The Representation of Negative Terms with Euler Diagrams

Reetu Bhattacharjee, Amirouche Moktefi, and Ahti-Veikko Pietarinen

Abstract In the common use of logic diagrams, the positive term is conveniently located inside the circle while its negative counterpart is left outside. This practice, already found in Euler's original scheme, leads to trouble when one wishes to express the non-existence of the outer region or to tackle logic problems involving negative terms. In this chapter, we discuss various techniques introduced by Euler's followers to overcome this difficulty: some logicians modified the data of the problem at hand, others amended the diagrams, and another group changed the mode of representation. We also consider how modern diagrammatic systems represent negation.

Keywords Negation · Euler diagram · Venn diagram · Syllogism

Mathematics Subject Classification: Primary 03A05; Secondary 01A55

1 Introduction

Euler diagrams are commonly used in logic. Although they are found in many earlier sources [22], it is generally admitted that Euler popularized them in the second volume of his *Letters to a German Princess*, first published in 1768 [17]. The idea is rather simple: a circle stands for the extension of a term. For instance, Fig. 1 represents a term *A*. Then, the logical relations between the terms are represented

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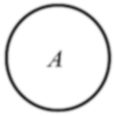


Fig. 1 Term *A*

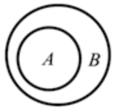


Fig. 2 All *A* are *B*

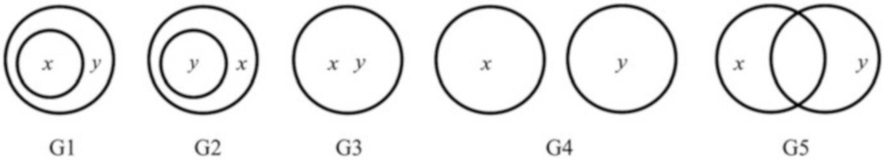


Fig. 3 Gergonne relations

by the topological relations between the circles. For instance, Fig. 2 represents the proposition "All A are B." Now, Euler introduced his diagrams to handle syllogistic problems where only positive terms occur, and these diagrams hardly lend themselves to the treatment of negative terms. Of course, each negative term *not-A* is incidentally represented by the space that is outside the circle that stands for *A*. But this indirect representation leads to difficulties when one attempts to tackle a reasoning involving those negative terms.

Let us for instance observe the various ways in which two terms relate to each other, as formulated by the French mathematician Joseph Gergonne in 1817 [46]. It is known that for two terms *x* and *y*, there are 5 such relations, as shown in Fig. 3.

In each case, the outer region stands for the negation of all the terms in the argument, i.e., here what is neither *x* nor *y*. We clearly see on the 5 diagrams (from G1 to G5) that this outer region always exists. Hence, it is not possible, without further amendments, to represent a proposition such as "Everything is *x* or *y*" which asserts the absence of that outer region. The aim of this chapter is precisely to discuss the solutions that were offered by Euler's followers to express information on negative terms with Eulerian diagrams. We will sketch some solutions based on a transformation of the data, an amendment of the diagrams, and a modification of the mode of representation. Before proceeding, it might be worth making a couple of remarks.

First, we consider here only extensional interpretations of Euler diagrams. If we were to consider intensions, as many early logicians did, the situation would be different and (far) more complex [1, 42]. Indeed, suppose a circle stands for the intension of a term A, as in Fig. 4. That means that the circle encloses the attributes that are predicated to *A* rather than the individuals that form the extension of *A*. It would then be incorrect to state that the outer region stands for *not-A* since the intensions of *A* and *not-A* may well share *some* attributes. Hence, the reader needs

Fig. 4 Intension of term *A*

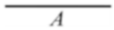


Fig. 5 Term *A*

to keep in mind that we are hereafter concerned with the extensions of the terms (commonly called classes in the considered historical period).

The second remark that needs to be made is that the difficulty of handling negative terms is not proper to Euler diagrams. It is also found in many early diagrammatic, and also algebraic, notations. For instance, if we were to consider linear diagrams that enjoyed some popularity in Euler's time [2], the problem would remain. Let a segment line stand for a term *A*, as in Fig. 5. The negative term *not-A* is indirectly represented by the infinite portions of the line that are beyond the two ends of segment *A*. Hence, as observed earlier with Euler diagrams, it is not possible to express the absence of the outer segment with traditional linear diagrams, without further amendments.

2 Transformation of the Data

Before we discuss some diagrammatic innovations to handle negative terms, we first consider in this section a strategy used by some logicians to tackle negative terms when they occur without the need to amend Euler diagrams. Indeed, an obvious solution consists in transforming the negative term into a positive one. For instance, if we are given a problem where a negative term *not-x* occurs, we might simply treat it as a positive term. For the purpose, it suffices to (mentally or really) replace *not-x* by a positive term *z* during the solution of the problem.

It is easy to see how this solution allows us to represent the problematic proposition "Everything is *x* or *y*" alluded to earlier. This proposition asserts that no *not-x* is *not-y*. To represent it with Euler diagrams, it suffices to situate the negative terms inside the circles and the positive terms outside. So, we simply draw two disjoint circles standing for *not-x* and *not-y*, respectively, as in Fig. 6.

This trick allowed logicians to handle arguments with negative terms, and even to find conclusions that followed from sets of premises that were previously held to be unproductive (i.e., yielding no conclusion). This is particularly the case when we face syllogistic forms that have two negative premises. Euler stated that from such premises, one cannot draw a conclusion ([17], p. 360). However, subsequent logicians challenged this rule. For instance, William S. Jevons argued that:

> [i]t would be a mistake, however, to suppose that the mere occurrence of negative terms in both premises of a syllogism renders them incapable of yielding a conclusion. The old rule [. . .] is actually falsified in its bare and general statement. In this and many other cases we can convert the propositions into affirmative ones which will yield a conclusion by substitution without any difficulty. ([20]: 63)

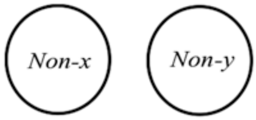


Fig. 6 Everything is *x* or *y*

Jevons states that a negative proposition can be transformed into a positive one. For instance, if we are told that "Some *x* are not *y*," we may convert it to "Some *x* are *not-y*." Here, we simply move the negation from the copula to the predicate. Hence, we obtain an affirmative proposition "Some *x* are *z*" (where $z = not\text{-}y$). Later, Lewis Carroll made a thorough use of this technique and systematically transformed propositions of the form "Some *x* are-not *y*" into "Some *x* are *not-y*" [16]. Carroll observed that logicians

> have somehow acquired a perfectly *morbid* dread of negative Attributes, which makes them shut their eyes, like frightened children, when they come across such terrible Propositions as "All not-*x* are *y*"; and thus they exclude from their system many very useful forms of Syllogisms. ([7], p. 172)

In order to understand the significance of this technique, as used by Jevons and Carroll, let us consider the following problem, which we will hereafter call *Carroll's problem* ([7], p. 180). Suppose we were given two premises:

No *x* is *m*
Some *m* are not *y*

And we were asked what conclusion follows from them.

Carroll proposed this problem to compare various logic methods (including Euler diagrams). Carroll himself introduced both symbolic and diagrammatic techniques for logical reasoning [29, 31]. In his time, a multitude of notations were introduced, and it was not rare to see logicians compare their methods by applying them to similar problems [14]. In the following, we will also use Carroll's problem to assess the various methods discussed hereafter.

Since both premises in *Carroll's problem* are negative, Euler would claim that no conclusion would follow from this pair. A direct application of his diagrams produces no conclusion. Indeed, these premises do not forbid any of the 5 Gergonne relations between *x* and *y*. Given that no traditional proposition is satisfied in all the cases, it follows that there is no conclusion to the problem. The introduction of negative terms changes the picture.

Suppose we follow Jevons' advice and transform the second premise "Some *m* are not *y*" into "Some *m* are *not-y*," and again into "Some *x* are *z*" (with $z = not\text{-}y$). Now, the pair of premises becomes:

No *x* is *m*
Some *m* are *z*

It is now easy to handle it with traditional Euler diagrams. We first represent the various possible combinations of the premises, as shown in Fig. 7.

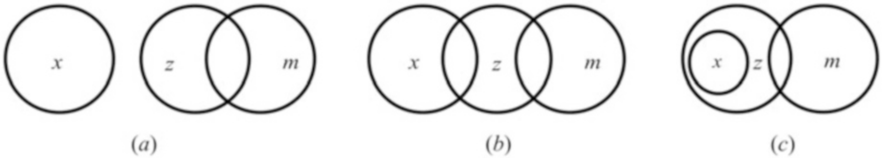


Fig. 7 'No *x* is *m*' and 'Some *m* are *z*'

Then, we eliminate the middle term *m*, by observing merely the relation between *x* and *y*. Among traditional propositions, only one follows:

Some *z* are not *x*,

which can be transformed into the final conclusion of the syllogism:

Some *not-x* are *not-y*.

The substitution of a positive term for the negative one allowed us to find the conclusion of *Carroll's problem*. An advantage of this method over the subsequent ones is that it can be worked out with traditional Euler diagrams, without further amendments. The transformation is achieved on the expression of the problem, rather than on the diagrams themselves. However, this method works merely when a term is not expressed twice with opposite signs in the problem. For instance, if the middle term *m* of a syllogism is affirmed in one premise and negated in the other, it would be useless to substitute the negative occurrence *not-m* by a positive term *z* since that would demand the replacement of the other occurrence *m* by a negative term *not-z*. In the following, we consider other methods that do not suffer from this shortcoming.

3 Amendment of the Diagrams

This second set of solutions proposes to introduce modifications to Euler diagrams in such a way as to make them suitable for negative terms. An attempt in this direction was made by John Neville Keynes. His idea consisted in enhancing Euler diagrams so that they would depict the actual relations between terms and their opposites, rather than positive terms alone ([21], p. 170–174). A step in this direction was previously achieved in 1846 by Augustus De Morgan who introduced the concept of Universes of discourse. Indeed, this innovation bounds the scope of our assertions, and hence, defines negative terms *not-x* as the complement of term *x* as to fulfill the universe ([13], p. 2). The universe of discourse is commonly represented in modern textbooks with a rectangle around our Euler diagram with a finite space standing for the outer region. This convention was known to Keynes' predecessors, notably Alexander Macfarlane ([23], p. 23; [24], p. 61).

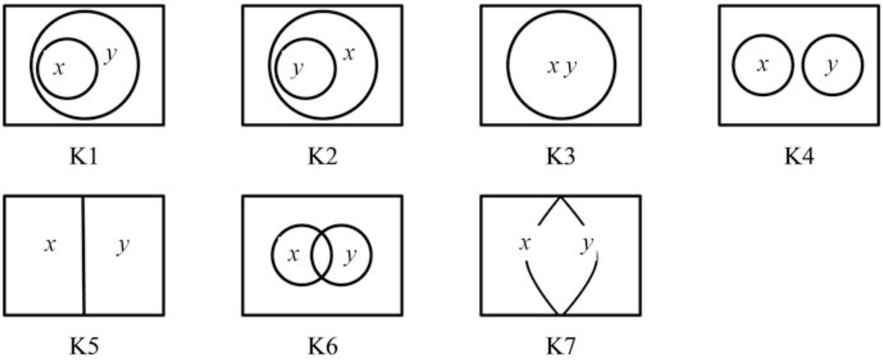


Fig. 8 Keynes relations

Keynes also restricted the universe in his diagrams. Then, he defined the various relations that two given terms and their opposites may have to each other. For the purpose, it suffices to consider each of the 5 Gergonne relations and subdivide it into 2 sub-relations: one in which the outer region is empty and one in which it is not. We would then get 10 sub-relations. However, Keynes excluded 3 cases infringing his assumption that all classes and their opposites must exist. Hence, he eventually obtained 7 actual relations between terms *x*, *y*, *not-x* and *not-y*, as shown in Fig. 8 (in K7, *x* and *y* intersect and fill the universe).

These Keynes relations provide the tools needed for a more accurate treatment of negative terms with Euler diagrams. For instance, they make it easy to represent the proposition "No *not-x* is *not-y*" (and its equivalent form "Everything is *x* or *y*"). This proposition is indeed depicted in the relations K5 and K7, depending on whether *x* and *y* intersect or not. These relations also are used to solve logic problems, involving negative terms without transforming the data. Let us consider again *Carroll's problem* where we are given the premises:

No *x* is *m*
Some *m* are not *y*

The first premise allows 2 Keynes relations between *x*, *not-x*, *m*, and *not-m*:

K4 and K5

The second premise permits 5 Keynes-relations between *m*, *not-m*, *y*, and *not-y*:

K2, K4, K5, K6, and K7

Then, a rather tedious process follows to merge the two premises and to identify all the distinct combinations of the three terms *x*, *m*, *y* and their opposites. The elimination of *m* (and its opposite) leaves 5 distinct relations between *x*, *y* and their opposites:

K1, K2, K3, K4, and K6

Fig. 9 Term *A*

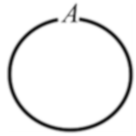
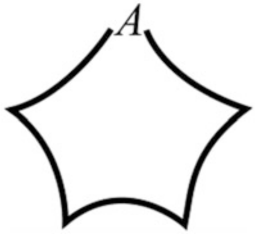


Fig. 10 Term not-*A*



All these relations depict the conclusion of the syllogism:

Some *not-x* are *not-y*

An advantage of Keynes's solution is that it represents the actual relations that are known between the terms, similarly to Euler's original practice. However, this solution suffers from the complexity and the multiplicity of the figures, especially when premises are merged, which increases the risk of misusing the diagrams.

Several other amendments of Euler diagrams were proposed by Charles S. Peirce [37]. In the following, we consider specifically an innovation introduced in the period 1896-1901 and that relates directly to the representation of negative terms. Rather than enclosing the Universe, Peirce reworked the shape of the curves in such a way as to convey the sign of the terms: positive terms are found of the concave side of the curve and negative terms on the convex side [4, 35]. This convention encompasses Euler's common use where positive terms are found inside the circle (which is the concave side). Hence, in Fig. 9, *A* is inside the curve and *not-A* outside it. There is no necessity to use circles, however, and Peirce introduces shapes that reverse the location of the terms. For instance, in Fig. 10, *A* is outside the curve and *not-A* inside it. The label 'A' is marked on both diagrams and indicates the differentia between the two spaces (inside vs. outside). Then, the shape of the curve is considered in order to locate terms. This practice differs from Euler's original diagrams where the shape of the curve had no logical meaning [28].

Peirce's innovation simplifies the representation of propositions involving negative terms. For instance, if we wish to represent the proposition "No *not-A* is *not-B*" (i.e., "Everything is A or B"), we can use any of the equivalent figures (*a*), (*b*), and (*c*) in Fig. 11. Neither has a space corresponding to Euler's outer region *not-A not-B*. In his treatment of syllogisms, Peirce favored forms (*b*) and (*c*) which transform the exclusion into an inclusion: either not-A is included in B (form *b*) or not-B is included in A (form *c*) [4].

Let us now consider again *Carroll's problem* and attempt to solve it with Peirce's method. We are offered two premises:

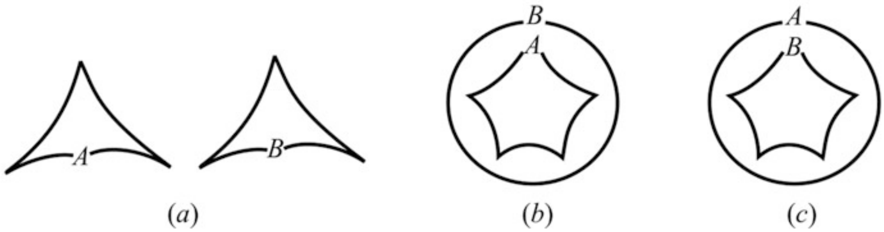
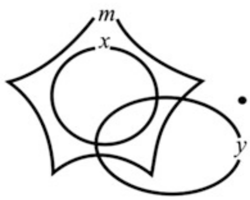


Fig. 11 Everything is *A* or *B*

Fig. 12 'No *x* is *m*' and 'Some *m* are not *y*'



No *x* is *m*
Some *m* are not *y*

There are various ways in which these premises can be *separately* drawn, depending on the shapes of the curves. However, for the purpose of merging the premises, it is crucial that the middle term *m* is represented with a similar shape in both premises. For the purpose, we will use some strategic rules that were used by Peirce to classify syllogisms into three groups [4]. The present problem belongs to the second group: it has one universal and one particular premise. Hence, the premises are better represented with a circle *x* inside a convex shape *m* and a circle *y* that intersects with both *x* and *m*, as shown in Fig. 12. The existential import of the second premise is indicated with a dot outside *m* and *y*.

The elimination of *m* (and its opposite) shows the conclusion of the syllogism:

Some *not-x* are *not-y*

Peirce's method offers great flexibility for the representation of terms and their opposites. This feature might be used with benefit for the treatment of logic problems but may also turn into an inconvenience for the untrained eye. In its principles of representation, it resembles earlier Eulerian methods in that it does not devote space to a class whose existence is denied. However, unlike Euler and Keynes, Peirce does not hold the existence of a space necessarily to entail the existence of the class. If one wishes to express existence, one needs to mark a space with a syntactic device, a dot for instance [34]. On this aspect, Peirce's approach rather resembles John Venn's that we discuss in the next section.

4 Modification of the Mode of Representation

Although each of the previous solutions by Jevons, Keynes, and Peirce had its strengths and weaknesses, all remained truthful to Euler's mode of representation: relations between classes are represented directly by the relations of their figures (an exception to this rule is Peirce's representation of existence, alluded to earlier). As such, all suffer to various extents from the shortcomings of this Eulerian mode of representation. Indeed, Eulerian diagrams represent relations as they are known and do not leave much room for uncertainty. Hence, new information may entail an entire redrawing of the diagram. Also, solving problems with Eulerian diagrams often require a multitude of diagrams and complex merging rules. Finally, Eulerian diagrams may work well for simple problems such as syllogisms but become impractical when the number of term increases. This was inconvenient for early symbolic logicians, such as Venn, who work out complex problems that involved several terms [33].

To overcome these shortcomings, Venn published in 1880 a new type of circle diagrams which differ in their mode of representation from Euler's [44]. Indeed, Venn first makes his circles (or whatever shape they may have) intersect in such a way as to form 2^n distinct compartments standing for the combinations of the n terms in the argument under consideration. For a syllogism, the 3-term diagram shown in Fig. 13 suffices. For any number of terms, only one Venn diagram is required. Then, Venn marks the compartments to indicate their states. For instance, shading indicates emptiness. Venn was more hesitant about occupation and introduced several conventions for the purpose [34]. Yet, the main idea was to have a distinctive device (a cross for instance) to express existence.

Venn located a negative term outside the circle standing for a positive term, as Euler and Keynes did. However, his modification of the mode of representation makes him handle better than they did problems involving negative terms. Let us consider again *Carroll's problem* (which was solved by Venn himself in ([7], p. 182]). Two premises are offered:

No x is m
Some m are not y

We need a 3-term Venn diagram. The first premise asserts the emptiness of compartment $x\,m$. Hence, all its subdivisions are shaded. The second premise asserts the existence of compartment m *not-y*. Hence, at least one of its subdivisions exists.

Fig. 13 3-term Venn diagram

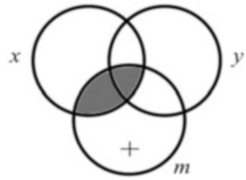


Fig. 14 'No *x* is *m*' and 'Some *m* are not *y*'

Since subdivision *x m not-y* is known to be empty, it follows that subdivision *not-x m not-y* exists, and thus is marked with a cross. We eventually obtain Fig. 14.

The elimination of *m* immediately shows that the outer region of the diagram is not empty, which gives the conclusion of the syllogism:

Some *not-x* are *not-y*

Although Venn diagrams were not designed to specifically tackle negative terms, his solution proves effective, simple, and unambiguous. However, this picture is spoiled by Venn's refusal to restrict the Universe of discourse arguing that its scope was an extra-logical issue ([45], p. 250). This decision complicates the shading of the outer region if one wishes to express the proposition "No *not-x* is *not-y*." Venn himself was confronted to this difficulty in one his examples and simply wrote that he did "not troubled to shade the outside of this diagram" ([45], p. 352). Venn was severely criticized on this ground by his contemporaries, notably Allan Marquand, Macfarlane, and Carroll. These logicians amended Venn's scheme and designed rectangular diagrams with a closed Universe (although Carroll's early familiarity with Venn diagrams is not established [30]).

The appeal to rectangular figures reintroduces the issue of the size of complementary terms, i.e., *x* and *not-x*. Symbolic logicians tended to consider these terms on the same footing, but this view was not necessarily reflected in their notations. Venn warned that considerations of shape and size should not be contemplated in the interpretation of his diagrams:

The compartments yielded by our diagrams must be regarded solely in the light of being bounded by such and such contours, as lying inside or outside such and such lines. We must abstract entirely from all consideration of their relative magnitude, as we do for their actual shape, and trace no more connection between these facts and the logical extension of the terms which they represent than we do between this logical extension and the size and shape of the letter symbols, A and B and C ([45], p. 527).

Naturally, if one wished to introduce quantitative considerations, circles hardly do the job and rectangular shapes are to be favored as Venn himself conceded ([45], p. 526) (See [15]). Another advantage of rectilinear shapes is that they ease the extension of diagrams by introducing additional curves while keeping regular figures [32].

Post-Venn tabular diagrams generally attribute equal space for opposite terms *x* and *not-x*, and as such reflect better than Venn did the formal symmetry between opposites. Carroll specifically insisted on this symmetry and the importance of

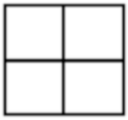


Fig. 15 2-term Carroll diagram

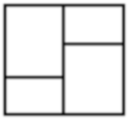


Fig. 16 Macfarlane diagram

treating opposites on the same footing [12]. The construction of his diagrams, where a square is divided by dichotomy, and their extension adequately reflect this symmetry [27]. For 2-terms, he used a symmetrical biliteral diagram such as Fig. 15. Marquand used a similar figure, although the justification of its symmetry is less enthusiastic: "The quantitative relation of the compartments being insignificant, they may for convenience be represented as equal" ([26], p. 267). Macfarlane also used symmetrical figures in his Logical Spectrum [25]. However, in an earlier treatise, he used an asymmetrical 2-term diagram such as Fig. 16 ([23], p. 54).

5 On Negation in Modern Diagrammatic Systems

Modern diagrammatic systems in logic are, to a large extent, based on Venn's scheme, with some amendments due to Peirce for the representation of existential statements and disjunctives [39]. Sun-Joo Shin played a decisive role in the transmission of these conventions to modern systems, through her two systems Venn-I and Venn-II published in 1994 [38]. Both systems, and most subsequent ones, represent negative terms in the path of Euler and Venn: the space is divided into two sub-spaces standing for complementary terms. The identification of the positive and negative terms is conventional but is conveniently indicated by the labeling of the spaces. Commonly, the positive term is inside the curve while the negative is left outside. In this chapter, we discussed the representation of negative terms with Euler diagrams and some other schemes. This subject should not be confused with the representation of negation itself. The latter might be understood and represented in a variety of forms, depending on its scope of application.

For instance, if we were to deny a proposition with Euler diagrams, it suffices to avoid ascribing a space to the classes forbidden by the proposition. For instance, the proposition "No *x* is *y*" is represented with two disjoint circles *x* and *y*, as in Fig. 17a, where no space is ascribed to a class of objects that would be both *x* and *y*. In this method, used by Euler and Keynes, only spaces known to exist are ascribed spaces. Peirce also eliminated the space of classes that were forbidden but did not grant the existence of those that were represented. As we saw, Venn had a different method and introduced a syntactic device to deny the existence of a class. This is achieved by

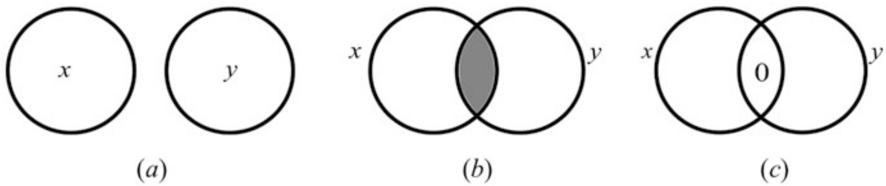


Fig. 17 No *x* is *y*

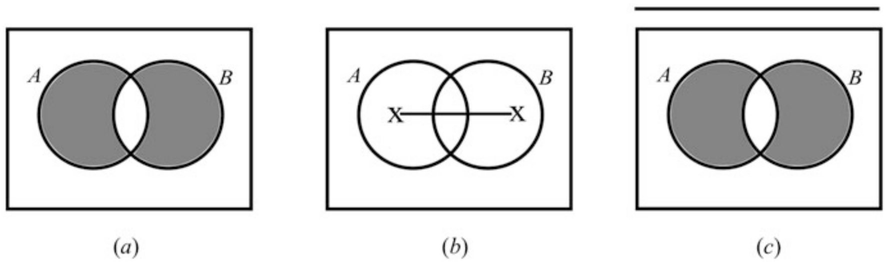


Fig. 18 A diagram and its negation

shading its corresponding compartment. For instance, the proposition "No *x* is *y*" is represented by shading the intersection between *x* and *y* to express its emptiness, as in Fig. 17b. Some of Venn's successors, like Marquand and Macfarlane, adopted his use of shadings. Others, such as Peirce and Carroll, designed different conventions, such as the insertion of a "0" on a compartment to indicate its emptiness, as shown in Fig. 17c. But shadings are dominant in modern systems such as Spider diagrams [19].

So far, we considered simple negative propositions that denied the existence of one or more classes and how this information was represented with Eulerian diagrams. Recent systems of diagrams felt the need to express the denial of complex expressions conveyed with Euler diagrams. Suppose that we are given the diagrammatic expression shown in Fig. 18a. Its negation is shown in Fig. 18b (the crosses indicate presence and the line connecting them disjunction).

Another method to represent the negation of that diagrammatic expression simply consists in adding a vinculum above it, as shown in Fig. 18c, in a manner similar to the practices of algebraic notations [41]. This strategy, used in Spider diagrams, requires an unambiguous definition of the scope of the negation. This scope is indicated here by the rectangle around the negated diagram. Early logicians, such as Peirce [36] and Gottlob Frege [18], anticipated such solutions on their graphs with greater success since their signs of negation incorporated a determination of the scope, and hence, did not necessitate an additional sign [3].

We may evoke another type of negation found in modern diagrammatic systems and that consists in expressing the absence of an individual. Except for few early attempts, notably by Peirce [34], the representation of individuals (and constants)

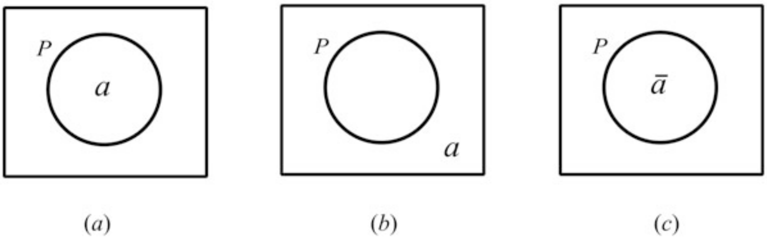


Fig. 19 '*a* is P', "*a* is not P', and 'the absence of *a*'

on Euler diagrams is recent [8, 43]. Interestingly, Lopamudra Choudhury and Mihir Chakraborty also introduced a representation for the absence of individuals [5, 8, 40]. Suppose we were given the singular proposition "*a* is P," shown in Fig. 19a, and are asked to represent its opposite "*a* is not P." For the purpose, one may simply insert *a* outside P, as shown in Fig. 19b, since its absence from P indicates its presence in not-P. However, they would rather represent the absence of *a* directly inside P, as shown in Fig. 19c, where $\bar{a}$ stands for "the absence of *a*."

This approach to the representation of the absence of individuals was apparently inspired by Nyāya-thinkers of ancient Indian tradition which led Choudhury and Chakraborty to consider an "absence" as a distinct ontological category that should be treated like an individual. In this tradition, it was believed that in one's cognition absence of an individual is directly perceived in a locus just as one perceives the presence of an individual in a locus. One does not generally infer the absence of something somewhere by observing its presence somewhere else. For example,

> The attendance register book of the students of a class is in one-to-one correspondence with the set of students. The representation of the absence of a student '*a*' is marked by a symbol just as is done in case of the presence. We are not pointing at the aspect of administrative convenience, but at the cognitive impact of this practice. No mark corresponding to some student would bring to our mind the message 'no information'. Thus in the register absence of a student in class is shown by a mark (and not by his/her presence somewhere else) [11].

Figure 19c depicts the absence of the individual *a* in P directly as Fig. 19a depicts the presence of *a* in P. Whereas in Fig. 19b, the information "*a* is absent in P" is a kind of derivative from the information depicted in Fig. 19b. Moreover, the cognitions of the absence of an individual *a* and the absence of another individual *b* are different. Depiction of these absences by $\bar{a}$ and $\bar{b}$ leads one towards perceiving this difference in a straightforward way [6]. Choudhury and Chakraborty also propose two other interpretations of absence. In one interpretation, Fig. 19b and Fig. 19c are not equivalent anymore, i.e., even if we can get Fig. 19c from Fig. 19b the converse is not possible generally [5, 10, 11]. In the other interpretation, there is no bounding rectangle, so there is no diagram corresponding to Fig. 19b [6, 9, 11]. In both of these cases, the notion "absence" becomes essential.

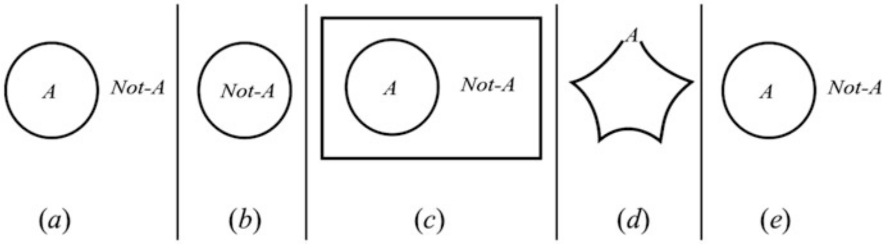


Fig. 20 Summary of solutions for negative terms

6 Conclusion

We surveyed various techniques used in the golden age of logic diagrams to treat negative terms. Figure 20 offers a summary of these solutions: (*a*) Euler (1768) keeps the negative term outside the circle that stands for the positive one, (*b*) Jevons [20] substitutes a positive term for the negative term and treats it accordingly, (*c*) Keynes (1894) encloses the universe and, thus, restricts the outer region that stands for the negative term, (*d*) Peirce (1896) reshapes the diagram to convey the sign of the term, and (*e*) Venn [44] returns to Euler's original plan but modifies the mode of representation.

It is seen that Venn actually uses the same convention as Euler: the negative term is outside the closed curve that stands for the positive term. The difference is that Euler's figure states the existence of both *A* and *not-A* while Venn's does not. It merely offers a framework on which marks will be inserted to indicate existence or emptiness. Given its effectiveness, Venn in a sense makes the subsequent attempts by Keynes and Peirce obsolete, on the condition that one accepts a change in the mode of representation. But within the Eulerian mode, Keynes' and Peirce's methods keep their advantages for the representation of negative terms and have other merits that are beyond the scope of this chapter.

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