam showed that the distinction between the two is at least unclear ([20], p. 259), because the notion of validity of a syllogism at once refers to second-order logic, so that almost all the classical logic would be included into set theory. Moreover, if one manages to identify something like "logic," our problem will not be that of the relation between logic and metalogic—namely, the "desire to discuss logic by means of logic," as Hintikka wrote ([11], p. 21), which characterizes reinterpretability in model theory.[2] Neither is it the relation between philosophical logic and philosophy of logic.[3] The relevant question is not between the normative evaluation of our reasoning ("how should we reason?") and the meta-normative approach ("how are these standards of correction justified?"), let alone their relation with actual reasoning. Nor is it a matter of connecting semantic principles and logical laws.[4]

Rather, the problem we shall consider is simply whether there is any connection between Peirce's "inventions" in logic and how he conceived of logic, that is, between logical theory and logical practice. Is logic philosophically neutral or does it involve at least a certain conception of itself? This question is called by the seeming heterogeneity between studies in logic and views on the nature of logic. The question itself already belongs to the philosophy of logic: Is a reflection on the

[1] Nevertheless, there is no doubt that the concept of possible practical consequence is closely related to logical analysis. For instance, in existential graphs, the meaning of indices is explained through their practical effects; see what Hilpinen wrote in ([10], p. 628).

[2] Hintikka quotes Peirce: "It is necessary that we should be able to reason in graphs about graphs" ([15] 4.527).

[3] For example, Grayling claimed that "when one does philosophy of logic, one is philosophizing about logic, but when one does philosophical logic, one is philosophizing" [6].

[4] Pataut asks: "More precisely, is a theory of meaning whose central concept is assertability, and which is furthermore molecularist only for logical constants, able to give a philosophical grounding to a Relevant Intuitionist Logic?" ([14], p. 144).

nature of logic useful to the logician? Does a particular formal system determine a certain conception of logic?[5] Does logic consist only in a practice, an art, or is it also a science (of its own foundations in particular)? What are the mutual contributions of philosophy of logic to logic in the Peircian framework?

2 An Overview of Peirce's Contributions to Logic[6]

Peirce was a pioneer of the logic of relations, propositional calculus, quantification, and diagrammatic logic. One could add a theory of lattices (nearly 20 years before Dedekind isolated the structure of lattices in group theory), studies in trivalent logic, an original interpretation of modalities,[7] a basis for game semantics, or a proof of the theorem of reducibility (according to which any n-adic relation can be described as a compound of triadic relations or less), not to mention his axiomatic arithmetic of natural numbers (8 years before Peano's axiomatization).

2.1 The Logic of Relatives

The "logic of relatives" is one of Peirce's most crucial contributions. Instead of using closed predicates (representing properties), it works with relational terms of one, two, or three valencies. It was put forth as an extension of Boolean algebra, which is nothing but computation on binary relations (with an additional structure) ([15] 3.45). Peirce was much influenced by DeMorgan, the author of an article on the logic of relations 10 years before and as such its true inventor. The young Peirce then attempted to "extend" Boole's algebra by introducing in 1867 the logical addition (the inclusive "or").[8] His father was also part of the picture: The calculus of relatives proposed by Peirce has analogies not only with the laws of Boole's algebra[9] but also with those of linear associative algebra, Benjamin Peirce's masterpiece. Peirce's fundamental article, "Description of a New Notation for the Logic of Relatives," appeared in 1870, the very year of his father's *Linear Associative Algebra*. He was

[5] Before such a meta-reflection on how formal logic is relevant to conceptualize logic, there could be a first-order study on the general consequences of formalism. This is what Bernard Williams intends to do when denying that deontic logic can express moral problems ([23], p. 311).

[6] This section owes to [3, 7, 24].

[7] Hilpinen [10] showed that Peirce interpreted modal propositions as a kind of quantified propositions and that quantification was interpreted in terms of game theory (or choice functions according to Hintikka).

[8] Thus, losing some purity in introducing a "minus" operator which is not properly a function. See ([3], p. 17).

[9] These laws beings themselves based on analogies that Boole noticed between laws of propositional logic, of set theory, and of arithmetic.

thereby merging the algebra of logic with the algebra of linear transformations (the simplest case of linear associative algebra, in which composition is analogous to the relative product for relations) while also relying on an analogy between the laws of exponentiation in arithmetic and of universal quantification. Geraldine Brady has underlined this productivity of analogy: The logic of relatives was conceived by analogy with Benjamin's work in linear algebra, individual term matching a coordinate, absolute term matching a vector, and relative term matching a matrix ([4], p. 48).

2.2 Propositional Logic and Quantification

The "logic of nonrelatives," that is to say propositional logic (of "first-order" logic, a phrase Peirce revived), made a leap in the 1880s, when Peirce's studies in logic entered a second phase. Freeing himself from the Boolean framework into which he had been locked, Peirce developed a system based on implication (more fundamental than identity and represented by an ambiguous symbol which also means inclusion), implication from which derives "illation" or deduction. The basic idea is a correspondence between inference and implication, since implication directly expresses the relation of deduction of the consequent from the antecedent. Thus, implication is introduced into a formal system syntactically from deduction (rather than semantically from truth tables). In adding negation, Peirce can rely on informal rules of introduction and elimination, which anticipate Prawitz's natural deduction and Gentzen's sequent calculus (as a set of rules for natural deduction). The rules of introduction and elimination of conjunction and disjunction are at the origin of the definition of propositional logic as a kind of lattice (whereas Dedekind approached them from a completely different angle, from lattices of subgroups of a group).

It was while developing first-order logic that Peirce proved that a single operator, the negation of disjunction ("non-or," named by Peirce "ampheck"), was enough to define all truth functions. This result would be rediscovered by Henry Sheffer with the other universal operator "non-and."

The introduction of quantification by Peirce is a long story, but it is worth investigating, since it may give some (or the whole of their) unity to his logical studies. As for the paternity of logical quantification, it was born or was not born in 1879 with Frege's *Begriffsschrift* according to which stage is considered decisive: In the works of Peirce, it happened somewhere between 1870 and 1885. It is reasonable to consider that quantification as we understand it today requires that quantifiers are no longer considered as relations but as separate operators. This revelation, when Peirce discovered the prenex normal form, occurred to him, thanks to his student O. H. Mitchell in 1883.[10] The previous attempts were initially constrained

[10] Even though O.H. Mitchell probably did not realize all the consequences of his own formalism, Peirce may be a little too generous to him.

by a too narrow frame: In 1867 and again in 1870, Peirce was still trying to combine Boolean algebra and Aristotelian syllogism to strengthen the expressive power of each. Aristotle had quantifiers but no propositional connectors, while Boole had propositional connectors without quantifiers. It is well known that the latter is incapable of expressing a particular proposition (quantified by "some"). The problem is that these two approaches are not reconcilable in the algebraic framework that Peirce was hoping to keep (by means of addition, multiplication, and exponentiation).

Yet, in 1870, Peirce managed to express universal quantification (by the exponential) and existential quantification (by the relative product) and to combine them; but they were relational operations, not separate entities. If, as he claimed, Peirce had already been able to handle quantification, he should have had a method for translating any quantified expression into his own formalism, which was not the case. Nevertheless, according to Geraldine Brady, "The most interesting feature of this approach is the hidden presence of existential quantification in the definition of relative product. We can understand something of Peirce's enterprise as a failed attempt to get full existential quantification out of relative product" ([4], p. 48). But since his concern was apparently to maintain the analogies with algebraic notations rather than to solve matters of quantification, it resulted in an insufficiently expressive system with rules that remained somehow opaque.

An important idea emerged, borrowed from DeMorgan, the notion of a universe of discourse. In contrast to the Frege-Russell tradition of universal language, where logic covers the range of all possible things without restriction, Peirce considered that reasoning is always at least implicitly limited to a certain domain. Therefore, quantification applies on defined domains of variables.

In 1883, following Mitchell's article, Peirce first made use of the symbols Π and Σ and in 1885 completed the theory with a reflection on the order of quantifiers to give the prenex form of predicate calculus of first order (not as quantifiers of a formal language on formal expressions, but as operations on propositional functions of a domain). It was still a kind of arithmetic reductionism, aiming at extending the power of logical algebra "over the whole of its own realm" and at illustrating the principles underlying any algebraic notation. Still, the article of 1885 is an accomplishment since it offers a full presentation of logic in its algebraic form.

2.3 Existential Graphs

The third phase of Peirce's logic is his theory of graphs, sketched as early as 1889 and made public for the first time in an 1897 article. He considered his system of existential graphs as his "masterpiece," which "ought to be the logic of the future" (Ms L224).[11] In a few words, logical relations are to be presented in diagrams

[11] Ms. refers to Peirce's manuscripts in Houghton Library, Harvard.

(which was very traditional after Euler and Venn), but based on iconic and almost topological properties. The "Alpha" system is a complete classical propositional calculus; "Beta" provides a complete theory of quantification with identity; and "Gamma," or modal logic, is like a book of assertions each leaf of which constitutes a possible universe of discourse.

Some rules allow one to read the existential graphs: the inscription on the assertion sheet is an affirmation; a circle denies what it surrounds; the identity is represented by a line; one begins by reading what is surrounded by the smallest number of circles.

In the example above, the first graph means:

$$\forall x\,(Catholic(x) \rightarrow \exists y\,(Adores\,(x, y) \wedge Woman(y)))$$

while the second one means:

$$\exists y\,(Woman(y) \wedge \forall x\,(Catholic(x) \rightarrow Adores\,(x, y)))\,.$$

Peirce also interpreted his diagrammatic logic in dialogical terms, as consisting of permission rules between a proposer and an opponent, prefiguring game theory. The interpretation of the second graph above by Peirce is: "A well-disposed person with sufficient means could find an index whose object should be a woman such that allowing an ill-disposed person to select an index whose object should be a Catholic, that Catholic would adore that woman" ([19], p. 168).

It is very remarkable that after having "invented" modern propositional logic, Peirce turned to a completely different, seemingly more archaic conception. This leads to the problem of the relation between two equivalent logical systems developed by a same man: Why did one of the founders of modern symbolic logic elaborate a system of diagrammatic representation? And do existential graphs bring a new logic, or do they only illustrate (so to speak) the previous achievements? Commentators disagree. Brady ([4], p. 10) sees existential graphs as a fusion of two logical systems, the calculus of relatives and natural deduction. On the contrary, Shin [22] emphasizes that if there are "two 'algebras'" ([15] 3.294), that is two diagrammatic representations which are in one case algebraic, in the other not (essentially) symbolic, they were "invented for the study of the logic of relatives" (idem) and only express a single logic. Why then this development to diagrammatization? The present study assumes that the answer to this question is provided by the philosophy of logic.

3 An Overview of Peirce's Philosophy of Logic

Peirce was quite loquacious about his conceptions of logic. Some affirmations invariably reappear throughout his works, while others seem to have evolved along with the logical formalisms he developed. In order to appreciate the real interaction between formalism and theoretical positions on logic, between logical representation and representation of logic, it is necessary to draw up a panorama of the Peircian philosophy of logic. To this end, we can isolate a set of more or less traditional questions, regarding the nature of logic, its foundations (psychological, physiological, or not), and its applications (mathematical, scientific). What is the nature of logic according to Peirce? Is it an art or a science? Is it normative? What is its function? How does it relate to mathematics?

3.1 Antipsychologism

Early in his life, Peirce provided many characterizations of logic and insisted particularly on what it is not. He advocated a "nonpsychological" view, that is, not preoccupied with the human mind, to the point that the most "objective" philosophers were suspect of psychologism—not to speak of John Stuart Mill or DeMorgan who erroneously defined logic as "the branch of inquiry (be it called science or art) in which the act of the mind in reasoning is considered" ([18] vol. 1, p. 164). Logic does not investigate on the way we reason ([18] vol. 1, p. 180), or the modes of cognition, but on forms. It bears on thought not in a psychological sense, but in sense of the thinkable: "Logic, it is true, does not deal with the matter of thought, but then it as certainly deals with thought as having matter that is as being a representation – true or false" (Ms 741).

What is a logical form? It will be the whole object of the theory of categories, hence of metaphysics, then phaneroscopy in Peirce's mature thought, to discover it. At least does Peirce state that "logic is an analysis of forms not a study of the mind. It tells why an inference follows not how it arises in the mind" ([18] vol. 1, p. 217) It would certainly be possible to look for forms in what Peirce called "anthropological logic," that is, to study logic by observing its development in the human mind; but it is healthier to follow a formal method, "by the comparison of the products of thinking" ([18] vol. 1, p. 361). Speaking of laws of thought, however, is ambiguous[12]: To say with Kant that logic is the science of the laws of thought, or of the pure form of thought in general, is to remain in the domain of human intellect, unless one means by thought not thought as it is, concrete and limited in time, but "nothing but a fiction which expresses merely the possibilities of discourse" ([18] vol. 1, p. 306). For it is irrelevant to the logician that the forms he studies have been

[12] Peirce was not always so precise. For instance, in 1898 he defined logic as the science "of the laws and forms of thought" ([19], p. 35–37).

or have not been actually thought: "Logic therefore deals with thought only in so far as the latter is a representation. And as I said every representation has its logical relations whether it is actually thought or not. So that it is more correct to say that logic is the science of the forms of representation than that it is the science of the forms of thought" ([18] vol. 1, p. 322).

In this respect, Peirce emphasizes the paradoxical importance of Locke, an empiricist who, through semiotics, managed to target representation as such, whether thought or not, material or psychic. He showed that logical forms are realized in symbols before they are understood by a mind. It should be noted, however, that Peirce defined logic as providing the laws of some *experience*: It is "the science of the laws of experience in virtue of its being a determination of the idea," that is, "the formal science of the logical world" ([18] vol. 1, p. 169).

But in spite of its relation to experience, logic is a theoretical science. It is not an art, nor a practical science, which differ like poetry from praxis: A practical science is not an art because art teaches how to make something, whereas practical science teaches only how to act or do something (Ms 607). Since Aristotle, logic has been reputed an art; it was held to be the art of disputing or arguing (Cicero, Ramus) and then the art of learning (Melanchton) and directing one's mind (Port-Royal), which tended to reduce logic to a collection of maxims. From an art, logic became an organon, a mere tool. Despite the use of the word, such a conception is not that of Aristotle. Thus, the view of logic as an art was that of the majority of logicians, from the Stoics to the English logicians (e.g., to Venn, it was a collection of more or less coherent precepts), through the scholastic opponents to Duns Scotus. To Peirce, on the contrary, as to the Subtle Doctor, logic was a theoretical science in its own right, closely related to mathematics.

Even in the early 1900s, when he placed the inquiry process in the heart of logic and paralleled it to ethics, Peirce firmly restated that logic is a science. The methodical conduct of inquiry being the most important part of logic, which may be considered a "special case of ethics," logic keeps the same theoretical status. Peirce readily admitted that there should be a practical science, or rather "a group of at least a dozen separate sciences," which would follow the principles of methodology (Ms 603), but it would no longer be logic.

3.2 Logic and Semiotics

As the reference to Locke indicates, the development of the semiotical conception of logic emanated from a meditation on representation. To represent is to be the symbol of a fact. "It is evident therefore that logic applies to thought only in so far as the latter is a symbol" ([18] vol. 1, p. 166). Logic endeavors to discover the formal laws of representation or symbolization, only to the extent that the parts of a representation are connected. A logician studies what constitutes a representation, that is to say the relation of symbols to their objects, and more precisely to possible objects: Logic considers symbols as possible objects of thought, regardless of their

actual understanding. It draws a map of the formal laws of objectivity in general. That is why a piece of reasoning expressed in a language that no one will ever understand anymore would remain valid (Ms 726). Thence a conception of logic as "the science of the conditions which enable symbols in general to refer to objects" ([18] vol. 1, p. 175) or later of "the laws of signs which determine what things they denote and what they do not" ([18] vol. 3, p. 98). Peirce emphasized that "these laws apply not merely to what can be thought but to whatever can be symbolized in any way" (Ms 340).

Does logic really need to mention denotation? On the contrary, it seems that interpreting signification does not belong to logic and that it ought to proceed like a "blind" calculus. Why did not Peirce define logic as simply the science of symbolic relations the ones with the others? Some scholars have suggested that the phrase is derived from Kant: Peirce interprets the conformity of a representation to its object, constitutive of its truth, as a reference of symbols to their object ([13], p. 86). This reason can be presented more specifically with reference to the three categories of "I, thou, and it" which structured the thought of the young Peirce. There are three formal systems of laws corresponding, respectively, to the relations of the symbol to the idea (the I), to consciousness (the thou), and to the object (the it) ([18] vol. 1, p. 174). If one of these codes is broken, the symbol no longer works as a symbol: When the relation of the symbol to the idea is interrupted, it becomes meaningless; when its relation to the object is broken, we get absurdity; when there is no relation with consciousness anymore, this is "quibbling." While grammar looks toward the idea and rhetoric toward consciousness, logic is what preserves from the absurd by guaranteeing that every symbol has an object: Logical laws only hold good, as conditions of the symbol's having an object ([18] vol. 1, p. 175). A typical example of absurdity due to the absence of an object is the liar paradox: The proposition "this proposition is false" cannot be either true nor false, because it has no other object itself, self-referentiality creating a vicious circularity.

In its relation with its object, the laws of logic are those of the symbol. The nature of the symbol acts directly on the mind: According to this nature, some principles of its use immediately regulate the action of the mind. As a result, the task of logic is to describe and catalog the laws of symbols. Logic is a classificatory science, analogous to chemistry ([15] 3.469),[13] a science in which Peirce had graduated at Harvard, botany, or comparative zoology ([18] vol. 1, p. 409). Peirce goes so far as to characterize logic as an inductive science ([18] vol. 1, p. 487). This very paradoxical statement obviously has nothing in common with John Stuart Mill's empiricist conception of logic, which views logical relations as emerging from the observation of natural objects.

If logic does not describe contingent facts of the world, the empirical part which it involves ought not to be underestimated: It "rests upon observations of

[13] For an analogy of the same kind although in a very different context, Russell ([21], p. 169) wrote that "logic is concerned with the real world just as truly as zoology, though with its more abstract and general features".

real facts about mental products" ([17], p. 267). Logic is indeed a science of facts ([15] 1.247) and is based on observations: "Then, *exact* logic will be that doctrine of the conditions of establishment of stable belief which rests upon perfectly undoubted observations and upon mathematical, that is, upon *diagrammatical*, or *iconic*, thought" ([15] 3.429).

Some specialists argue that, for instance, Kronecker's naturalistic conception of number may be considered as a justification for his use of certain mathematical tools.[14] Should we speak of the same kind of influence of Peirce's conception over his logical practice?

In addition, logic classifies symbols in general according to the reason for their reference to their objects ([18] vol. 1, p. 329). It is to say that logic does not aim only at a typology of symbols, but at a classification of reasons, that is, of logical relations—even though saying so defines logic by the logical character of its objects. In the end, "it is the business of logic to classify arguments" ([18] vol. 1, p. 370). This definition is perennial, since in 1879 Peirce refers to the essential purpose of logic as "the analysis of argument, not the art of drawing inferences" ([18] vol. 4, p. 23), and 30 years later, its task is again "to get possession of a method for determining the values of arguments" ([16], p. 298) and to produce "the comparative anatomy of arguments" ([16], p. 287). In his conception of normative sciences, Peirce would confirm the essentially classificatory status of logic, but as a classification of ends (Ms 602). Is this function equivalent to classifying the value of arguments? It is not to be excluded, but Peirce says nothing about it.

A classificatory science of reasons ([18] vol. 1, p. 359), the goal of logic is to make reasoning intelligible ([18] vol. 1, p. 486), to provide a test for reasoning ([18] vol. 1, p. 361):

> "Logic is the science needed in order to test arguments. The science required for any testing is one which merely divides its object into its natural kinds and describes the characters of each kind. [. . .] Such a knowledge will be termed a classificatory in opposition to a causal or demonstrative science." ([18] vol. 2, p. 294)

To test and classify, to describe the laws of symbols, and to give reasons for the arguments are one and the same task according to Peirce. A paradoxical consequence of the development of an objective semiotics in the name of antipsychologism is that it finally leads to the study of inference procedures. In 1910, logic is said to be the science that studies reasoning, its principal kinds, and their different modes and conditions of trustworthiness (Ms 655). We must of course understand reasoning as an objective logical procedure and not as a psychological activity. But are we

[14] Boniface [2] showed that Kronecker insisted on the concept of representation, whereas to Dedekind it was important not to introduce any particular representation that would have reduced generality. To her, such a belief comes from Kronecker's conception of mathematical objects, which would be known by experience. To Kronecker, mathematics were to be treated as a natural science, for its objects are as real as those of its sister-sciences. The idea that phenomena are at the bottom of mathematical knowledge is close to Peirce's conception—*pace* ([3], p. 210)—arguing that Kronecker was writing against Peirce and Peano.

dealing with only one notion of logic, or should we distinguish between "objective symbolistic," the science of classifying symbols, and the study of reasoning?

It appears that logic fulfills several tasks. However, since in the present state of knowledge one and the same group of men inquires on the classification of signs, the structure of propositions, and the validity of arguments, logic (in a broad sense) must include all these domains ([15] 4.9). As for the 1867 "logic proper," the ancestor of the "critic" of arguments, it is the "central department" of this science which, turning away from the particular state of things, studies the nature of the confidence to be placed in the different kinds of reasoning (Ms 602) or "the ways in which a sign can be related to the object independent of it that it represents" ([19], p. 327). Alongside this refoundation of scholastic logic are also grammar and methodeutics. The three of them are united in the broad acceptation of logic synonymous with semiotics. Speculative grammar (or stechiology) "studies the ways in which an object can be a sign" ([19], p. 327). It is a "general theory of the nature and meaning of signs" ([19], p. 260; see [1]). Now "one of the very first discoveries of stechiology" is that all reasoning is of the nature of a sign: Logic is interested in thought only insofar as it is a representation of reality (or falsely claims to represent reality) (Ms 602). Last, methodology, the new theory replacing speculative rhetoric, is the science of the essential conditions in which a sign determines an interpretant of itself and of what it signifies. The fact remains, however, that logical critic looks dual: On the one hand, it is the theory of the relation of a sign to its object, but on the other hand it is supposed to evaluate the probability and the certainty of deductions, inductions, and abductions. The equivalence between these two tasks is not obvious.

3.3 Normativity

The normativity of logic raises a difficult matter. That logic is normative has generally been admitted, since Wundt at least: Logic is one of the three normative sciences with ethics and esthetics. This point is supported by the Peircian classification of normative sciences: Logic, the summit of the normative edifice, is based on ethics, which itself is based on esthetics considered as the science of the ultimate ends or *summum bonum*. Nevertheless, a more detailed examination reveals a confused situation.

First, Peirce's theory of normative sciences is very late (it was born around 1902). Previously, not only did Peirce not speak in favor of the normative character of logic, but he criticized the conceptions (probably connected to Herbart and Überweg) who saw in logic the science of the normative laws of human cognition ([18] vol. 1, p. 164) or thought ([18] vol. 4, p. 378). He went so far as to assert that, if normativity is understood as an obligation or a duty, "the idea that [the laws of logic] are 'normative' laws is false" ([18] vol. 1, p. 166). We must therefore acknowledge a reversal of Peirce's views: At some point between 1880 and 1900, Peirce conceived the need to revise the claim that logic is nonnormative. Was there a motivation for this change in Peirce's logical formalism?

A misinterpretation is often made about the meaning of normativity according to Peirce,[15] although he explained most clearly that "normative" tends to replace, under the influence of Überweg in particular, the adjective "directive" ([15] 2.7). "Normative" refers to what is guided by an end. It is by no means the empirical (underivable) Humean "ought," nor a moral imperative, nor a duty in accordance with a rule, nor some kind of necessity.[16] Therefore, it is not because logic would enjoin us to think in a certain way—for example, by following the norm of truth—that Peirce regarded it as a normative science. Indeed, logic does not mention any duty, but only the reality of symbolic relations, which are not subject to discussion: "These may be regarded as laws of the symbol itself which it cannot as a symbol transgress" ([18] vol. 1, p. 173). Since "the objects of these laws cannot but comply with the laws," "the whole idea of their being 'normative' laws is false" ([18] vol. 1, p. 166). The laws of the reference of symbols to their objects are descriptive, indicating the different possible ways for the symbols to refer:

> "It has been supposed that the laws of logic might be broken. That they say 'Thou ought' not 'thou shalt,' that in short they are statements not of *fact* but of *debt*. But what page of man's ledger does this 'ought' refer to? Thought *debtor* to what? It is impossible to say." ([18] vol. 1, p. 166)

One might think that this non-deontic or anti-prescriptivist conception of logic disappeared around 1900 along with anti-normativism. But it was not so: If the facts are by themselves (logically or morally) binding, it is enough to describe them, without relying on an inexplicable "ought":

> "Logical treatises never say anything about what 'ought to be thought' as long as there is any compulsion of thought or reflection. In those cases they only speak of how the facts are. It is where there is no such compulsion that the 'ought' finds room." ([15] 2.50)

Logic is therefore normative only to the extent that it does not prescribe, because the facts described are binding on their own. When they are not, then the uncertain domain of choice, of more or less good conduct and of duty, opens up.

However, Peirce did not stay at this point and made more precise the modality of the "ought to be." Since normative sciences do not consider thought as it is actually present in the universe, logic is the science of what thought should be, not of what it is ([15] 2.7)—or, with a very subtle sense of nuance, of what "must be and ought to be true representation" ([15] 1.539). Properly understood, such a duty referred to by the normative sciences contrasts both what is and what could be: It is "between the two" (Ms 602). A normative science is "simply the theory of a dual distinction

[15] For example, "Logic is not the science of how we do think; but, in such sense as it can be said to deal with thinking at all, it only determines how we ought to think; nor how we ought to think in conformity with usage, but how we ought to think in order to think what is true. That a premise should be pertinent to such a conclusion, it is requisite that it should relate, not to how we think, but to the necessary connections of different sorts of fact ([15] 2.52). Logic, then, is not just a science, but a *normative* science" [24].

[16] Not many scholars noticed this point. See ([13], p. 88n9): "Although a method is a prescription, a norm, according to Peirce, is not."

between a 'may' and an 'ought not'" (ibid.): All that one can do is not to be done. The norm introduces the idea of a possible deviation, hence the possibility of self-control and control of the ends that one wants to reach. The concept of a "must be" can only be clarified in relation to an end. Therefore, if Peirce agreed that logic is normative in the sense of a duty, and that normative sciences determine "what conceptions and theories we ought to entertain" ([15] 5.594), it is ultimately because "the word 'ought' has no meaning except relatively to an end" ([15] 5.594). It is in so far as it indicates an end, namely, correct thought, that logic is normative.

This subtle characterization of logical normativity fits into a picture of the origin of logical thought. Since the "quintessence" of the normative sciences lies in their dualism ([19], p. 379)—dualism of the conditions for being true or false, for a "wise or crazy" conduct, for ideas attractive or repulsive—it is a true "mathematical form" ([19], p. 378) that appears there. Now, Peirce accounts for the duality of truth and falsehood by the structure of the semiotic process: A sign, to actualize itself, is compelled by its object; such compulsion does not go without resistance; a "quarrel" follows, thence a clash of two parties ([19], p. 379). The poles of the true and the false do not correspond to the distinction between object and sign, but to their reciprocal interactions—actions and reactions.

As for the origin of the logical conduct itself, it comes from self-control. In domains where a choice is possible, and especially when we have to take (more or less) rational decisions, the possibility of a deliberate choice is decisive. Since normativity aims at an ideal, it only bears on practices that can be deliberately directed toward a goal. What we have no control over has no normative criteria. Self-control is what makes thinking logical by distinguishing between the logical and the illogical ([15] 4.540). But this process of logical control "takes precisely the same quite complicated course" as ethical control, so that the logical norm of thought is based on ethics and is intrinsically ethical ([15] 5.533). Therefore, "just as Moral Conduct is Self-controlled conduct so Logical Thought is Moral, or Self-controlled, thought" ([15] 8.40). Logic may be called a special kind of ethics, if by ethics we mean the theory of self-control of conduct in order to achieve a deliberately adopted goal (Ms 602). Consequently, logical agents "must have all the characters of personal intellects possessed of moral natures" (Ms 280).

This theory may be contrasted with an 1880 text from the important article "The Algebra of Logic." It is one of the rare parts where Peirce mentions, albeit allusively, a link between logical operations and extralogical knowledge about logic: "In order to gain a clear understanding of the origin of the various signs used in logical algebra and the reasons of the fundamental formulae, we ought to begin by considering how logic itself arises" ([18] vol. 4, p. 163). Then follows a rather puzzling development (at least to whom takes the antipsychological fight seriously), explaining the origin of logical laws through the general laws of nervous action, the stimulation of the ganglia, and the creation of habits. Since a judgment is only the representation of a cerebral habit, it is easy to draw the whole syllogistic process from our human organic constitution. The dichotomy of the valid and the invalid corresponds to the practical success of a habit or its failure, which accounts for the raison d'être of logic ([18] vol. 4, p. 165).

3.4 The Functions of Logic

Peirce assigns several functions to logic. In his early texts, it is supposed, along sharing with metaphysics, to analyze conceptions in order to reveal the ultimate categories of reality. This objective would later be left to a science prior to logic, phenomenology, because categories help in discovering the divisions of logic (e.g., semiotic trichotomies). Logic also aims at discovering "a method for determining the values of arguments" ([16], p. 298) and to produce a "comparative anatomy of arguments" ([16], p. 287). In the meantime, Peirce would have defined logic as "the science of the laws of the stable establishment of beliefs" ([15] 3.429). Nevertheless, if logic has a function, a role, or a goal, it is certainly not external to itself. The purpose and end of a system of logical symbols "is simply and solely the investigation of the theory of logic, and not at all the construction of a calculus to aid the drawing of inferences. These two purposes are incompatible [. . .]" ([15] 4.373). Logic must be maximally analytic, that is to say, reveal all the stages of inference by decomposing them, whereas mathematics on the contrary search for rapid methods of resolution. The benefit of the graphical method in logic is precisely to literally make visible each step in reasoning: It is as simple as possible, as iconic, and as analytical as possible ([15] 4.561n).

It provides some insights into the relationship between logic and mathematics according to Peirce. Their methods diverge. Logic is not required at all in mathematical deductions and is even only exceptionally required in reasoning ([17], p. 272). On the other hand, logic, like any other science, has its mathematical branch ([15] 1.247). And not only is there a mathematical logic, but without mathematics, logic would be unable to solve its problems, to the point that "all formal logic is merely mathematics applied to logic" ([15] 4.228). "Thus logic must appeal to mathematics, or else, what amounts to the same thing, must invade the domain of mathematics, in order to make certain of the truth that it essentially seeks" ([17], p. 194).

Was Peirce defending a program symmetrical to that of logicism, and was he proposing to reduce logic to mathematics, and especially, in the wake of the symbolic school of British analysts, to build an arithmetic of logic?[17] Unlike Frege claiming that "arithmetic thus becomes simply a development of logic, and every proposition of arithmetic a law of logic, albeit a derivative one" ([5], p. 99), Peirce did not claim any foundational program. When working in algebraic formalism, he did not worry about the possible philosophical motivations of Boole. Hobbes' proposition that reasoning is computation was recognized as original and fruitful, but it was not clear what he meant by a calculus ([18] vol. 1, p. 163). Peirce later seemed to regret that science was not yet able to reduce entirely mind to calculation: "In the present state of our knowledge," it is not possible "to apply the Calculus to psychological or moral problems" ([18] vol. 3, p. 109). In 1902, however, he

[17] As for a Peircean logicism as supposed by [8], [12] convincingly answered in the negative.

would explicitly contradict Hobbes: "Although not all reasoning is computation, it is certainly true that numerical computation is reasoning" ([15] 2.56).

4 An Outline of Some Connections Between Peirce's Logic and His Philosophy of Logic

Finding out causal or explanatory links between the two previously mentioned aspects, symbolic-iconic and philosophical, may seem implausible. However, two strategies are available to connect them. One is to consider the successive steps of each technical choice or invention, as well as the characteristics generally attributed to logic, and view them as the symptoms of micro-interactions between Peirce's logical practice and his ideas as a philosopher of logic. The other approach relies on the hope that there is a general key for both the evolution of Peirce's logic and his philosophy of logic, that is to say, very roughly, the transition from symbolic logic to a logic of graphs paralleling the adoption of a normative conception of logic.

The limited framework of this article does not allow for the first strategy. One can at most attempt to propose a general guideline that accounts for the coevolution of Peirce's logic and his philosophy of logic (presupposing that such a coevolution really took place). A first hypothesis must be rejected: that of "arithmeticism," a program opposite to logicism. Peirce did not attempt to derive the set of logical connectors from strictly mathematical tools. This idea was all the more foreign to him since foundation is a problem mainly in the frame of a "lingua characteristica," as Hintikka noticed: To Frege, the basic principles and axioms are inexplicable, and the unfortunate one who does not understand first-order logic understands nothing about logic. "By contrast, far from taking any set of accustomed logical principles for granted, Peirce was constantly trying to give them a deeper foundation or extending their range" ([11], p. 17). "Now to say that the graphical procedure is more analytical than another is to say that it demonstrates what the other virtually assumes without proof" ([17], p. 319). There is a variety of (more or less grounding) models, but no ultimate, axiomatic or universal basis.

Other hypotheses are available. The most commonly shared—more or less implicitly—is that the semiotic approach was decisive for Peirce. Having established that logic is nothing but the science of signs, he could not restrict his formalism to algebraic calculus. More precisely, it is by becoming aware that logic is not only "symbolistics" but semiotics, or that it deals with all kinds of signs, that Peirce would have revised his notation system. For formal logic is traditionally symbolic, Peirce discovered that it must integrate icons and indices. The transition toward diagrammatic logic, which results from a dissatisfaction with symbolic logic, would reflect his acknowledging all kinds of signs. Peirce was particularly interested in iconicity: Since logic is a science of observation, it is legitimate for logical writing to show the actual operations produced. Therefore, all mathematical reasoning in general being diagrammatic, and mathematical logic in particular, the

algebraic notation gave room to a graphical notation. This is the point of Zeman, who concluded: "Thus does the very notation which the mature Peirce prefers for the mathematics of his logic tell much about his view of the nature of logic" [24]. Philosophical reasons based on Peirce's sign theory would have led him to get rid of the algebraic notation for a more iconic exposition. However, the semiotic track has seemed insufficient to many, because what Peirce says about icons stands very far from his whole system of diagrams.

A second keystone could be Peirce's interest in relations. This is Shin's hypothesis, who argued that "Peirce's invention of a different kind of representation system is not just an accidental product of a logician's mind, but a clear reflection of his philosophy of logic, which differed from that of contemporary logicians" ([22], p. 13). Phrased in a somewhat emphatic way, her view amounts to the following: Peirce developed a new logical notation because he discovered a new categorization of the real into relations of one, two, and three valences, and correlatively, he entrusted logic with the task of developing all types of possible relations, thanks to the analysis of signs. This is why logic can only be descriptive: It is the science that enumerates the possible relations of a sign with an object. In fact, the paper of 1870 inaugurating the logic of relations is largely a classification of relations. And the logical article of 1885 gives as an object to logic "the enumeration of the essentially different kinds of necessary inference" ([18] vol. 3, p. 165). The Beta system would be the culmination of the long journey that Peirce started no later than 1870. The same goal that of developing a new logic of relatives, guided both the technical innovations of Peirce "the symbolist"—motivating in particular the introduction of quantification—and the transition toward a diagrammatic logic more apt to represent relational states of fact. It supposes, like Shin [22] does, that the logic of existential graphs is not only a different formalization but an extension of logic, just as the theory of quantification introduces not only a different formalism but a logic qualitatively different from that of Aristotle. Brady went even further back in the common genealogy of the new logic and Peirce's surprising philosophy of logic: According to her, Peirce's interest for relations (rather than sets, e.g., in Frege's work) would be due to geometry. Geometric reasoning, which is almost entirely dependent on the two binary relations of incidence and congruence, requires that logic be able to grasp relations:

> "We surmise that this is what Peirce meant by the 'various facts' that led him to desire a more perfect logic, and that reasoning in geometry was thus Peirce's route for discovering the necessity of using relations as well as sets, and of using some kind of algebra of relations." ([4], p. 22)

An alternative hypothesis would zero in on quantification rather than on relations and on the frustrating impossibility of uniting Aristotelian syllogistics and Boolean algebra. It is well known that Boole did not have the means to properly express a proposition as simple as "some As are B," that is, an existential statement. The work on monadic predicates, and therefore on classes, would have led Peirce to a study of Euler and Venn diagrams, hence the development of a personal system of graphs. The Peircian quest would therefore be that of a logical expressivity large enough

to account for all kinds of existence, hence the view of logic as describing forms of existence and the ambiguity between formal science and science of inductive observation.

As sensible as they are, these proposals may be disappointing for at least two reasons. First, they keep a somewhat teleological conception of diagrams. In the hypothesis of iconicity like in the hypothesis of relations, existential graphs are actually considered as Peirce's masterpiece, under the implicit rule that showing is better than symbolizing, ostension better than speech. Why? It is justified by Peirce's theory of reasoning: Reasoning is already observing and experimenting on diagrams. But is not this theory a consequence of diagrammatic logic rather than its basis? If so, the previous readings only explain logical graphs by presupposing their value. Second, they hardly justify the most striking feature of Peirce's philosophy of logic, the transition from an anti-normative conception to a normative though anti-prescriptive conception of logic. This is why I will eventually suggest a third hypothesis.

Instead of supposing that Peirce's views on logic map his logical practice, it starts from the observation of their relative independence. Even though he was claiming that the task of logic is to classify the kinds of signs according to their relation to an object, Peirce worked on something different, the improvement of Boolean calculus. When he wrote that the logician has to do a "comparative anatomy of the arguments," he drew circles on sheets of paper. Despite its characterization as an inductive science, logic remained a deductive practice. Logic ought not to be based on psychological thinking but has its origin in nervous excitement, Peirce also claimed. Examples of such "schizophrenia" could be multiplied. It may be in reaction to these inconsistencies that Peirce produced what he wanted to be a great synthesis, his graphical and normative logic. Chronologically, the turning point took place between the first, algebraic and semiotic phase and the first years of the twentieth century. In this interval, Peirce developed his theory of knowledge as inquiry. The search for truth begins with the irritation of real doubt; peace is restored only by a belief which the scientific method is the best way to fix in the long run. Moreover, this research is community-based, up to the point that truth is nothing but the ultimate agreement of opinions. This is why we must take seriously the definition of logic as the science of the laws of the stable fixation of beliefs. Logic gives its laws to inquiry. Epistemology and science are logical throughout, especially in their human and social aspects. Logic observes objective facts, but its practice is embodied and even requires behaving in a virtuous manner, hence the conception of normativity as reaching ends, which are ultimately those of the good life. The formalist, symbolic conception of logic erases this aspect that Peirce intended to value. It is also this character that is expected in diagrams: The investigator has his own place; he dialogs with an opponent, offers evidence, tests, and why not hates, gets angry, despairs, or triumphs! What graphs show is the ethical relationship in logic.

The foregoing hypotheses are not exclusive. Logic and philosophy of logic have developed reciprocal influences that privileged a semiotic, iconic, relational, dialogical, inquiry-based formalism and thought—in one word, a pragmaticist

conception. Stressing the importance of the theory of inquiry has the merit of simultaneously accounting for the architecture of normative sciences and the adoption of a graphical logic. It also emphasizes the insufficiency of deductive logic, which was for Peirce only the least interesting part of logic. The inquiry is above all inductive and abductive, which no formalism (except perhaps, in a very limited and awkward way, syllogism) can express. This is why it is finally illegitimate to separate the contemporary meaning of logic from its Peircian use: Logic and philosophy of science are not separable, and it may paradoxically be this faith which motivated most of the formal advances of Peirce's logic.

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